Antiheavy-Antiheavy-Light-Light Four-Quark Bound States

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Motivation (1)

Why study heavy-light tetraquarks?

- Large number of four-quark states has been found in the last years
- Start of this year: LHCb found a bound tetraquark $T_{cc}^{+}(cc\bar{u}\bar{b})$

  R. Aaij *et al.* [LHCb], Nature Commun. **13**, 3351 (2022)
- Lattice study predict virtual bound state for $cc\bar{u}\bar{b}$

  M. Padmanath and S. Prelovsek, Phys. Rev. Lett. **129**, 032002 (2022)

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- We study similar $\bar{Q}\bar{Q}'qq'$ systems in the bottom sector ($\bar{Q} = \bar{b}$)
- For $\bar{b}\bar{b}ud$ bound state has been predicted:
  - *Born-Oppenheimer approximation* at $E_{\text{bind}} \approx -90\,\text{MeV}$

    Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012)
    P. Bicudo *et al.* [ETMC], Phys. Rev. D **87**, 114511 (2013)
    P. Bicudo *et al.*, Phys. Rev. D **92**, 014507 (2015)
    P. Bicudo *et al.*, Phys. Rev. D **95**, 034502 (2017)
    P. Bicudo *et al.*, Phys. Rev. D **96**, 054510 (2017)
  - *Full Lattice QCD using Non-Relativistic QCD* at $E_{\text{bind}} \approx -130\,\text{MeV}$

    A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017)
    P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D **99**, 034507 (2019)
    L. Leskovec, S. Meinel, M.P. and M. Wagner, Phys. Rev. D **100**, 014503 (2019)

Motivation (2)

Further Lattice calculations for tetraquarks in the bottom sector:

- For $\bar{b}\bar{b}us$ bound state has been predicted at $E_{\text{bind}} \approx -80\,\text{MeV}$
- For $\bar{b}\bar{c}ud$ inconsistent results if bound state exists

A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017)
P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D **99**, 034507 (2019)
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D **99**, 054505 (2019)
R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, Phys. Rev. D **102**, 114506 (2020)

In this talk...

... I will present our studies with an improved operator basis using

- Scattering operators at the *sink* for
  - $\bar{b}\bar{b}us$ with $J^P = 1^+$
  - $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$ and $0(1^+)$

  ➡ Results published in S. Meinel, M. P., M. Wagner, 2205.13982 [hep-lat]
- Scattering operators at the *sink* and the *source* for
  - $\bar{b}\bar{b}ud$ $I(J^P) = 0(1^+)$ with preliminary results

Scattering Operators at the Sink

Lattice Setup for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$

- Gauge link configuration generated by **RBC and UKQCD collaboration**

  Y. Aoki *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **83**, 074508 (2011)
  T. Blum *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **93**, 074505 (2016)
- $2+1$ flavours domain-wall fermions and Iwasaki gauge action
- Five different ensembles which differ in

lattice spacing	a	$\approx 0.083\,\text{fm} \ldots 0.114\,\text{fm}$,
lattice size	L	$\approx 2.65\,\text{fm} \ldots 5.48\,\text{fm}$,
pion mass	m_π	$\approx 139\,\text{MeV} \ldots 431\,\text{MeV}$

- Explore dependence on m_π for bound states

- Smeared point-to-all propagators for light quarks
- Anisotropic clover action for charm quarks

  A. X. El-Khadra, A. S. Kronfeld, and P. B. Mackenzie, Phys. Rev. D **55**, 3933 (1997)
  P. Chen, Phys. Rev. D **64** 034509 (2001)
  RBC and UKQCD Collaboration, Y. Aoki *et al.*, Phys. Rev. D **86**, 116003 (2012)
- NRQCD action for bottom quarks

  B. Thacker and G. Lepage, Phys. Rev. D **43**, 196 (1991)
  G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, K. Hornbostel, Phys. Rev. D **46**, 4052 (1992)

Interpolating Operators for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$

- Up to now, *only* local operators have been used in studies
  ➡ All quarks at the same space-time position
- We use *additionally* scattering operators
  ➡ Two mesons separated in space-time position
- Due to the usage of point-to-all propagators, no scattering operators at the source

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- Local operators: good overlap to bound four-quark states
- Scattering operators: good overlap to higher excitations at or above threshold (2 meson states)

