Optimized meson operators for charmonium spectroscopy and mixing with glueballs

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Meson spectroscopy with Distillation

Main focus: Meson operators $\mathcal{O} = \bar{\psi}\Gamma\psi$ ($\Gamma = \gamma_5, \gamma_i, \nabla_i, ...$) at zero spatial momentum.

Main method: **Distillation**. Replace $\psi \to VV^\dagger\psi$, where V contains the N_v lowest eigenmodes of the 3D Laplacian operator.

[M. Peardon et al. Phys. Rev. D80 (2009) arXiv:0905.2160]

Building blocks

- Laplacian eigenvectors $V[t]$
- Perambulators $\tau[t_1, t_2] = V^\dagger[t_1]D^{-1}V[t_2]$
- Elementals $\Phi[t] = V^\dagger[t]\Gamma V[t]$

Advantages

- ✓ Perambulators/elementals have manageable sizes.
- ✓ Perambulators are independent from elementals.

Disadvantages

- × N_v scales with 3D physical lattice volume.
- × Many inversions required.

Developing an improvement

How to choose N_v?
Too small:

- **Neglects** significant low energy modes.

Too large:

- **Expensive**.
- Can include **non-significant** modes.

Is a given N_v equally good for all states?

- Different Γ correspond to different J^{PC} with **different** spatial properties.
- **Excited states** of a same J^{PC} can also further differ.

Starting point: Distillation profiles

- Use $\psi \rightarrow VJV^{\dagger}\psi$ with $J[t]_{ij} = \delta_{ij} g(\lambda_i[t])$.
- Quark distillation profile $g(\lambda)$ modulates contributions from each of the N_v eigenvectors.

Exploiting the quark distillation profiles

Our case: Fix Γ and study ground/excited states via a GEVP formulation.

[C. Michael & I. Teasdale, Nucl. Phys. B215 (1983)] [M. Lüscher & U. Wolff, Nucl. Phys. B339 (1990)] [B. Blossier et al. JHEP 04 (2009) arXiv:0902.1265]

- Variational basis: Quark distillation profiles $g_a(\lambda)$.
- Correlation matrix $C_{ab}(t) = \left\langle \mathcal{O}_a(t)\bar{\mathcal{O}}_b(0) \right\rangle$
  - Pruning via SVD recommended for numerical stability.
    [J. Balog et al. Phys.Rev. D60 (1999) arXiv:hep-lat/9903036], [F. Niedermayer et al. Nucl.Phys. B597 (2001) arXiv:hep-lat/0007007]
- Solve GEVP $C(t)u_e(t,t_0) = \rho_e(t,t_0)C(t_0)u_e(t,t_0)$.
  - Eigenvalues $\rho_e(t,t_0)$ give access to masses of the different states.
  - Eigenvectors $u_e(t,t_0)$ allow to build an operator $\mathcal{O}_e$ with the largest overlap with the wanted energy eigenstate from the basis elements.

The optimal meson distillation profiles

For a fixed Γ and energy level e one can build an optimal elemental given by

$$\tilde{\Phi}^{(\Gamma,e)}[t]_{\substack{ij \\ \alpha\beta}} = \tilde{f}^{(\Gamma,e)}(\lambda_i[t], \lambda_j[t]) v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t]$$

which includes the optimal meson distillation profile given as

$$\tilde{f}^{(\Gamma,e)}(\lambda_i[t], \lambda_j[t]) = \sum_k \eta_k^{(\Gamma,e)} g_k(\lambda_i[t])^* g(\lambda_j[t]).$$

