Scale Setting for RBC-UKQCD 2+1 flavor Domain Wall Fermion Lattices

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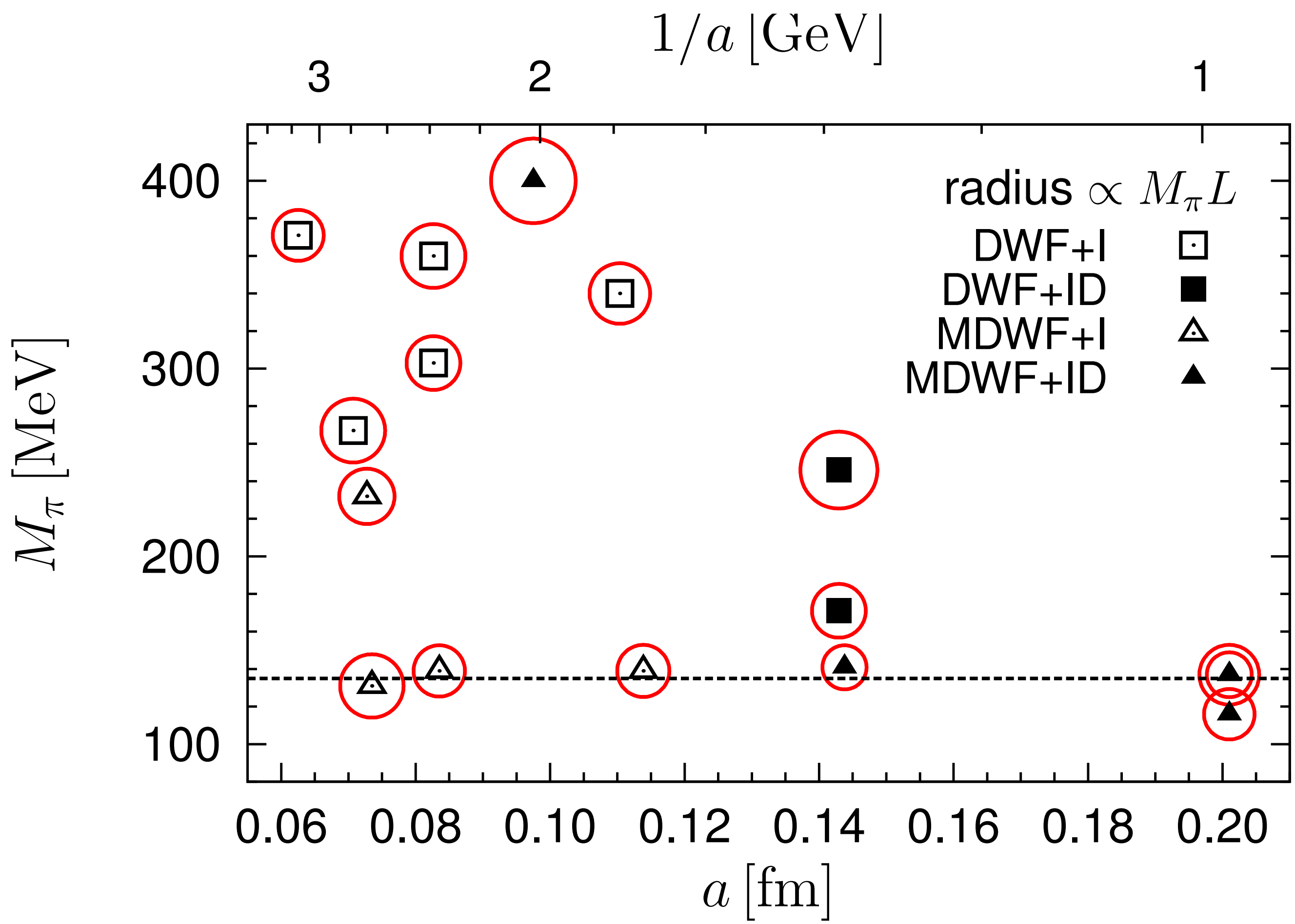
Most of the results reported here were produced by Yong-Chull Jang

RBC-UKQCD 2+1 flavor
Domain Wall Fermion Lattices



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RBC-UKQCD Ensembles

The gauge and fermion (G+F) action abbreviations used are:

