Nucleon transverse quark spin densities

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with

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Extended Twisted Mass Collaboration

STIMULATE
European Joint Doctorates

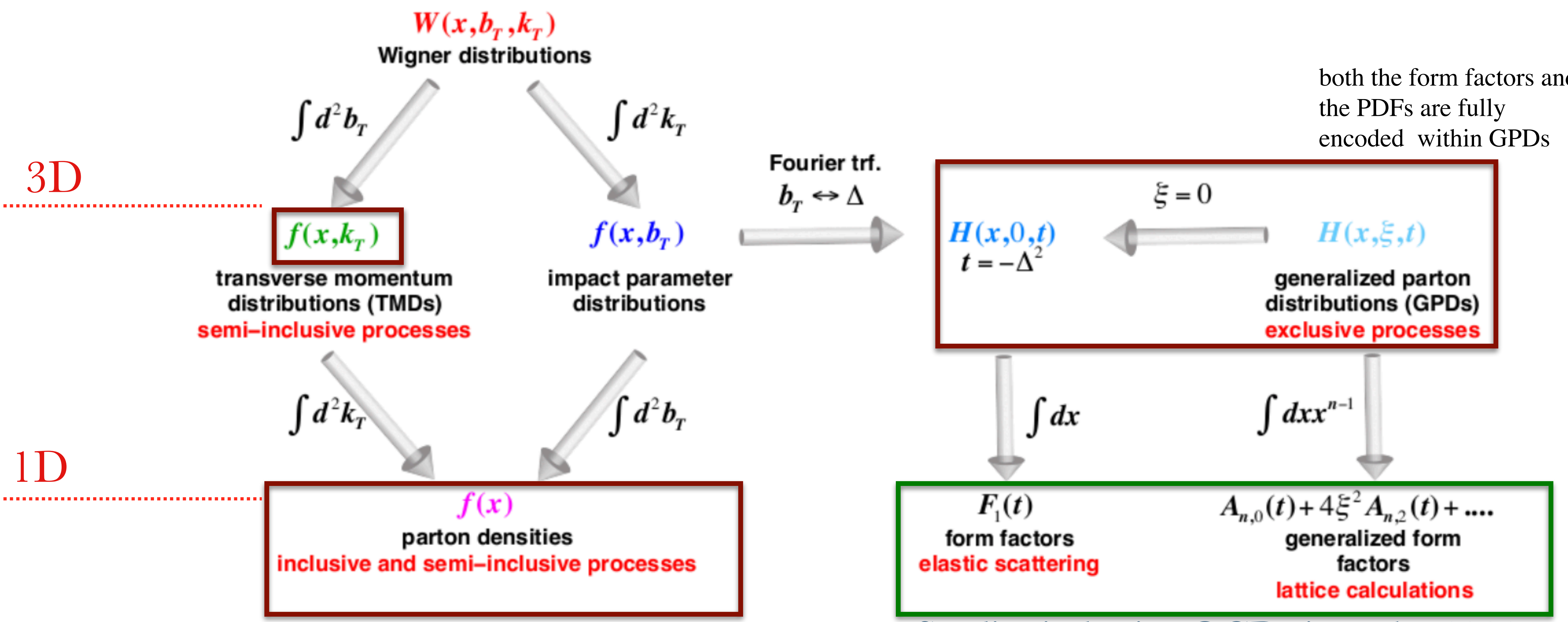
LATTICE 2022

The 39th International Symposium on Lattice Field Theory

Motivation

- Understand the 3D-structure of the nucleon from its fundamental constituents, the quarks and the gluons - major goal of nuclear physics and a key aim of EIC
- Significant progress in revealing the longitudinal spin structure
- But transverse spin structure is much less known - focus of this talk
- Very relevant for JLab and EIC physics

EIC white paper, arXiv:1212.1701



Studies in lattice QCD since the 1980s

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- ❋ But transverse spin structure is much less known - focus of this talk
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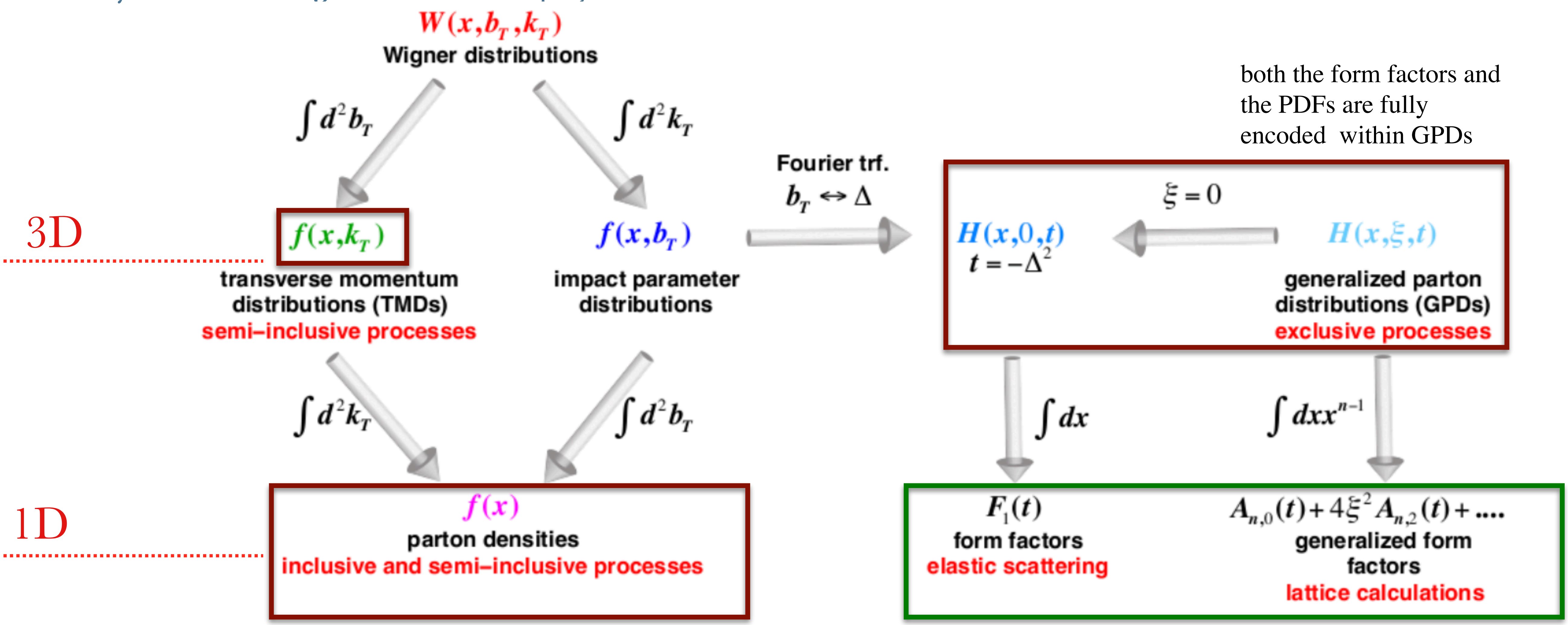


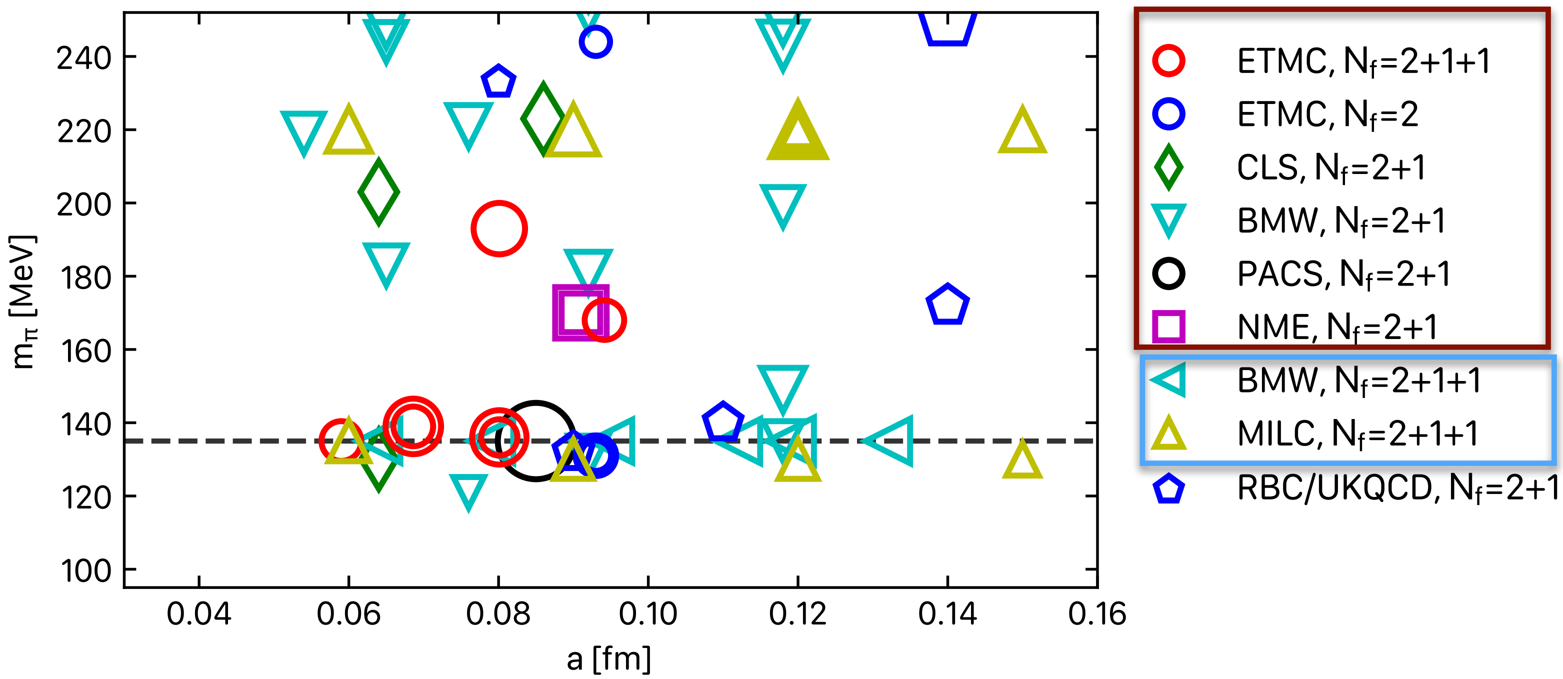
Studies in lattice QCD since the 1980s

- ❋ Compute moments of the transverse density

$$\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) = \frac{1}{2}\Big[H(x,b_\perp^2)+\frac{\mathbf{b}_\perp^j\epsilon^{ji}}{m_N}\left(\mathbf{S}_\perp^i E'(x,b_\perp^2)+\mathbf{s}_\perp^i \bar{E}'_T(x,b_\perp^2)\right)+\mathbf{s}_\perp^i\mathbf{S}_\perp^i\left(H_T(x,b_\perp^2)-\frac{\Delta_{b_\perp}\tilde{H}_T(x,b_\perp^2)}{4m_N^2}\right)+\mathbf{s}_\perp^i(2\mathbf{b}_\perp^i\mathbf{b}_\perp^j-\delta^{ij}b_\perp^2)\mathbf{S}_\perp^j\frac{\tilde{H}''_T(x,b_\perp^2)}{m_N^2}\Big]$$

