Ab-initio study of dibaryons with highest bottom number#

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Dibaryons and baryon-baryon interactions

Understanding baryon-baryon interactions from first-principles is of prime interest in nuclear and astrophysics, as they form the foundation towards a fundamental explanation of why atomic nuclei exist. Dibaryons are the simplest systems with baryon number 2, in which such interactions can be studied transparently. Despite decades of experimental efforts, only two dibaryons are known (Deuteron and d*(2380)) to this date. Even so, there might be other bound or resonant dibaryons, particularly in the strange and heavy sectors, that exist in Nature that are yet to be discovered.

Among the heavy baryons, Ω_{ccc} and Ω_{bbb} serves a promising domain to study the nonperturbative features of QCD, free of the light quark dynamics. On the same ground, a system of two Ω_{ccc} and two Ω_{bbb} baryons are interesting as they serve as useful platforms in understanding the baryon-baryon interactions in chiral dynamics free environment.

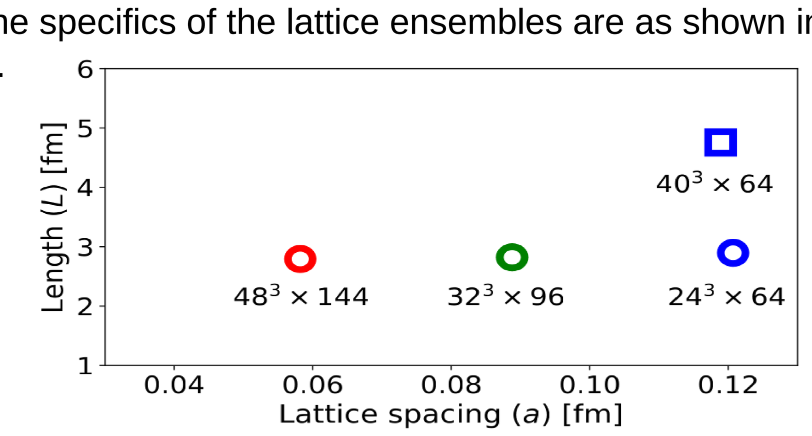
Dibaryons from lattice QCD

As of today, there are a handful of lattice QCD investigations of dibaryons in the light and the strange sector, mostly performed at unphysically heavy quark masses, covering channels such as Deuteron, dineutron, H-dibaryon, etc. An important lesson learned from these studies is that discretization effects could be substantial in these systems, which when unaddressed lead to large discrepancies [1]. Two Ω_{sss} system has been addressed in two lattice QCD works [2,3] following different methodologies with conflicting observations, whereas a recent study indicated a shallow bound state in two Ω_{ccc} system [4]. Another recent study of deuteron-like heavy dibaryons indicated possible deeply bindings in the bottom sector [5]. Naturally, an investigation of two Ω_{bbb} would be very timely, as this can shed light on the behaviour of interactions between two Ω baryons in the heavy quark sector.

In this work, we perform a lattice QCD calculation of scalar dibaryons with highest number of bottom quarks. We name such a dibaryon system as $\mathcal{D}_{6b}$. The elastic threshold in this channel is twice the mass of a spin 3/2 Ω_{bbb} baryon.

Lattice setup

Ensembles: We utilize four ensembles with dynamical u/d, s, and c quark fields generated by the MILC collaboration, with HISQ fermion action [6]. The specifics of the lattice ensembles are as shown in the below figure.



Bottom quark propagators: Quark propagators were computed using the time evolution of an NRQCD Hamiltonian, including imporvements up to $\mathcal{O}(\alpha_s v^4)$ on Coulomb gauge fixed wall sources and multiple source timeslices. The bare quark mass was tuned using the spin averaged kinetic mass of 1S bottomonia states and was found to reproduce the 1S bottomonia hyperfine splitting.

