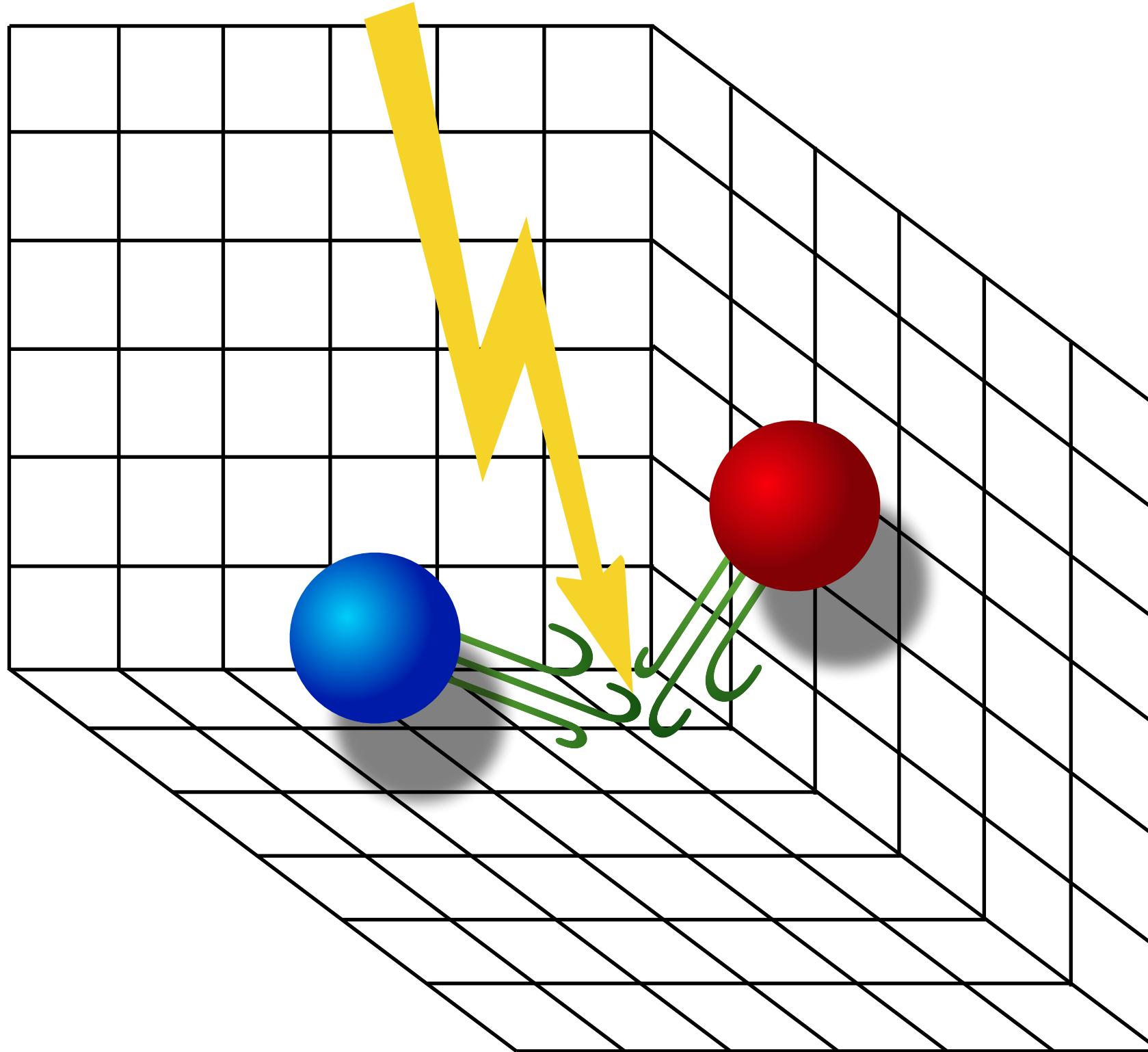


Finite volume corrections for form factors of two-nucleon systems



Felipe Ortega-Gama

William & Mary

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Lattice 2022 - August 8th



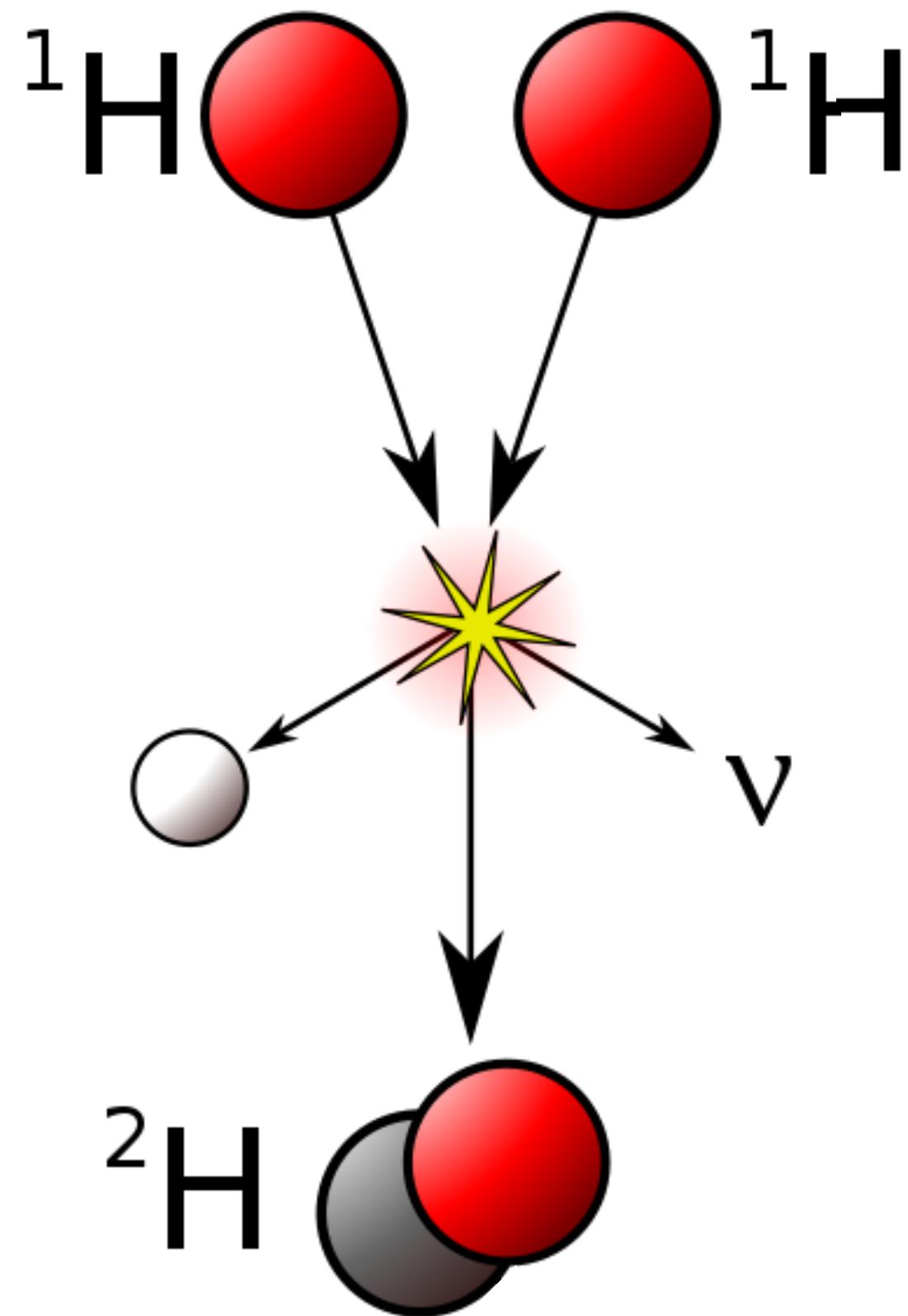
WILLIAM & MARY
CHARTERED 1693

In collaboration with:



Dr. Andrew Jackura
ODU/JLAB

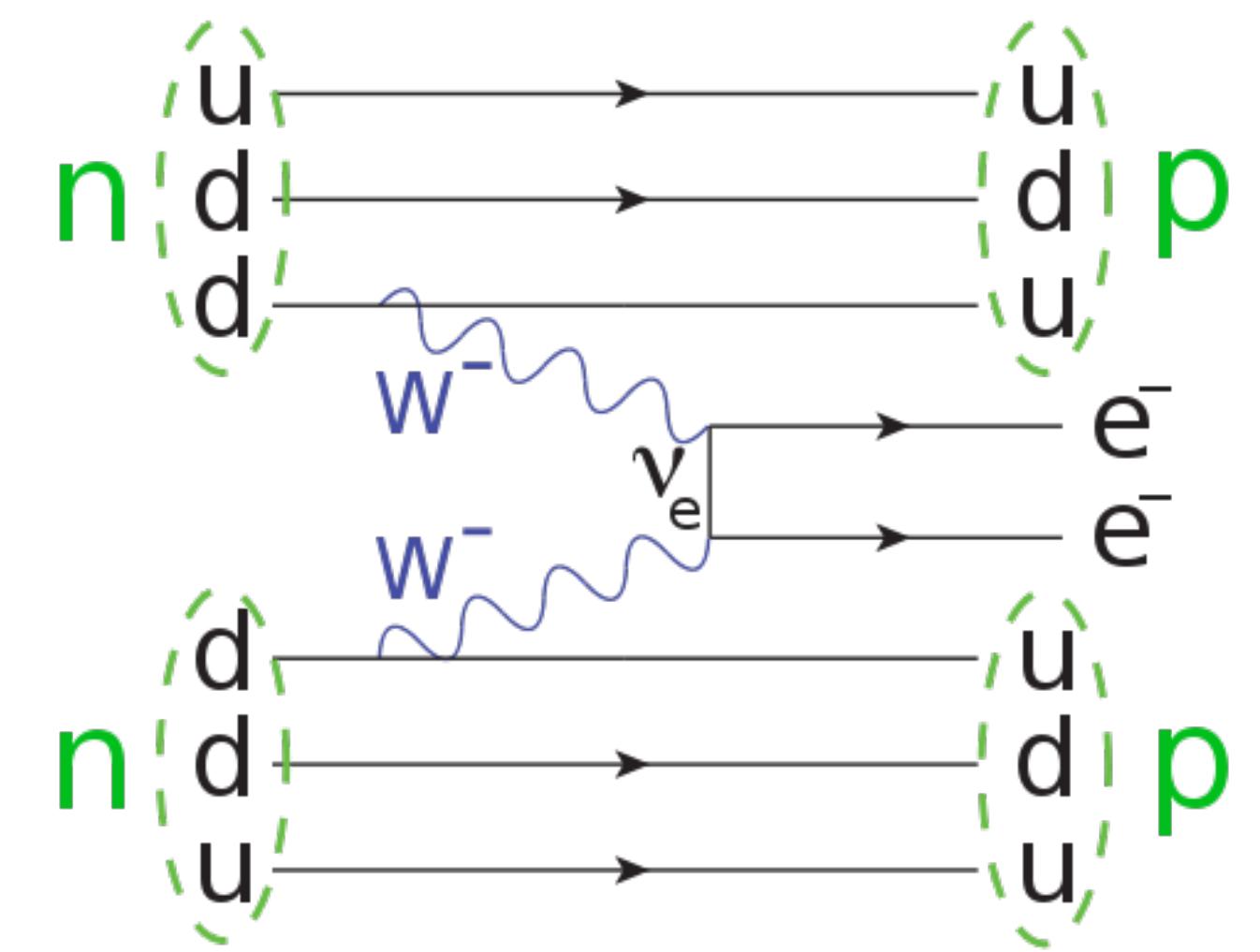
Describing hadrons



Nuclear fusion



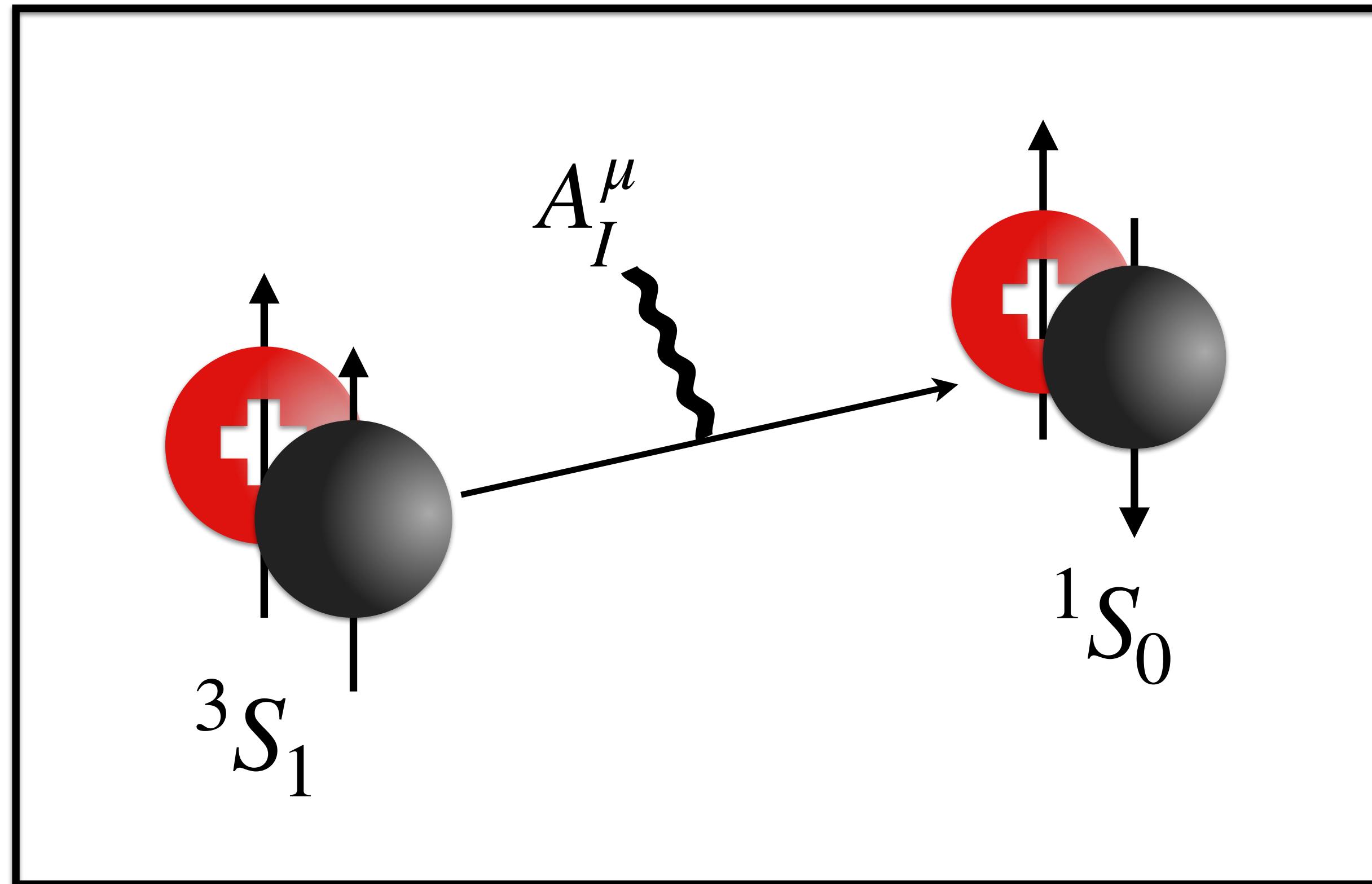
Proton radius puzzle



Neutrinoless double-beta decay

Nuclear processes in a box

[Phys. Rev. Lett. **119**, 062002] by the NPLQCD Collaboration.