- DWF = domain wall fermions
- MDWF = Mobius domain wall fermions,
- GMDWF = G-parity Mobius domain wall fermions,
- W = Wilson gauge action
- I = Iwasaki gauge action
- ID = Iwasaki plus Dislocation Suppressing Determinant Ratio (DSDR) gauge action.
- WE = Wilson plus Dislocation Enhancing Determinant (DED) gauge action.
- o following time extent = open boundary conditions in time

The total light quark mass (in lattice units) is $m_l + m_{\mathrm{res}}$ and the total strange quark mass is similarly $m_s + m_{\mathrm{res}}$.

2+1 Flavor RBC-UKQCD Ensembles

Early ensembles with heavy pions								
Ens.	Action (F+G)	$1/a$ (GeV)	Lattice volume	m_l (in lattice units)	m_s	$m_{\rm res}$	m_π (MeV)	Size (fm)
1	DWF+I	1.785(5)	$24^3\times64\times16$	0.005	0.04	0.00308	340	2.6
2	DWF+I	1.785(5)	$24^3\times64\times16$	0.01	0.04	0.00308	432	2.6
3	DWF+I	1.785(5)	$24^3\times64\times16$	0.02	0.04	0.00308	560	2.6
4	DWF+I	1.785(5)	$24^3\times64\times16$	0.03	0.04	0.00308	670	2.6
5	DWF+I	2.383(9)	$32^3\times64\times16$	0.004	0.03	0.000664	303	2.6
6	DWF+I	2.383(9)	$32^3\times64\times16$	0.006	0.03	0.000664	360	2.6
7	DWF+I	2.383(9)	$32^3\times64\times16$	0.008	0.03	0.000664	412	2.6
8	DWF+ID	1.378(7)	$32^3\times64\times32$	0.0042	0.045	0.00184	246	4.6
9	DWF+ID	1.378(7)	$32^3\times64\times32$	0.001	0.045	0.00184	171	4.6

Table 1: Early ensembles with heavy pions.

2+1 Flavor RBC-UKQCD Ensembles

Ensembles including those with physical pions								
Ens.	Action (F+G)	$1/a$ (GeV)	Lattice volume	m_l (in lattice units)	m_s	$m_{\rm res}$	m_π (MeV)	Size (fm)
10	MDWF+I	1.730(4)	$48^3\times96\times24$	0.00078	0.0362	0.000614	139	5.5
11	MDWF+I	2.359(7)	$64^3\times128\times12$	0.000678	0.02661	0.000314	139	5.4
12	DWF+I	3.15(2)	$32^3\times64\times12$	0.0047	0.0186	0.000631	371	2.0
13	MDWF+ID	0.98(4)	$32^3\times64\times24$	0.00022	0.05960	0.00217	117	3.8
14	MDWF+ID	2.02(1)	$32^3\times64\times24$	0.00478	0.03297	0.00447	401	6.2
15	GMDWF+ID	1.37(1)	$32^3\times64\times12$	0.0001	0.045	0.00184	141	4.6
16	MDWF+ID	0.98(4)	$32^3\times64\times24$	0.00107	0.0850	0.00217	137	6.4
17	MDWF+ID	0.98(4)	$24^3\times64\times24$	0.00107	0.0850	0.00217	137	4.8
18	MDWF+ID	0.98(4)	$48^3\times64\times24$	0.00107	0.0850	0.00217	137	9.6
19	MDWF+ID	1.37(1)	$32^3\times64\times12$	0.0001	0.045	0.00189	141	4.6
20	DWF+I	2.785	$48^3\times96\times12$	0.002144	0.02144	0.000968	267	3.5
21	MDWF+I	2.708	$32^3\times64\times12$	0.00054	0.02132	0.000233	140	2.3
22	MDWF+I	2.708	$96^3\times192\times12$	0.00054	0.02132	0.000233	140	6.9
23	MDWF+I	2.708	$48^3\times96\times12$	0.002144	0.02144	0.000236	232	3.5
24	GMDWF+ID	1.723	$40^3\times64\times12$	0.0003	0.0342	0.00101	135	4.6
25	GMDWF+ID	2.068	$48^3\times64\times12$	0.00074	0.02775	0.000276	135	4.6

Table 2: Ensembles including those with physical pions.