(Di)baryon interpolators

A quasi-local nonrelativistic operator with $J^P=3/2^+$ was utilized for Ω_{bbb} baryon. The color space is trivially antisymmetric for a baryon. The flavor and the spatial structure of the operator is trivially symmetric. The remaining spin space for a nonrelativistic operator allows only J=3/2 spin, which is symmetric as shown below.

$$\chi_m = |3/2, m\rangle; \quad |3/2,3/2\rangle = |1\,1\,1\rangle$$
$$|3/2,1/2\rangle = (|1\,1\,2\rangle + |1\,2\,1\rangle + |2\,1\,1\rangle)/\sqrt{3}$$
$$|3/2,-1/2\rangle = (|1\,2\,2\rangle + |2\,1\,2\rangle + |2\,2\,1\rangle)/\sqrt{3}$$
$$|3/2,-3/2\rangle = |2\,2\,2\rangle$$

Here 1 and 2 refer to $|1/2,+1/2\rangle$ and $|1/2,-1/2\rangle$ nonrelativistic quark spins respectively.

Assuming dominance of *s*-wave scattering in the Ω_{bbb} interactions near the two baryon threshold, we build the two hadron operator relevant for scalar $\mathcal{D}_{6b}$ channel using simple spin algebra as follows.

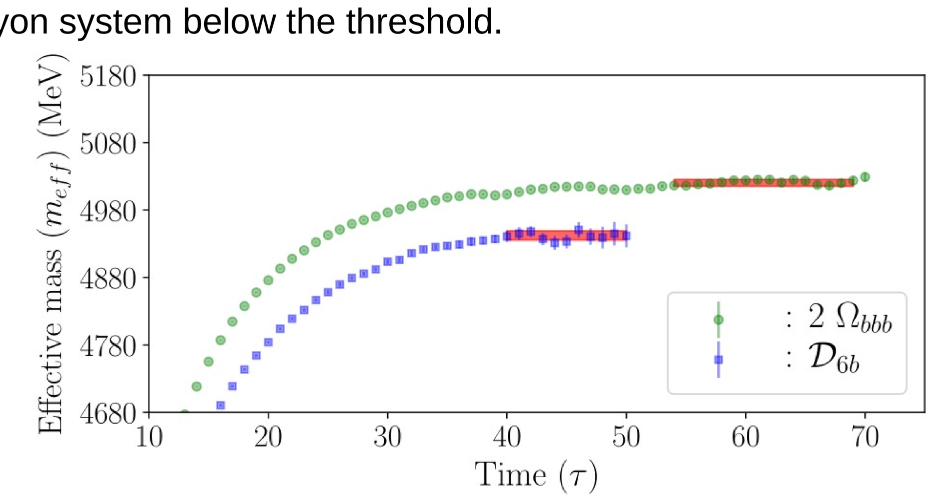
$$\mathcal{O}_{\mathcal{D}_{6b}^{J=0}} = \frac{1}{2}\left[\chi_{\frac{3}{2}}\chi_{-\frac{3}{2}} + \chi_{-\frac{1}{2}}\chi_{\frac{1}{2}} - \chi_{\frac{1}{2}}\chi_{-\frac{1}{2}} - \chi_{-\frac{3}{2}}\chi_{\frac{3}{2}}\right]$$

Two point correlation functions are computed using these baryon and dibaryon interpolators as

$$C_{\mathcal{O}}(t_f - t_i) = \sum_{\vec{x}_f} e^{-i\vec{p}.\vec{x}_f}\langle 0|\mathcal{O}(\vec{x}_f, t_f)\bar{\mathcal{O}}(t_i)|0\rangle.$$

Signal quality and fit estimates

We present the signal quality and saturation in baryon and the dibaryon correlation functions in terms of $am_{eff}=log[\bar{C}(\tau)/\bar{C}(\tau+1)]$, referred to as the effective mass, in the figure below. Here $\bar{C}$ is the ensemble average of correlation function C. Signal saturation and a clear energy gap between the noninteracting two-baryon level and the interacting dibaryon system in the finest ensemble is evident from the figure below. This pattern is observed on all four ensembles we study, which unambiguously point to a finite volume energy level in the interacting dibaryon system below the threshold.



