The 33th International Symposium on Lattice Field Theory

Towards $K\pi$ scattering at physical point using distillation

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THE UNIVERSITY *of* EDINBURGH

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Background

Motivations

- Resonances and Standard Model
- Multi-hadron states
  - rare decays, e.g. $B \to \boldsymbol{K^*} l^+ l^- (\to \boldsymbol{K\pi} l^+ l^-)$
  - multibody decays, e.g. $B \to \boldsymbol{K\pi\pi}$, $D \to \boldsymbol{\pi\pi}, \boldsymbol{K\bar{K}}$
- Insights into new physics (e.g. CP violation)
- Study hadronic resonances **non-perturbatively** on the lattice

[Briceño, Dudek, Young - RevModPhys.90.025001, 2018]

$\to$ Status of P-wave, $I = 1/2$, $K\pi$ scattering - resonant

$\to$ Dynamical strange quark

$\to$ ***Physical*** m_π

Lüscher method: workflow

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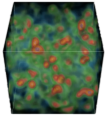
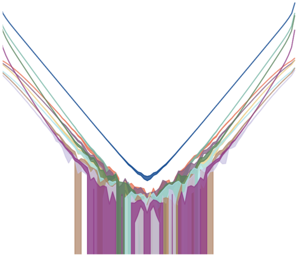
Lattice Data

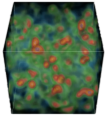


- correlators



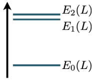
Lüscher method: workflow

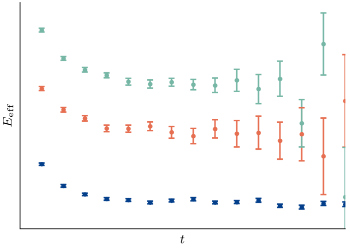
Lattice Data



- correlators
- variational method

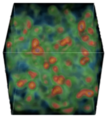
Finite-volume Spectrum



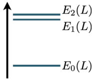


Lüscher method: workflow

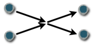
Lattice Data

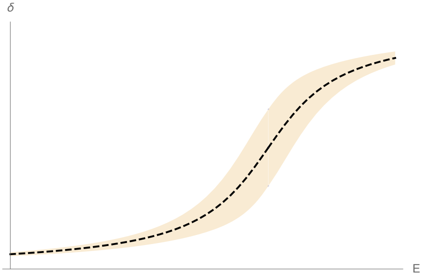


Finite-volume Spectrum



Finite-volume Analysis





- correlators
- variational method
- quantisation condition

Lattice Setup

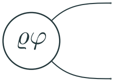
[Peardon, Bulava, Foley, Morningstar, Dudek, Edwards, Joó, Lin, Richards, Juge - PRD.80.054506, 2009] [Morningstar, Foley, Juge, Lenkner, Peardon, Wong - PRD.83.114505, 2011]

- (Stout) link smearing + **Covariant** $3D$**-Laplacian quark smearing**

$$\Box_{\mathbf{xy}}(t) = \sum_{k=1}^{N_{\mathrm{vec}}} v_k(\mathbf{x};t) v_k(\mathbf{y};t)^\dagger, \qquad v_k : \text{ } 3D\text{-Laplacian eigenvectors}$$

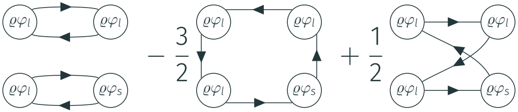
- **Meson fields** are the computational blocks

$$M_\Gamma(\varrho\varphi;\mathbf{p},t) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \varrho(\mathbf{x},t)^\dagger \Gamma \varphi(\mathbf{x},t) =$$



- For example

$$\left\langle O_{K\pi}(t_f)\; O_{K\pi}(t_0)\right\rangle_{I=1/2} =$$



Code

Lattice Framework

- Open-source and free software

github.com/paboyle/Grid

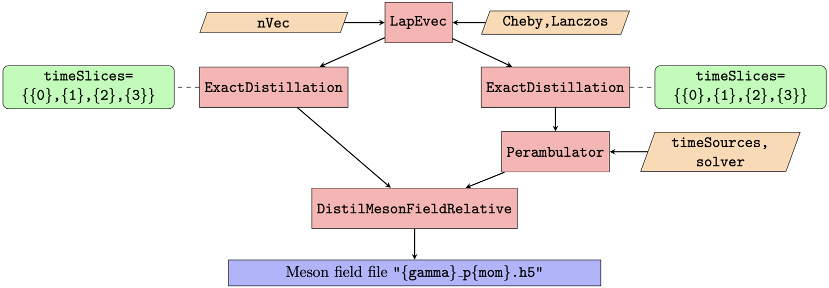
github.com/aportelli/Hadrons

Distillation within **Grid** and **Hadrons** [https://aportelli.github.io/Hadrons-doc/#/mdistil]