2+1 Flavor RBC-UKQCD Ensembles

Iwasaki Physical Point Ensembles

Ensembles including those with physical pions

Ens.		Action (F+G)	$1/a$ (GeV)	Lattice volume	m_l (in lattice units)	m_s	$m_{\rm res}$	m_π (MeV)	Size (fm)
10	48I	MDWF+I	1.730(4)	$48^3\times96\times24$	0.00078	0.0362	0.000614	139	5.5
11	64I	MDWF+I	2.359(7)	$64^3\times128\times12$	0.000678	0.02661	0.000314	139	5.4
12		DWF+I	3.15(2)	$32^3\times64\times12$	0.0047	0.0186	0.000631	371	2.0
13		MDWF+ID	0.98(4)	$32^3\times64\times24$	0.00022	0.05960	0.00217	117	3.8
14		MDWF+ID	2.02(1)	$32^3\times64\times24$	0.00478	0.03297	0.00447	401	6.2
15		GMDWF+ID	1.37(1)	$32^3\times64\times12$	0.0001	0.045	0.00184	141	4.6
16		MDWF+ID	0.98(4)	$32^3\times64\times24$	0.00107	0.0850	0.00217	137	6.4
17		MDWF+ID	0.98(4)	$24^3\times64\times24$	0.00107	0.0850	0.00217	137	4.8
18		MDWF+ID	0.98(4)	$48^3\times64\times24$	0.00107	0.0850	0.00217	137	9.6
19		MDWF+ID	1.37(1)	$32^3\times64\times12$	0.0001	0.045	0.00189	141	4.6
20		DWF+I	2.785	$48^3\times96\times12$	0.002144	0.02144	0.000968	267	3.5
21		MDWF+I	2.708	$32^3\times64\times12$	0.00054	0.02132	0.000233	140	2.3
22	96I	MDWF+I	2.708	$96^3\times192\times12$	0.00054	0.02132	0.000233	140	6.9
23		MDWF+I	2.708	$48^3\times96\times12$	0.002144	0.02144	0.000236	232	3.5
24		GMDWF+ID	1.723	$40^3\times64\times12$	0.0003	0.0342	0.00101	135	4.6
25		GMDWF+ID	2.068	$48^3\times64\times12$	0.00074	0.02775	0.000276	135	4.6

Table 2: Ensembles including those with physical pions.

2+1 Flavor RBC-UKQCD Ensembles

Ensembles probing effects near physical pion ensembles								
Ens.	Action (F+G)	$1/a$ (GeV)	Lattice volume	m_l (in lattice units)	m_s	m_{res}	m_π (MeV)	Size (fm)
26	MDWF+I	1.73	$32^3 \times 64 \times 24$	0.0025	0.0362	0.00063	208	3.7
27	MDWF+I	1.73	$24^3 \times 48 \times 32$	0.0055	0.0368	0.00046	284	2.8
28	MDWF+I	1.73	$32^3 \times 64 \times 24$	0.0025	0.05	0.00065	210	3.7
29	MDWF+I	1.74	$24^3 \times 48 \times 24$	0.0049	0.0362	0.00062	279	2.8
30	MDWF+I	2.37	$32^3 \times 64 \times 12$	0.00372	0.0257	0.00030	281	2.7
31	MDWF+I	1.76	$24^3 \times 48 \times 8$	0.002356	0.03366	0.00415	303	2.7
32	MDWF+I	1.73	$32^3 \times 64 \times 24$	0.00078	0.0362	0.00061	139	3.7
33	MDWF+I	1.73	$64^3 \times 128 \times 24$	0.00078	0.0362	0.00061	139	7.4
34	MDWF+I	1.74	$32^3 \times 64 \times 24$	0.0049	0.0362	0.00062	279	3.7
35	MDWF+I	3.50	$48^3 \times 192o \times 12$	0.0026	0.0176	0.00014	280	2.7

Table 3: Ensembles probing effects near physical pion ensembles

New ensembles generated by Christoph Lehner. Not used in these fits.