Moments of ρ can be determined from generalised form factors (GFFs)

Status of current simulations



Wilson-type

Staggered

Domain wall

Plenary, Sat. by J. Finkenrath

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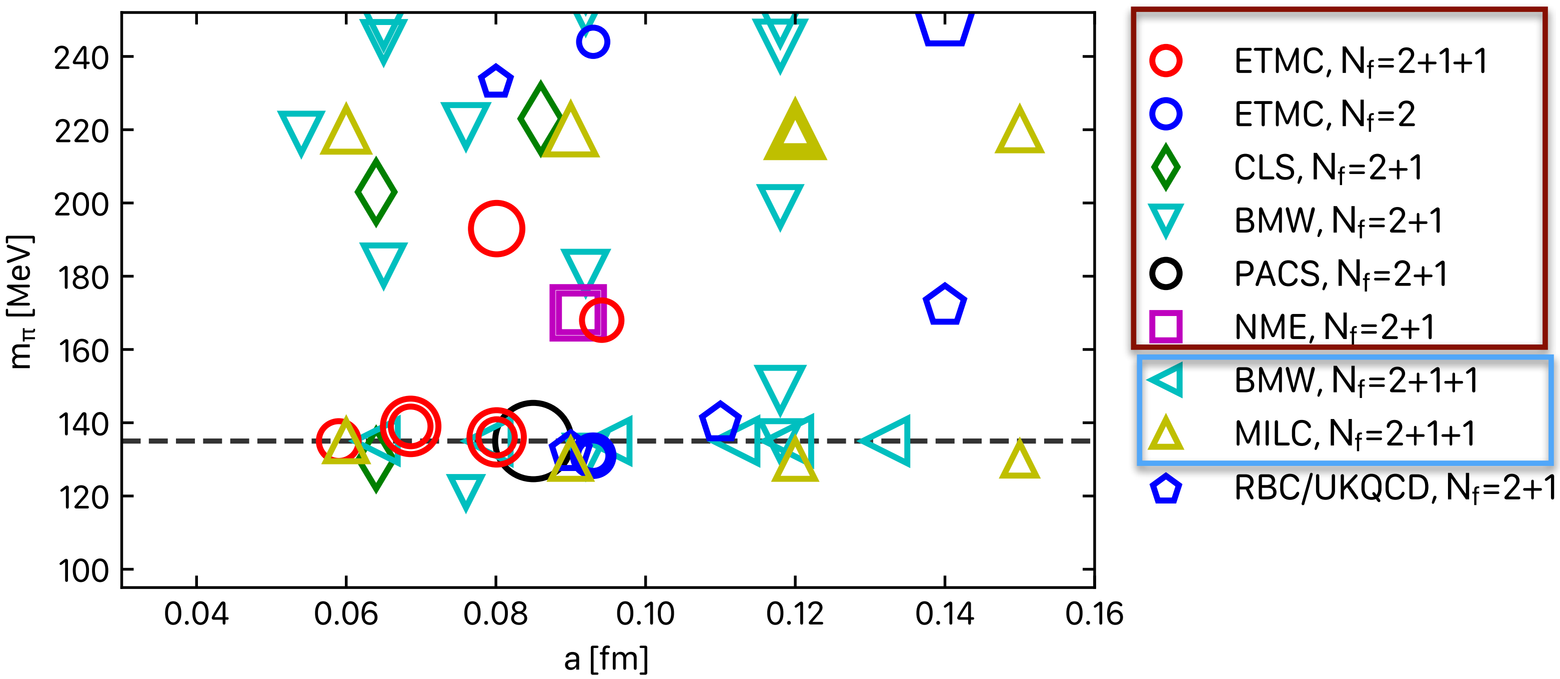

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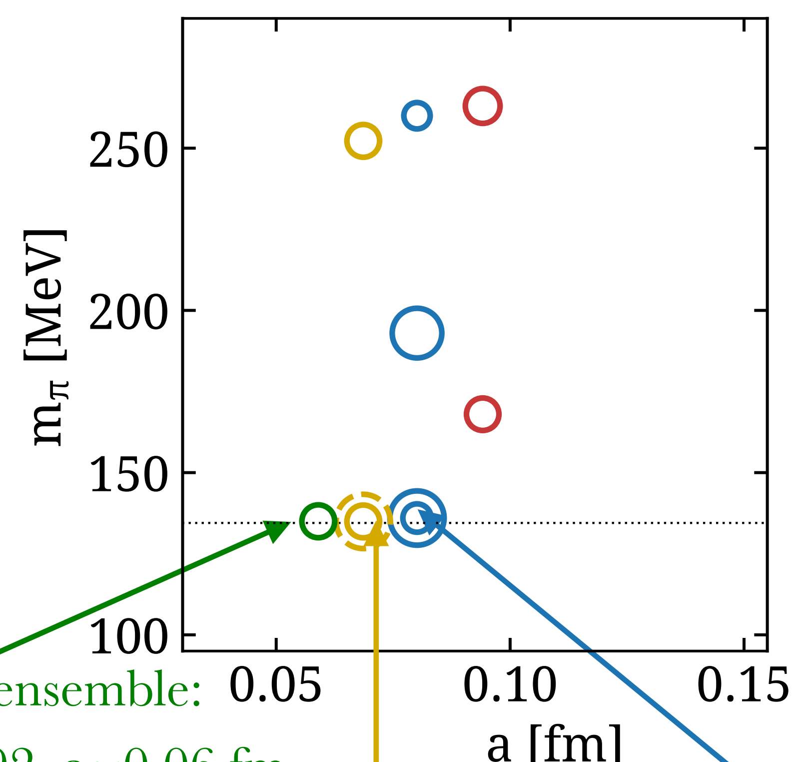
❋ Ensembles generated by ETMC:



D-ensemble: 96^3x192, a~0.06 fm

C-ensemble: 80^3x160, a=0.07 fm

B-ensemble: 64^3x128 a=0.08 fm

We use 3 ensembles generated with physical values of the u, d, s, and c quark masses

ensemble	$(L/a)^3.T/a$	a (fm)	m_π (MeV)	L (fm)	$m_\pi L$
cB211.072.64	$64^3 \cdot 128$	0.07961 (13)	140.2 (2)	5.09	3.62
cC211.060.80	$80^3 \cdot 160$	0.06821 (12)	136.7 (2)	5.46	3.78
cD211.054.96	$96^3 \cdot 192$	0.05692 (10)	140.8 (2)	5.46	3.90

C. A. *et al.* (ETMC), Phys. Rev. D98 (2018) 054518

Talk, Tues. by B. Kostrzewa

Mellin moments

- Moments for small n are readily accessible on the lattice from matrix elements of local operators
- Computation of the low Mellin moments has a long history, G. Martinelli and Ch. Sachrajda Phys. Lett. B217 (1989) 319
- Only recently we have results directly at the physical point (i.e. simulations with $m_\pi \sim 135$ +/-10 MeV)
- To access the transverse spin densities one needs to compute the twist-two matrix elements of the chiral-even unpolarized and chiral-odd transversity GPDs

$$\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) = \frac{1}{2}\Bigg[H(x,b_\perp^2) + \frac{\mathbf{b}_\perp^j\epsilon^{ji}}{m_N}\left(\mathbf{S}_\perp^i E'(x,b_\perp^2) + \mathbf{s}_\perp^i \bar{E}_T'(x,b_\perp^2)\right) + \mathbf{s}_\perp^i\mathbf{S}_\perp^i\left(H_T(x,b_\perp^2) - \frac{\Delta_{b_\perp}\widetilde{H}_T(x,b_\perp^2)}{4m_N^2}\right)$$
$$+\mathbf{s}_\perp^i(2\mathbf{b}_\perp^i\mathbf{b}_\perp^j - \delta^{ij}b_\perp^2)\mathbf{S}_\perp^j\frac{\widetilde{H}_T''(x,b_\perp^2)}{m_N^2}\Bigg]$$

$$F' \equiv \frac{\partial F}{\partial b_\perp^2} \qquad \bar{E}_T \equiv E_T + 2\widetilde{H}_T$$

M. Diehl and P. Haegler, Eur.Phys.J.C44:87-101,2005, arXiv:hep-ph/0504175

- Consider moments of GPDs at zero skewness, e.g.

$$A_{Tn0} = \int_{-1}^{1} dx\, x^{n-1} H_T,\ \ B_{Tn0} = \int_{-1}^{1} dx\, x^{n-1} E_T,\ \ \widetilde{A}_{Tn0} = \int_{-1}^{1} dx\, x^{n-1}\widetilde{H}_T$$

Mellin moments

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- Only recently we have results directly at the physical point (i.e. simulations with $m_\pi \sim 135$ +/-10 MeV)
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Nucleon isovector charges (first Mellin moments)

$$g_V = \langle 1 \rangle_{u-d}$$
$$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$$
$$g_T = \langle 1 \rangle_{\delta u - \delta d}$$

- $g_V = 1$
- $g_A = 1.2723 \pm 0.0023$ reproduce
- $g_T = 0.53 \pm 0.25$ M. Radici and A. Bacchetta. PRL 120 (2018) 192001