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- Local operators: good overlap to bound four-quark states
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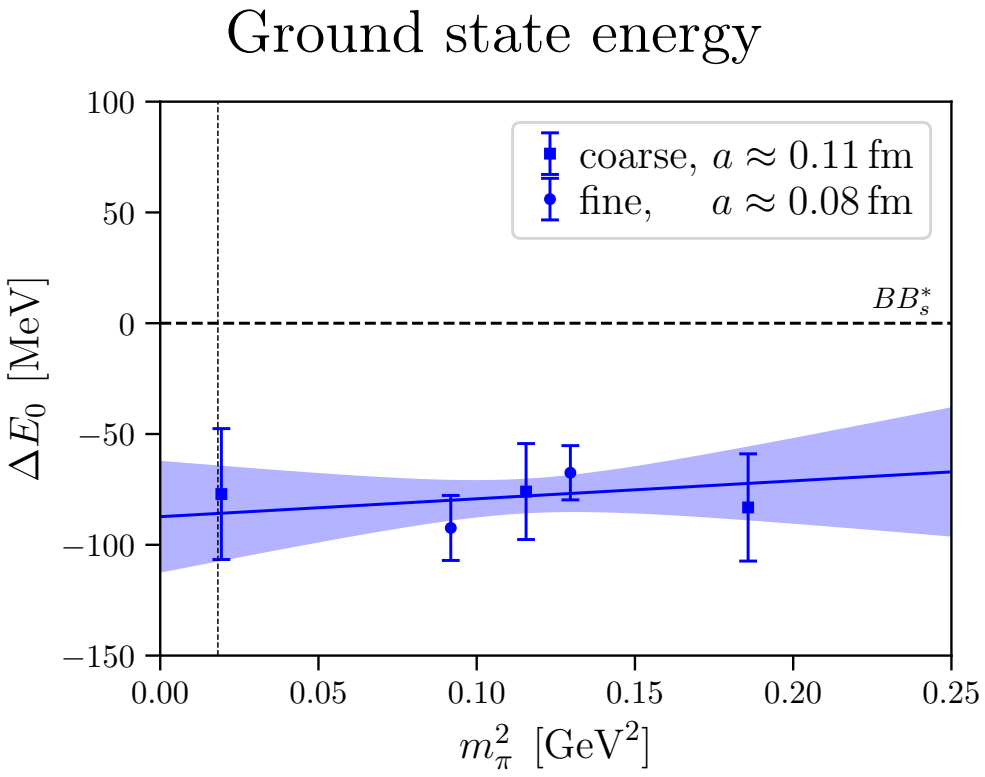
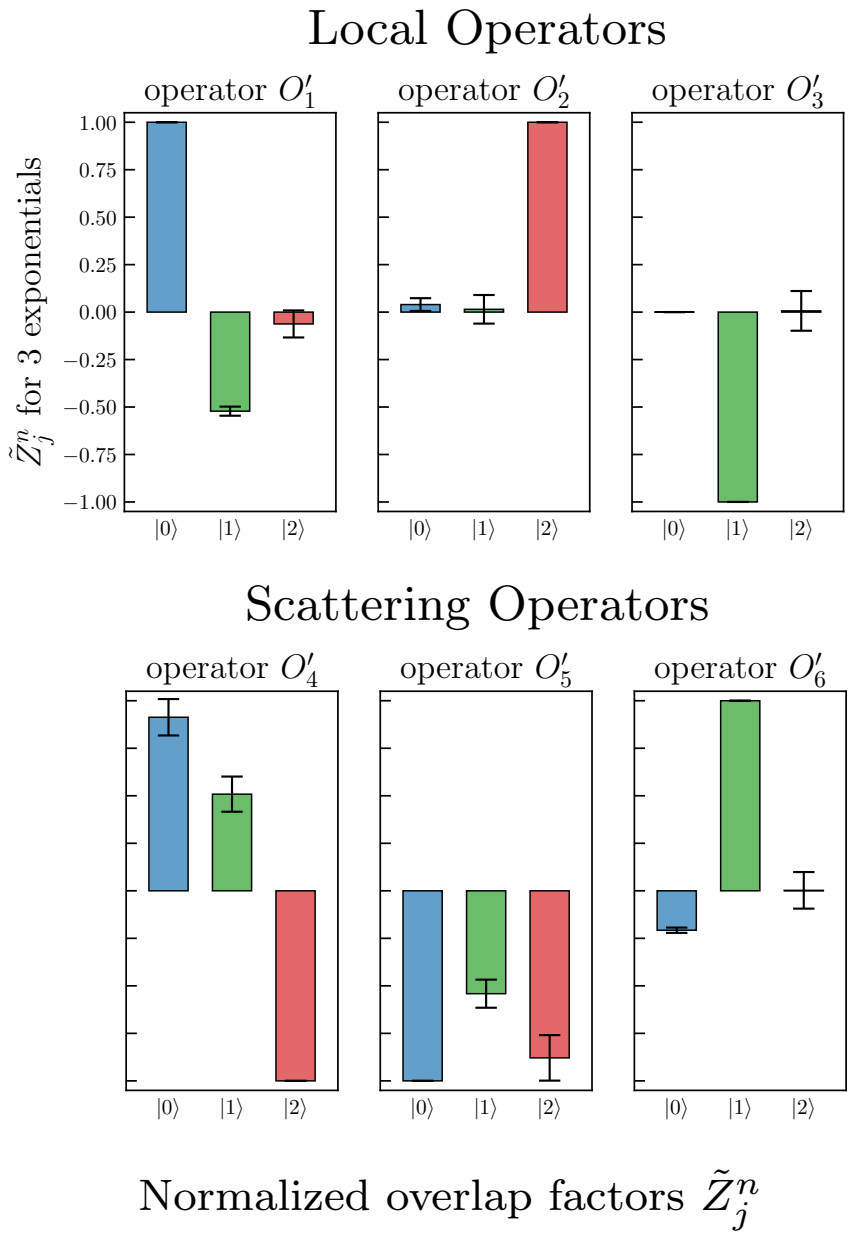
➡ We found for all systems that including scattering operators decreases the lowest energy level significantly!

Results for $\bar{b}\bar{b}us$, $J^P = 1^+$



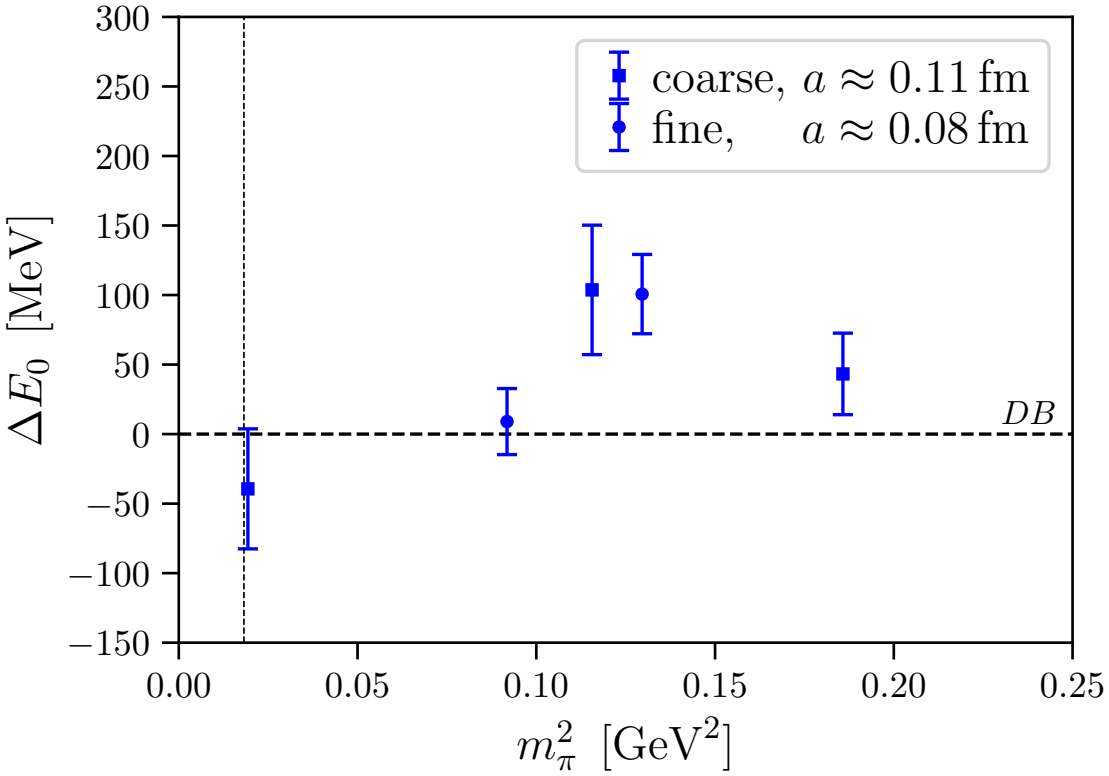
$$\Delta E_0 = (-86 \pm 22 \pm 10)\,\text{MeV}$$