- First version in 2019 [Boyle, Marshall, Portelli, Erben, Tsang, HÓgáin - Lattice2019 PoS, arxiv:1912.07563]
- Stochastic and Exact distillation
- Dirac prop. inversions on CPU and GPU (same code)
- Meson fields (CPU only for now) - high time-sparsity of ϱ vectors

Running Distillation

[github.com/aportelli/Hadrons/tree/develop/Hadrons/Modules/MDistil][https://aportelli.github.io/Hadrons-doc/#/mdistil]



Distillation with $\varrho\varphi$ meson field workflow. Exact distillation on a $N_t = 4$ lattice with non-displaced sources at $t = 0, 2$. Laplacian eigenpack module (`LapEvec`) encodes N_{vec}.

- Covariant under `ExactDistillation` $\leftrightarrow$ `InterlacedDistillationNoise`

Ensemble

- Single RBC-UKQCD physical point lattice [Blum, Boyle, Christ - PRD.93.074505, 2016]
- $N_f = 2 + 1$
- Domain-wall fermion action (Möbius + Iwasaki)

volume	$48^3 \times 96$
a	≈ 0.114 fm
L	≈ 5.5 fm
$m_\pi L$	≈ 3.8
m_π	≈ 139 MeV
m_K	≈ 499 MeV

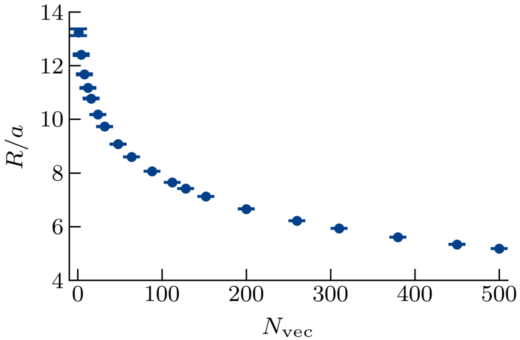
Smearing Radius

- Smearing profile

$$\Psi(r) = \sum_{\hat{r}=1}^{3} \sum_{\mathbf{x},t} \sqrt{\mathrm{tr}\, \square_{\mathbf{x},\mathbf{x}+\mathbf{r}}(t) \square_{\mathbf{x}+\mathbf{r},\mathbf{x}}(t)}$$

- Define R

$$\frac{\int_0^R \Psi(r)dr}{2\int_0^{aL/2} \Psi(r)dr} = 0.341$$



- Roughly a power-law decay exponent $c \approx -0.3$ [NPL, Marshall, Boyle, Portelli, Erben - Lattice2021 PoS, arxiv:2112.09804]
- Explored $N_{\mathrm{vec}} \sim 100$ on various setups (low statistics) [see also F. Joswig's talk]
  - looked at simple correlators and different moving frames
  - decided for $N_{\mathrm{vec}} = 64$ and exact distillation

Physical Point Run Status

- Exact distillation ($N_{\text{vec}} = 64$)
- Strange and light perambulators production at $\sim 40\%$ of total ensemble (35 configs.)
- Inversions on every time slice
  - no γ^5-hermiticity
  - more time sources
  - more flexibility for other projects

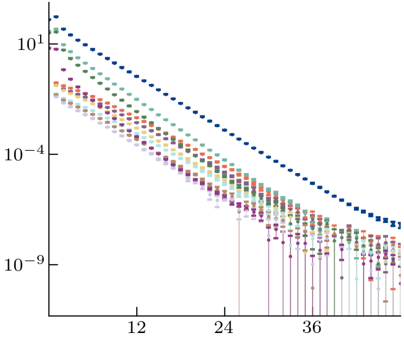


Illustration: T_{1u} correlator dump

* done using DiRAC Extreme Scaling HPC Services Tesseract and Tursa [https://dirac.ac.uk/resources/#ExtremeScaling]