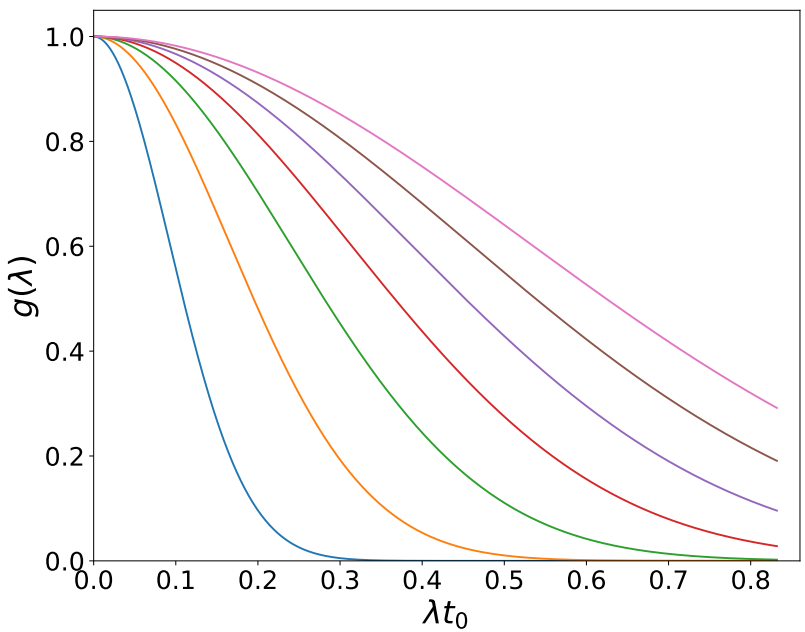
[F. Knechtli, T. Korzec, M. Peardon, J. A. Urrea-Niño, Phys. Rev. D106 (2022) arXiv:2205.11564]

Advantages:

- ✓ Correlation matrix and optimal elementals can be built with very small extra cost.
- ✓ $\tilde{f}^{(\Gamma,e)}(\lambda_i[t], \lambda_j[t])$ tells us if N_v is large enough and how to use the N_v eigenvectors **for each Γ and energy state**.

Testing the method

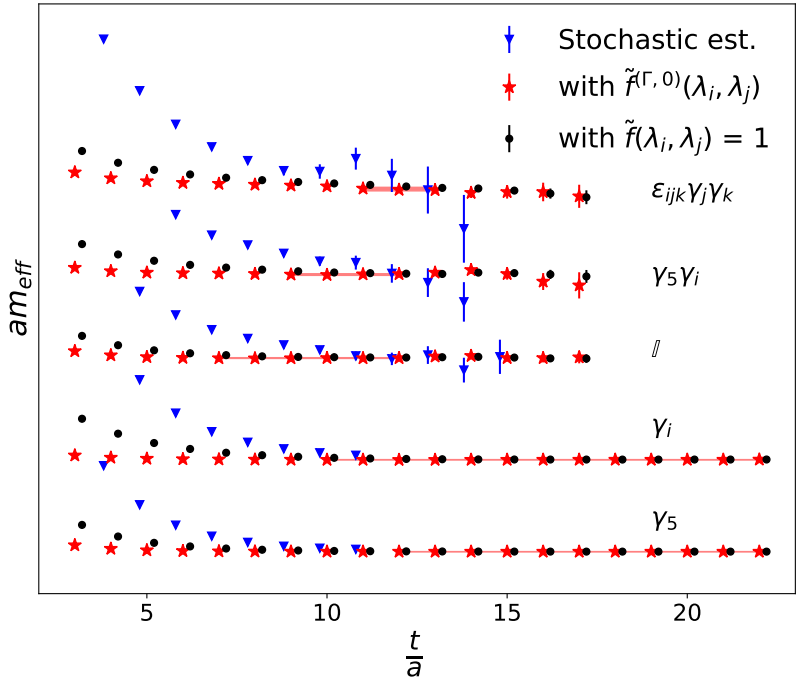
- QCD with $N_f = 2$ at half the physical charm quark mass.
  No light quarks. Clover-improved Wilson fermions.
- 48×24^3 and 96×48^3 lattices with $a \approx 0.0658, 0.049$ fm.
  Check effectiveness at smaller resolutions
- Both local and derivative Γ. [J. J. Dudek et al. Phys. Rev. D77 (2008) arXiv:0707.4162]