➡ Bound State!



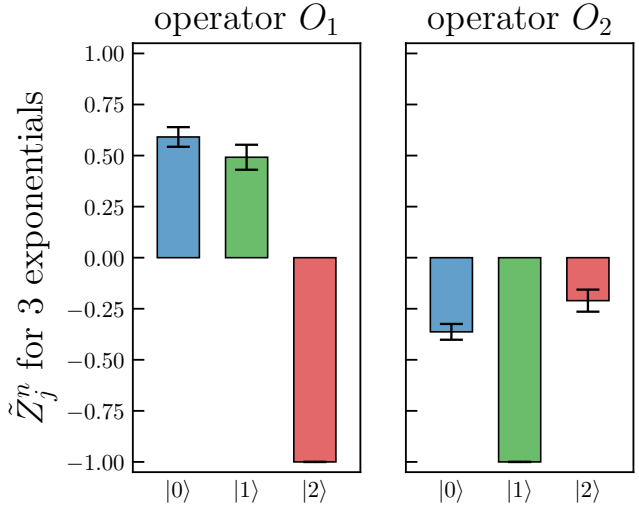
Results $\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$

Ground state energy

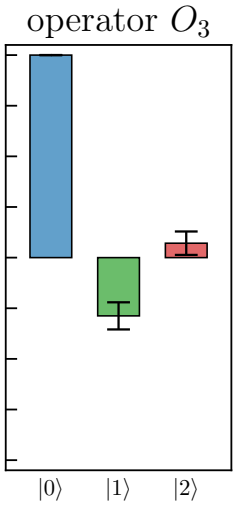


➡ No indication that stable tetraquark exists!

Local Operators



Scattering Operators



Normalized overlap factors $\tilde{Z}_j^n$

Results $\bar{b}\bar{c}ud$, $I(J^P) = 0(1^+)$

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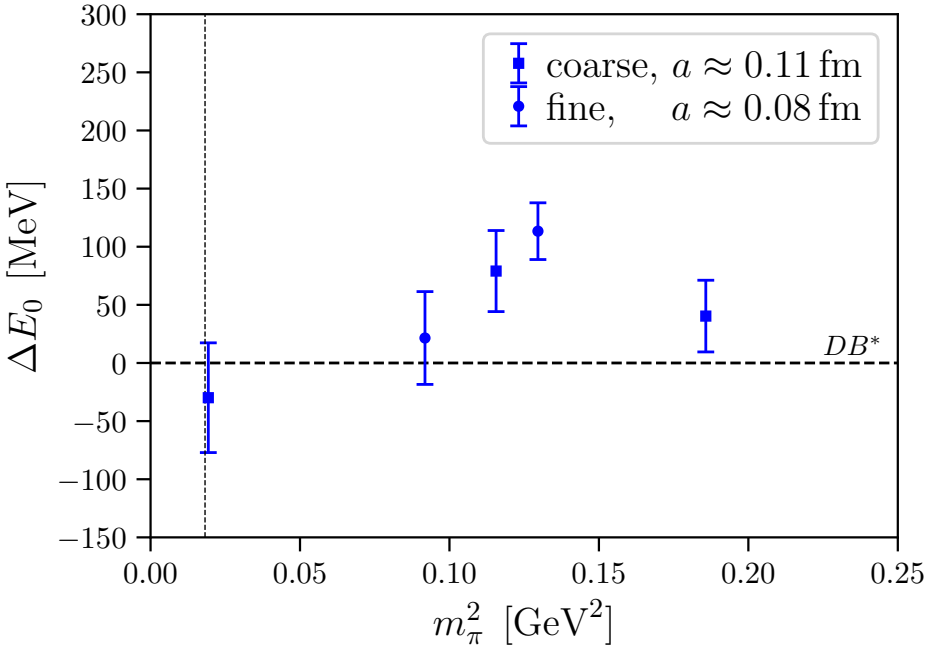

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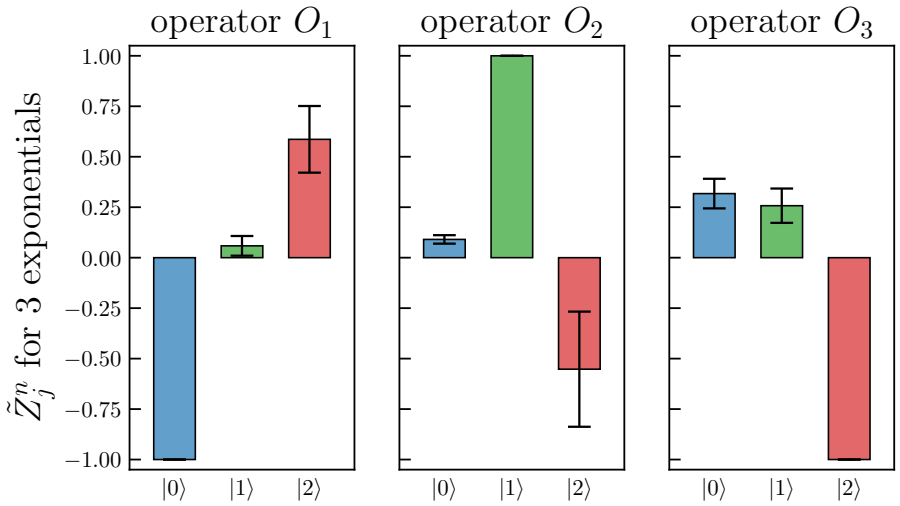
Local Operators