- Nucleon matrix elements of the vector operator $\bar{q}\gamma_\mu q$ yield $A_{10}(Q^2) = F_1(Q^2), \quad B_{10}(Q^2) = F_2(Q^2)$

$$\int_{-1}^{1} dx\, H(x,0,Q^2) = F_1(Q^2)$$
$$\int_{-1}^{1} dx\, E(x,0,Q^2) = F_2(Q^2)$$

Dirac and Pauli FFs

Tensor form factors

- Nucleon matrix elements of the tensor operator

$$\langle N(p',s')|\bar{q}\sigma^{\mu\nu}q|N(p,s)\rangle = \bar{u}_N(p',s')\left[\sigma^{\mu\nu}\,A^q_{T10}(Q^2) + i\frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}\,B^q_{T10}(Q^2) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\,\widetilde{A}^q_{T10}(Q^2)\right]u_N(p,s)$$

$$A_{T10}(0) = g_T \qquad \bar{B}^q_{T10}(0) \equiv B^q_{T10}(0) + 2\widetilde{A}^q_{T10}(0)$$

$$\Delta^\mu = p'^\mu - p^\mu,\;\; \bar{P}^\mu = \frac{p'^\mu + p^\mu}{2}\,,\;\; Q^2 = -(p'-p)^2$$

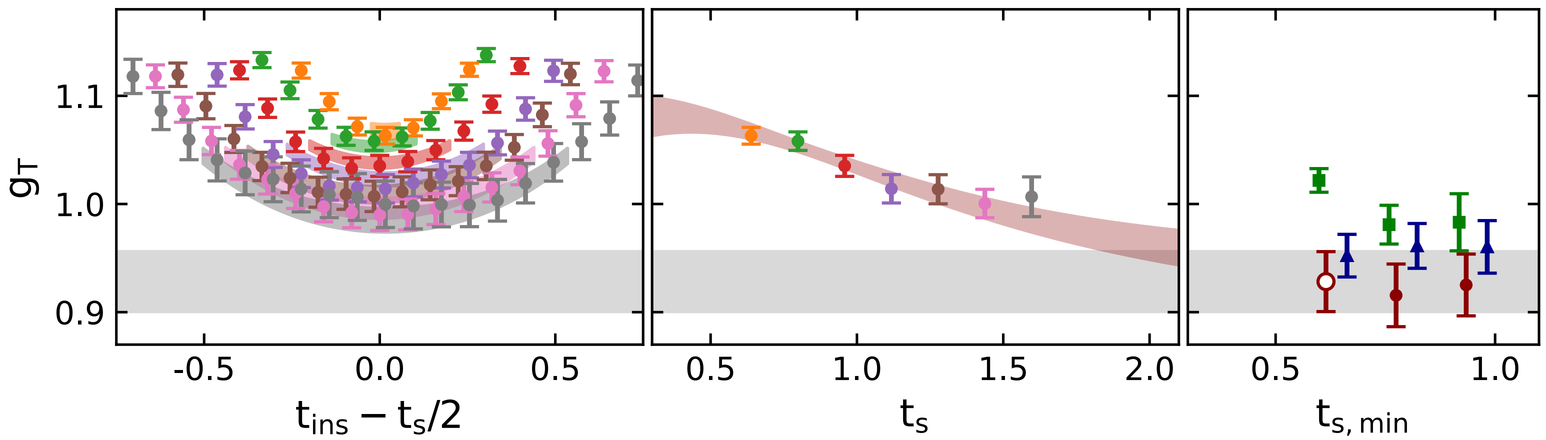
$$\bar{B}^q_{T10}(0) = \kappa_T \sim -h^\perp$$

Boer- Mulders function

anomalous tensor magnetic moment

Isovector tensor charge

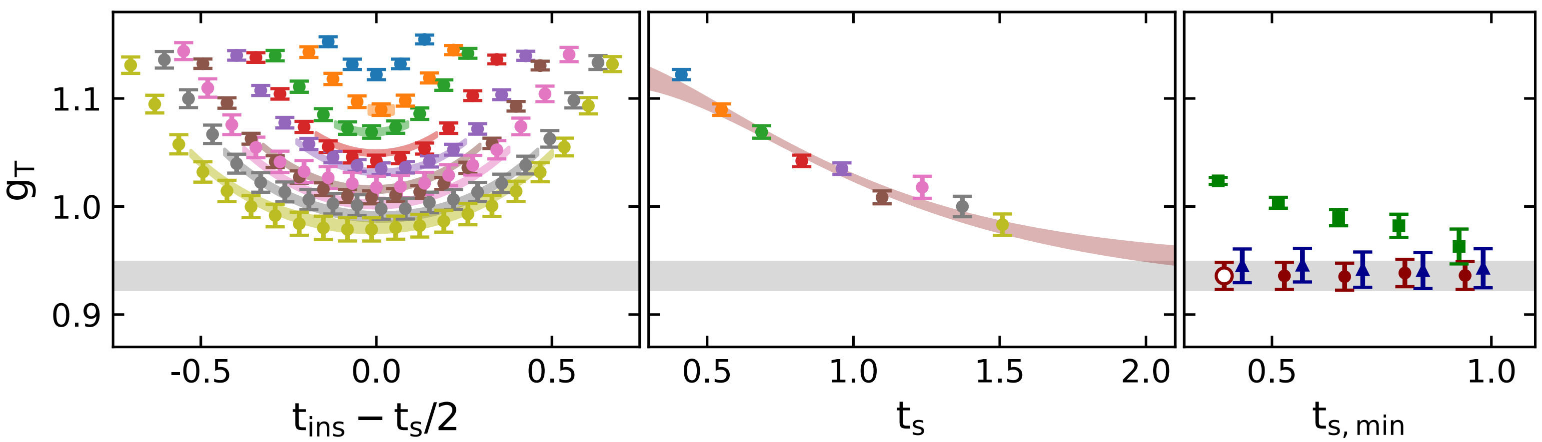
B-ensemble: 64^3 x 128, a~0.08 fm



t_s/a	t_s [fm]	n_{srcs}
8	0.64	1
10	0.80	2
12	0.96	4
14	1.12	6
16	1.28	16
18	1.44	48
20	1.60	64
2-point		264

x 750 configurations =
15 M inversions!

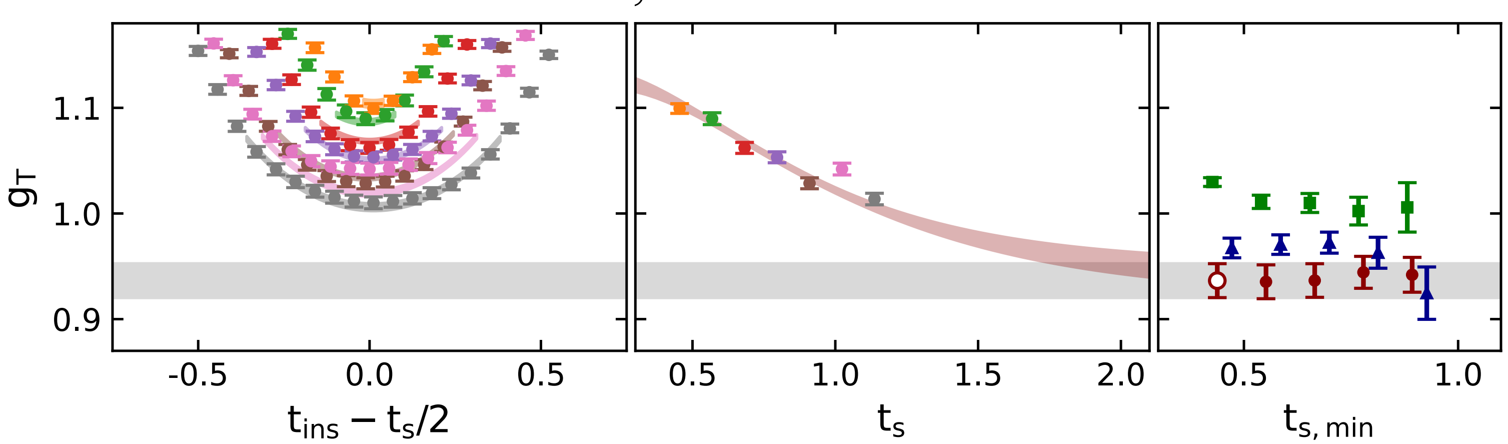
C-ensemble: 80^3 x 160, a~0.07 fm



t_s/a	t_s [fm]	n_{srcs}
8	0.549	1
10	0.686	2
12	0.823	5
14	0.961	11
16	1.098	24
18	1.235	45
20	1.372	116
22	1.509	246
2-point		650

x 401 configurations =
23.6 M inversions!

D-ensemble: 96^3 x 192, a~0.06 fm



t_s/a	t_s [fm]	n_{srcs}
8	0.456	1
10	0.570	2
12	0.684	4
14	0.798	8
16	0.912	16
18	1.026	32
20	1.140	64
22	1.254	16
24	1.368	32
26	1.482	64
2-point		368

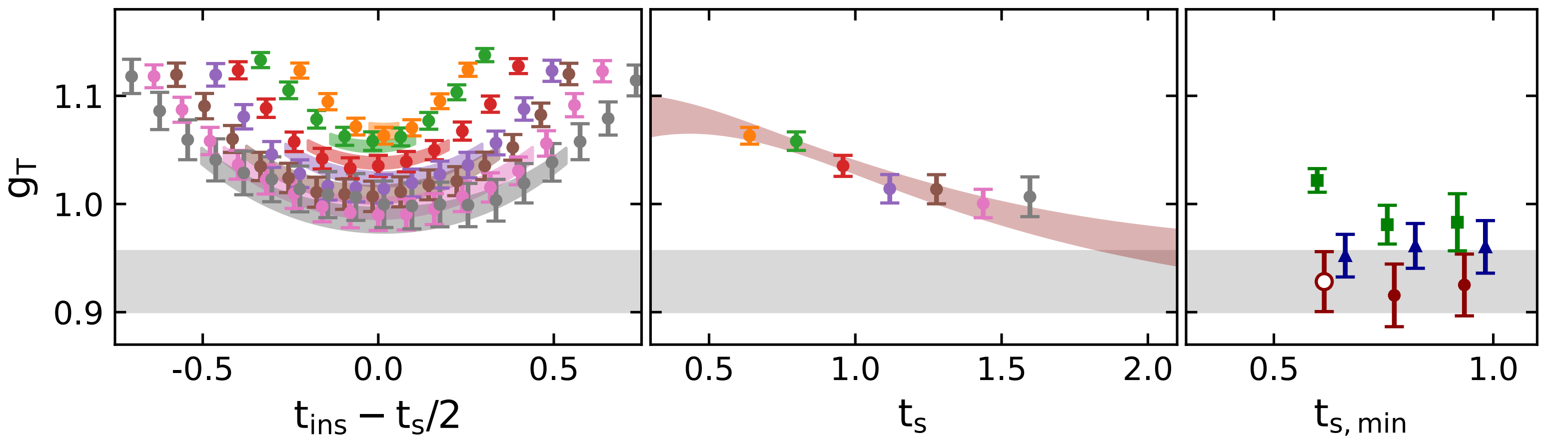
on-going

x 500 configurations =
15.9 M inversions!