Global Fits

- Global fits (PRD 83 (2011) 074508, PRD 87 (2013) 094514, PRD 93 (2016) 074505) are an expansion:
  - * About the continuum limit, $a^2 = 0$:
    - ◊ Different $O(a^2)$ coefficients for different actions for same observable
  - * About the chiral limit, $m_l = 0$, for light quarks:
    - ◊ Separate dependence on valence and dynamical light quarks
    - ◊ Use ChPT for m_π, f_π and light quark dependence of m_K and f_K
    - ◊ Linear dependence for m_Ω, w_0, $t_0^{1/2}$, M_{ss}
  - * About the physical m_s for dynamical and valence strange quarks
    - ◊ Use separate linear dependence for dynamical and valence
- Choose m_π, m_K and m_Ω to set the scale and to have no $O(a^2)$ dependence
  - * With functional form of quark mass dependence known from fit, determine quark masses which give physical values for m_π/m_K and m_K/ m_Ω
  - * Then lattice spacing is determined by any one of m_π, m_K, and m_Ω

Global Fits: More Details

- SU(2) NLO example for m_π and f_π:

$$(m_{ll}^{\mathbf{e}})^2 = \chi_l^{\mathbf{e}} + \chi_l^{\mathbf{e}} \cdot \left\{ \frac{16}{f^2}\Big((2L_8^{(2)} - L_5^{(2)}) + 2(2L_6^{(2)} - L_4^{(2)})\Big)\chi_l^{\mathbf{e}} + \frac{1}{16\pi^2 f^2}\chi_l^{\mathbf{e}} \log\frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\}$$

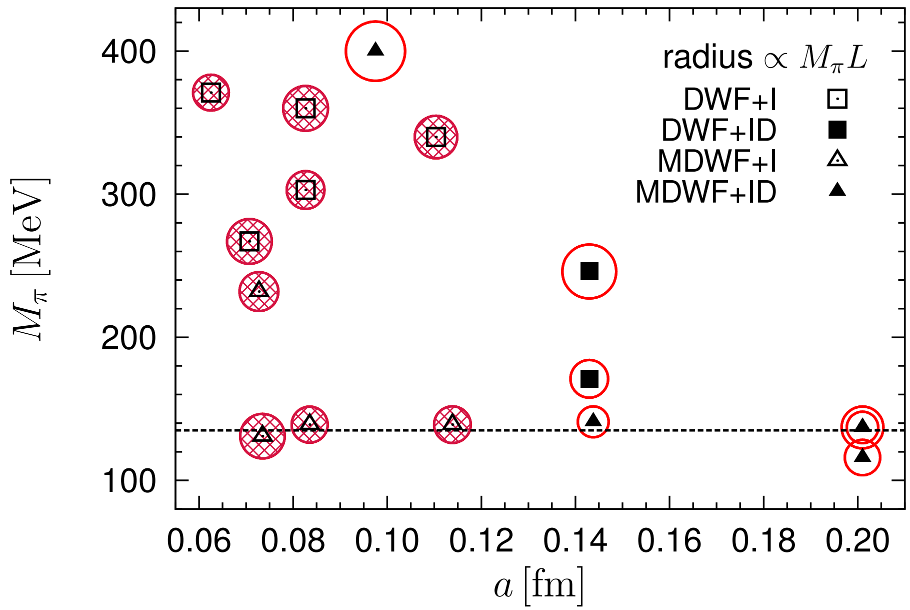
$$f_{ll}^{\mathbf{e}} = f\left[1 + c_f (a^{\mathbf{e}})^2\right] + f \cdot \left\{ \frac{8}{f^2}(2L_4^{(2)} + L_5^{(2)})\chi_l^{\mathbf{e}} - \frac{\chi_l^{\mathbf{e}}}{8\pi^2 f^2} \log\frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\}$$