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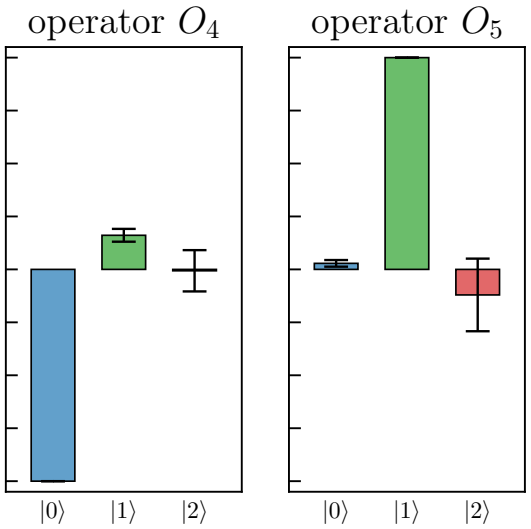

Normalized overlap factors $\tilde{Z}_j^n$

Scattering Operators at the Sink and Source

New Operator Basis

- Up to now: Scattering operators only at the *sink*
- Now: Computing scattering operators also at the *source*
- Realized via stochastic timeslice-to-all propagators

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Why is this necessary?

- Most tetraquarks are close to the relevant threshold or even resonances
  - ➡ Finite volume effects might be significant
  - ➡ Scattering analysis must be done
  - ➡ Requires full information from scattering operators
- We study finite volume effects in $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$

See also poster by Marc Wagner in Poster Session!

Lattice Setup

- Gauge link configurations from the **MILC collaboration**
  A. Bazavov *et al.* [MILC], Phys. Rev. D **87**, 054505 (2013)
- Highly improved staggered quark (HISQ) action for sea quarks
- Wilson-Clover action for the valence quarks
  T. Bhattacharya *et al.* [PNDME], Phys. Rev. D **92**, 094511 (2015)
  R. Gupta *et al.* [PNDME], Phys. Rev. D **98**, 034503 (2018)
- Six ensembles with

lattice size	L	$\approx 2.84\,\mathrm{fm} \ldots 4.76\,\mathrm{fm}$,
pion mass	m_π	$\approx 220\,\mathrm{MeV},\ 310\,\mathrm{MeV}$,
lattice spacing	a	$\approx 0.09\,\mathrm{fm},\ 0.12\,\mathrm{fm}$

- We will perform ...
  - Scattering analysis to obtain the infinite volume energy
  - Chiral extrapolation to the physical pion mass

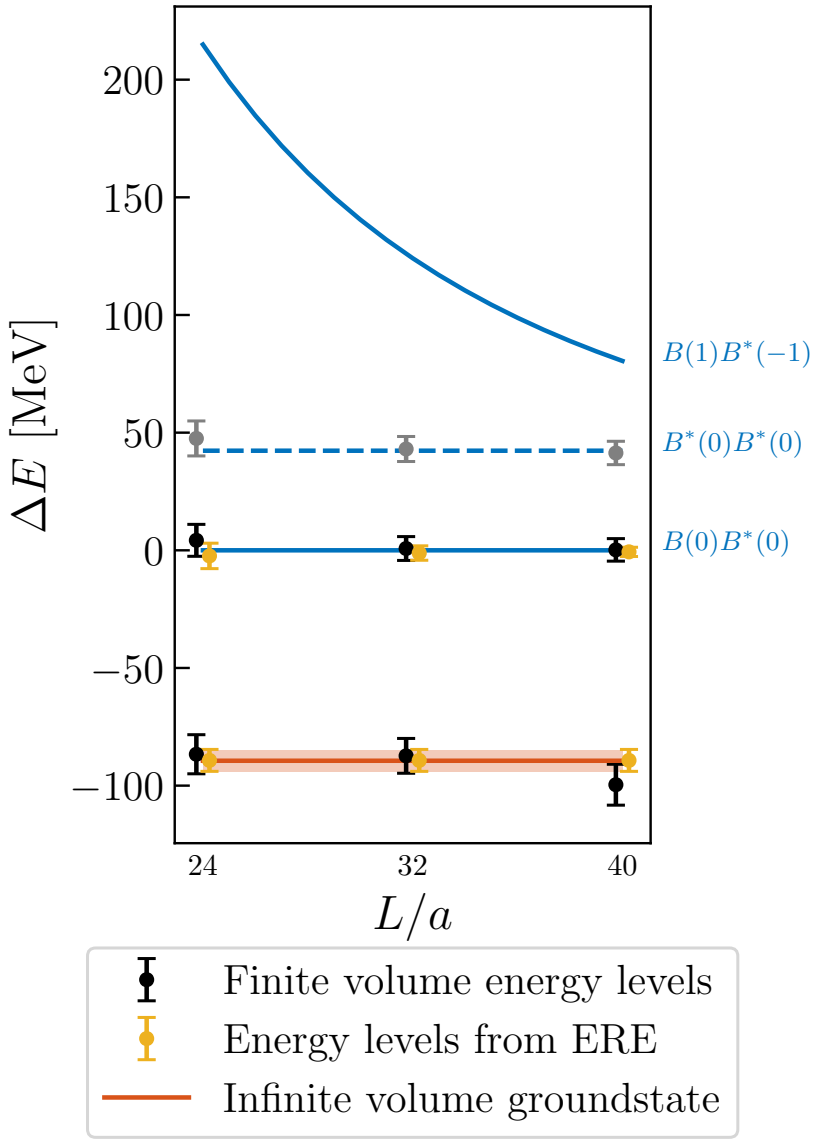
Results $\bar{b}\bar{b}ud$, $I(J^P) = 0(1^+)$ (1)