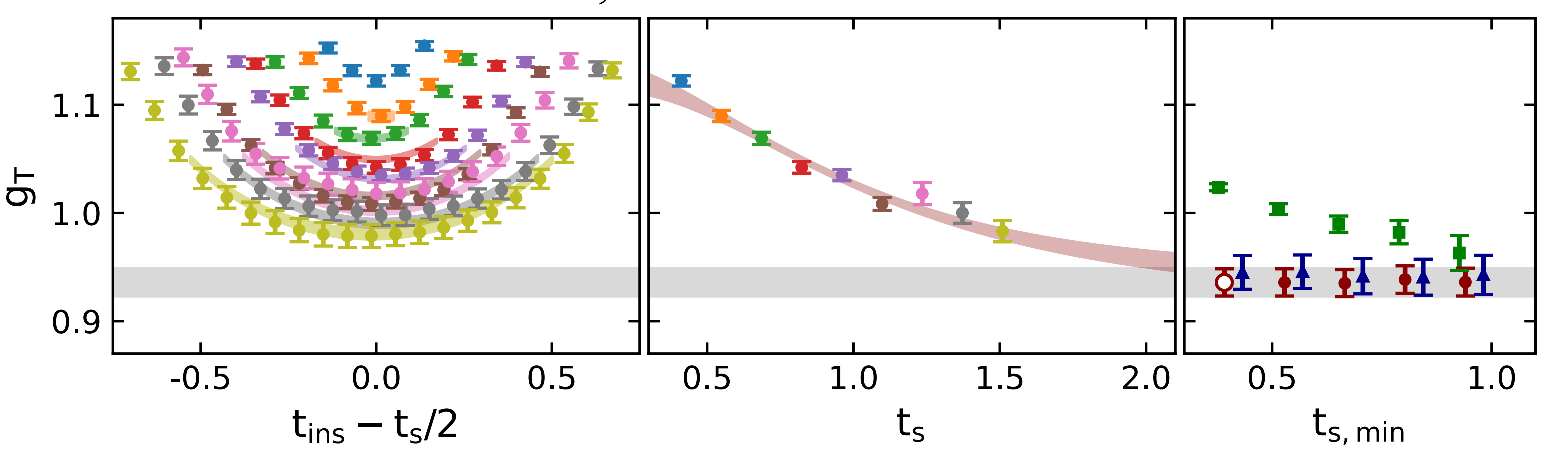
- Important to probe large t_s values keeping error approximately the same to reliably eliminate contributions from excited states

Isovector tensor charge

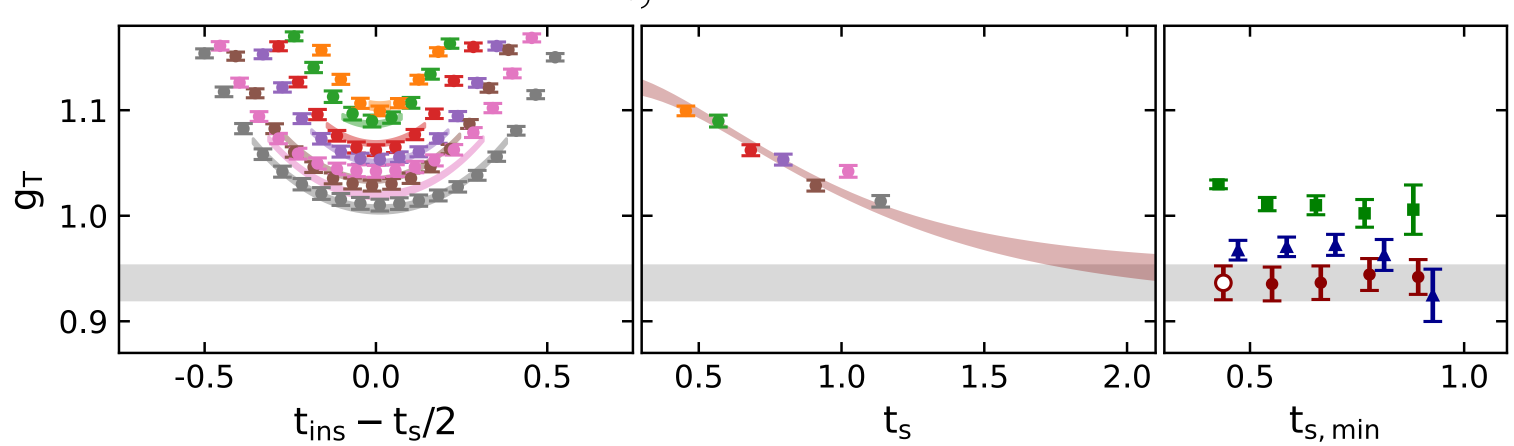
B-ensemble: 64^3 x 128, a~0.08 fm



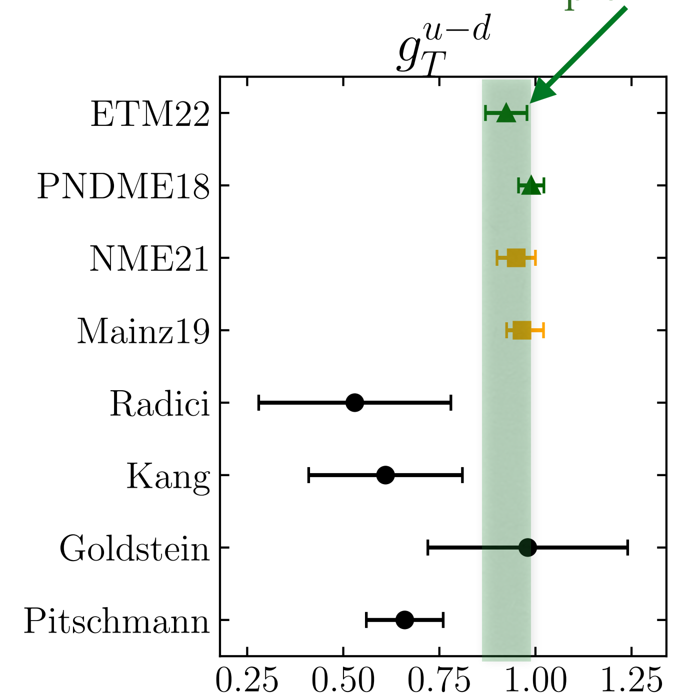
C-ensemble: 80^3 x 160, a~0.07 fm



D-ensemble: 96^3 x 192, a~0.06 fm



Continuum limit directly at physical pion mass



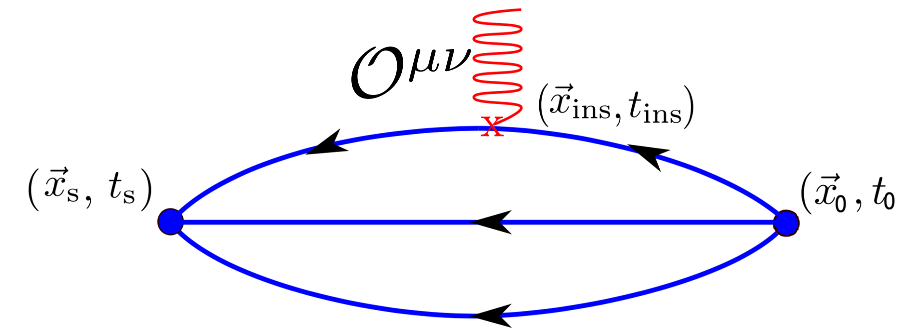
- We find g_T=0.924(54) more accurate than what extracted from phenomenology.

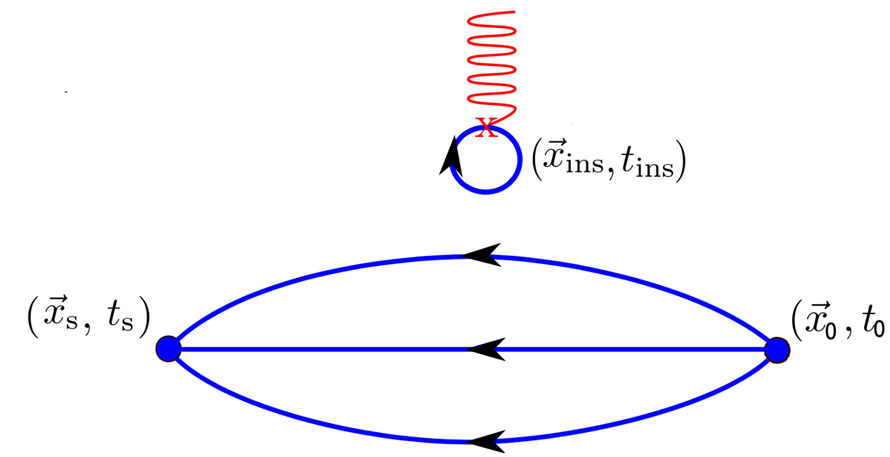
—> precision era of lattice QCD

Tensor charge for each quark

N_f=2+1+1 twisted mass fermions with a clover term

- Lattice size 64^3 x 128
- a=0.08 fm determined from the nucleon mass
- m_π=139 MeV
- Lm_π=3.6