Single exponential fits to $C(\tau)$ yield the mass estimates in lattice units for the baryon and dibaryon ground states, which also includes an additive normalization proportional to the number of heavy quarks within the hadron realized using NRQCD Hamiltonian. We determine the energy splittings $\Delta E = M_{\mathcal{D}_{6b}} - 2M_{\Omega_{bbb}}$ from the baryon and dibaryon ground state mass estimates, $M_{\Omega_{bbb}}$ and $M_{\mathcal{D}_{6b}}$, respectively. Note that by taking such energy differences, it automatically takes care of the additive normalization in the energy estimates intrinsic to the NRQCD formulation. ΔE in physical units as determined on different lattice are tabluated in the below table.

Ensemble	ΔE	Ensemble	ΔE
$24^3\times 64$	$-64(11)$	$40^3\times 64$	$-65(7)$
$32^3\times 96$	$-72(9)$	$48^3\times 144$	$-75(7)$

Scattering length and binding energy

The existence and the properties of a hadron from finite volume spectrum is determined in terms of pole singularities in the relevant scattering amplitudes across the complex Mandelstam *s*-plane. To this end, we follow Lüscher's finite-volume formalism. The s-wave scattering amplitude is given by $t=(cot\delta_0-i)^{-1}$, and a pole in t related to a bound state happens when $k.cot\delta_0 = -\sqrt{-k^2}$. Here $\delta_0(k)$, the phase shift in two spin 3/2 Ω_{bbb} scattering leading to $J^P=0^+$ is related to the finite-volume energy eigenvalues as

$$k\ cot(\delta_0(k)) = \frac{2Z_{00}(1;(\frac{kL}{2\pi})^2)}{L\sqrt{\pi}}$$

Here k is the momentum of the Ω_{bbb} baryon in the centre of momentum frame. k is given by

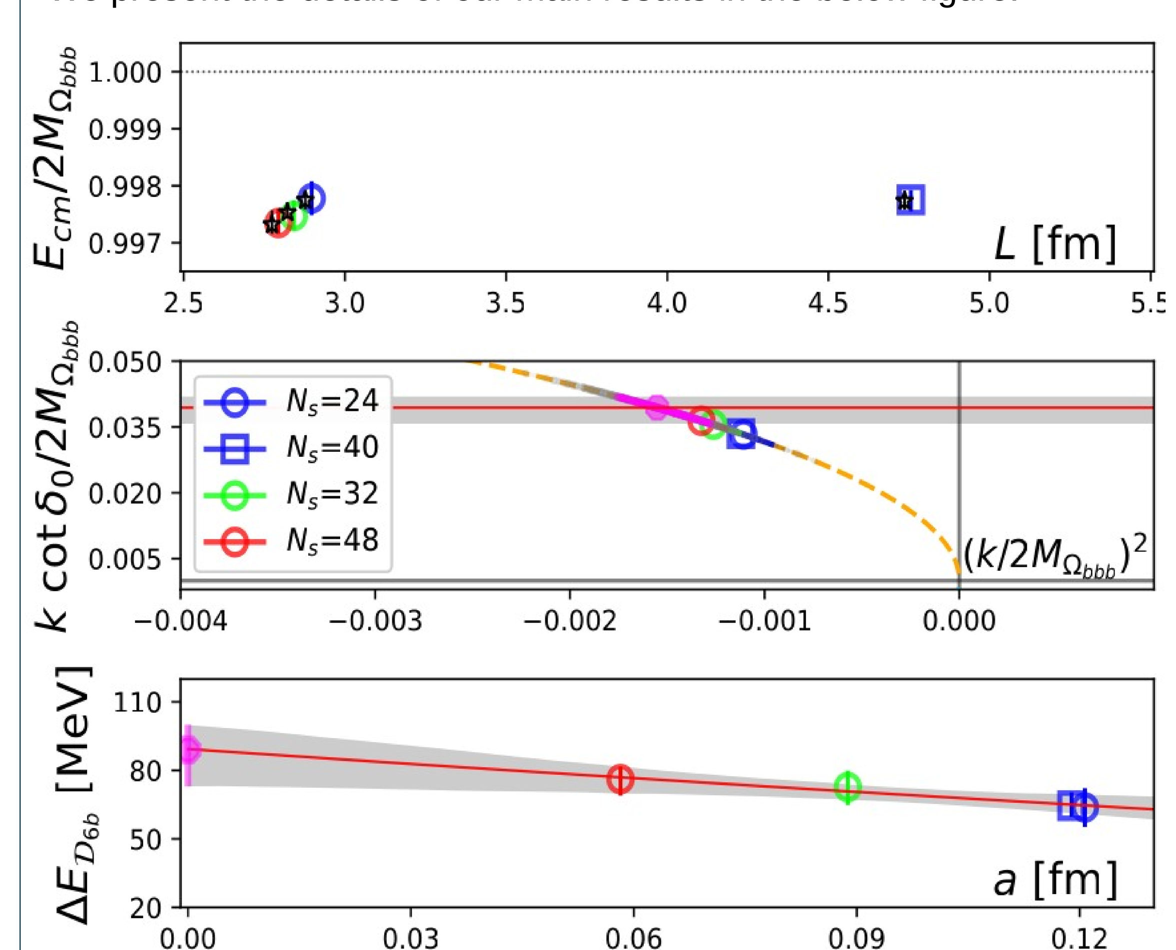
$$k^2 = \frac{\Delta E}{4}(\Delta E + 4M^{phys}_{\Omega_{bbb}})$$