- Determine the *finite* volume energy via a *GEP*
- Perform a scattering analysis
- Compute $\cot(\delta)$ via Lüscher's formula
- Use an effective range expansion (ERE) to parametrize $\cot(\delta)$
- Determine *infinite* volume energy from T-Matrix

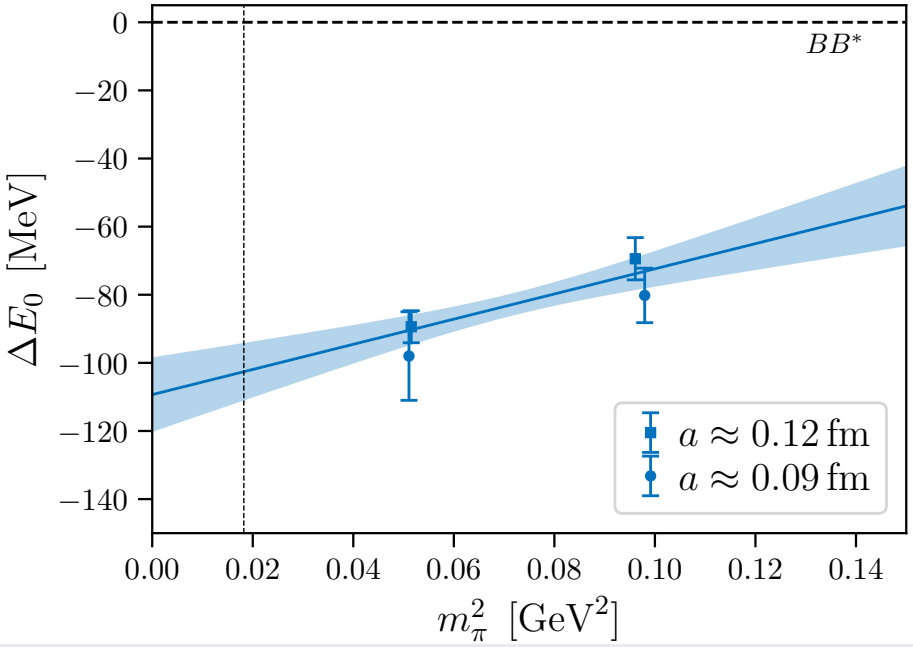
➡ Ground state energy essentially unaffected

M. Lüscher, Nucl. Phys. **B354**, 531 (1991)
R. A. Briceño, J. J. Dudek, and R. D. Young, Rev. Mod. Phys. **90**, 025001 (2018)

$a \approx 0.12\,\text{fm}$, $m_\pi \approx 220\,\text{MeV}$



Results $\bar{b}\bar{b}ud$, $I(J^P) = 0(1^+)$ (2)



- We recognize m_π dependence of ground state energy
- Perform a chiral extrapolation to the physical point
- Final binding energy for $\bar{b}\bar{b}ud$ in $I(J^P) = 0(1^+)$ channel:

$$E_{\text{binding}} = (-103 \pm 8)\,\text{MeV}$$

➡ For more details visit the poster by Marc Wagner!

Summary

- Study bound states in doubly heavy tetraquarks
- Consider local and additionally scattering interpolating operators at the *sink*
- Predict a bound state in the $\bar{b}\bar{b}us$, $I(J^P) = \frac{1}{2}(1^+)$ channel with $E_{\text{binding}} = (-86 \pm 22 \pm 10)\,\text{MeV}$
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- Consider local and additionally scattering interpolating operators at the *sink* **and** at the *source*
- Study finite volume effects for $\bar{b}\bar{b}ud$ more rigorously
- Found bound state at $E_{\text{binding}} = (-103 \pm 8)\,\text{MeV}$

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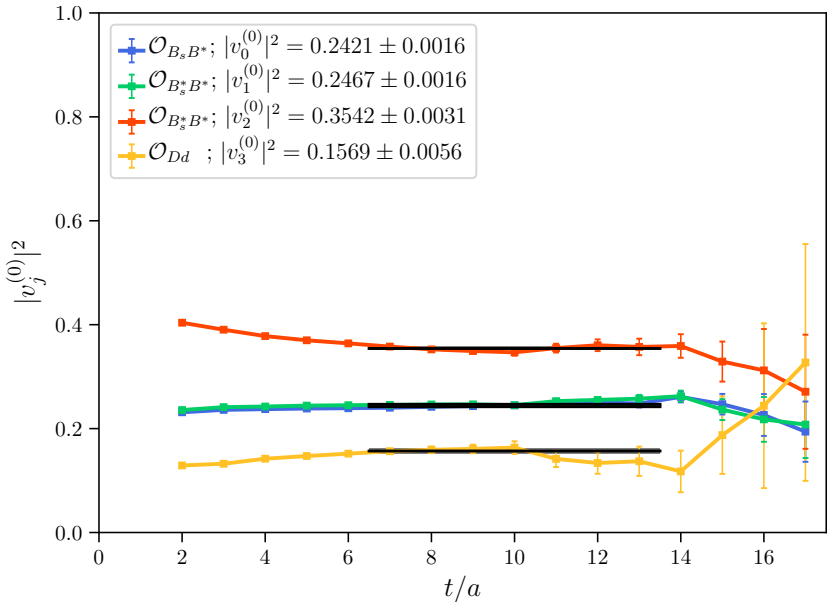
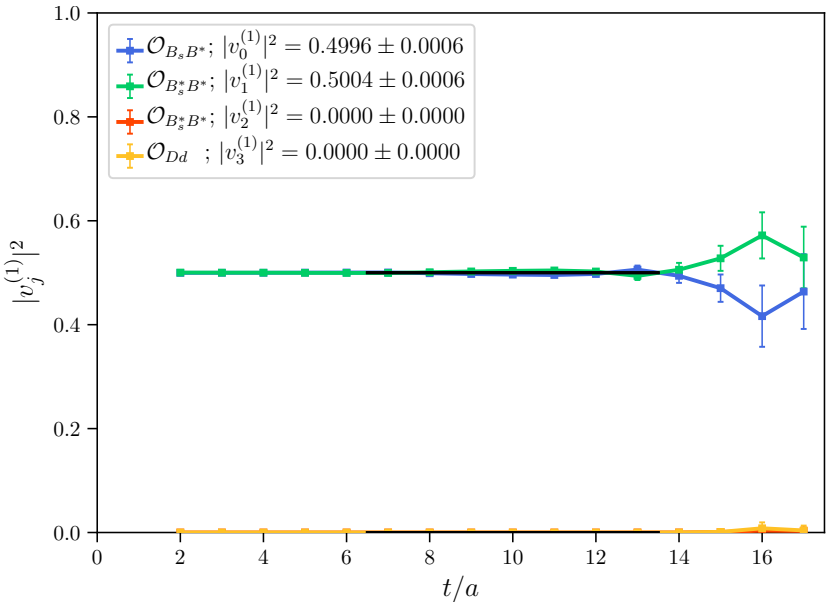
Outlook