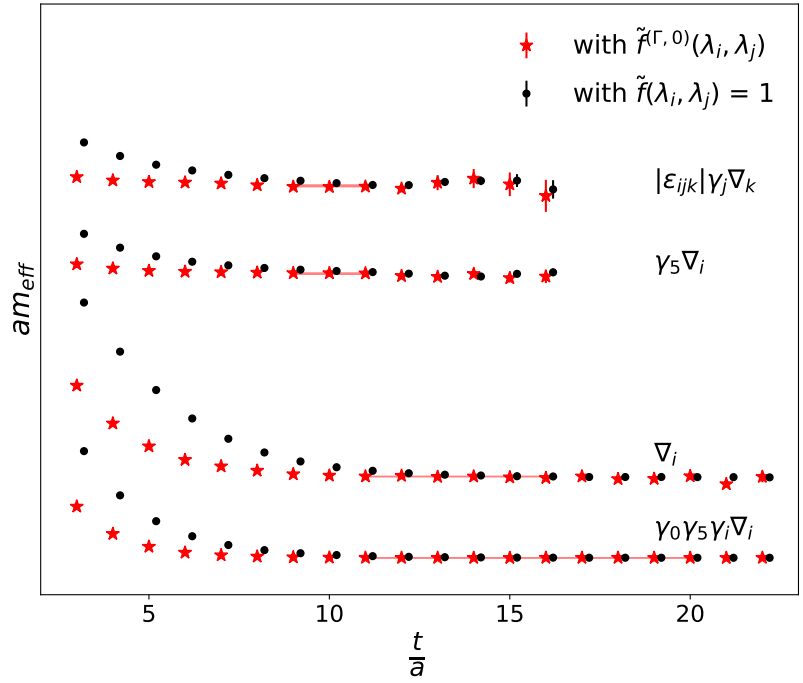
- $g_i(\lambda) = e^{-\frac{\lambda^2}{2\sigma_i^2}}$ in this work.
  Suppression of large λ follows distillation intuition.
- $g_i(\lambda) = \lambda^i$ was tried too.
  Same result but less numerical stability. Avoided basis bias.

Coarser lattice results: $N_v = 200$

Local iso-vector operators



Derivative iso-vector operators

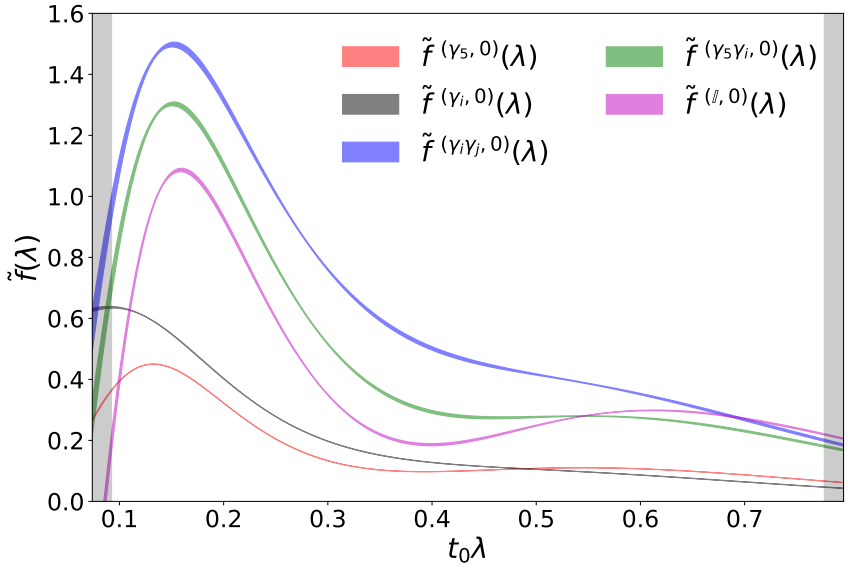


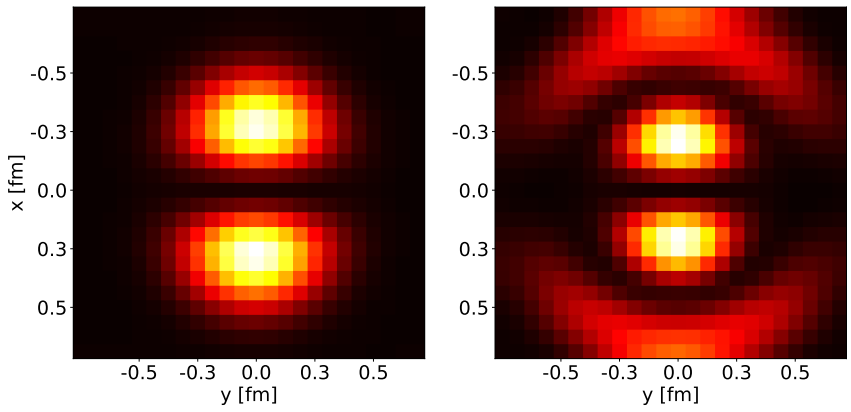
Fractional overlaps:

- γ_5: $0.9272(3) \rightarrow 0.9858(2)$
- γ_i: $0.8743(10) \rightarrow 0.9900(5)$
- $\epsilon_{ijk}\gamma_j\gamma_k$: $0.77(7) \rightarrow 0.93(1)$

Fractional overlaps:

- ∇_i: $0.4758(7) \rightarrow 0.742(2)$
- $\gamma_5\nabla_i$: $0.84(1) \rightarrow 0.970(5)$
- $\mathbb{Q}_{ijk}\gamma_j\nabla_k$: $0.858(8) \rightarrow 0.981(3)$

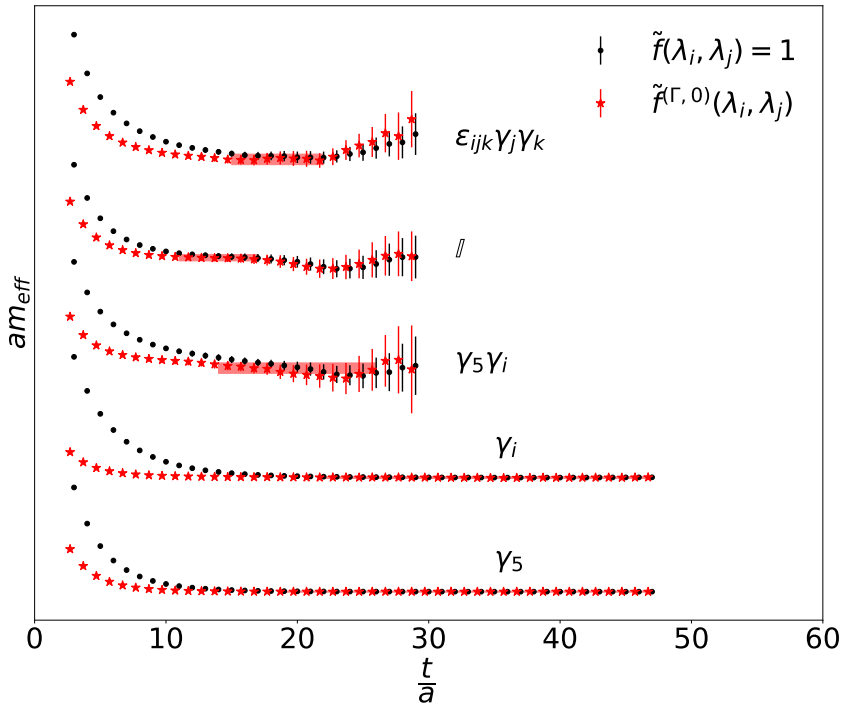




- $\tilde{f}^{(\Gamma,0)}(\lambda_i,\lambda_j) \neq 1$ always $\rightarrow$ Improvement over orthogonal projection.
- Suppression of large λ remains $\rightarrow$ Distillation intuition still holds.
- Different profile for different Γ $\rightarrow$ Profiles are unique.
- $\tilde{f}^{(\Gamma,0)}(\lambda_i,\lambda_j)$ at large λ_i,λ_j tells us if we have enough eigenvectors $\rightarrow$ More systematic criterion for choosing N_v.
- Spatial profile can be recovered, e.g $\Psi^{(\gamma_5\nabla_1,e)}(\vec{x}) = \frac{1}{N_t}\sum_t ||\, Tr\left(\gamma_5 V[t]\tilde{\Phi}^{(\gamma_5\nabla_1,e)}[t]V[t]^\dagger\right)\phi_0||^2$ with ϕ_0 a 3D point source. Profiles dictate spatial structure.