with

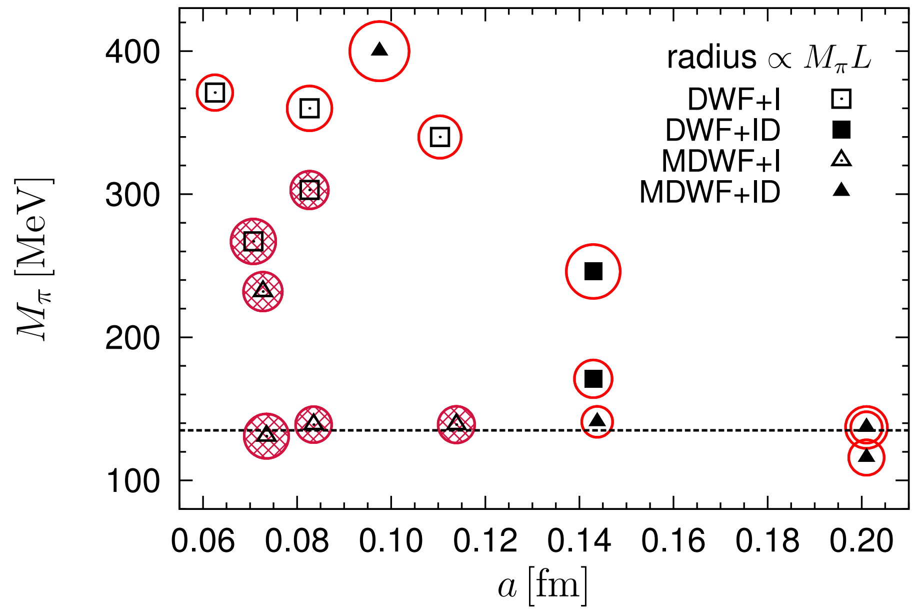
$$\chi_l^{\mathbf{e}} = \frac{Z_l^{\mathbf{e}}}{R_a^{\mathbf{e}}} \frac{B^{\mathbf{1}}\widetilde{m}_l^{\mathbf{e}}}{(a^{\mathbf{e}})^2}$$

- We include NLO ChPT finite volume effects in our formula.
- Input physical values
  - * m_π = 135.0 MeV
  - * m_K = 495.7 MeV
  - * m_Ω = 1672.45 MeV

Global Fit Cuts



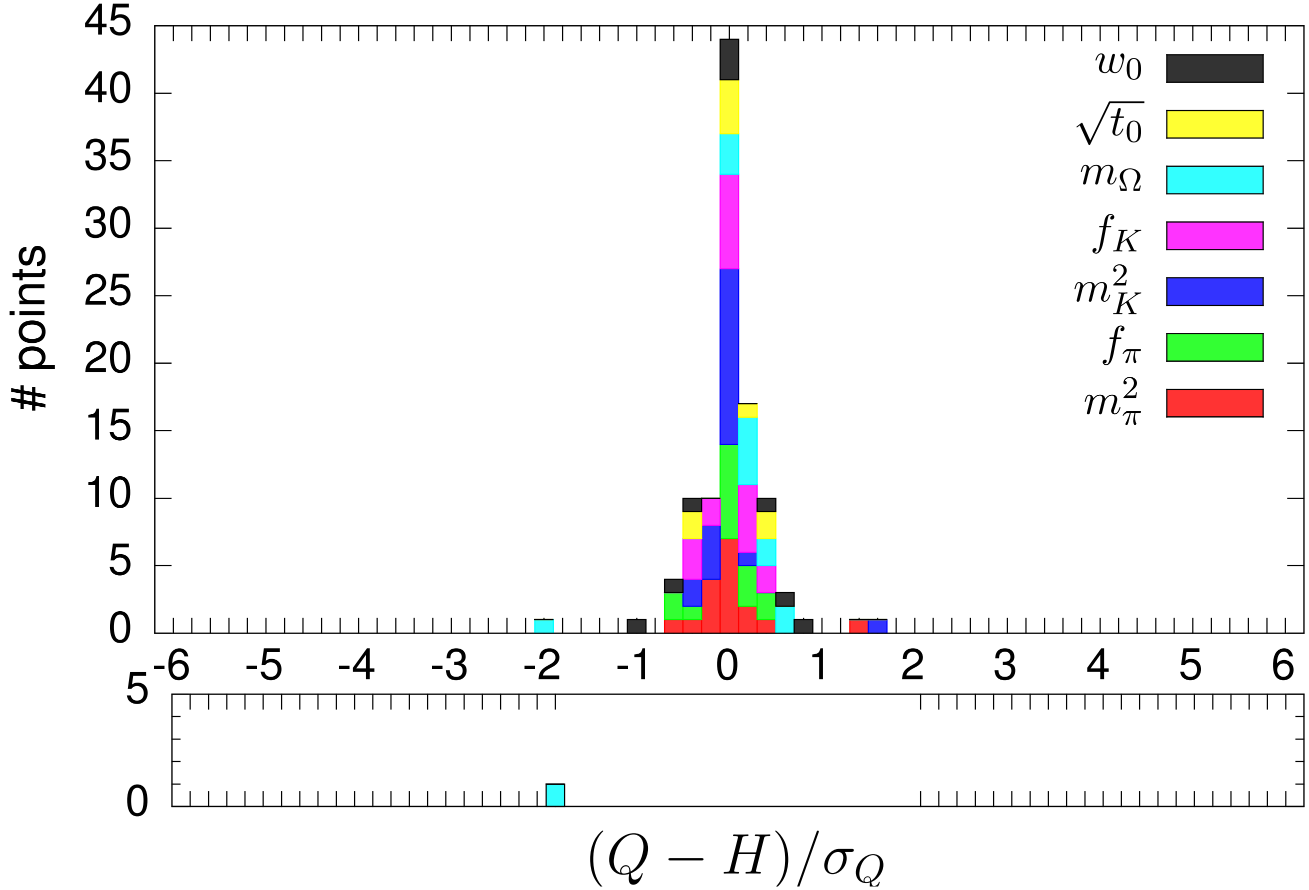
A: $a_{\mathrm{cut}} = 0.12\,\mathrm{fm},\ M_{\pi,\mathrm{cut}} = 370\,\mathrm{MeV}$