Unphysical mass: deeply bound states

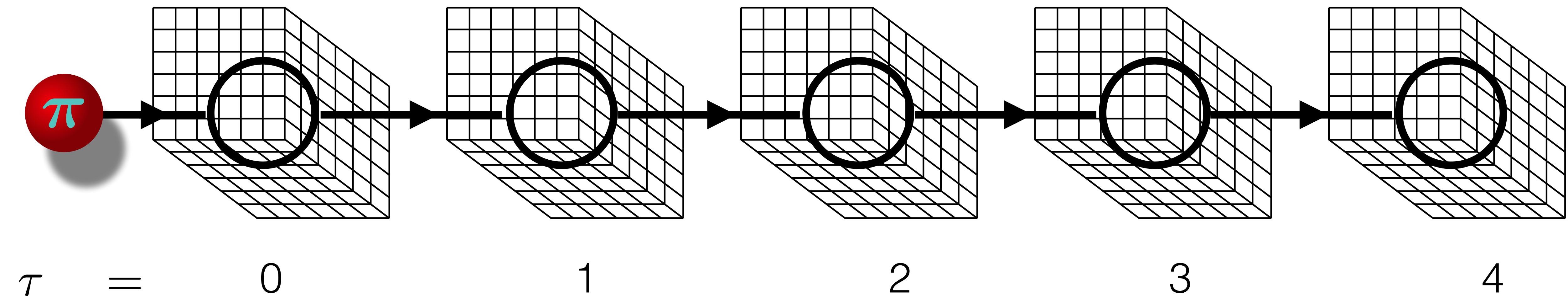
$B \sim 30$ MeV

pionless EFT

$$pp \rightarrow d + e^+ \bar{\nu}_e$$

Finite Volume discrete spectrum

Single hadron propagation



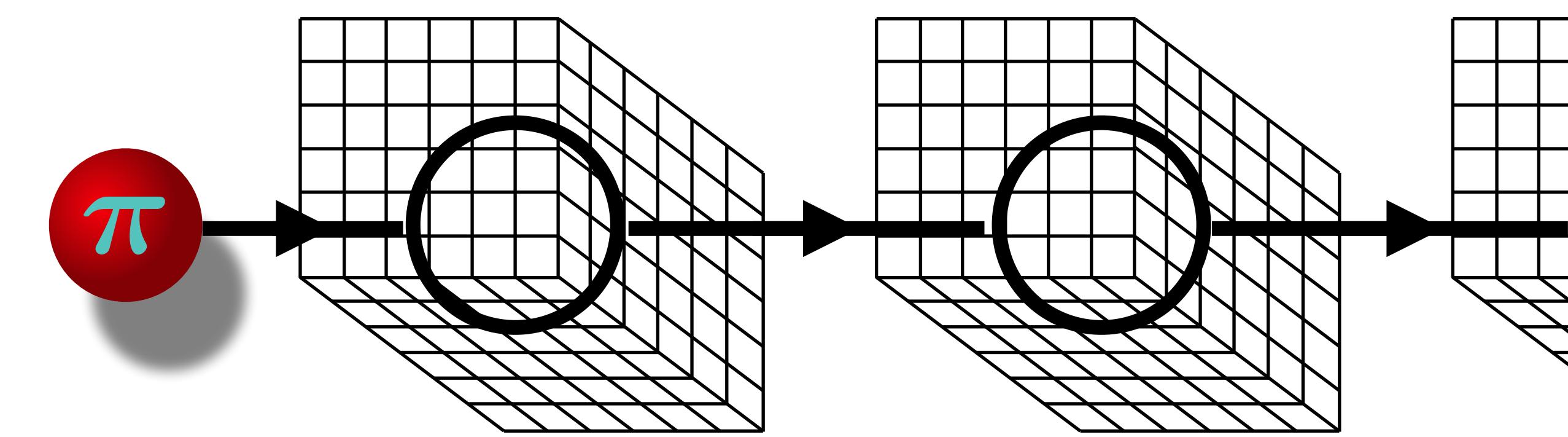
$$C_{2pt}^\pi(P) \sim \frac{1}{P^2 - m(L)^2}$$

$$m(L) \approx m(\infty) + c e^{-m(\infty)L}$$



Finite Volume discrete spectrum

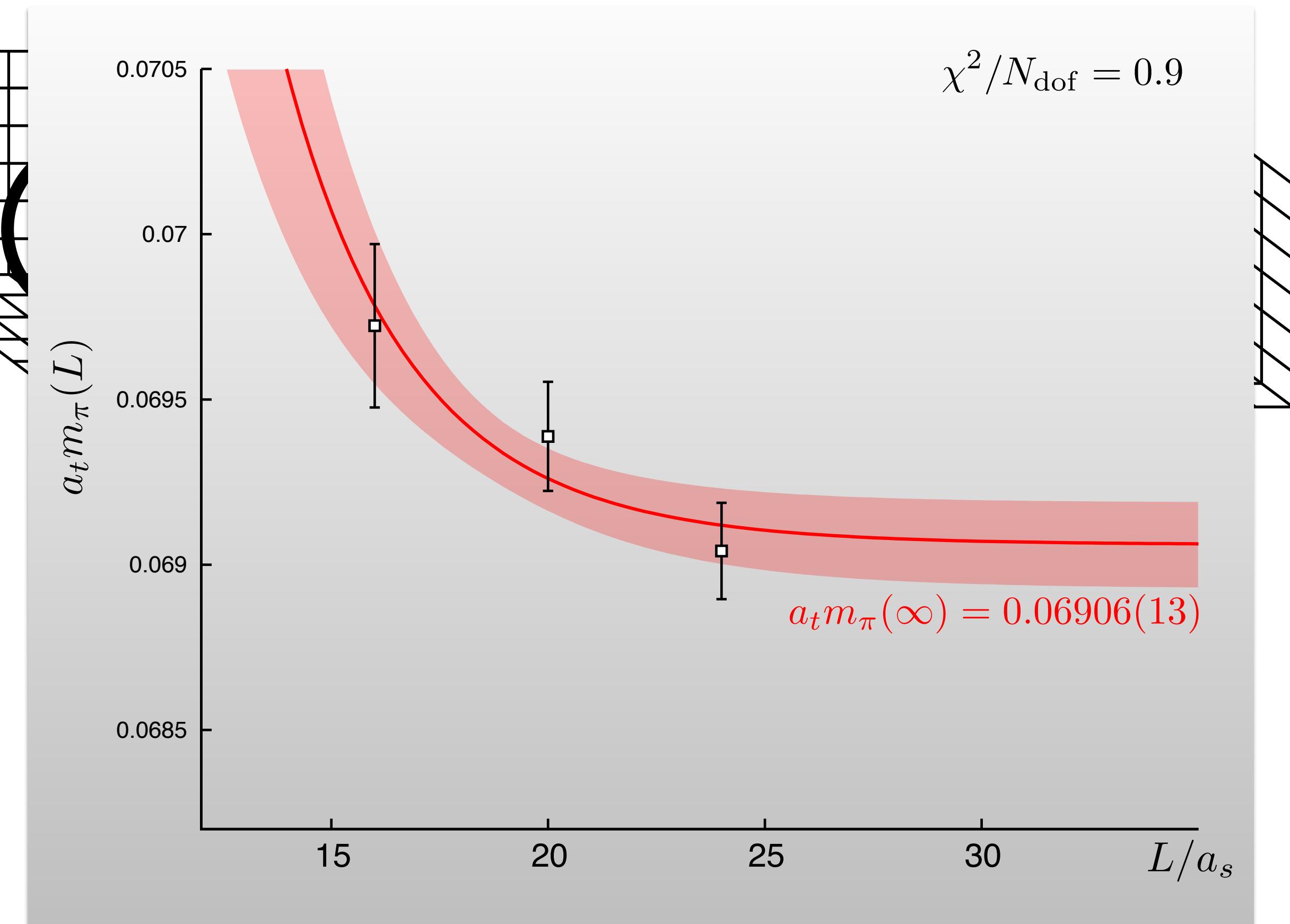
Single hadron propagation



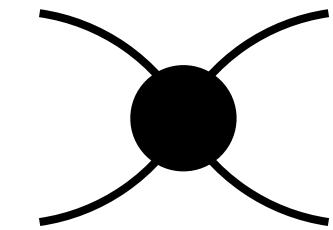
$$\tau = 0$$

$$C_{2pt}^\pi(P) \sim \frac{1}{P^2 - m(L)^2}$$

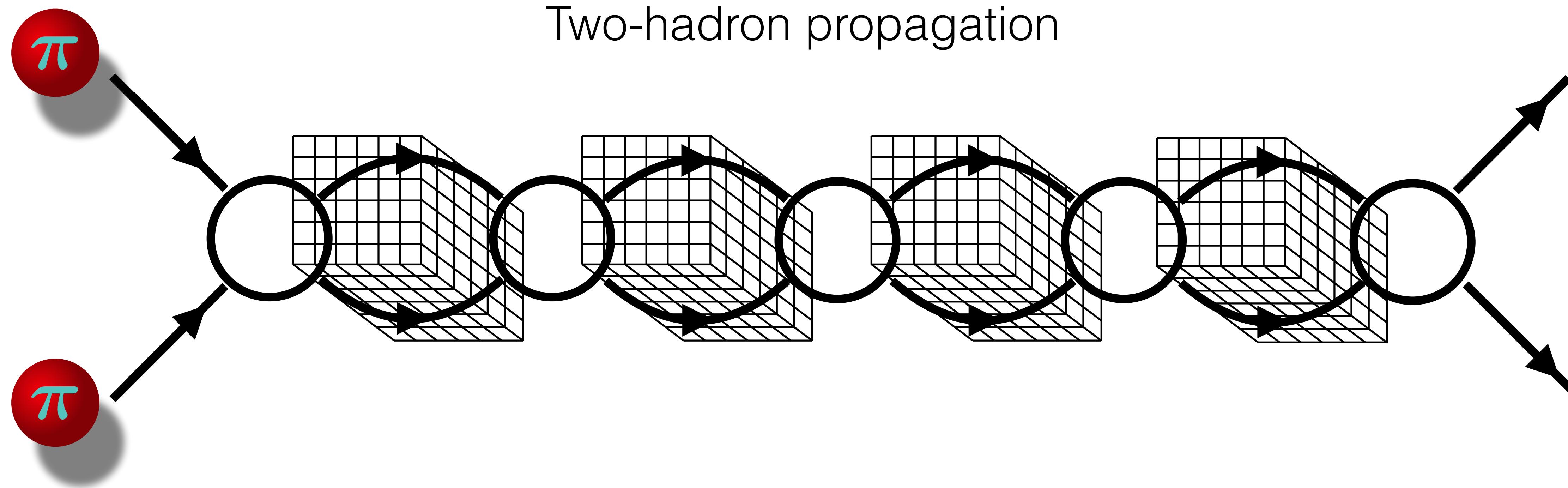
$$m(L) \approx m(\infty) + c e^{-m(\infty)L}$$



[HadSpec Collaboration, 2012]

$$i\mathcal{M} =$$


Finite Volume discrete spectrum



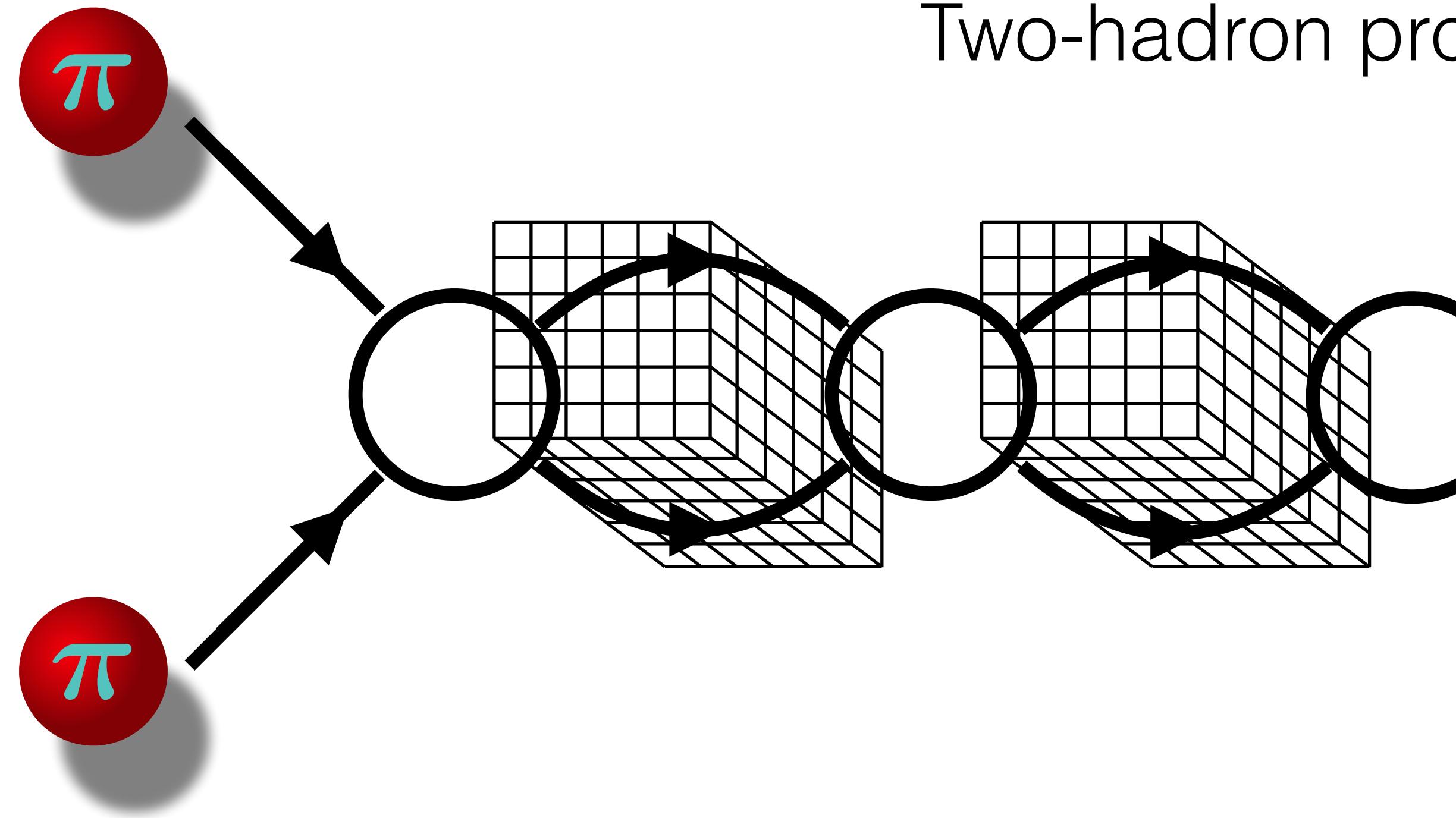
$$C_{2pt}^{\pi\pi}(P) \sim \frac{1}{F^{-1}(P, L) + \mathcal{M}(P^2)}$$