$K\pi$ scattering - Preliminary Results

Spectrum: variational method

- Simple choice of operator basis

$$O_{\gamma^j}(\mathbf{P}) = \left(\bar{s}\gamma^j u\right)(\mathbf{P}), \qquad \mathbf{P}^2 = 0, \ldots, 4$$

$$O_{K\pi}(\mathbf{p}_1, \mathbf{p}_2), \qquad \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}, \quad \mathbf{p}_{1,2}^2 \leq 4$$

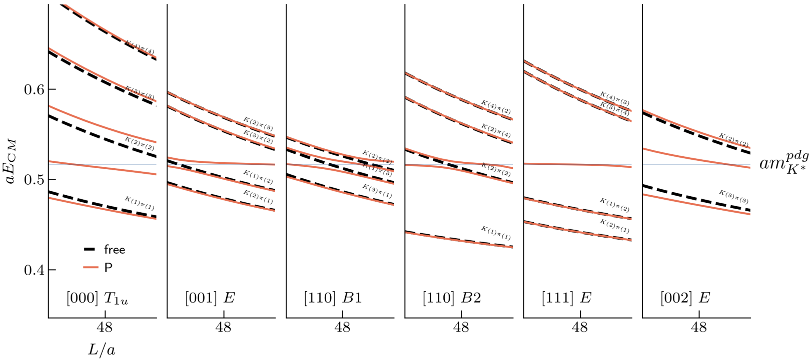
- Build correlator matrix $[C(t)]_{nm} = \langle O_n(t) O_m(0)^\dagger \rangle$ and use fixed-t_0 GEVP

$$C(t) u_n(t, t_0) = \lambda_n(t, t_0) C(t_0) u_n(t, t_0)$$

- For now only irreps without even-odd partial wave mixing [Leskovec & Prelovsek - PRD.85.114507, 2012]

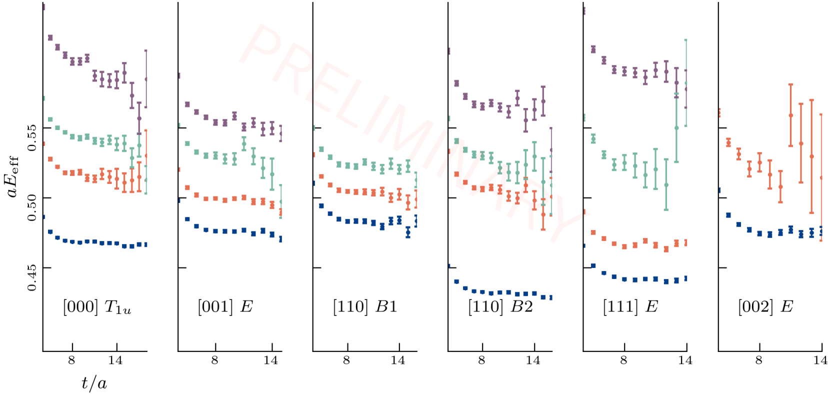
$$[000]T_{1u},\ [001]E,\ [110]B_1,\ [110]B_2,\ [111]E,\ [002]E$$

- Two-particle QC + PDG $K^*(892)$ data + relativistic Breit-Wigner
- Possible levels on our $L = 48$ lattice for now ($\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P},\ \mathbf{p}_{1,2}^2 \leq 4$)



Non-interacting energies labelled by $K(p_K^2)\pi(p_\pi^2)$, p's in units of $2\pi/L$

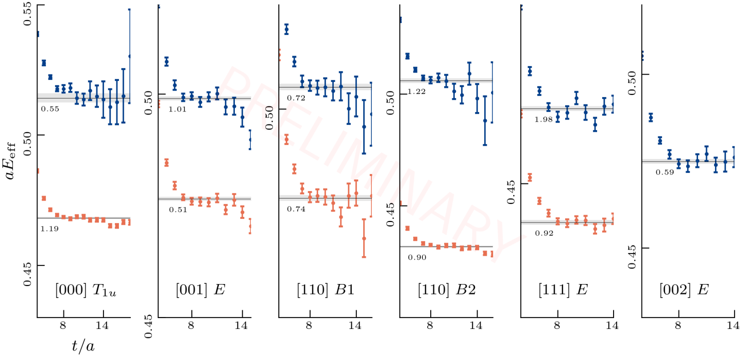
- Spectrum overview in center-of-mass frame



Summary of log effective mass from GEVP eigenvalues. Highest level omitted.