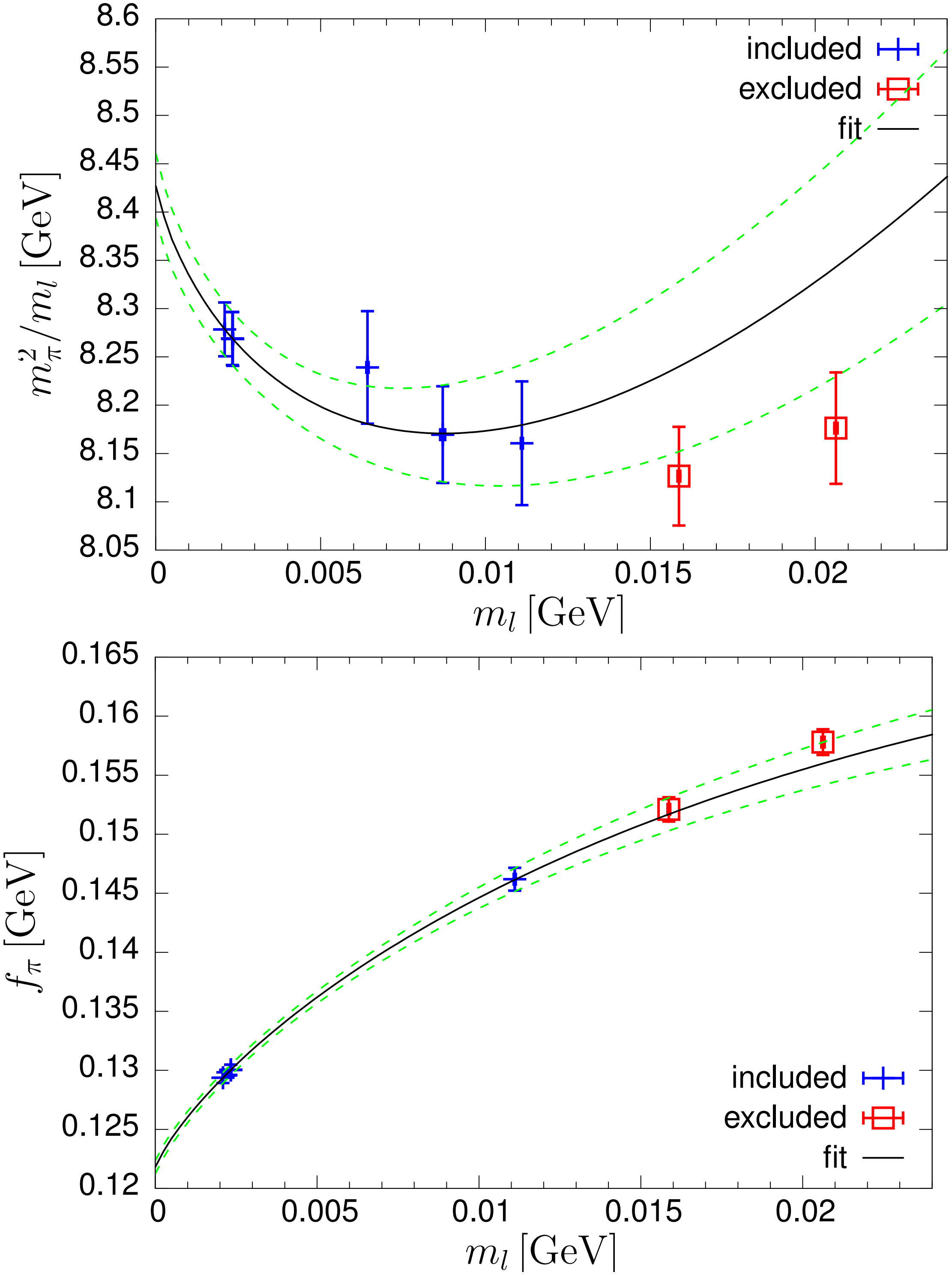


B: $a_{\mathrm{cut}} = 0.12\,\mathrm{fm},\ M_{\pi,\mathrm{cut}} = 310\,\mathrm{MeV}$

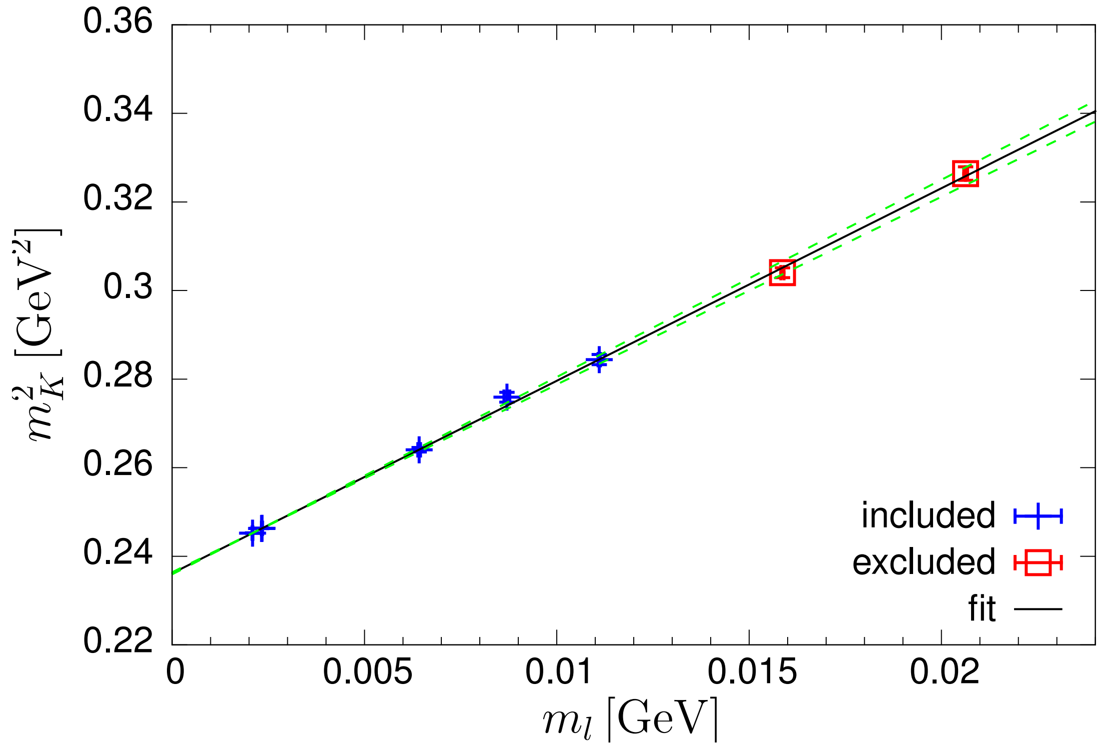
- Will consider two fits, with cuts as listed
- Shaded points represent ensembles included in the fits
- Global fits are uncorrelated fits