Statistics for connected contribution

t_s/a	$N_{\rm cnfs}$	$N_{\rm srcs}$	$N_{\rm meas}$
8	750	1	750
10	750	2	1500
12	750	4	3000
14	750	6	4500
16	750	16	12000
18	750	48	36000
20	750	64	48000
2-pt	750	264	198000

0.64 fm

Needed for studying excited states

1.6 fm

Increase statistics to keep approx. constant error

Use hierarchical probing

no. of Hadamard vectors

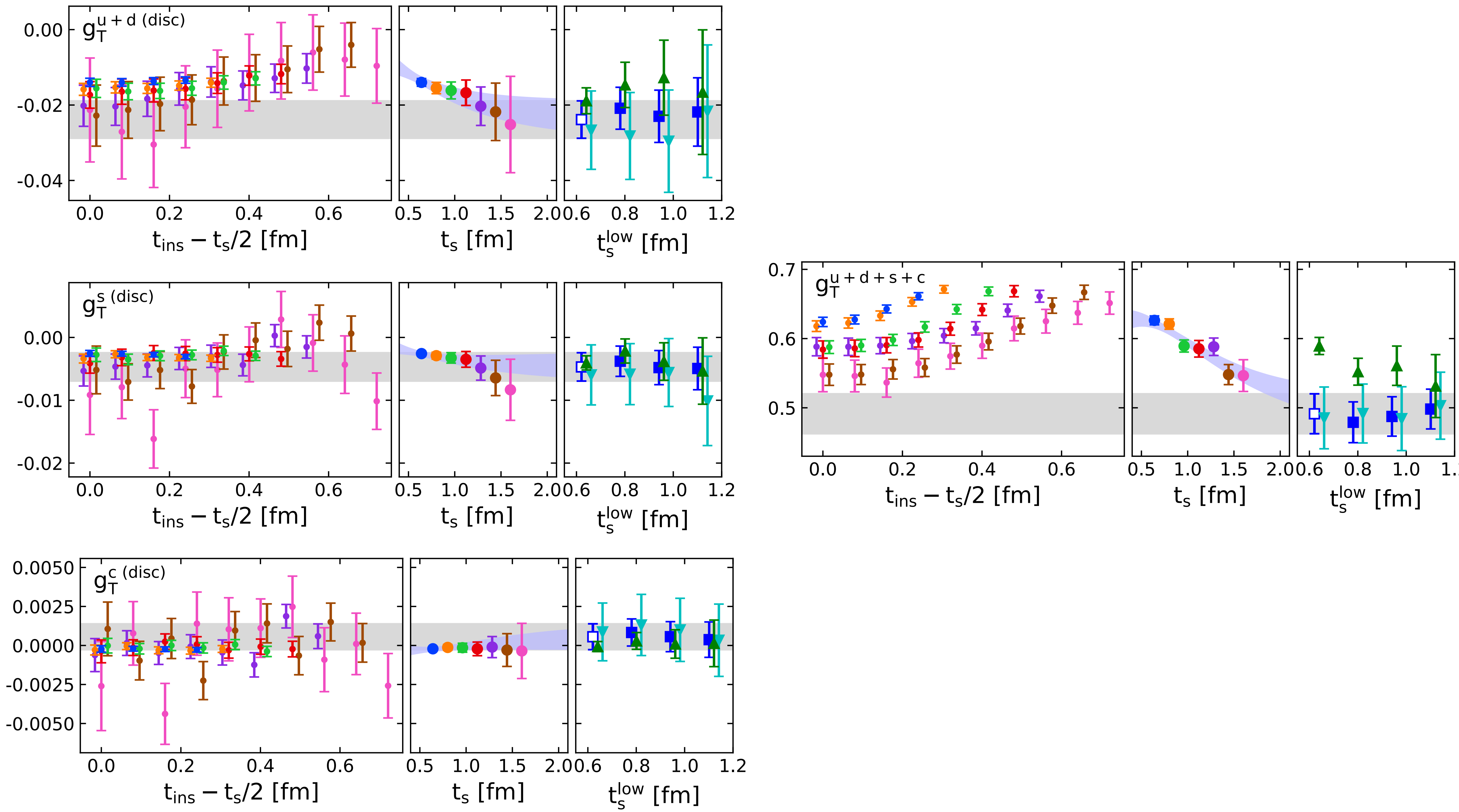
Statistics for disconnected contribution

2pt	(u+d)-quark loop	s-quark loop	c-quark loop
600000	750×512 + deflation of 200 modes	750×512	9000×32

no. of stochastic vectors

Isoscalar and flavour diagonal tensor charge

B-ensemble: 64^3 x 128, a~0.08 fm



❋ Disconnected small but non-zero

	u-d	u	d	s	c
g_T	0.936(25)	0.737(23)	-0.217(23)	-0.0041(12)	-0.0060(37)

C. A., S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou and A. Vaquero Aviles-Casco, *Phys.Rev.D* 102 (2020) 5, 054517, arXiv:1909.00485

Second Mellin generalized unpolarised and tensor form factors

- Unpolarized: $\mathcal{O}_{\mathrm{V}}^{\mu\nu} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$

$$\langle N(p',s')|\mathcal{O}_{\mathrm{V}}^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}(Q^2)\gamma^{\{\mu P^{\nu}\}}+B_{20}(Q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m}+C_{20}(Q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\Big]u_N(p,s)$$

$$\langle x\rangle_q = A_{20}^q(0) \qquad J_q = \frac{1}{2}\left[A_{20}^q(0)+B_{20}^q(0)\right]$$

- Tensor: $\mathcal{O}_T^{\mu\nu\rho} = i\,\bar{q}\sigma^{[\mu\{\nu]}\,i\,D^{\rho\}}\,q$

$$\langle N(p',s')|\mathcal{O}_T^{\mu\nu\rho}|N(p,s)\rangle = u_N(p',s')\left[i\sigma^{\mu\nu}\overline{P}^\rho\,A_{T20}(Q^2)+\frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}\overline{P}^\rho\,B_{T20}(Q^2)+i\,\frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\overline{P}^\rho\,\widetilde{A}_{T20}(Q^2)+\frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\Delta^\rho\,\widetilde{B}_{T21}(Q^2)\right]u_N(p,s)$$

$$\langle x\rangle_{\delta q} = A_{T20}^q(0)$$

Unpolarized Mellin moment

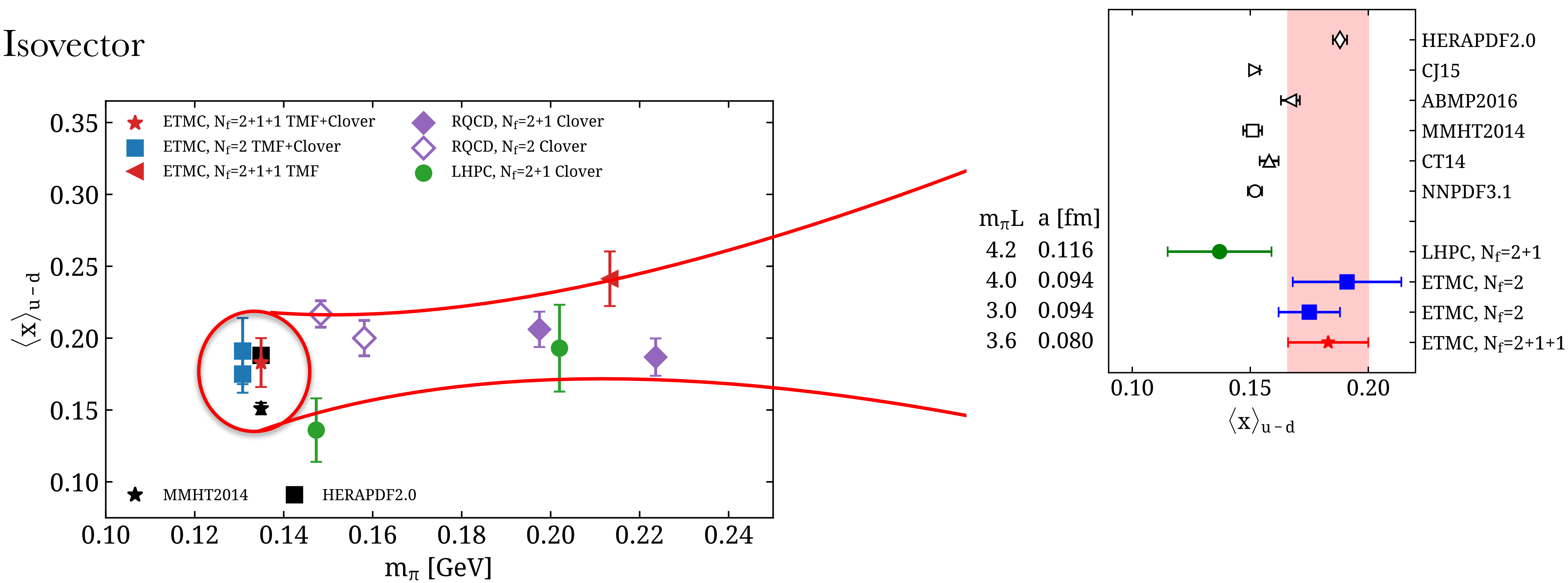
Momentum fraction - best measured

$$\mathcal{O}_{\mathrm{V}}^{\mu\nu} = \bar{q}\gamma^{\{\mu} iD^{\nu\}} q$$

$$\langle N(p',s')|\mathcal{O}_{\mathrm{V}}^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^{\mathrm{q}}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{\mathrm{q}}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^{\mathrm{q}}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

$$\langle x\rangle_q = A_{20}^q(0) \qquad J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