$M^{phys}_{\Omega_{bbb}}$ is the mass of Ω_{bbb} baryon in the continuum limit determined independently from the Ω_{bbb} baryon correlation functions.

We parametrize $k.cot\delta_0$ either as a constant or as a constant plus a linear term in the lattice spacing to determine possible cutoff systematics in our finite volume treatment. We perform several different fits involving different subsets of the four energy splittings listed earlier, with the two different fits forms. All of the fits indicate the existence of a deeply bound state pole in this channel. The best fit is found to be the one that incorporates all the energy splittings, and incorporates the lattice spacing dependence of the scattering length a_0 with a linear parametrization $k\ cot\delta_0 = -1/a_0^{[0]} - a/a_0^{[1]}$. The fit quality turns out to be $\chi^2/d.o.f = 0.8/2$, and the estimates in this case and the resultant binding energy are

$$a_0^{[0]} = 0.17(^{+0.01}_{-0.02})\ \text{fm}, \quad a_0^{[1]} = -0.14(^{+0.18}_{-0.06})\ \text{fm}^2$$
$$\text{and}\quad \Delta E_{\mathcal{D}_{6b}} = -89(^{+16}_{-12})\ \text{MeV}.$$

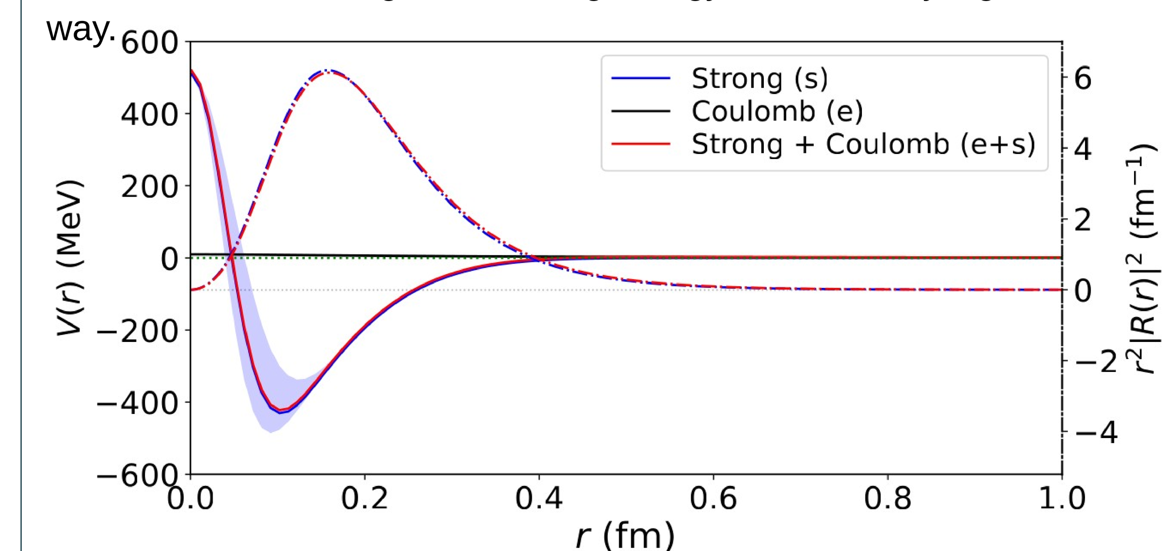
We present the details of our main results in the below figure.