Fits

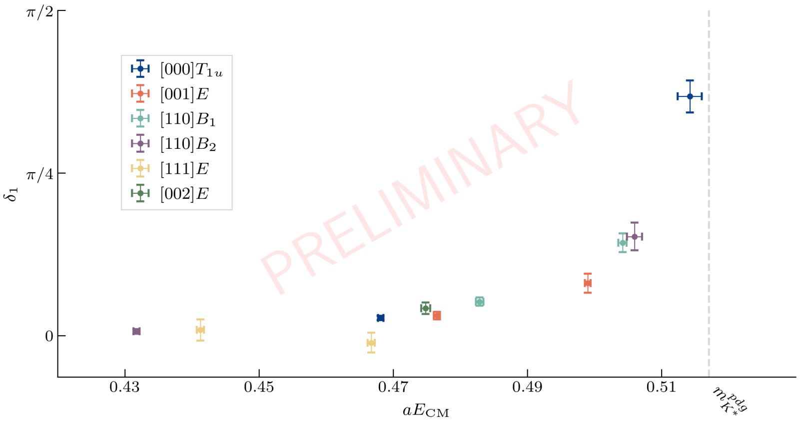
- Correlated exponential one-state fits to correlators
- Fit range scanning in t_i, t_f + AIC statistical criterion



Energy fit result on top of effective masses. χ^2_{dof} below the levels.

Preliminary $K\pi$ scattering Phase Shift

- *P*-wave - $I = 1/2$
- Only statistical errors included



Conclusions and Outlook

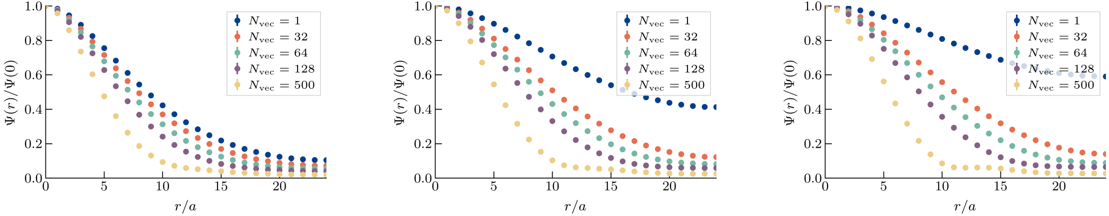
Conclusions and Outlook

- `Grid/Hadrons` distillation code for large simulations (open source)
- Multi-operator correlators and first GEVP results
- Preliminary P-wave - $I = 1/2$ - $K\pi$ scattering phase shift
- Next:
  - Full statistics (90 configs)
  - Additional operators and GEVP fine tuning
  - Other irreps (A_{1g}, A_1's) and amplitude fits

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 813942 and No 757646.

Backup

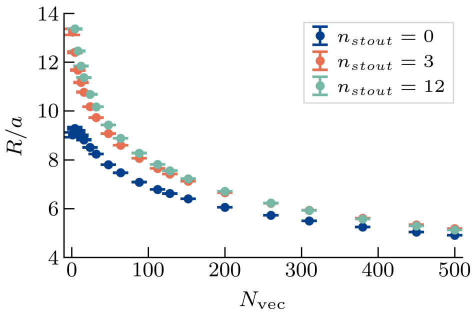
Smearing spatial distribution for $n_{\text{stout}} = 0, \mathbf{3}, 12$ ($\rho = \mathbf{0.2}$)



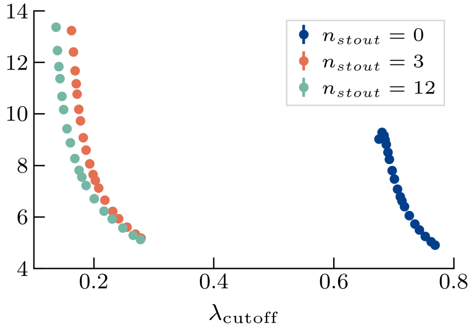
$n_{\text{stout}} = 0$ $\quad$ $n_{\text{stout}} = \mathbf{3}$ (chosen) $\quad$ $n_{\text{stout}} = 12$