Isovector



Unpolarized Mellin moment

Momentum fraction - best measured

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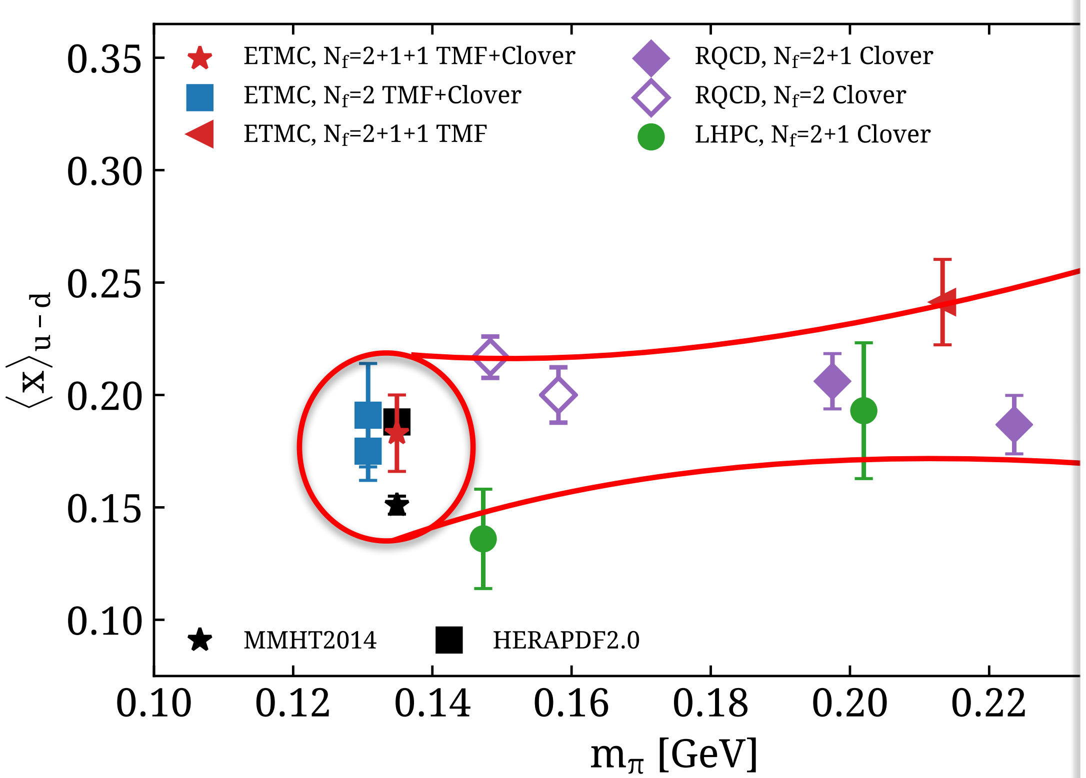
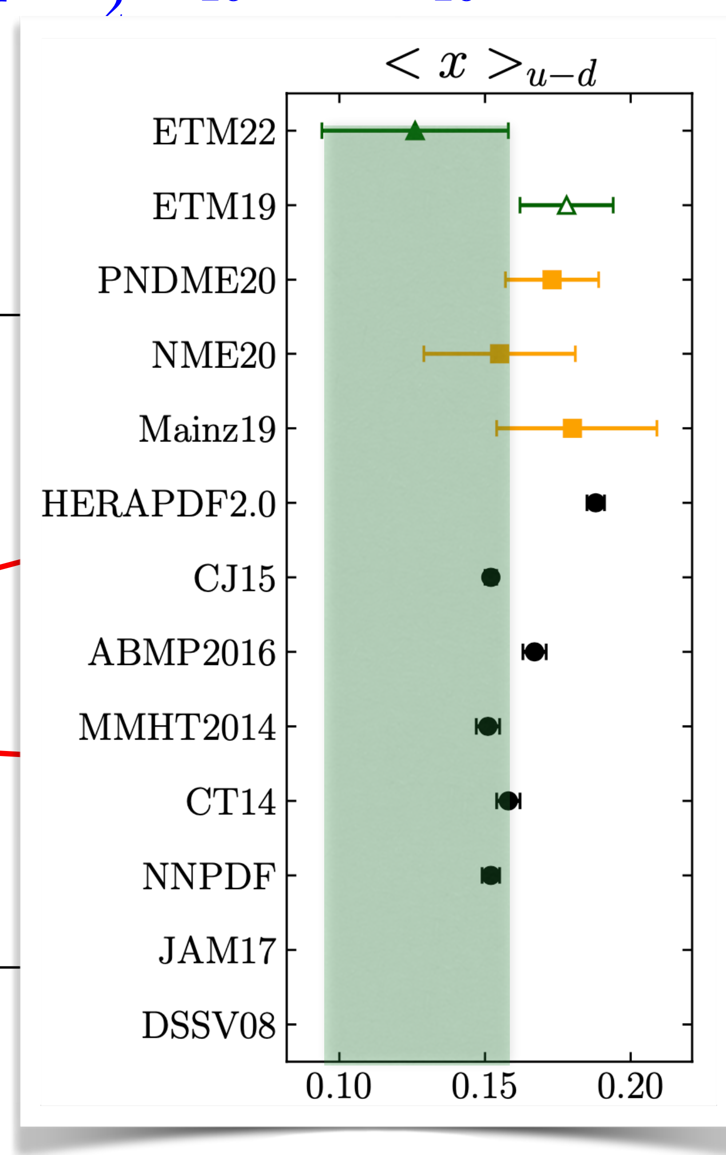
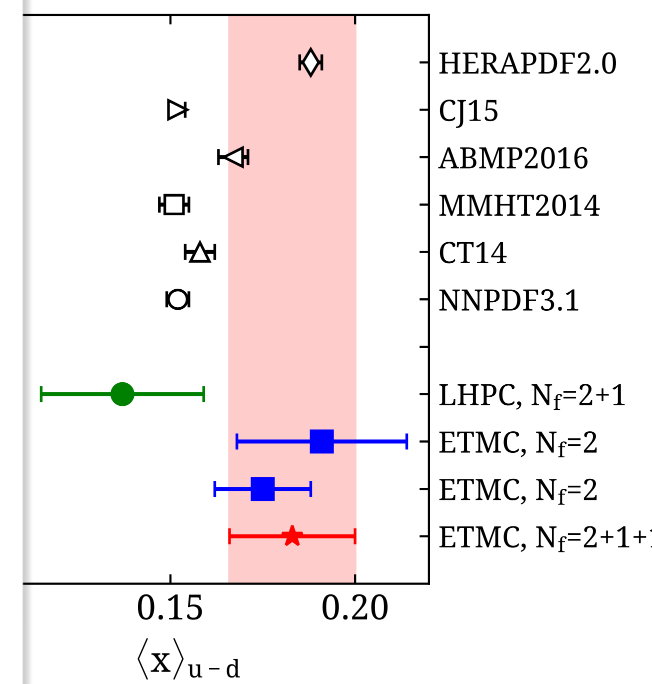
$$\langle N(p',s')|\mathcal{O}_{\rm V}^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}^{\rm q}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{\rm q}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^{\rm q}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\Big]u_N(p,s)$$

$$\langle x\rangle_q = A_{20}^q(0) \qquad J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

Isovector







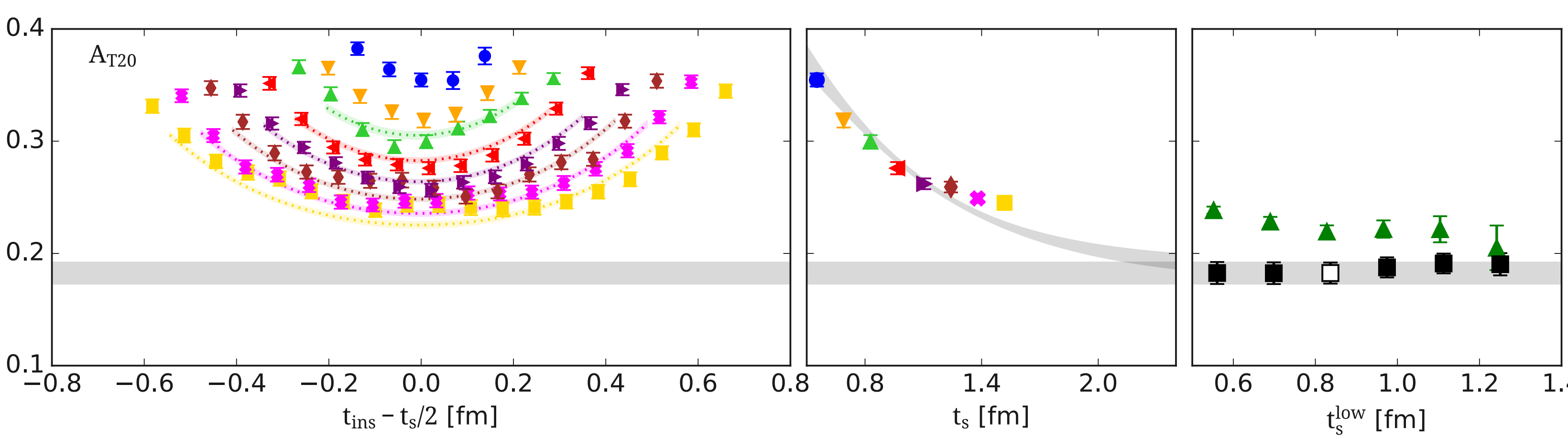
Second Mellin of generalized tensor form factors