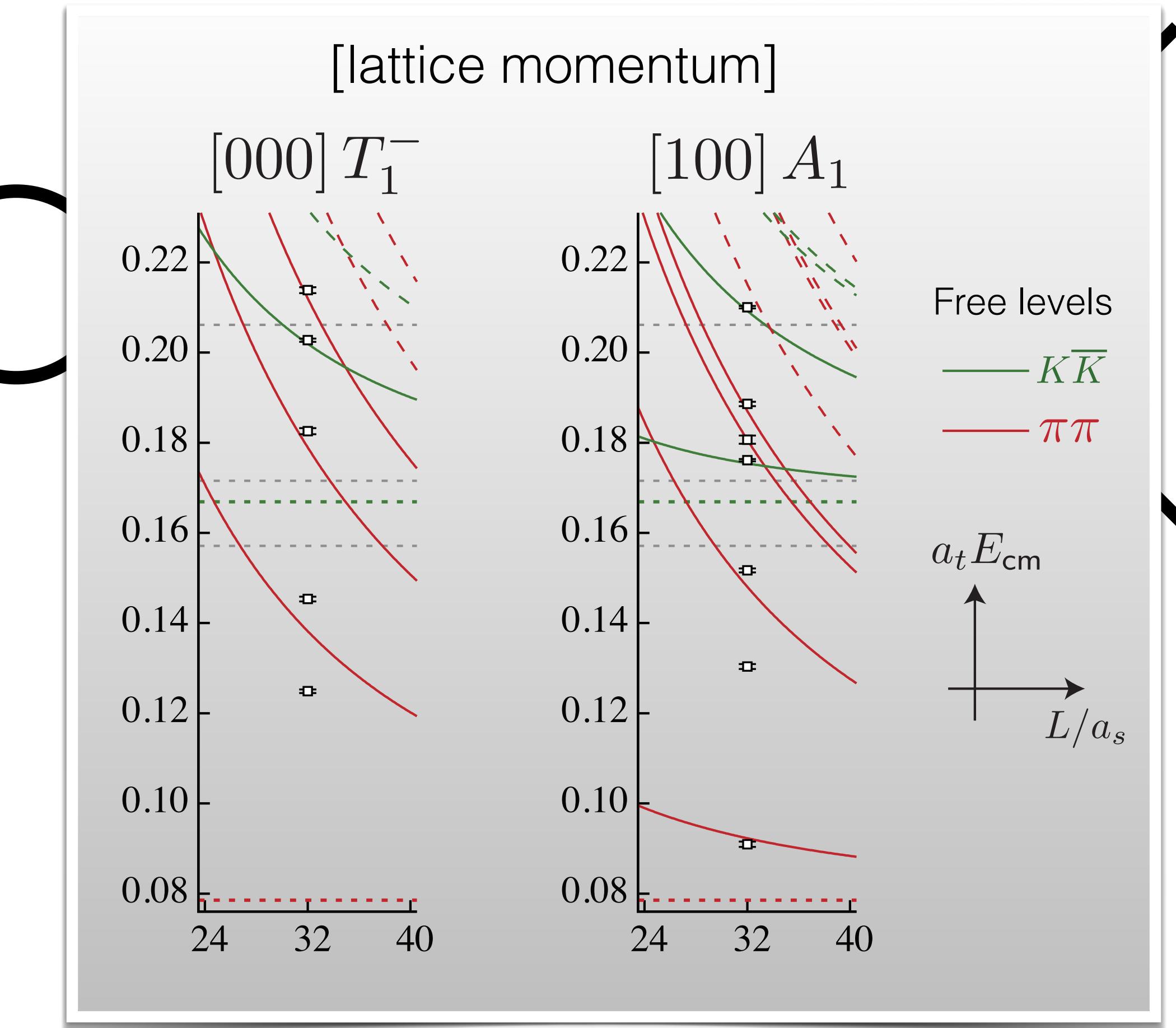
$$i\mathcal{M} = \text{Diagram}$$

Finite Volume discrete spectrum

Two-hadron propagation



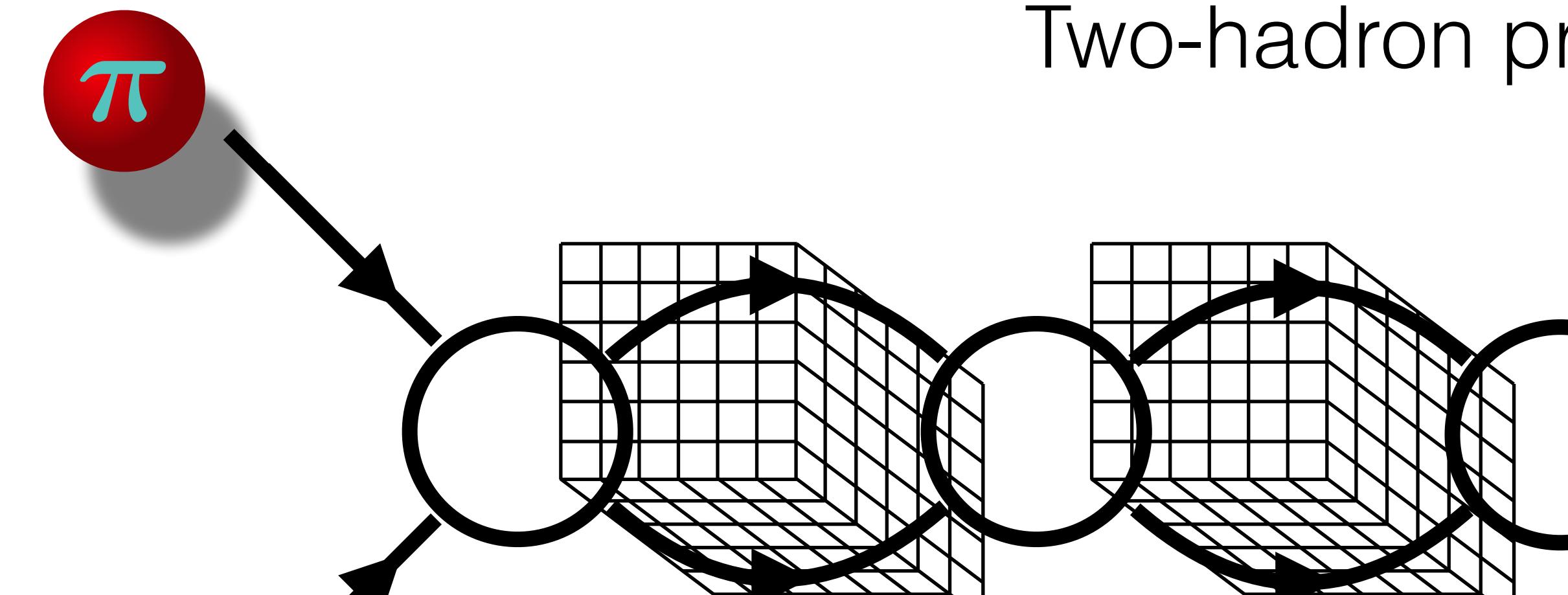
$$C_{2pt}^{\pi\pi}(P) \sim \frac{1}{F^{-1}(P, L) + \mathcal{M}(P^2)}$$



[HadSpec Collaboration, 2015]