Backup

Smearing radius for $n_{\text{stout}} = 0, 3, 12$ ($\rho = 0.2$)

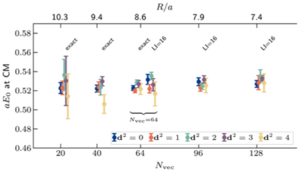


radius vs N_{vec}

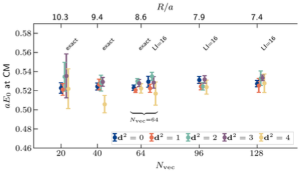


radius vs λ_{cutoff}

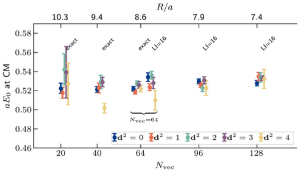
Varying bin size (E_0 vs N_{vec})



bin size=1



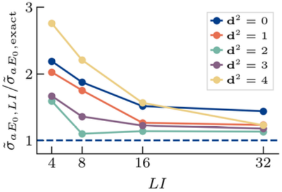
bin size=2



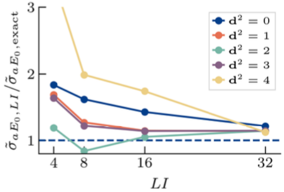
bin size=4

Backup

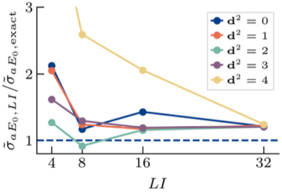
Varying bin size ($\frac{\sigma_{E_0,LI}\sqrt{N_{\mathbf{inv},LI}}}{\sigma_{E_0,exact}\sqrt{N_{\mathbf{inv},exact}}}$ vs LI)



bin size=1



bin size=2

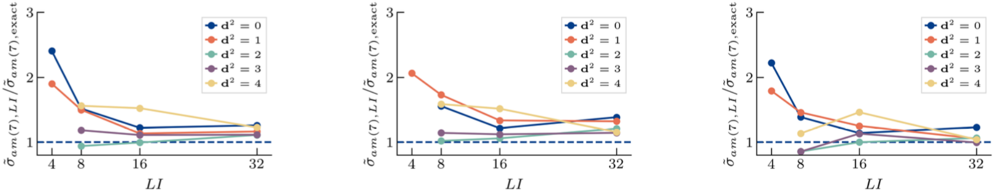


bin size=4

Fluctuates a bit but it is not changing conclusions

Backup

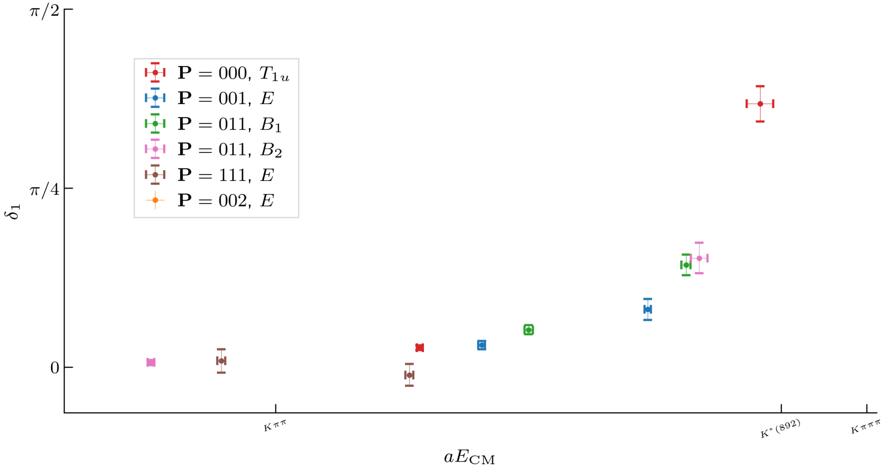
Varying bin size $\left(\frac{\sigma_{m_{\mathrm{eff}}(t=7),LI}\sqrt{N_{\mathrm{inv},LI}}}{\sigma_{m_{\mathrm{eff}}(t=7),exact}\sqrt{N_{\mathrm{inv},exact}}}\right)$ vs LI)



bin size=1 bin size=2 bin size=4

Fluctuates a bit but it is not changing conclusions

Backup



$\pi/2$
$\pi/4$
0
δ_1
$\mathbf{P} = 000,\ T_{1u}$
$\mathbf{P} = 001,\ E$
$\mathbf{P} = 011,\ B_1$
$\mathbf{P} = 011,\ B_2$
$\mathbf{P} = 111,\ E$
$\mathbf{P} = 002,\ E$
$K\pi\pi$
$K^*(892)$
$K\pi\pi\pi$
aE_{CM}