- Study also finite volume effects for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$

Eigenvector Components for $\bar{b}\bar{b}us$



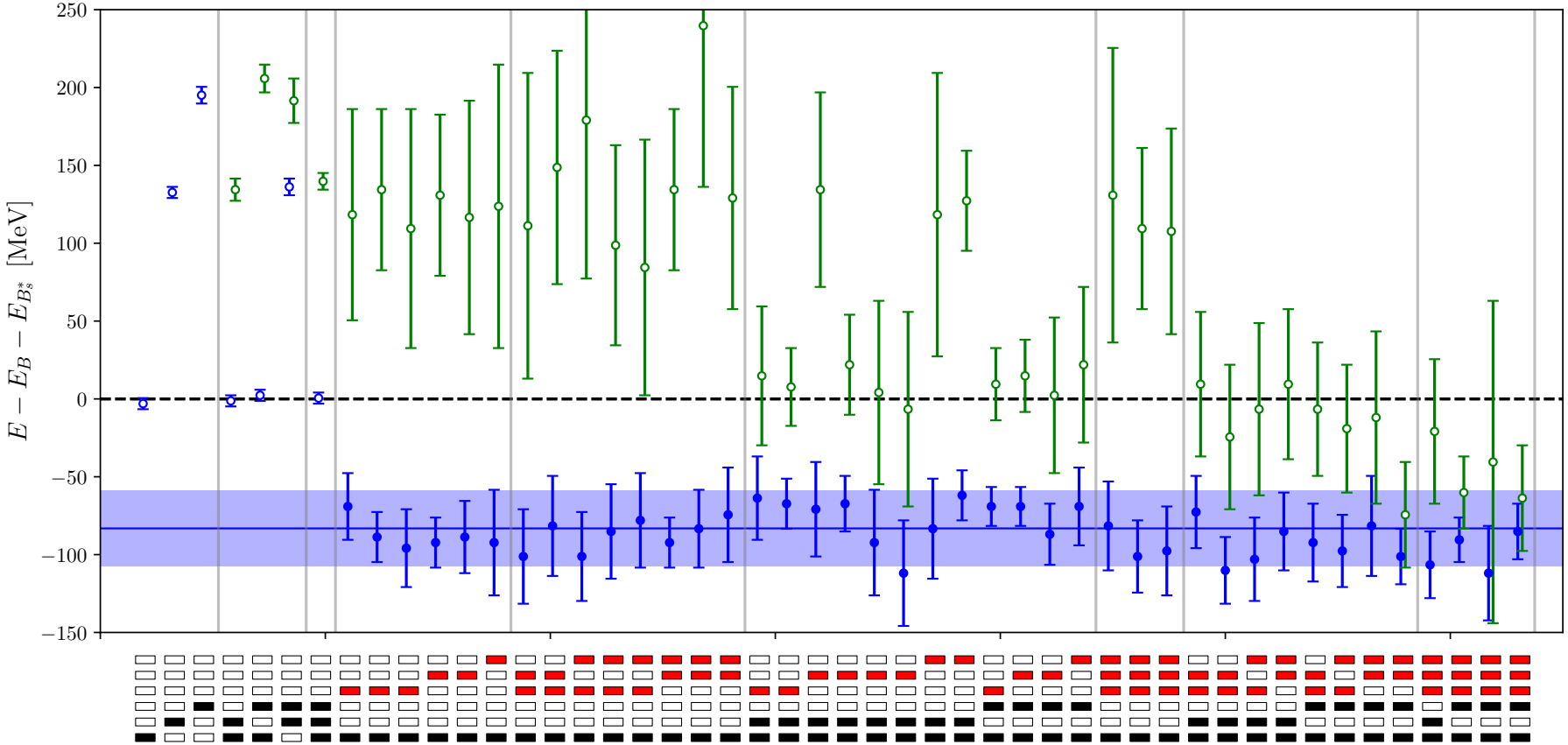


Normalized eigenvector components for the 4×4 matrix using only local operators. **left:** Ground state. **right:** First excited state.