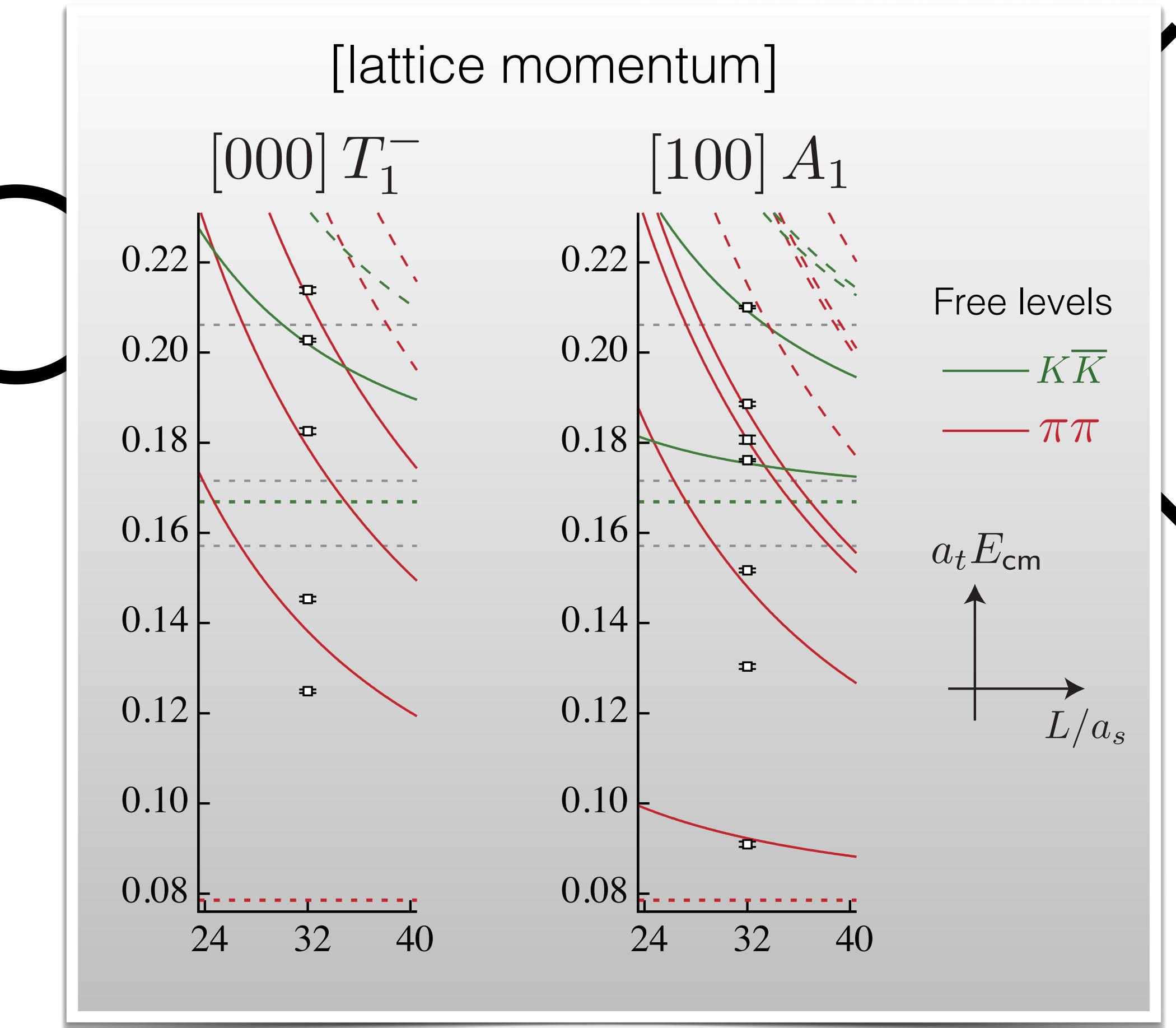
$$i\mathcal{M} = \text{---}$$

Finite Volume discrete spectrum



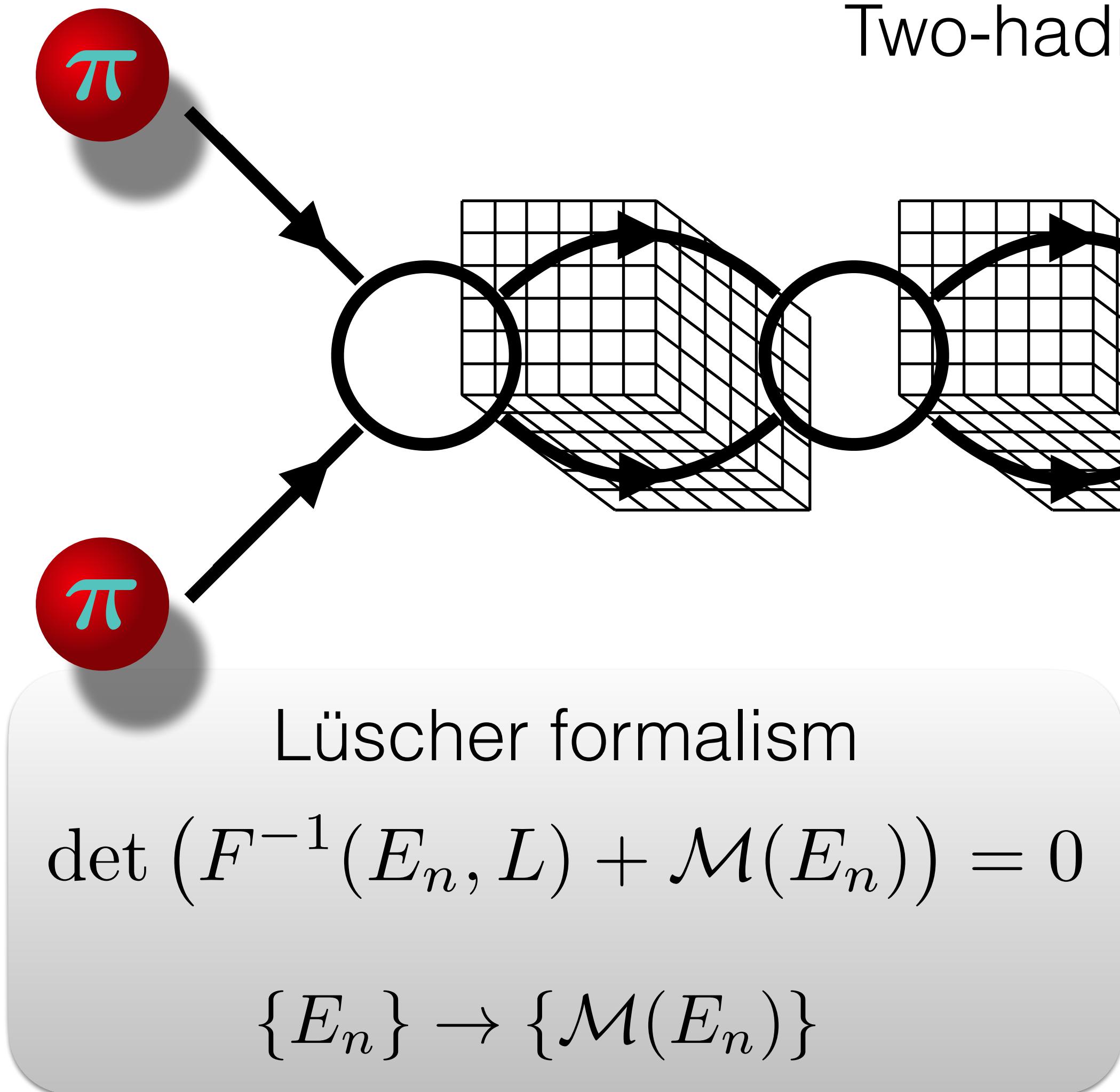
Lüscher formalism

$$\det(F^{-1}(E_n, L) + \mathcal{M}(E_n)) = 0$$

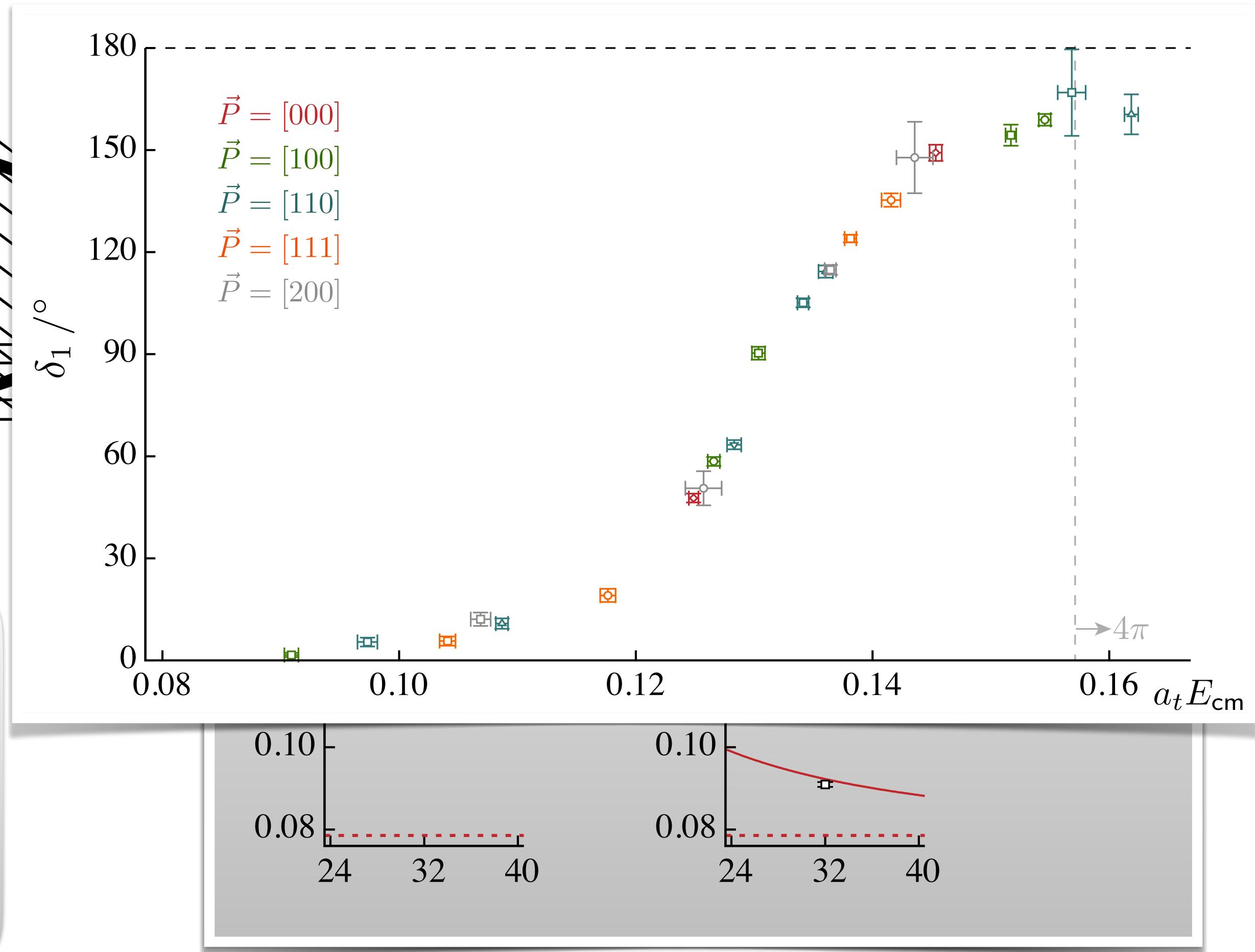
$$\{E_n\} \rightarrow \{\mathcal{M}(E_n)\}$$


$$i\mathcal{M} = \text{Diagram}$$

Finite Volume discrete spectrum



Two-hadron propagation

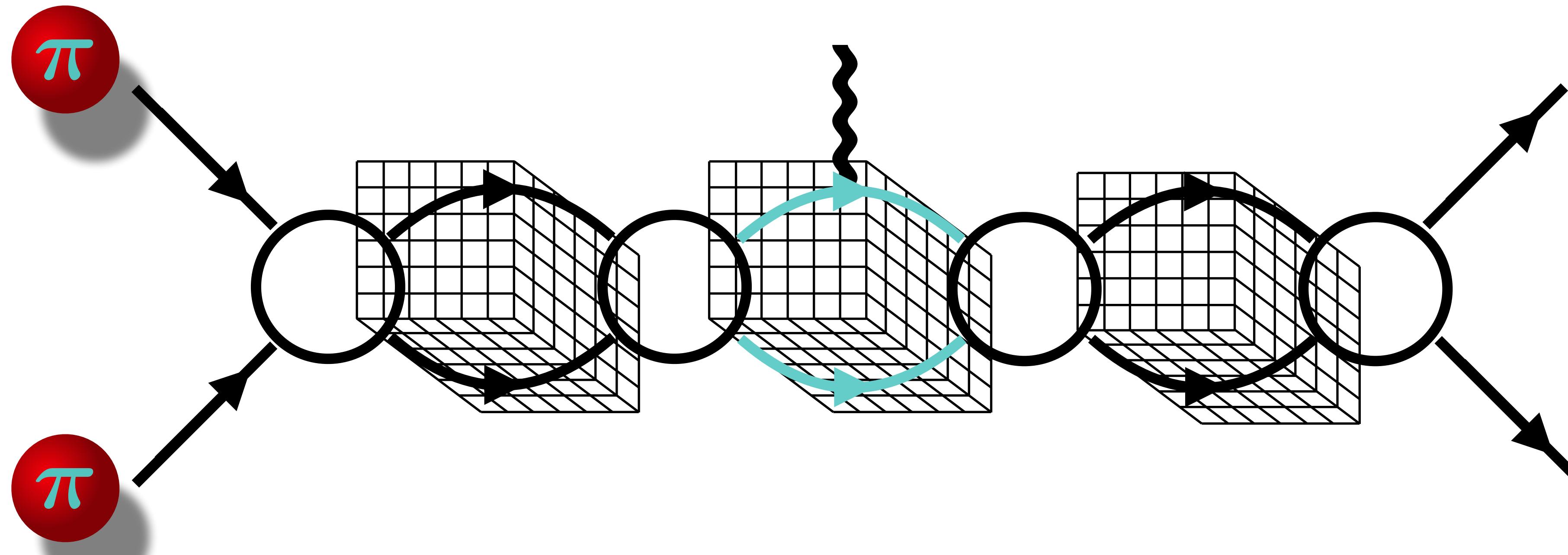


[HadSpec Collaboration, 2015]

$$i\mathcal{W}^A = P_f \leftarrow \text{[Feynman diagram with a wavy line and two external lines labeled } P_f \text{ and } P_i\right]$$

Finite Volume: matrix elements

Two-hadron current insertion

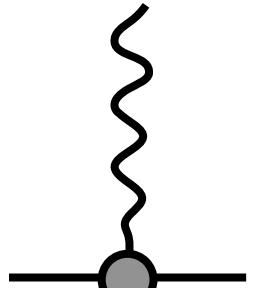


FV Pole in $E_f \times (\mathcal{W} + \text{triangle FV effects}) \times \text{FV Pole in } E_i$

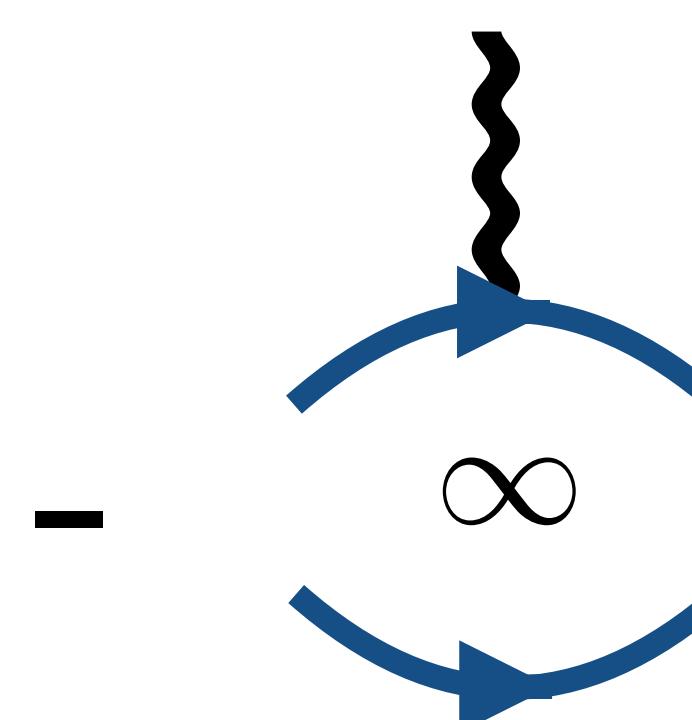
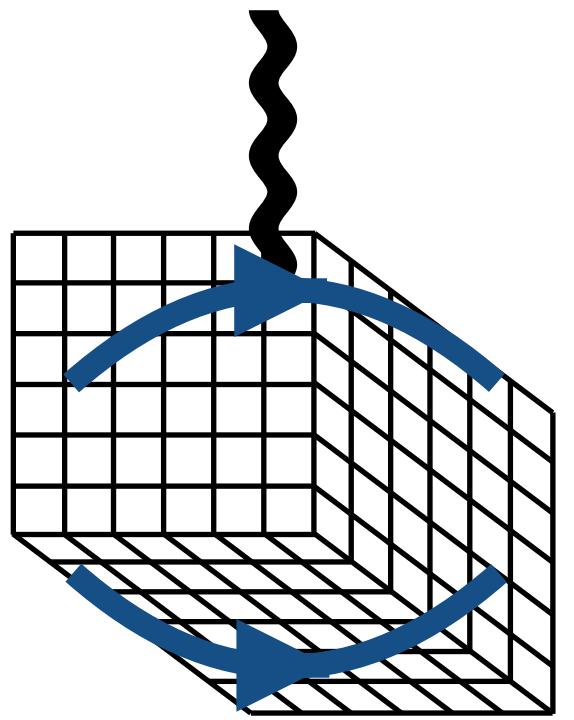
$$C_{3pt}^{\pi\pi}(P_f, P_i) \sim \frac{1}{F^{-1}(P_f, L) + \mathcal{M}(P_f^2)} \mathcal{W}_L \frac{1}{F^{-1}(P_i, L) + \mathcal{M}(P_i^2)}$$

[Briceño, Hansen, 2015]