Finer lattice results: $N_v = 325$

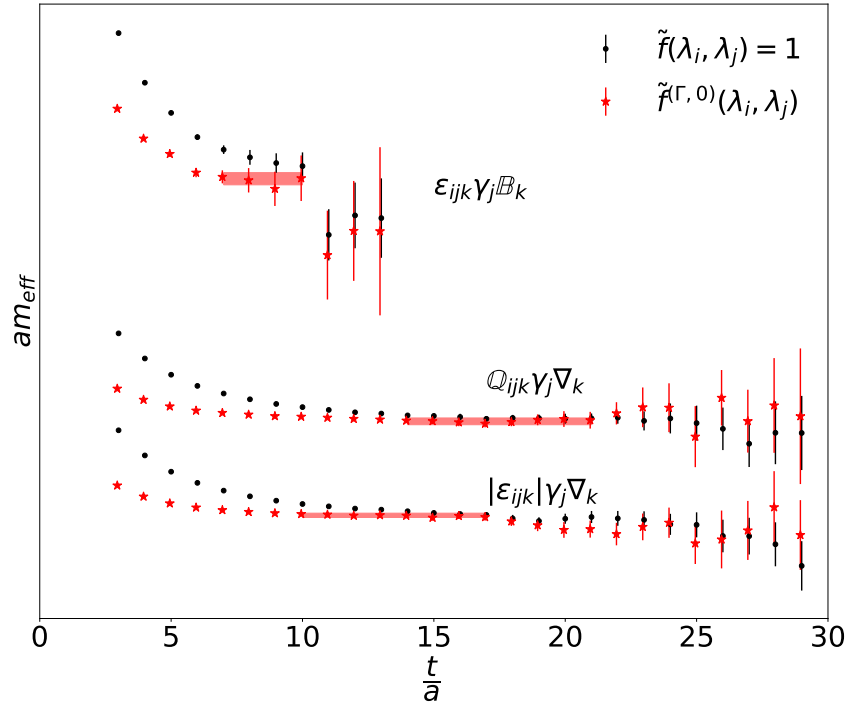
Local iso-vector operators



Fractional overlaps:

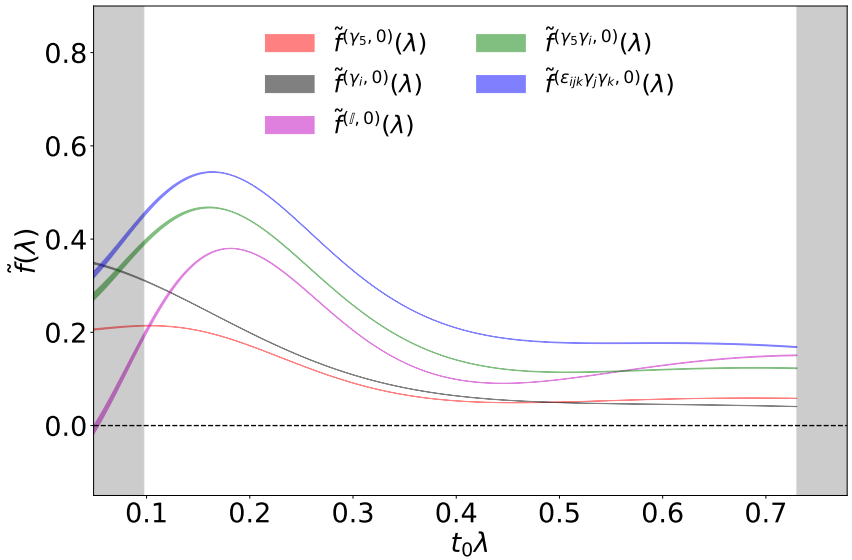
- γ_5: 0.8765(7) $\rightarrow$ 0.9555(5)
- γ_i: 0.825(3) $\rightarrow$ 0.969(2)