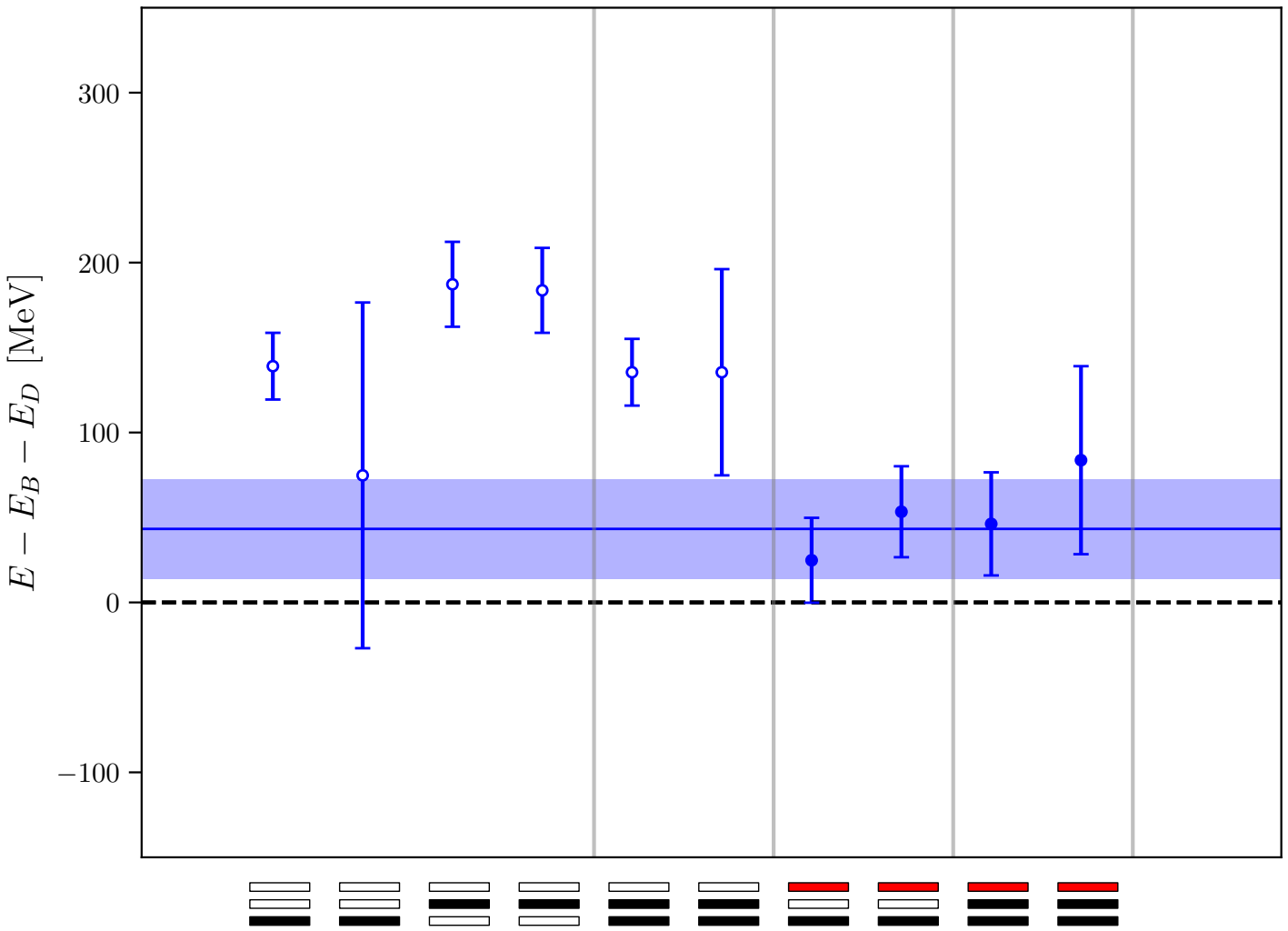
- Determine the eigenvectors for 4×4 correlation matrix with local operators $\mathcal{O}_k$
- New optimized operators $\mathcal{O}'_j$ are given by