$$\mathcal{W}_L = \mathcal{W} + \mathcal{M}[G \cdot w]\mathcal{M}$$

$$w_{\sigma,\sigma'}^{\lambda} =$$


$$G \cdot w =$$

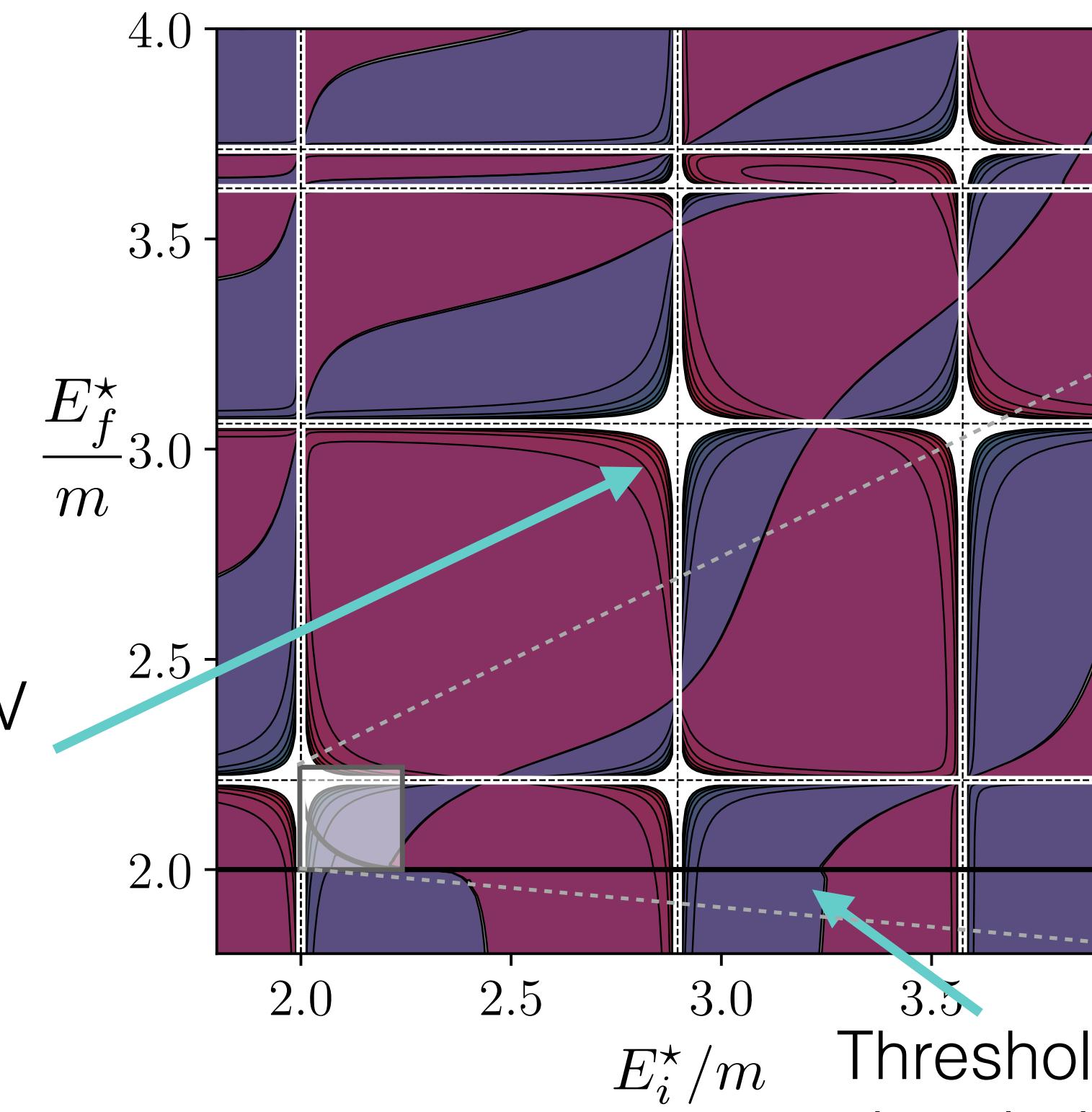


$$\sim \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \right]$$

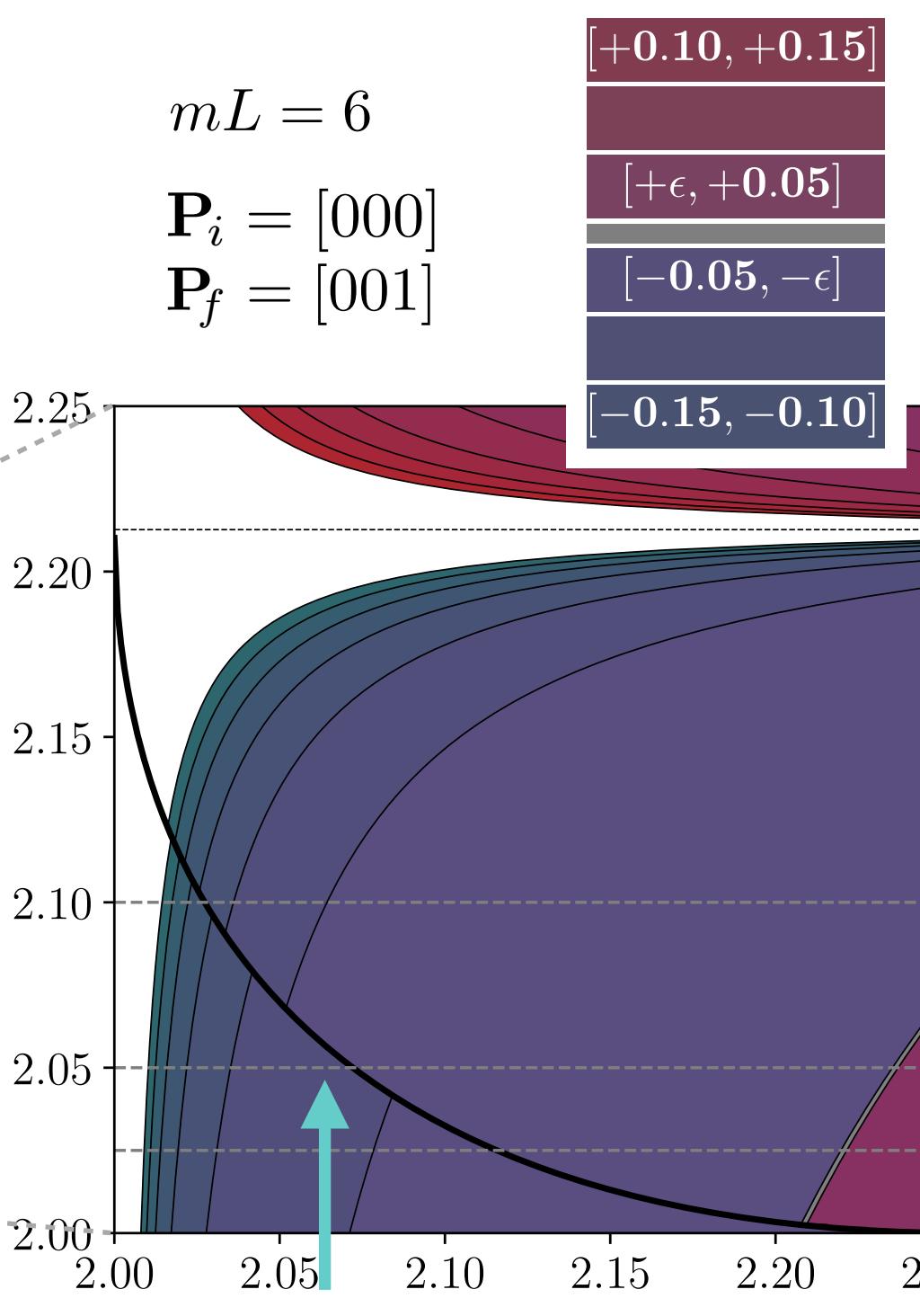
$$i \frac{i}{(P_f - k)^2 - m^2 + i\epsilon} w_{\sigma\sigma'} \frac{i}{(P_i - k)^2 - m^2 + i\epsilon}$$

Scalar G :

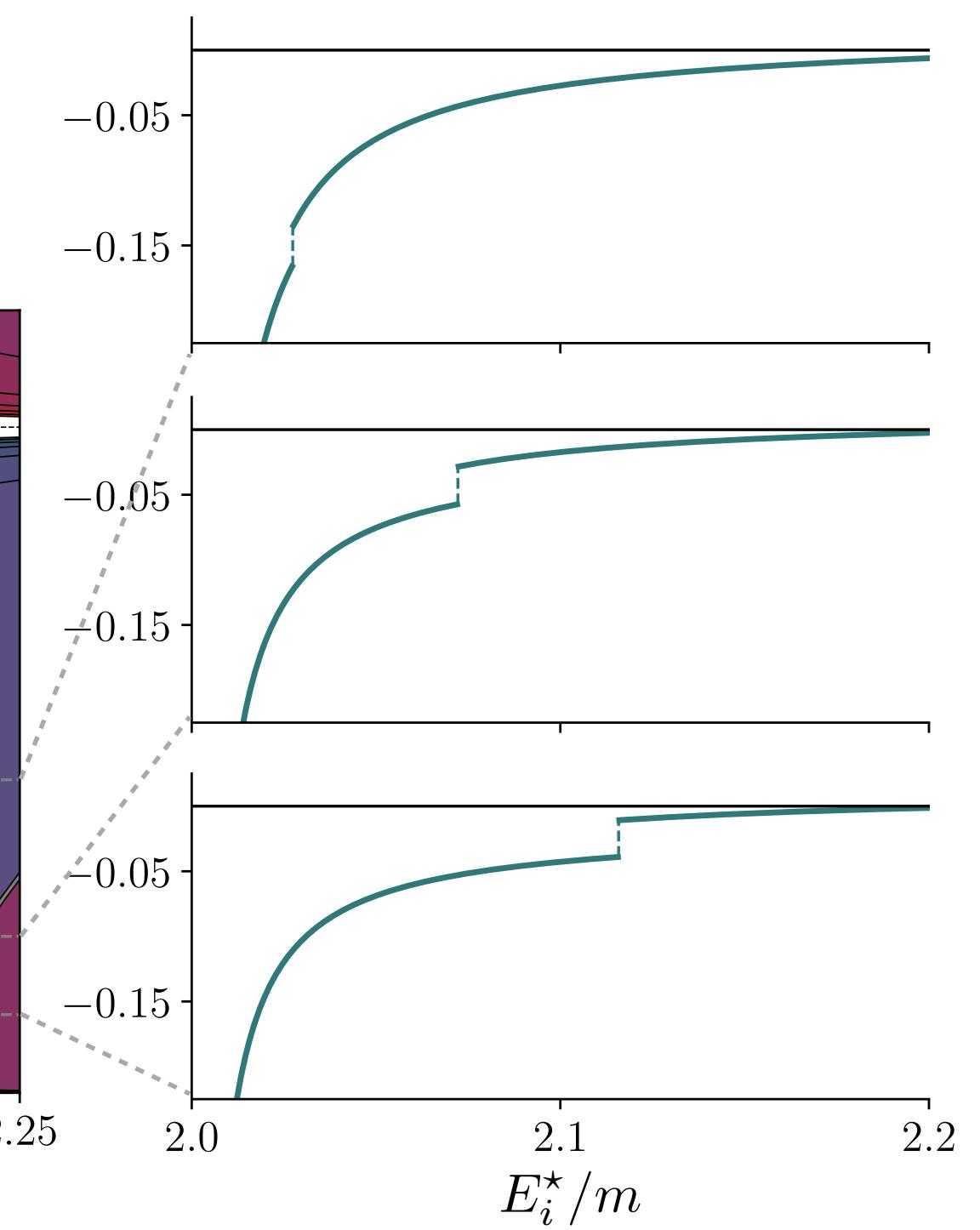
Poles at FV
free
energies.



Threshold
singularities.



Triangle
singularities.



On shell projection of the triangle loop

$$\begin{aligned}
 & \text{Diagram showing the subtraction of two triangle loops:} \\
 & \quad \text{Left side: } (\mathcal{L} \text{--- } V \text{--- } \mathcal{R}^\dagger) - (\mathcal{L} \text{--- } \infty \text{--- } \mathcal{R}^\dagger) \\
 & \quad \text{Right side: } (\mathcal{L} \text{--- } V \text{--- } \mathcal{R}^\dagger) + [(\mathcal{L} \text{--- } V \text{--- } \mathcal{R}^\dagger) - (\mathcal{L} \text{--- } V \text{--- } \mathcal{R}^\dagger)] \\
 & \quad + [(\mathcal{L} \text{--- } V \text{--- } \mathcal{R}^\dagger) - (\mathcal{L} \text{--- } V \text{--- } \mathcal{R}^\dagger)]
 \end{aligned}$$

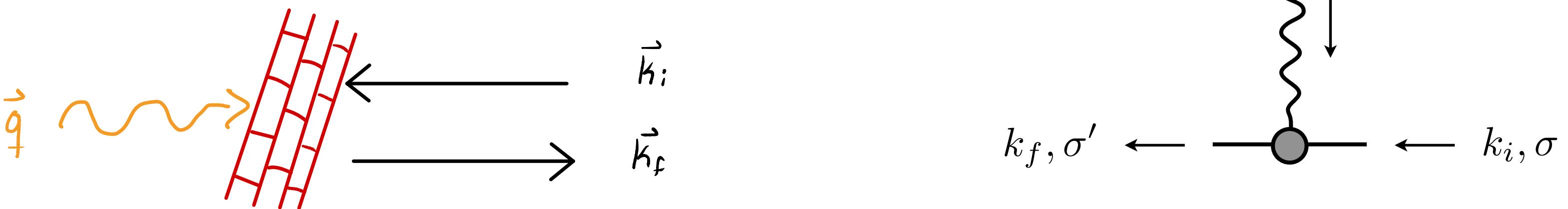
Dotted lines: place on-shell the quantities at the end of the propagator.

Prescription is needed
for on shell expansion

$$w_{\sigma, \sigma'}^{\lambda} =$$

$$w_{\sigma, \sigma'}^{\lambda}(Q^2, k_f^2, k_i^2) = w_{\sigma, \sigma'}^{\lambda}(Q^2) + \delta w_{\sigma, \sigma'}^{\lambda}(Q^2, k_i^2) + w_{\sigma, \sigma'}^{\lambda}(Q^2, k_f^2)\delta + \delta w_{\sigma, \sigma'}^{\lambda}(Q^2, k_f^2, k_i^2)\delta.$$

Brick-Wall frame: one-body FFs



The multipole expansion of currents in the BW frame:

[Durand et al., 1962]

$$w_{\sigma\sigma'}^{\lambda, BW}(k_f, k_i) = 2m_N \sum_J \begin{pmatrix} 1/2 & J & 1/2 \\ \sigma & \lambda & \sigma' \end{pmatrix} M_J^\lambda(Q^2),$$

*Spinors + EM current:
Charge, and magnetic dipole*

The relationship between multipoles and Lorentz FFs is well known for arbitrary spin

[Lorce, 2009]

$$F_1(Q^2) = \frac{1}{1 + Q^2/(4M^2)} \left(C(Q^2) + \frac{Q^2}{4M^2} M_1(Q^2) \right),$$

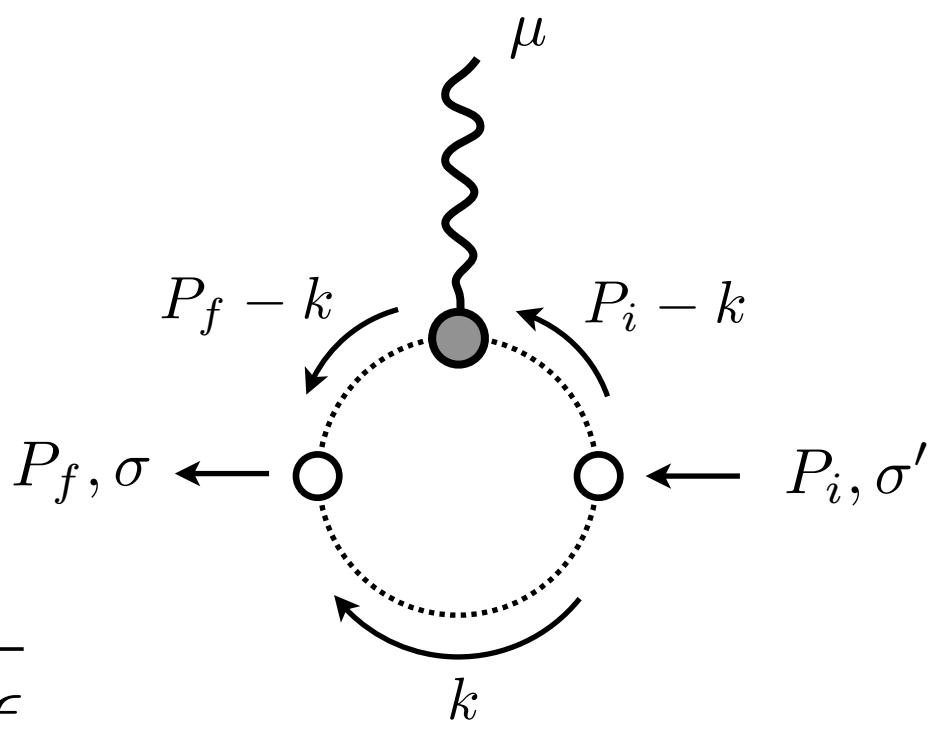
Dirac and Pauli FFs

$$F_2(Q^2) = \frac{1}{1 + Q^2/(4M^2)} (-C(Q^2) + M_1(Q^2)),$$

*Sachs FFs: multipole charge
and magnetic dipole*

G with spin 1/2 and an EM current

$$[G \cdot w]_{\sigma;\sigma'}^{\mu}(P_f, P_i, L) = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \right] \frac{1}{2\omega_k} \frac{1}{(P_f - k)^2 - m^2 + i\epsilon} w_{\sigma\sigma'}^{\text{BW},\mu} \frac{1}{(P_i - k)^2 - m^2 + i\epsilon}$$