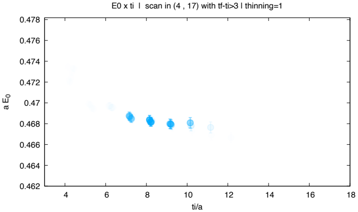
Backup

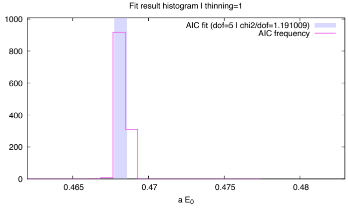
Akaike Information Criterion (AIC) -based weighting

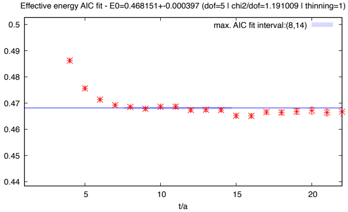
$$w_{AIC} = \mathcal{N}_{AIC} \exp\left[-\frac{1}{2}(\chi^2 - 2n_{dof})\right], \quad 0 \leq w_{AIC} \leq 1 \tag{1}$$

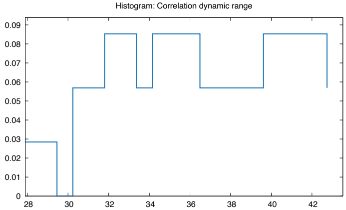
Backup

Rest frame $P = (000)$ - irrep T_{1u} - $I = 1/2$ - State 0

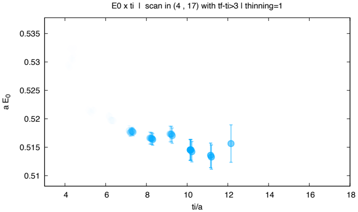


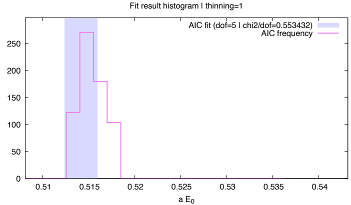


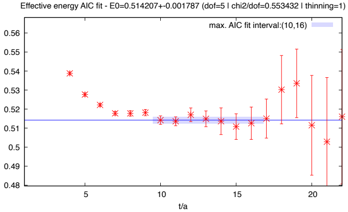


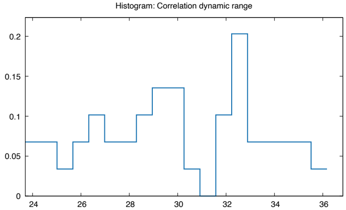


Rest frame $P = (000)$ - irrep T_{1u} - $I = 1/2$ - State 1



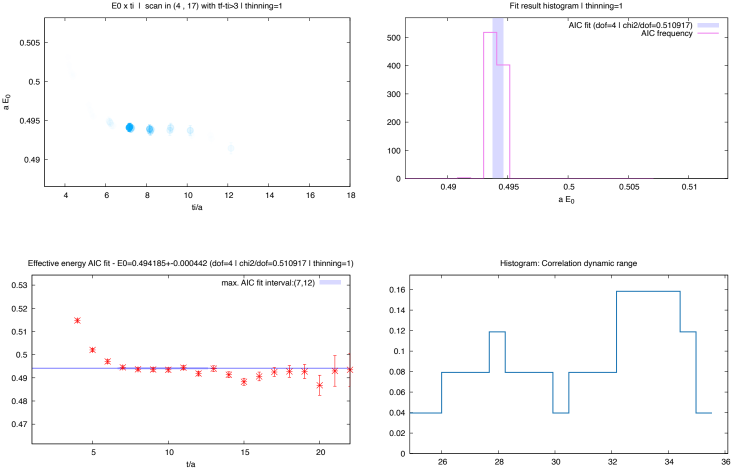






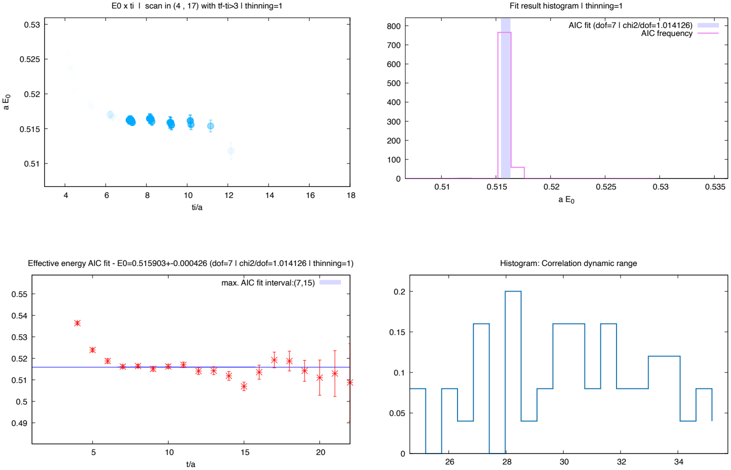
Backup

Rest frame $P = (001)$ - irrep E - $I = 1/2$ - State 0

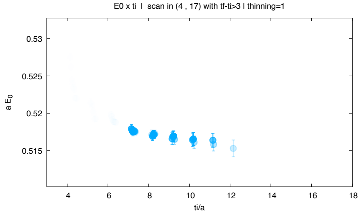


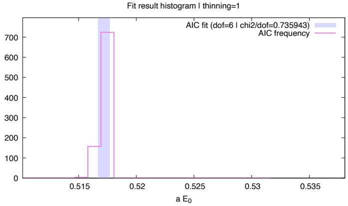
Backup

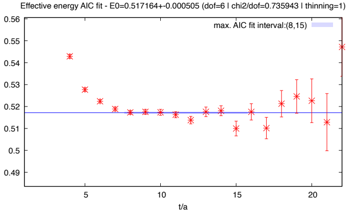
Rest frame $P = (001)$ - irrep E - $I = 1/2$ - State 1