$$\mathcal{O}'_j = \sum_{k=1}^{4} \bar{v}_k^{j-1} \mathcal{O}_k$$

Fit results for $\bar{b}\bar{b}us$ with $J^P = 1^+$



Fit results for $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$



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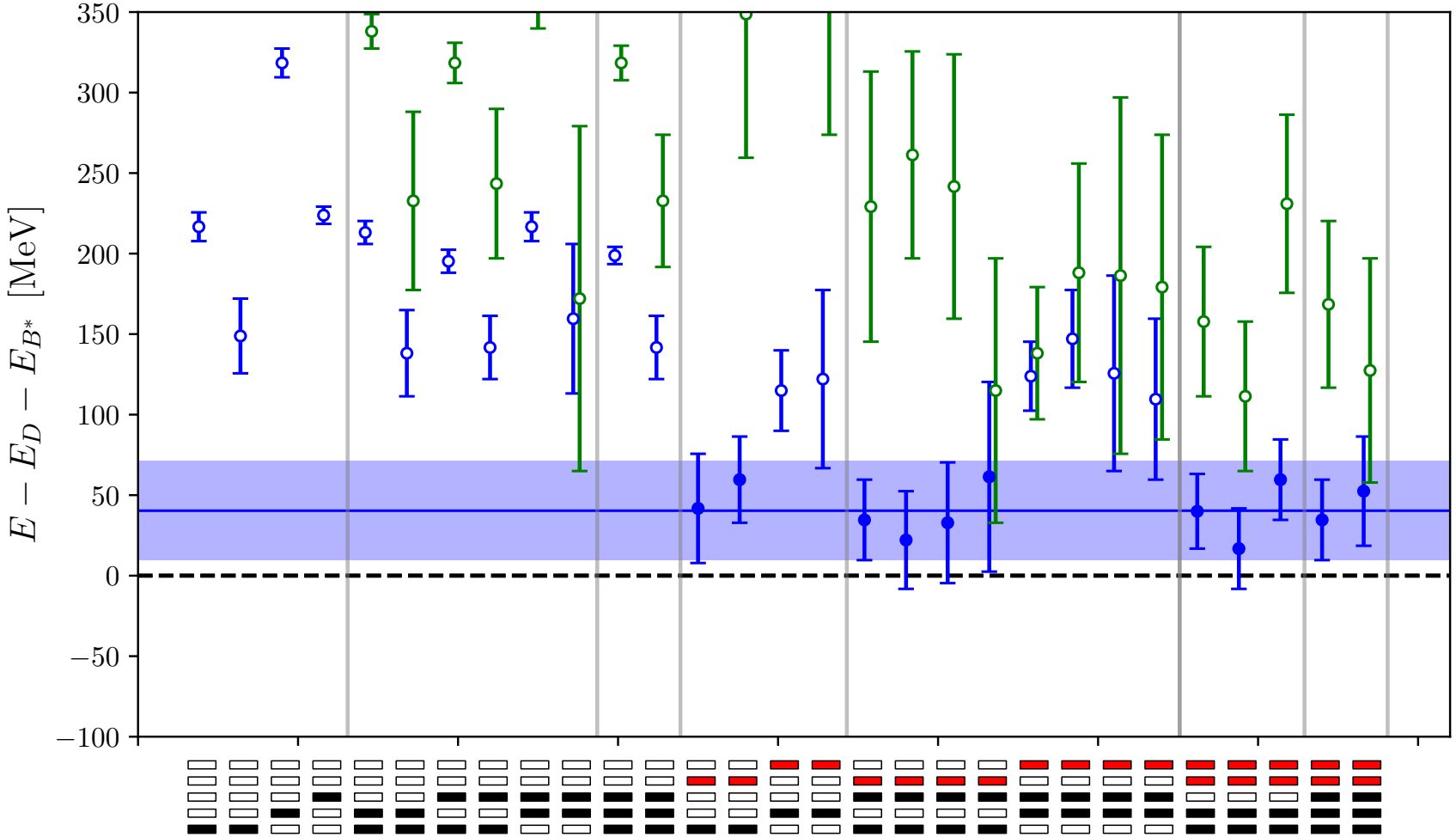
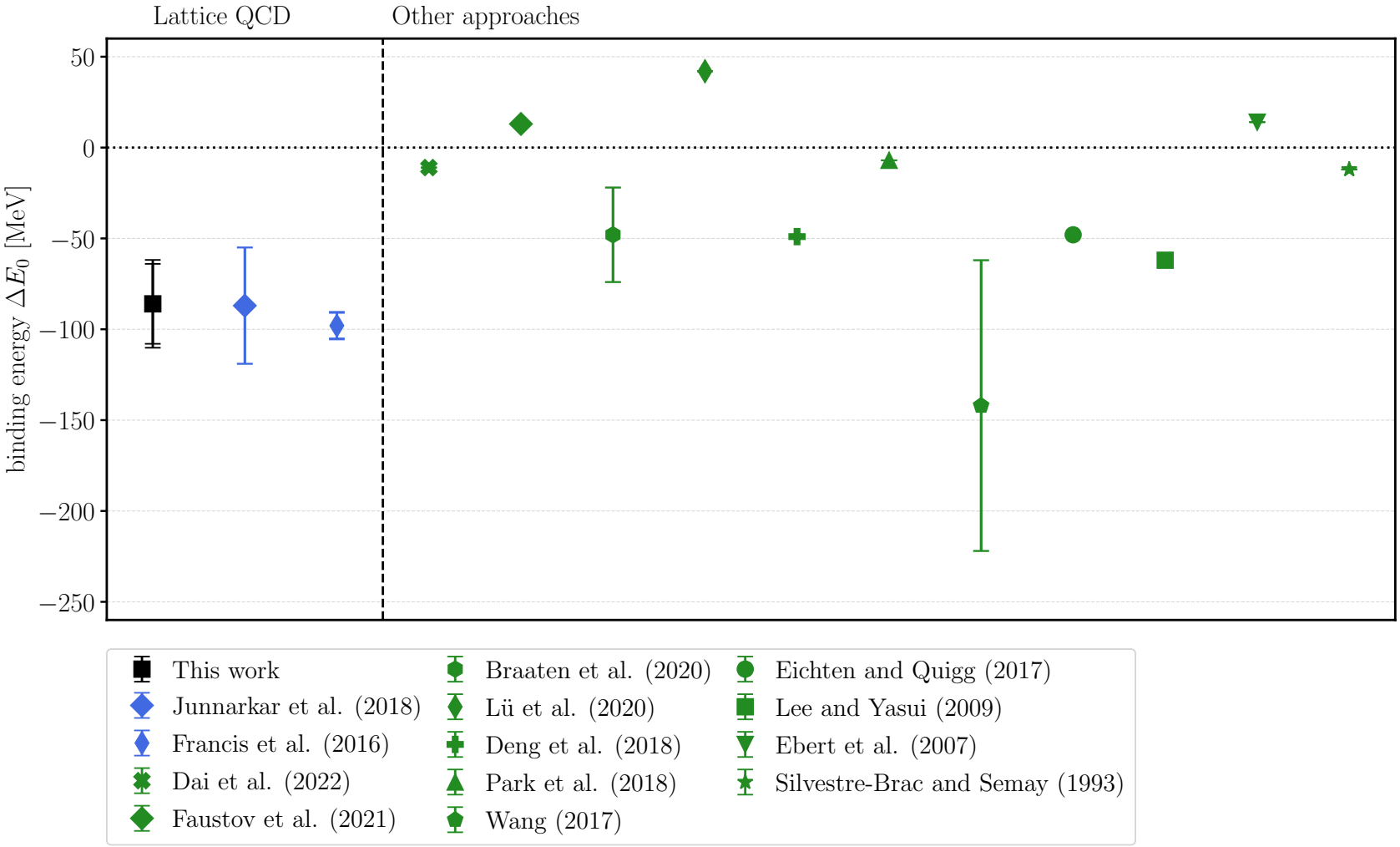


Comparison of $\bar{b}\bar{b}us$ results



Comparison of $\bar{b}\bar{b}us$ tetraquark binding energies with $J^P = 1^+$ (black: this work; blue: lattice NRQCD; green: effective field theories and potential models).

Scattering Analysis

- Relate *finite volume* energy spectrum E_n to *infinite volume* scattering amplitude for 2 energy levels per ensemble in T_1^+ irrep
- Use Lüscher's formula and scattering momenta k_n^2 to determine phase shift (or $\cot\delta_0(k_n)$)
- Apply effective-range-expansion (ERE)

$$k\cot\delta_0(k) = \frac{1}{a_0} + \frac{1}{2}r_0k^2 + \mathcal{O}(k^4).$$

- and determine a_0 and r_0 by fitting the result for the k_n's
- Search bound state pole of scattering amplitude below threshold at

$$\cot\delta_0(k_{\mathrm{BS}}) = i, \quad \text{so:} \quad -|k_{\mathrm{BS}}| = \frac{1}{a_0} - \frac{1}{2}r_0|k_{\mathrm{BS}}|^2$$

- Compute binding energy E_{bind} from binding momenta k_{BS}