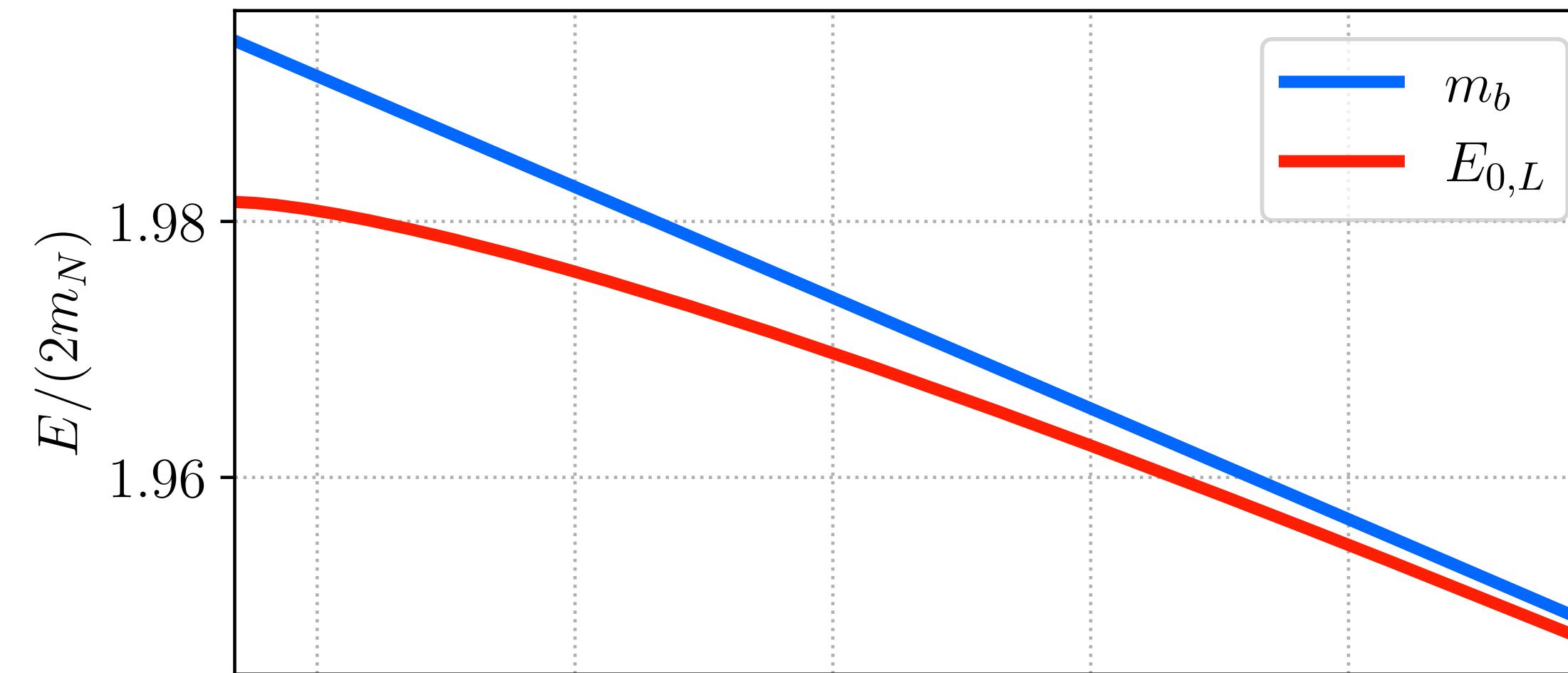
- Boost to from BW to lattice frame
- Thomas precession
- $SO(3)$ to cartesian

$$w_{\sigma\sigma'}^{\text{BW},\mu} = [\Lambda_{BW}(-\mathbf{k}/\omega_k)]^{\mu}_{\nu} D_{\tau\sigma}^{1/2}(R_T(\hat{k}_f)) 2m \left[\sum_{J,\lambda=0,1} \epsilon_{\lambda}^{\nu} \begin{pmatrix} 1/2 & J & 1/2 \\ \tau & \lambda & \tau' \end{pmatrix} M_J^{\lambda}(Q^2) \right] D_{\tau'\sigma'}^{1/2*}(R_T(\hat{k}_i))$$

$$\begin{aligned} \epsilon_0^{\mu} &= \delta_0^{\mu} & M_J^0(Q^2) &= \delta_{J0} C(Q^2) \\ \epsilon_{\pm 1}^{\mu} &= \epsilon_{\pm 1}^{\mu}(0) & M_J^1(Q^2) &= \delta_{J1} M_1(Q^2) \end{aligned}$$

A toy example: the Deuteron electric charge

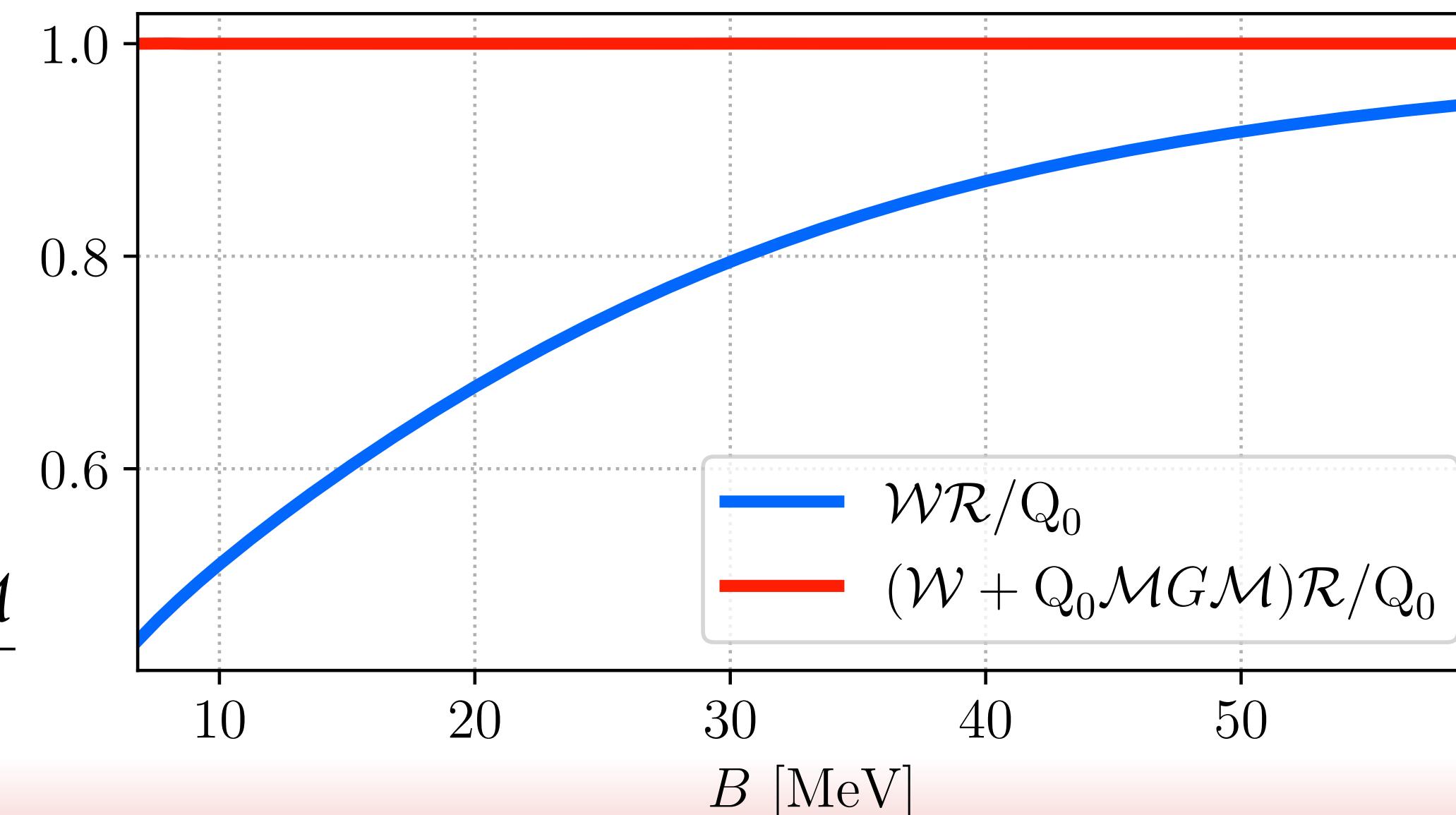
- Energies: Finite volume effects vanish for deeply bound states



- Matrix elements: more significant volume dependence from G even below threshold for

[Briceño et al., 2019]

$$\mathcal{W}^0(P_f = P_i = (E, \mathbf{0})) = Q_0 \frac{\partial \mathcal{M}}{\partial E}$$



Summary and Outlook

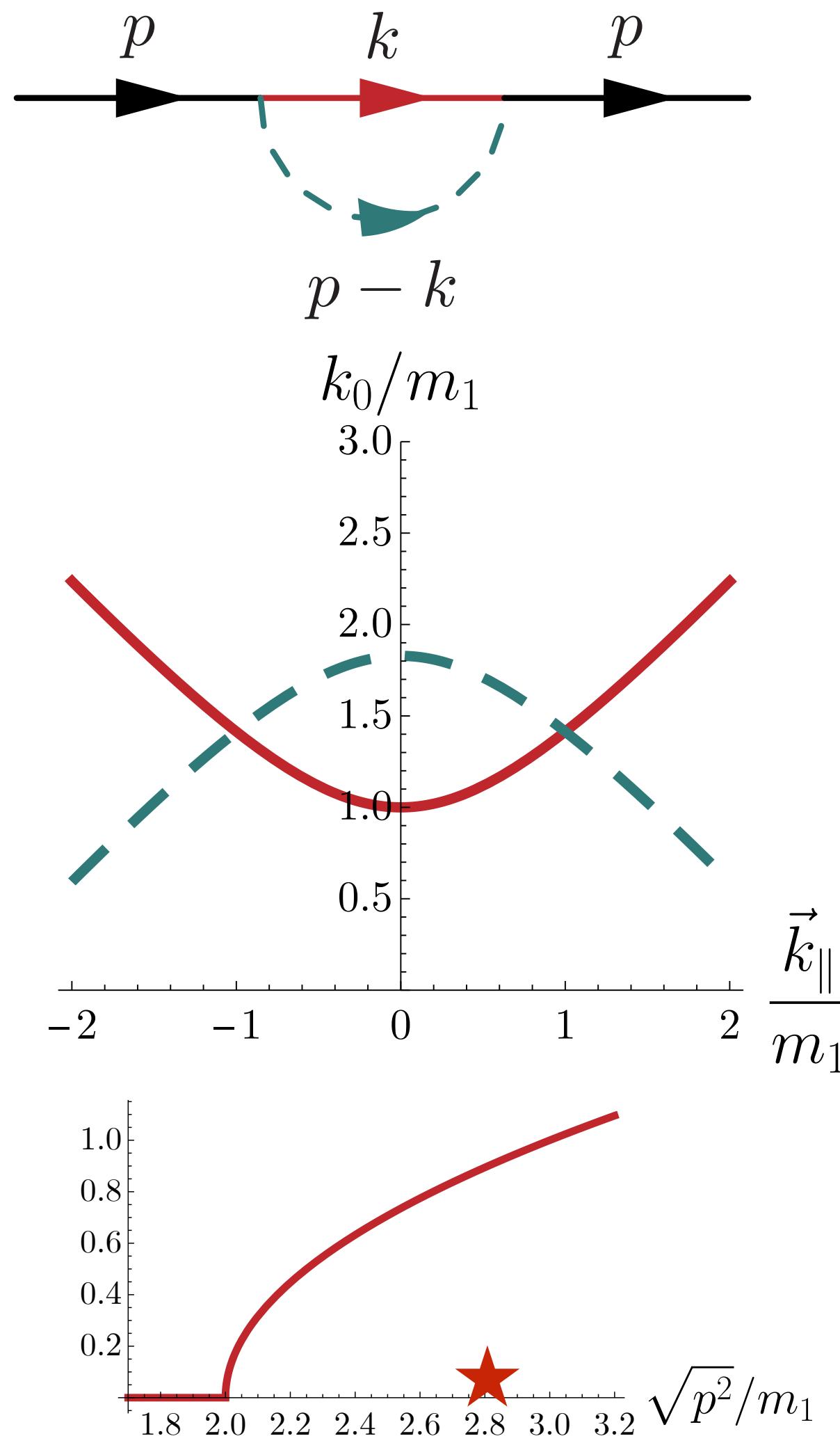
- Formulation of a prescription to calculate the power-law finite volume corrections of particles with spin in triangle loop diagrams.
- These corrections can be significant even when binding energies have small finite volume effects.
- Future path for QCD predictions of two-nucleon matrix elements.