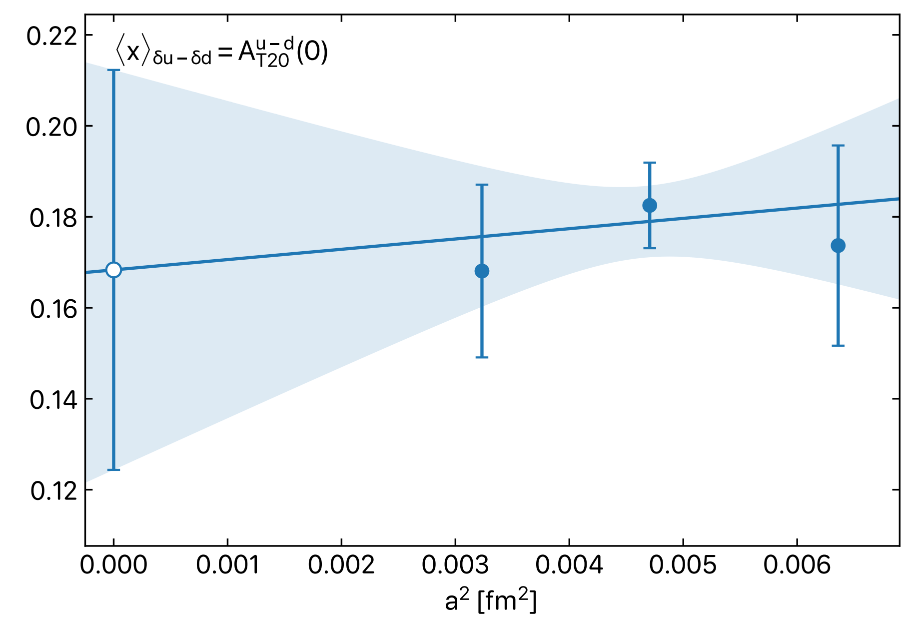
- Tensor: $\mathcal{O}_T^{\mu\nu\rho} = i\,\bar{q}\sigma^{[\mu\{\nu]}\,i\,D^{\rho\}}\,q$

$$\langle N(p',s')|\mathcal{O}_T^{\mu\nu\rho}|N(p,s)\rangle = u_N(p',s')\left[i\sigma^{\mu\nu}\overline{P}^{\rho}\,A_{T20}(Q^2) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}\overline{P}^{\rho}\,B_{T20}(Q^2) + i\,\frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\overline{P}^{\rho}\,\widetilde{A}_{T20}(Q^2) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\Delta^{\rho}\,\widetilde{B}_{T21}(Q^2)\right]u_N(p,s)$$

$$\langle x\rangle_{\delta u-\delta d} = A_{T20}^{u-d}(0)$$

C-ensemble: 80^3 x 160, a~0.07 fm
Isovector

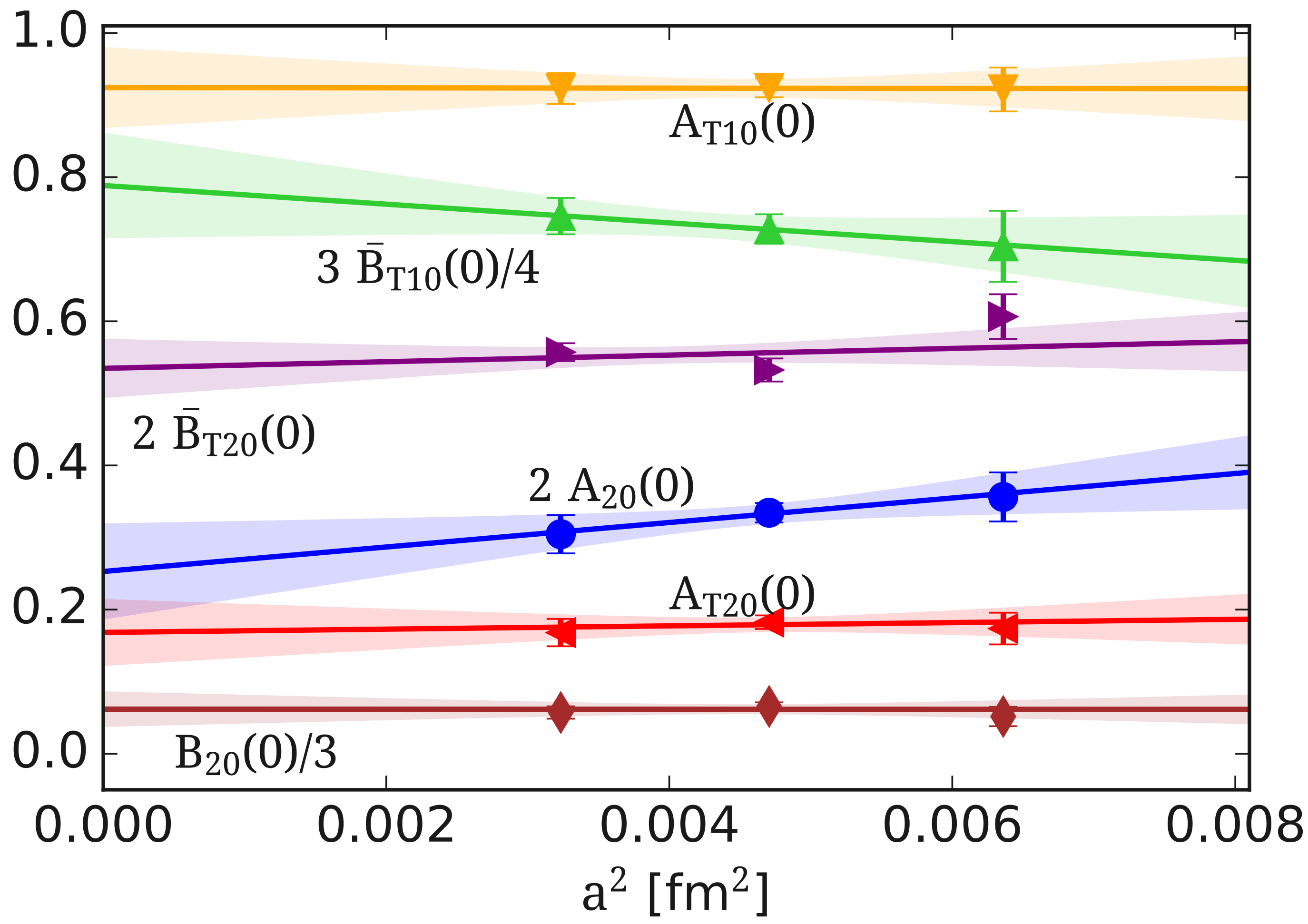




- In the continuum limit we predict: $\langle x\rangle_{\delta u-\delta d} = 0.168(44)$

Continuum extrapolation of Q^2=0 generalised form factors

- For the first we extrapolate to the continuum limit using 3 gauge ensembles directly at the physical pion mass
- All quantities shown from now on are for the isovector combination

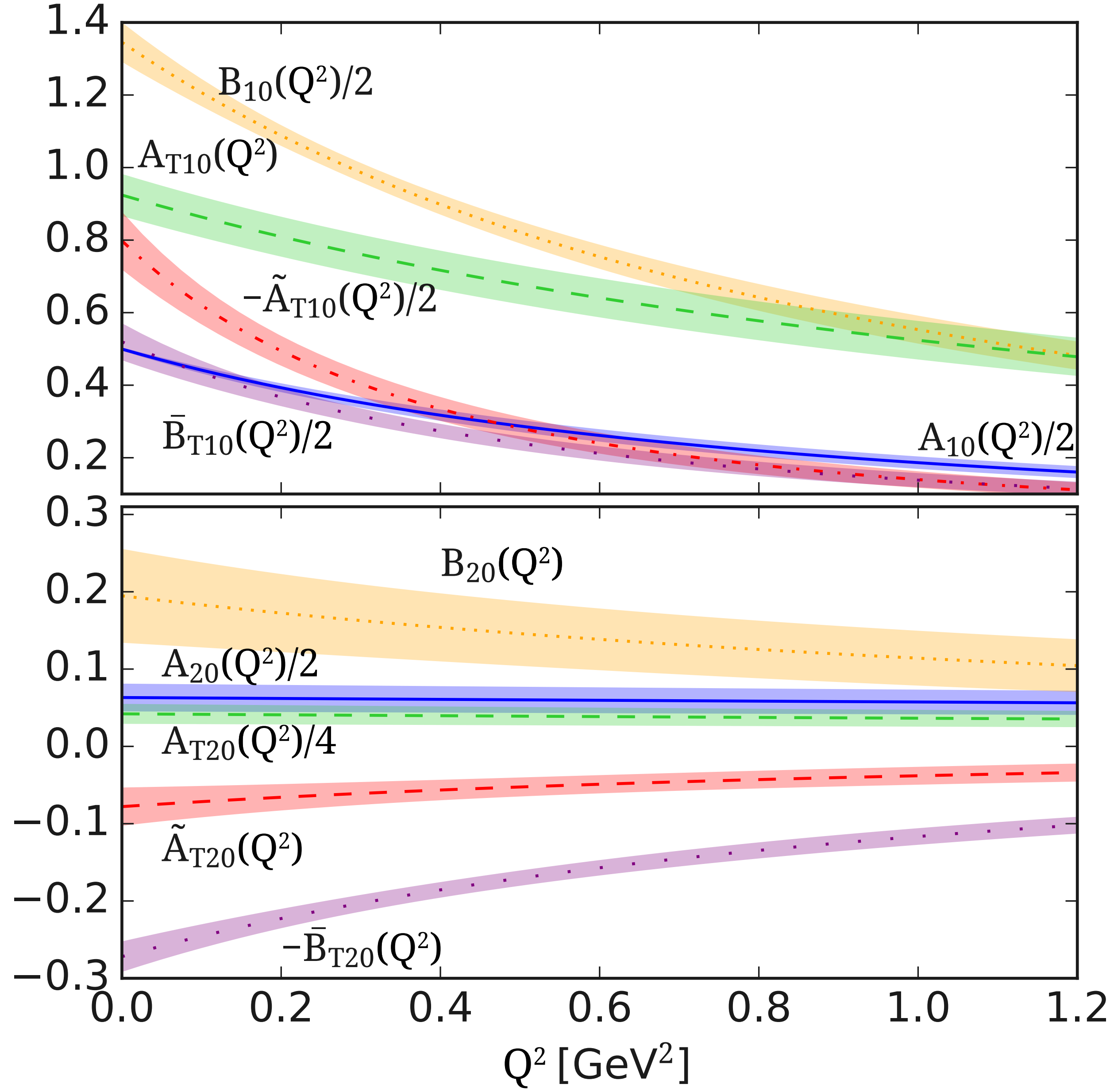


- We find for the anomalous tensor magnetic moment κ_T=1.05(9) —> Boer-Mulders functions negative and sizeable

C. A., S. Bacchio, M. Constantinou, P. Dimopoulos, J. Finkenrath, R. Frezzotti, K. Hadjiyiannakou, K. Jansen, B. Kostrzewa, G. Koutsou, G. Spanoudes, and C. Urbach, arXiv:2202.09871

Continuum extrapolation of generalised form factors

- For the first time, we extrapolate to the continuum limit using 3 gauge ensembles directly at the physical pion mass
- For a fixed value of Q^2 we extrapolated to a=0 and then fit to $F(Q^2) = \frac{F(0)}{(1+Q^2/m^2)^p}$