Top: The simulated energy levels (large symbols) together with the analytically reconstructed energy levels (black stars) from the fit results above. Middle: $k.cot\delta_0$ versus k^2 in units of the elastic threshold. The orange dashed line is the bound state constraint ($-\sqrt{-k^2}$), the horizontal red line and error band indicate the fitted estimate for scattering length in the continuum limit. The crossing of these two curves, indicated by the magenta symbol, refers to the bound state pole. Bottom: Continuum extrapolation of the reconstructed binding energies $\Delta E_{\mathcal{D}_{6b}}$ determined from the fitted scattering amplitude.

Coulombic repulsion and other systematics

Coulomb repulsion: Being very heavy, the $\mathcal{D}_{6b}$ system could be very compact and hence the Coulombic repulsion effects could be nonnegligible. To account for this, we model the strong interactions between two Ω_{bbb} baryons with a multi-Gaussian attractive potential V_s. The parameters are constrained to reproduce a binding energy of -89 MeV. Assuming the electric charge distribution $\rho(r) = 12\sqrt{6}/(\pi r_d^3)e^{-2\sqrt{6}r/r_d}$ with r_d chosen as the root-mean-square radius of the Quantum Mechanical ground state of V_s, we determine the Coulombic potential V_e. We look for the Quantum Mechanical solutions of the effective potential $V_{eff}=V_s+V_e$, and then determine the scattering length a_0^{e+s}. In the figure below, we present the strengths of each potentials and the ground state radial probability densities for V_s and V_{eff} potentials. It is evident that the Coulombic repulsion serves only as a perturbation and hence does not change the binding energy of $\mathcal{D}_{6b}$ in any significant way.



The lattice setup being used in this work is identical to several of our published works in the recent past [4, 7, 8, 9]. Statistical and fit window errors are added in quadrature and then convolved through the Lüscher's analysis and continuum extrapolation, yielding a total error of about 18% for the binding energy. Other possible sources of errors are related to the continuum extrapolation fit forms, scale setting, quark mass tuning and electromagnetic corrections. In the table below, we present a summary of various systematic uncertainties in the binding energy estimate.

Source	Error (MeV)
Statistical + Fit-window + Continuum extrapolation	$(^{+16}_{-12})$
Discretization	8
Scale setting	3
m_b tuning	2
Electromagnetism	8
Total systematics	12

Summary

We present the first lattice QCD investigation of dibaryons composed of all six quarks with bottom flavor. We find a pole in the $^1S_0$ channel of Ω_{bbb}-Ω_{bbb} scattering. Following the Lüscher's finite volume formalism, and after considering different possible systematic uncertainties, we identify the deeply bound state with binding energy $-89(^{+16}_{-12})(12)$ MeV. Considering the small magnitudes of lattice estimates for binding energy in equivalent systems ($\mathcal{D}_{6s}$ and $\mathcal{D}_{6c}$) [2, 3, 4], it would be interesting to investigate the quark mass dependence of such systems, which could shed further light into the onset of possible light and strange quark dynamics at lighter quark masses.

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 # Computation was performed using XC30 of TIFR.

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