$$\mathcal{W}_L = \mathcal{W} + \mathcal{M}[G \cdot w]\mathcal{M}$$

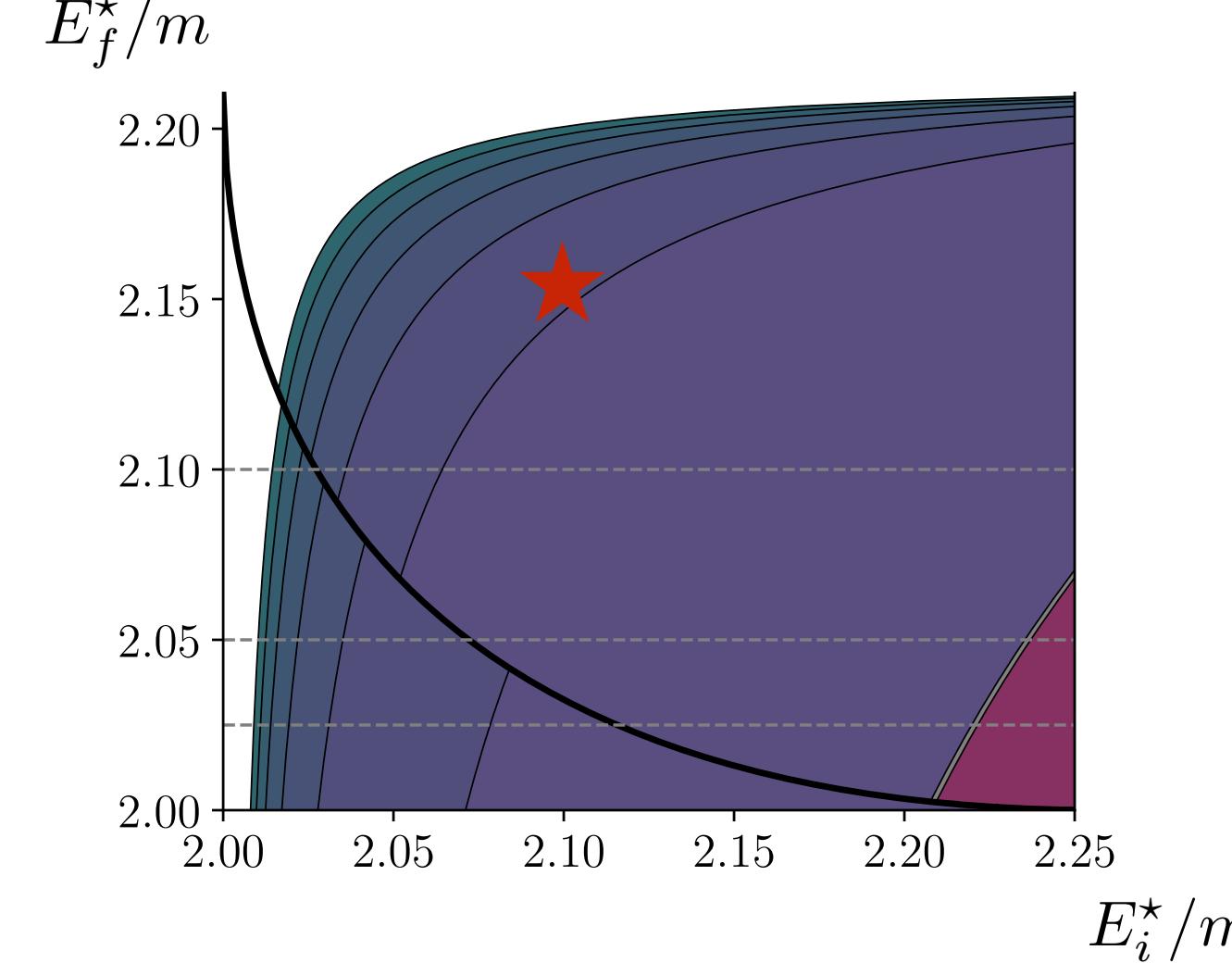
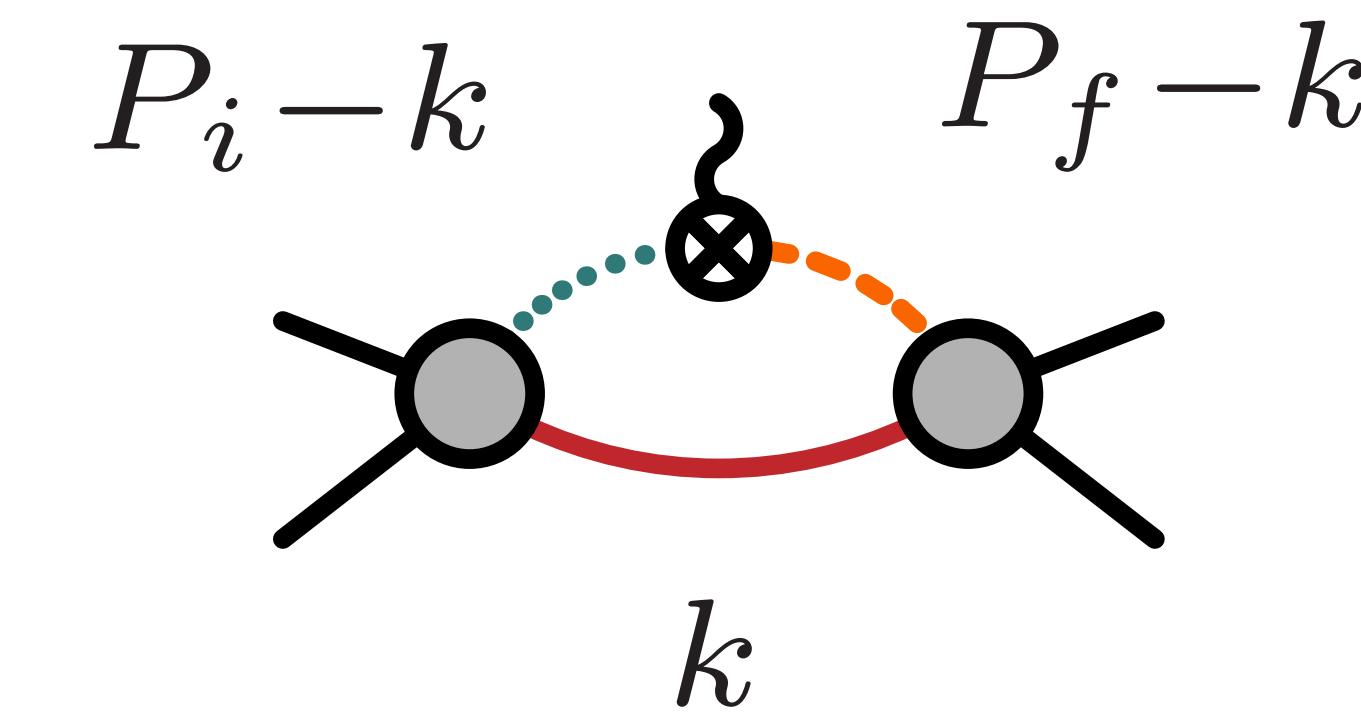
$$w_{\sigma\sigma'}^{\text{BW},\mu} = [\Lambda_{BW}(-\mathbf{k}/\omega_k)]^{\mu\nu} D_{\tau\sigma}^{1/2}(R_T(\hat{k}_f)) 2m \left[\sum_{J,\lambda=0,1} \epsilon_{\lambda}^{\nu} \begin{pmatrix} 1/2 & J & 1/2 \\ \tau & \lambda & \tau' \end{pmatrix} M_J^{\lambda}(Q^2) \right] D_{\tau'\sigma'}^{1/2*}(R_T(\hat{k}_i))$$



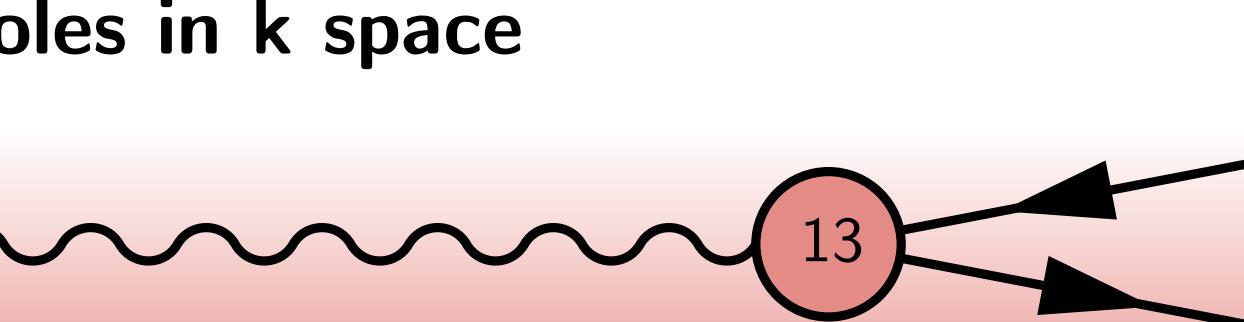
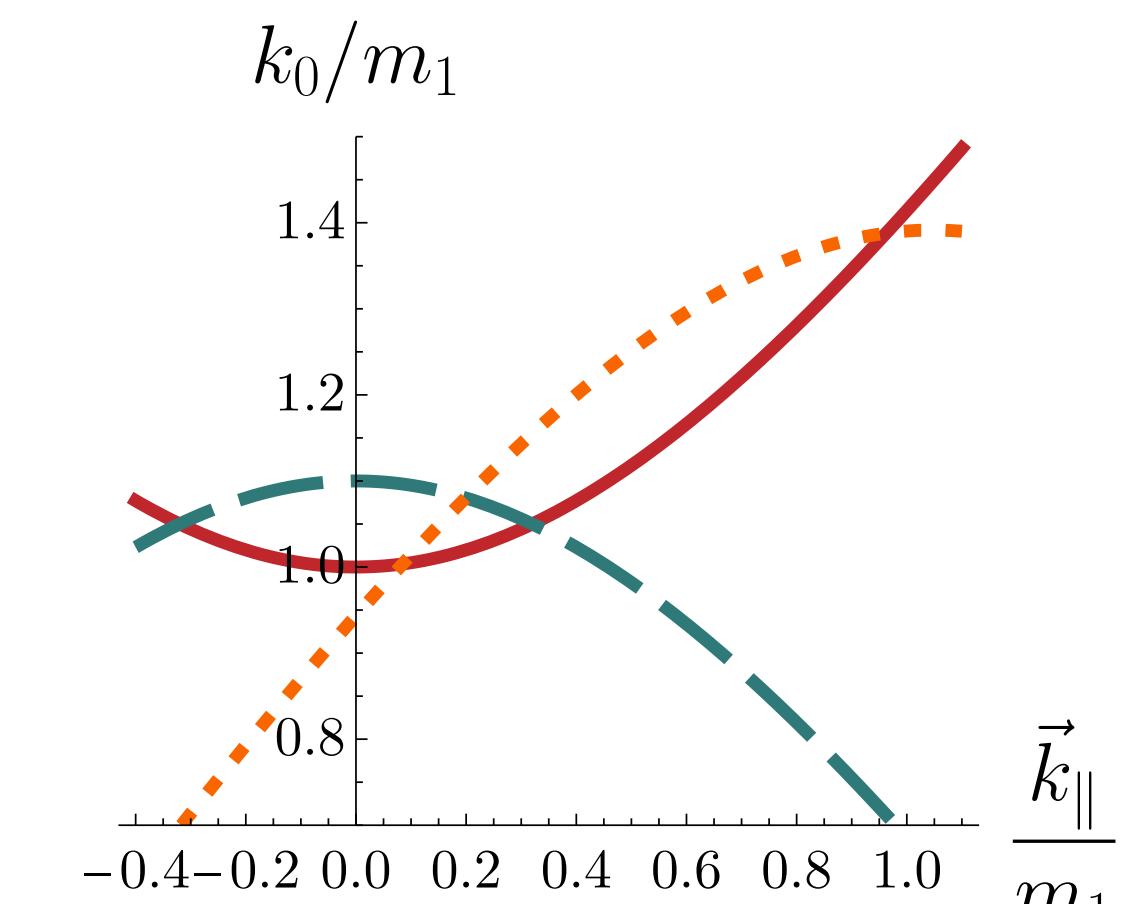
Singularities of loop diagrams