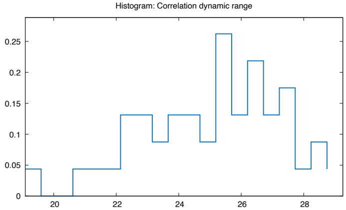


Rest frame $P = (110)$ - irrep B_1 - $I = 1/2$ - State 0

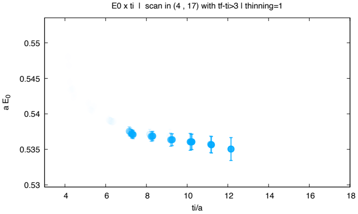


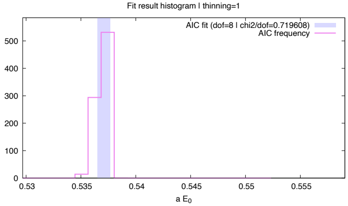


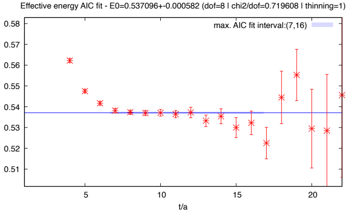


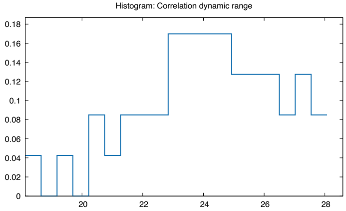


Rest frame $P = (110)$ - irrep B_1 - $I = 1/2$ - State 1



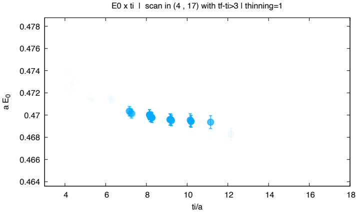


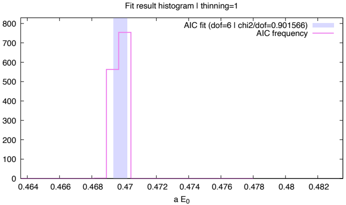


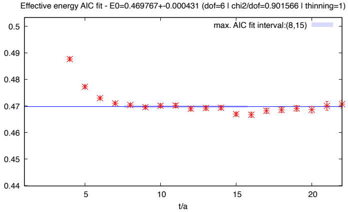


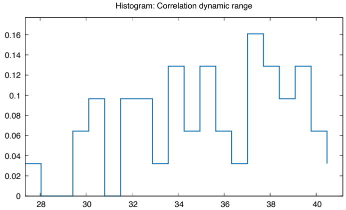
Backup

Rest frame $P = (110)$ - irrep B_2 - $I = 1/2$ - State 0

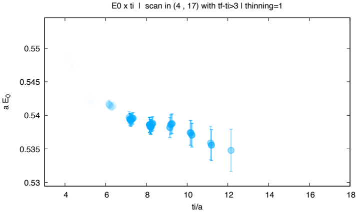


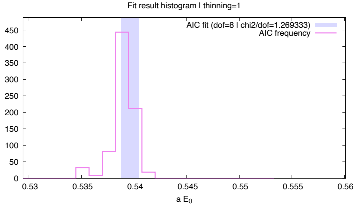