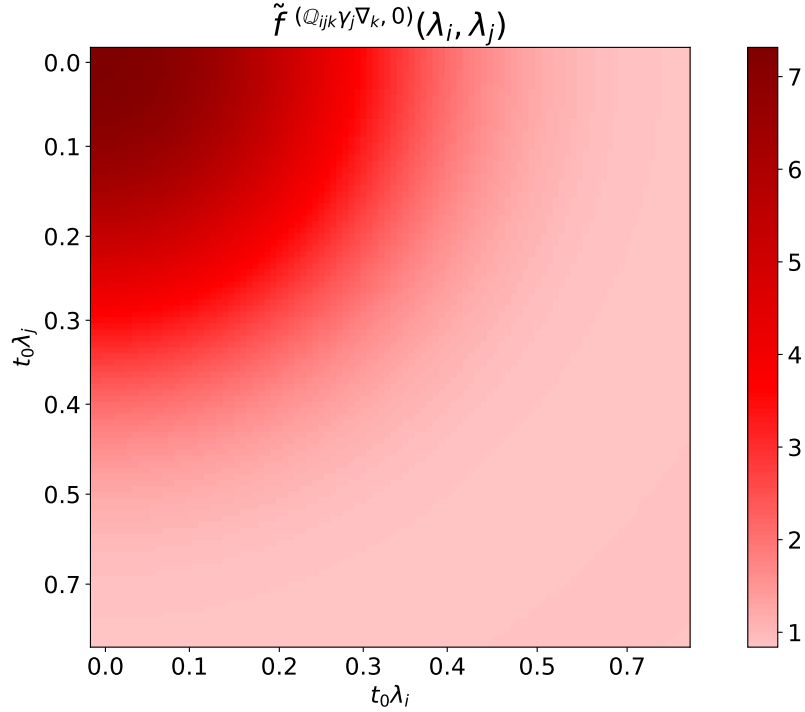
Derivative iso-vector operators



Fractional overlaps:

- $\mathbb{Q}_{ijk}\gamma_j\nabla_k$: 0.82(2) $\rightarrow$ 0.92(1)
- $\epsilon_{ijk}\gamma_j\mathbb{B}_k$: - $\rightarrow$ 0.91(1)



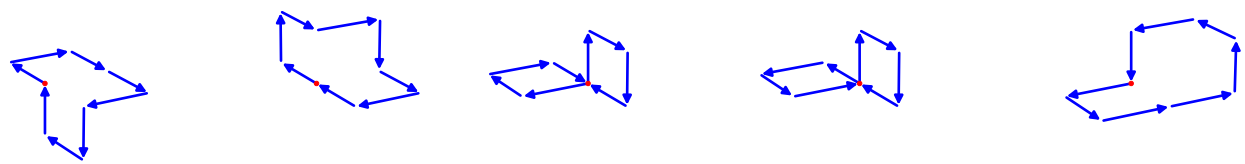


- $N_v^{fine} = 325 \leftrightarrow N_v^{course} = 100 \rightarrow$ Volume scaling is a good initial guide.
- Scaling of σ_i with lattice spacing $\rightarrow$ Keep same pattern of suppression.
- Same observations as in coarse lattice $\rightarrow$ Profiles serve their intended purpose.

Charmonium-Glueball mixing

To keep in mind:

- Iso-scalar meson operators require disconnected pieces in correlation function. Feasable thanks to distillation.
- Glueballs are hard to find in un-quenched QCD. Optimal operators must be found via GEVP
  - Different loop shapes and windings. [C. J. Morningstar & M. Peardon, Phys. Rev. D60 (1999) arXiv:9901004] [B. Berg & A. Billoire, Nucl. Phys. B221 (1983)