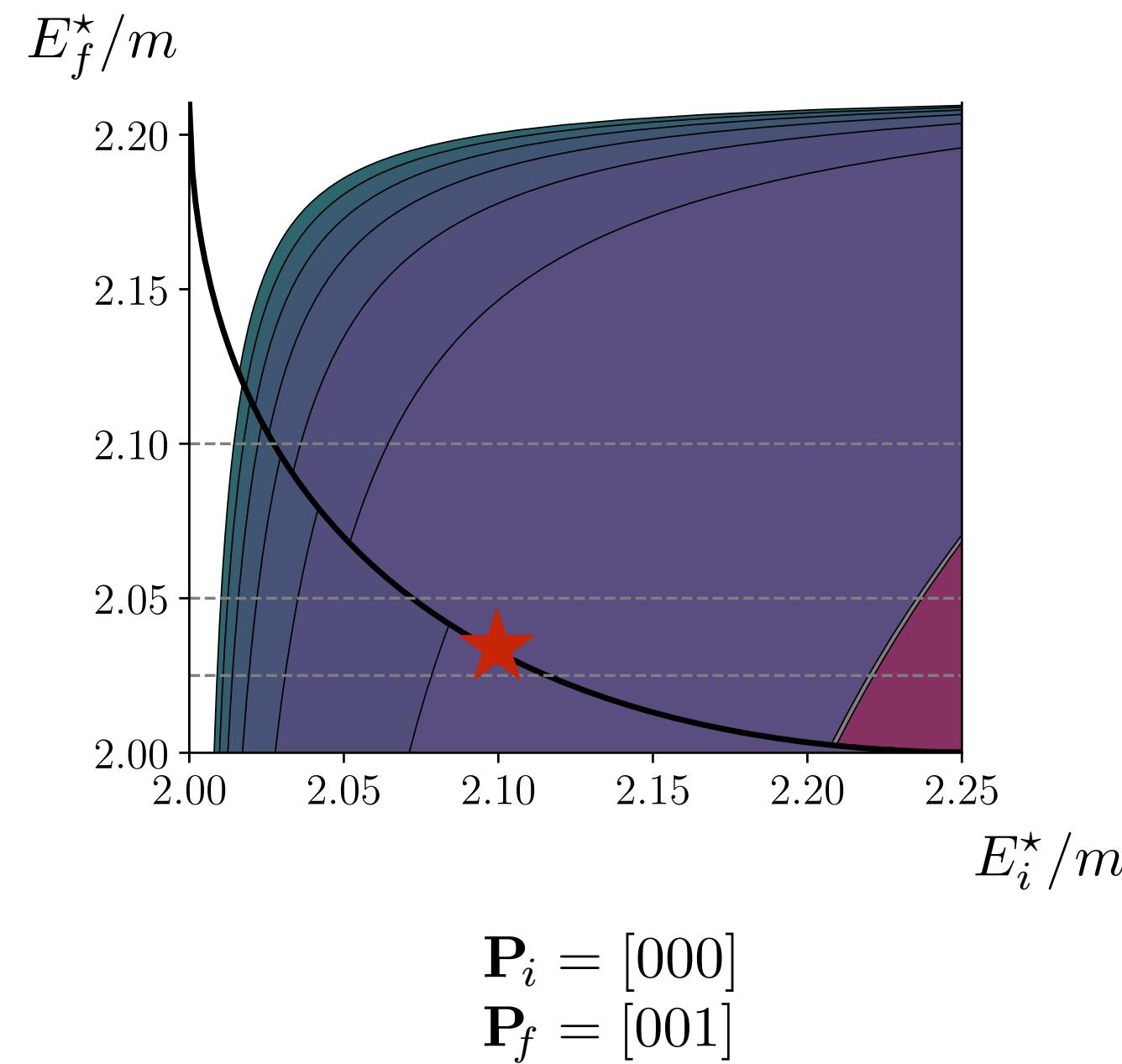
Branch cut: square root singularity



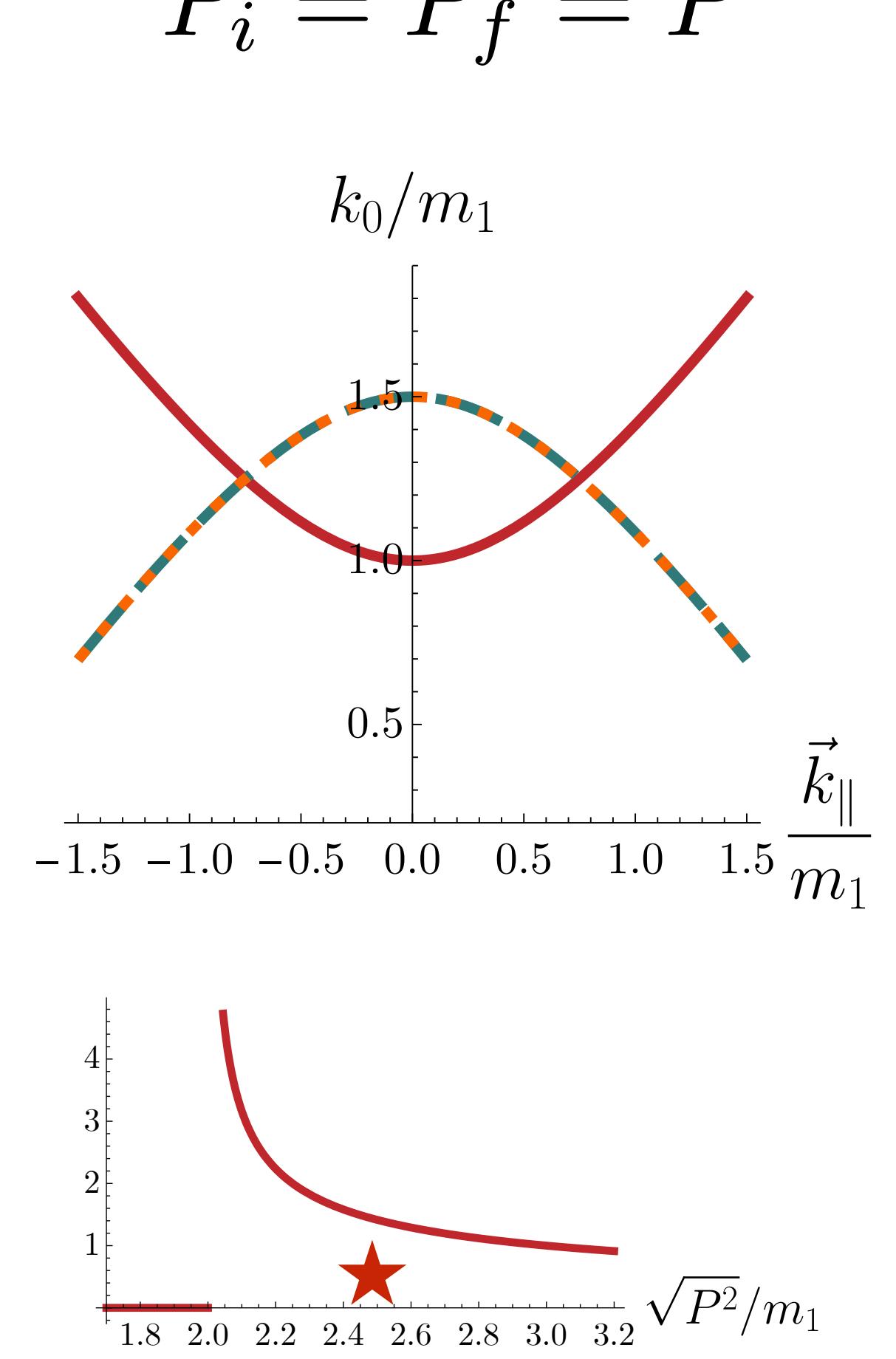
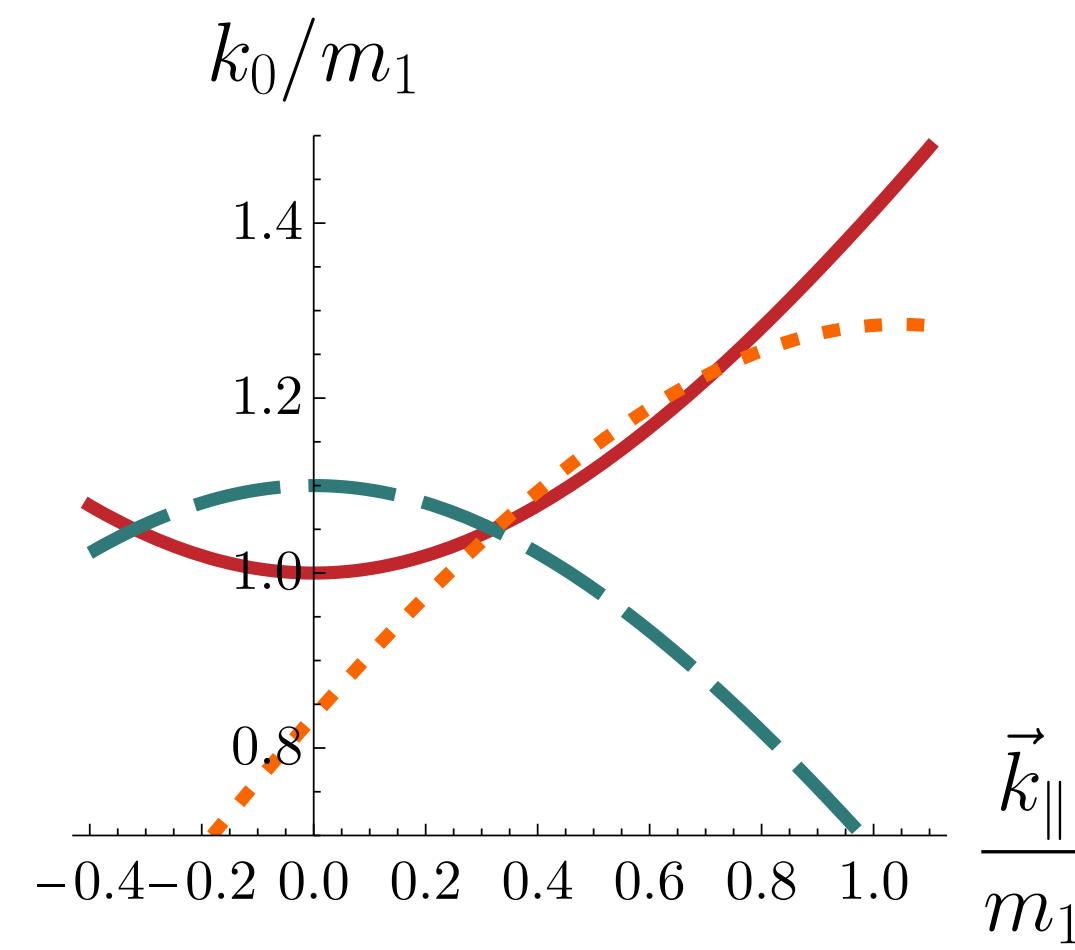
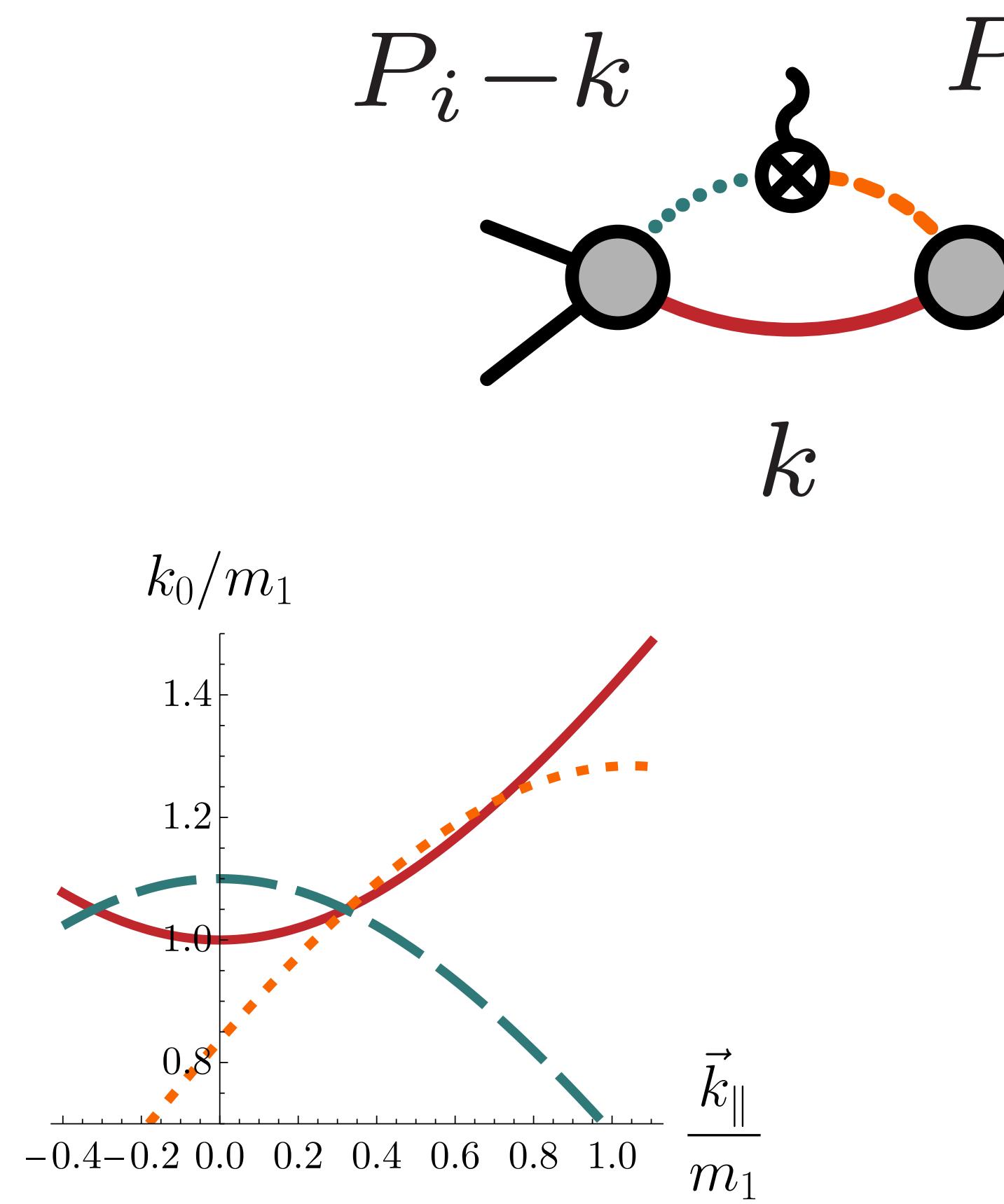
Triangle diagram: poles in k space



Triangle diagram singularities

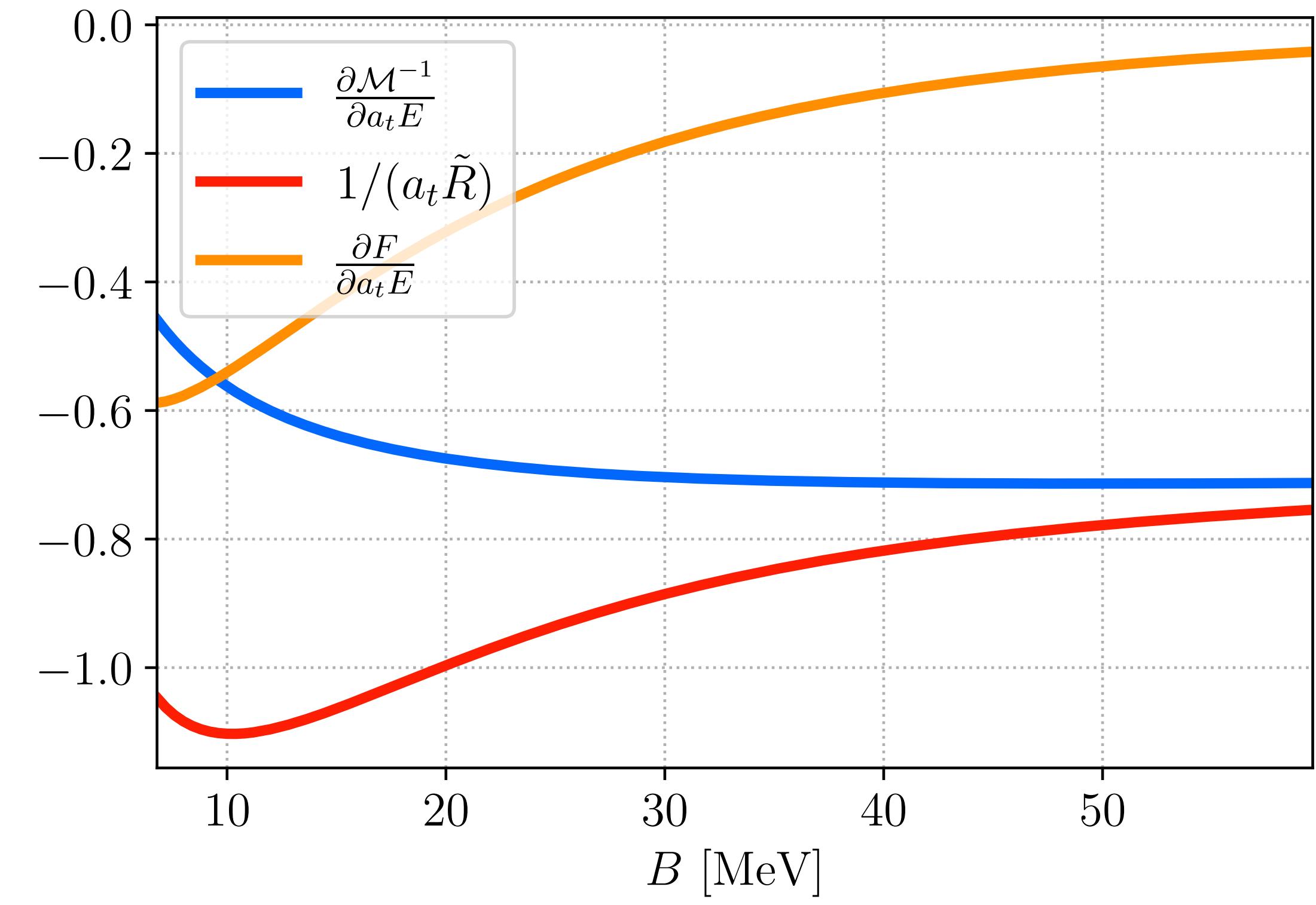


Logarithmic singularity



Inverse square root singularity

$$\mathcal{R}(E_0, L) = -\mathcal{M}^{-1} \left(\tilde{\mathcal{R}}(E_0, L) \right) \mathcal{M}^{-1}$$



- The Lellouch-Luscher factor tends towards the value of the residue of the scattering amplitude.
- However for smaller binding energies the F function becomes significant.
- The F-function does not tend to a double-pole behavior as naively expected because it is evaluated at the FV energy, which at low B also deviates strongly from the deuteron pole.