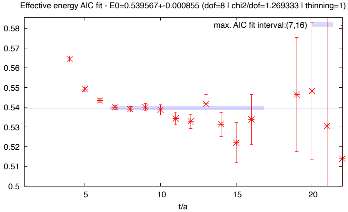


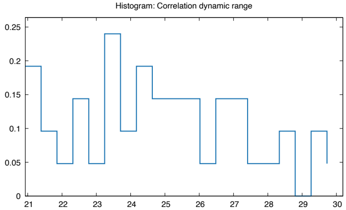


Rest frame $P = (110)$ - irrep B_2 - $I = 1/2$ - State 1



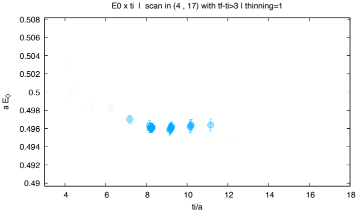


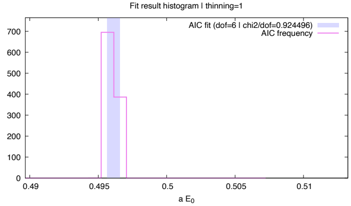


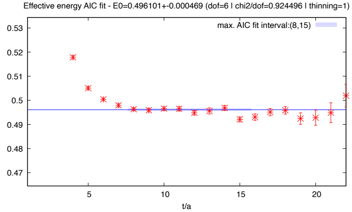


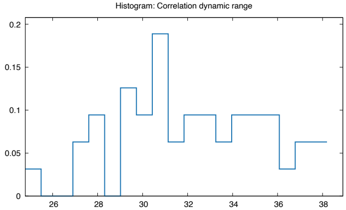
Backup

Rest frame $P = (111)$ - irrep E - $I = 1/2$ - State 0



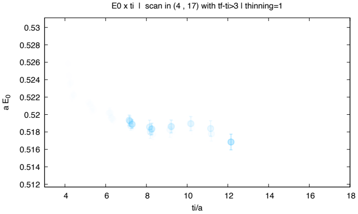


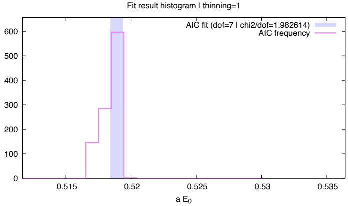


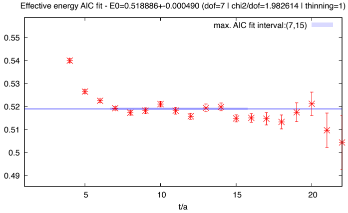


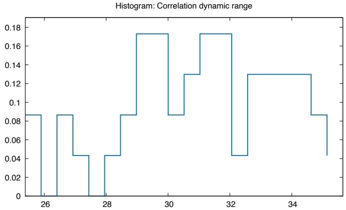
Backup

Rest frame $P = (111)$ - irrep E - $I = 1/2$ - State 1









Backup

Rest frame $P = (002)$ - irrep E - $I = 1/2$ - State 0