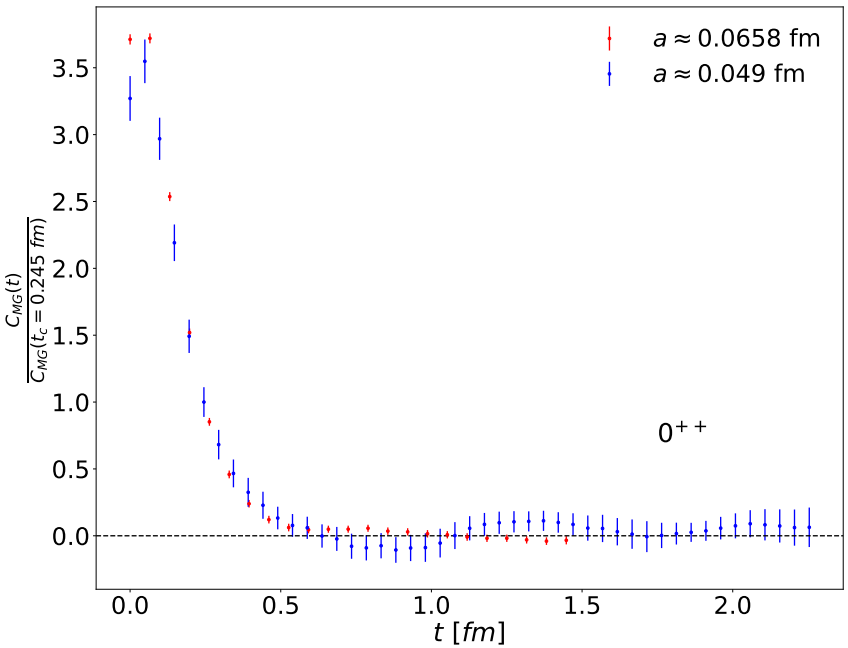


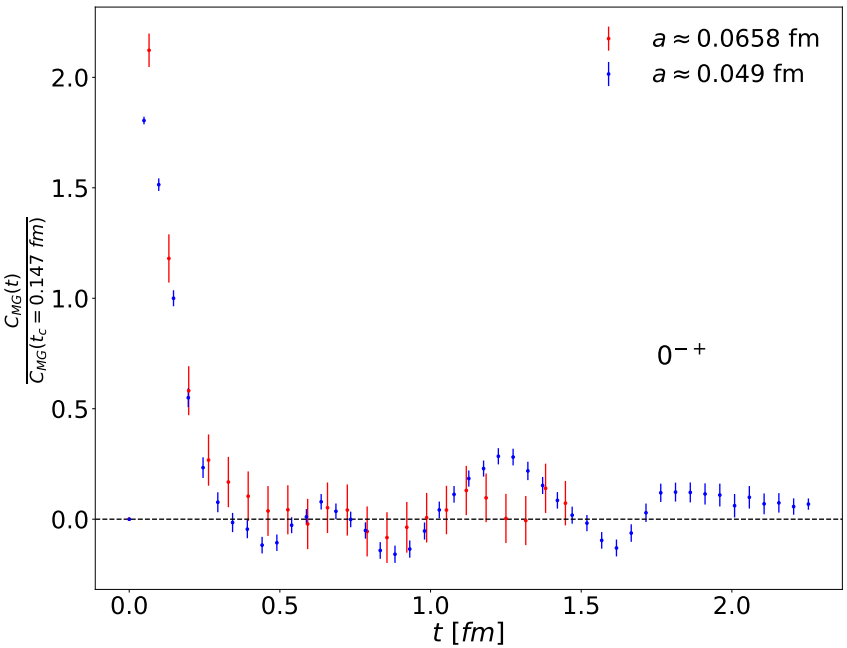
  - Different smearing schemes and levels:
    - 3D-HYP [A. Hasenfratz & F. Knechtli, Phys. Rev. D64 (2001) arXiv:0103029]
    - 3D improved APE [B. Lucini et al. JHEP 06 (2004) arXiv:hep-lat/0404008]

Mixing results

$0^{++} \rightarrow \Gamma = \mathbb{I},\ \tilde{f}(\lambda_i, \lambda_j) = 1$

$0^{-+} \rightarrow \Gamma = \gamma_5,\ \tilde{f}^{(\gamma_5,0)}(\lambda_i, \lambda_j)$





- $C_{MG}(t) = Tr\left(\Phi^{(\Gamma)}[t]\tau[t,t]\right) G^{(R^{PC})}(0)$.
- Correlators normalized at fixed time in physical units.
- Noise is dominated by the glueball. Glueballs require more statistics than mesons.

Conclusions

- Meson distillation profiles were introduced and used for a wide variety of operators at two different lattice spacings.
  - ✓ Excited state contamination is **significantly** reduced at **no** additional inversion cost.
  - ✓ The meson profile serves as an **additional** degree of freedom for a GEVP formulation.
  - ✓ Profiles for individual energy eigenstates reveal additional **spatial** information
  - ✓ These profiles can be applied to other hadron operators and to stochastic distillation.
- The improved operators were used for meson-glueball mixing.
  - ✓ Reduction of contamination at early times is convenient for such mixing.
  - GEVP formulation with glueball and meson operators is an ongoing work.

Outlook: Study of an ensemble with physical charm quark and 3 degenerate light quarks with this method.