- Fourier transform to impact parameter space $\mathbf{\Delta}_\perp \leftrightarrow \mathbf{b}_\perp$

C. A., S. Bacchio, M. Constantinou, P. Dimopoulos, J. Finkenrath, R. Frezzotti, K. Hadjiyiannakou, K. Jansen, B. Kostrzewa, G. Koutsou, G. Spanoudes, and C. Urbach, arXiv:2202.09871

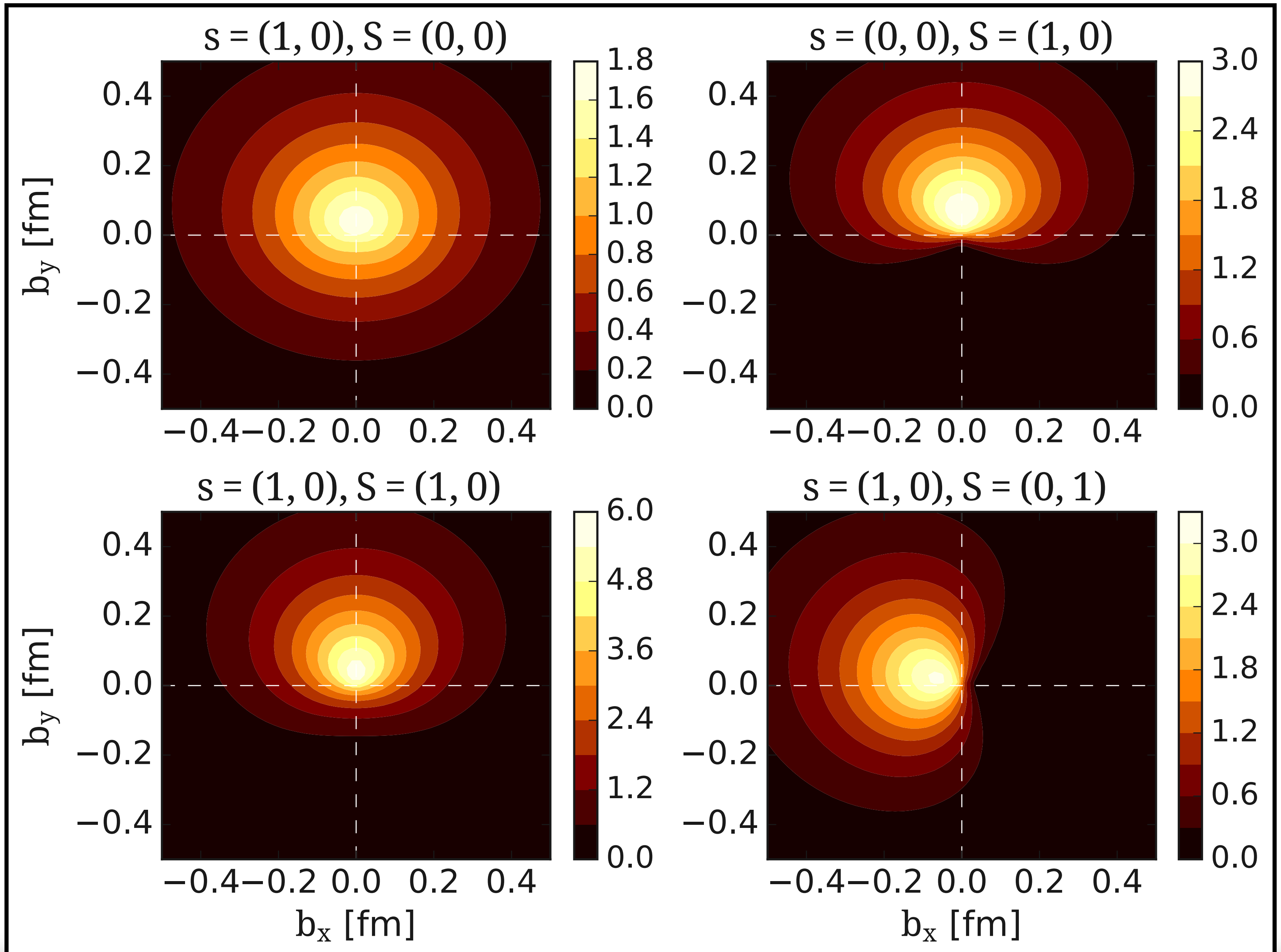
Transverse density distributions (isovector)

$\langle x^{n-1}\rangle_\rho(\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) \equiv \int_{-1}^{1} dx\; x^{n-1}\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp),$

transversely polarized quarks in an unpolarized nucleon

unpolarized quarks in a transversely polarized nucleon

Large effect due to B_{10} (Sivers effect)



transversely polarized quarks in a transversely polarized nucleon

transversely polarized quarks in a perpendicularly polarized nucleon

Contours of the first moment (n=1) of the probability density, as a function of b_x and b_y

$$\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) = \frac{1}{2}\Big[H(x,b_\perp^2)+\frac{\mathbf{b}_\perp^j\epsilon^{ji}}{m_N}\left(\mathbf{S}_\perp^i E'(x,b_\perp^2)+\mathbf{s}_\perp^i \bar{E}'_T(x,b_\perp^2)\right)+\mathbf{s}_\perp^i\mathbf{S}_\perp^i\left(H_T(x,b_\perp^2)-\frac{\Delta_{b_\perp}\tilde{H}_T(x,b_\perp^2)}{4m_N^2}\right)+\mathbf{s}_\perp^i(2\mathbf{b}_\perp^i\mathbf{b}_\perp^j-\delta^{ij}b_\perp^2)\mathbf{S}_\perp^j\frac{\tilde{H}''_T(x,b_\perp^2)}{m_N^2}\Big]$$

M. Diehl and Ph. Hägler, Eur. Phys. J. C44 (2005) 87, hep-ph/0504175

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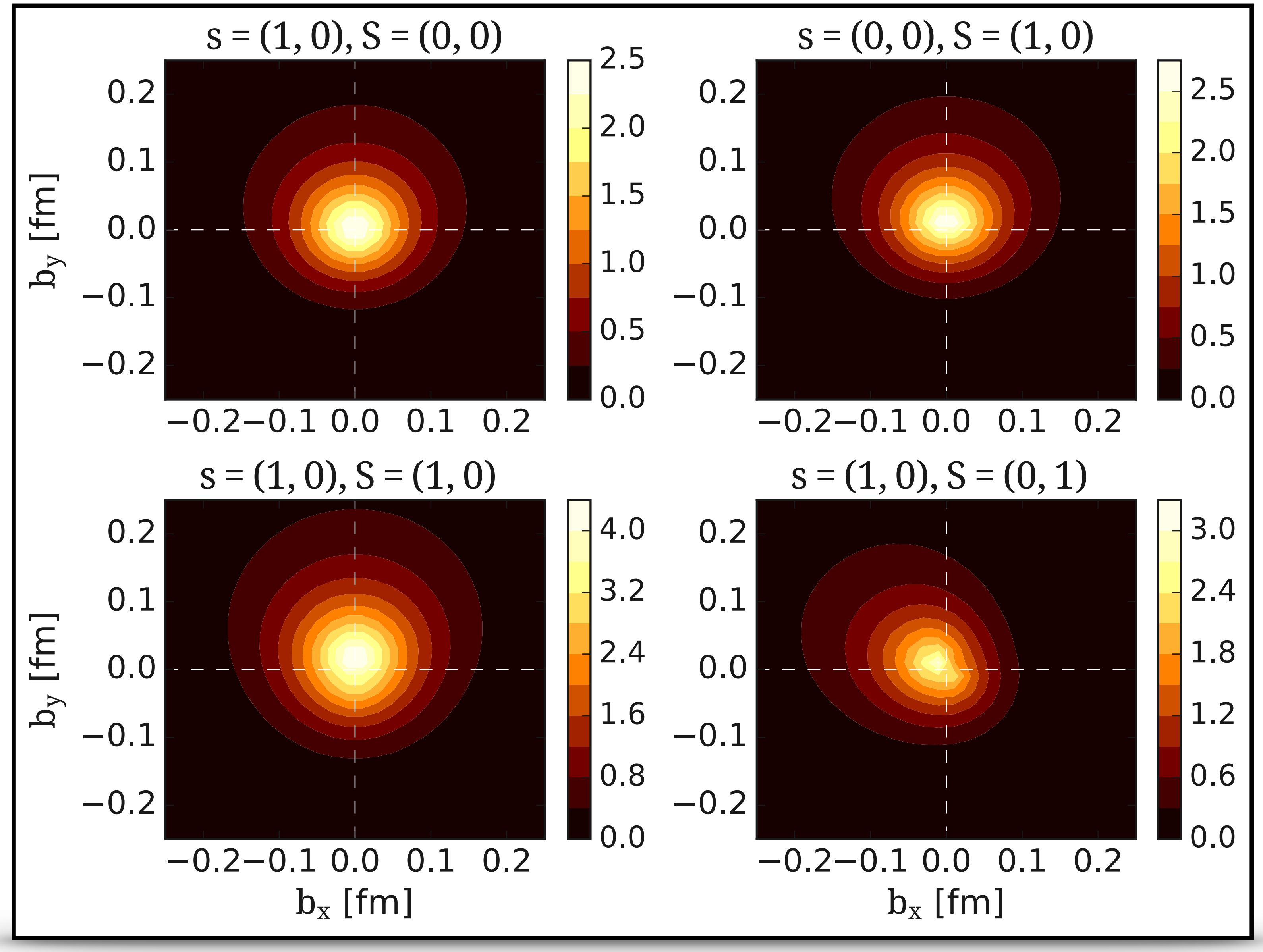
transversely polarized quarks in an unpolarized nucleon

unpolarized quarks in a transversely polarized nucleon



transversely polarized quarks in a transversely polarized nucleon

transversely polarized quarks in a perpendicularly polarized nucleon

Contours of the second moment (n=2) of the probability density, as a function of b_x and b_y

Distortion is milder than for n=1 due to the milder dependence of $A_{20}(Q^2)$ compared to $A_{10}(Q^2)$

Conclusions

- **Precision era of lattice QCD**: Moments of PFDs can be extracted precisely and directly at the physical pion mass including the continuum limit
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