Fractional overlap

Correlation function: Ground state + Excited state contamination

$$C(t) = 2c_0 e^{-m_0 \frac{T}{2}} \cosh\left(\left(\frac{T}{2} - t\right) m_0\right) + B_1(t)$$

Normalized correlator:

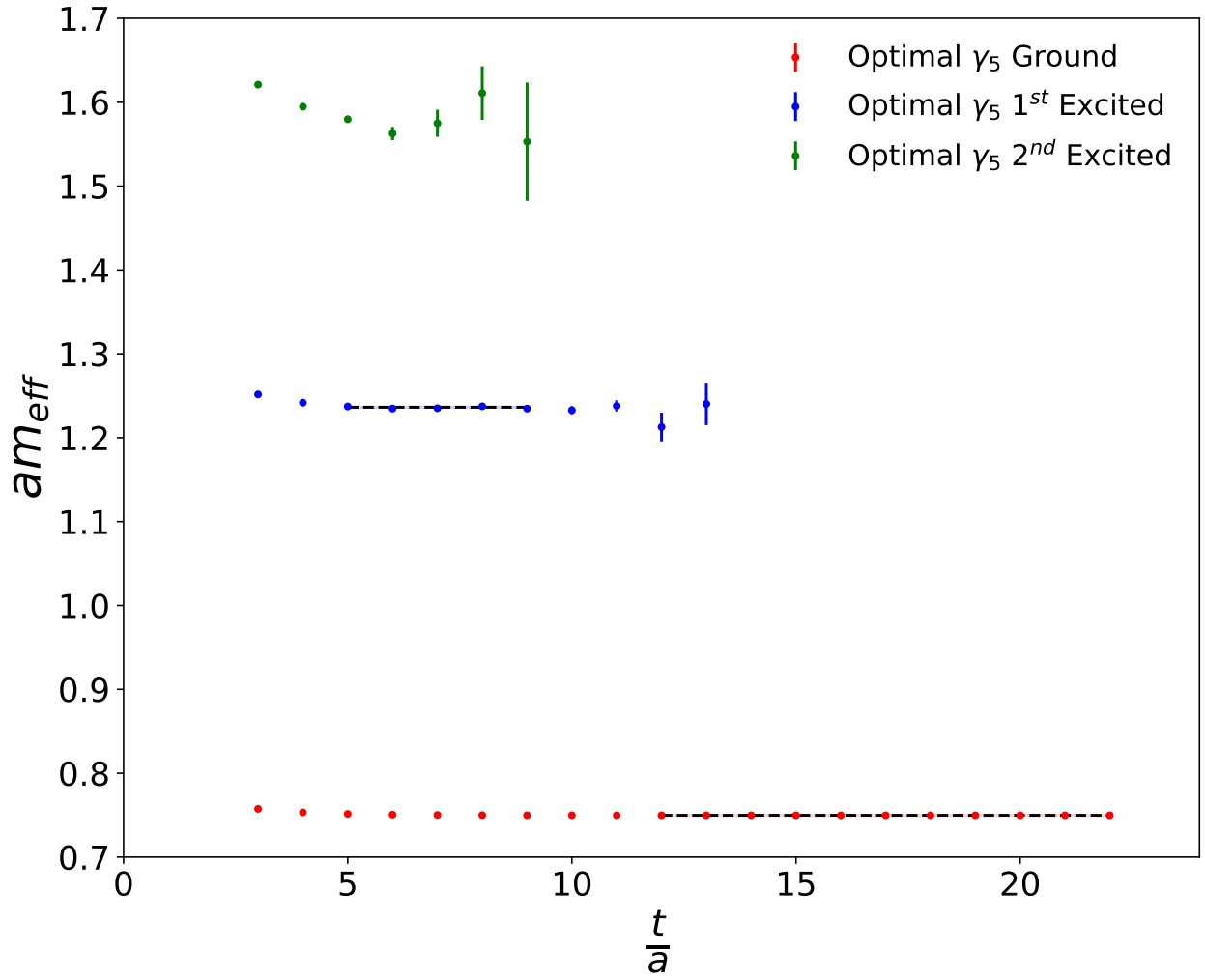
$$C'(t) = \frac{C(t)}{C(t_0)} = \left(\frac{1 + B_2(t)}{1 + B_2(t_0)}\right) \frac{\cosh\left(\left(\frac{T}{2} - t\right) m_0\right)}{\cosh\left(\left(\frac{T}{2} - t_0\right) m_0\right)}$$

$$B_2(t) = \frac{B_1(t) e^{m_0 \frac{T}{2}}}{2c_0 \cosh\left(\left(\frac{T}{2} - t\right) m_0\right)}$$

At mass plateau $B_1(t)$ is 0 and the fractional overlap can be fitted:

$$A_G = \frac{1}{1 + B_2(t_0)}$$

Access to excited states



Excited states can be accessed via inclusion of meson distillation profiles.