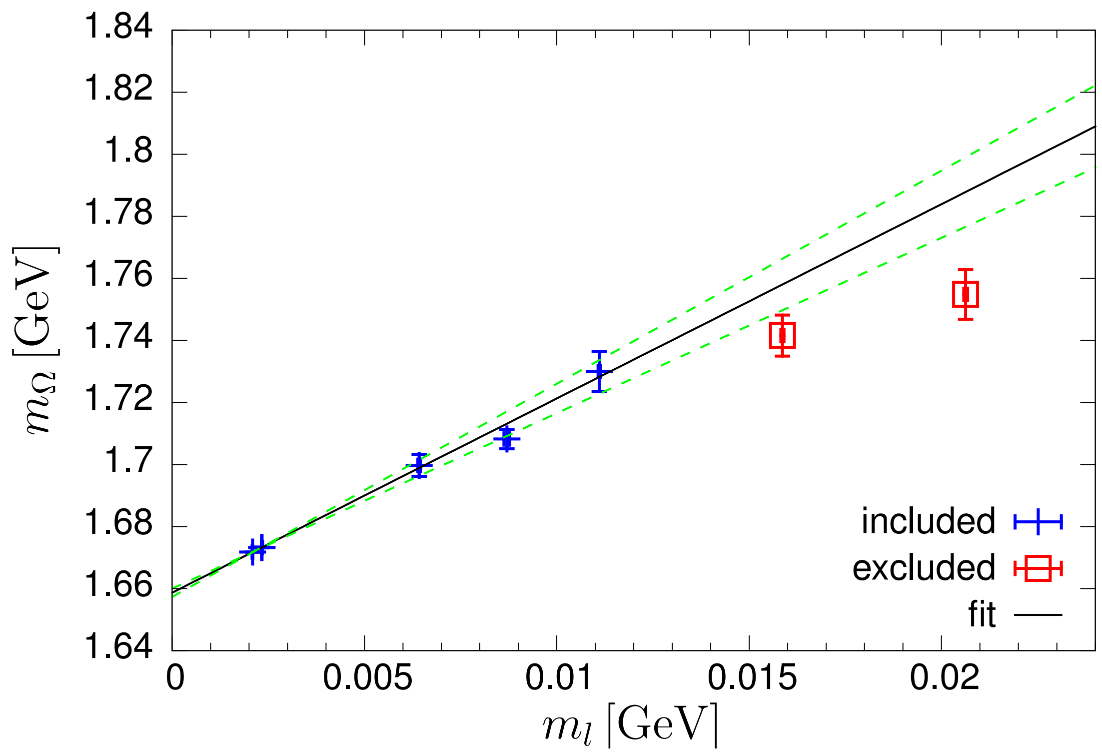
Plots for Fit B





included
excluded
fit
m_π^2/m_l [GeV]
m_l [GeV]
included
excluded
fit
f_π [GeV]
m_l [GeV]


0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
m_K^2 [GeV^2]
0
0.005
0.01
0.015
0.02
m_l [GeV]
included
excluded
fit


1.84
1.82
1.8
1.78
1.76
1.74
1.72
1.7
1.68
1.66
1.64
m_Ω [GeV]
0
0.005
0.01
0.015
0.02
m_l [GeV]
included
excluded
fit

Some Results

	with χ^2 weight on physical ensembles					
	A + 32ID		A		B	
f_π	0.12969(44)	0.12969(44)	0.12969(44)	0.12969(44)	0.12969(44)	0.12969(44)
f_K	0.15496(42)	0.15496(42)	0.15496(42)	0.15496(42)	0.15496(42)	0.15496(42)
$t_0^{1/2}$	0.7331(21)	0.7331(21)	0.7331(21)	0.7331(21)	0.7331(21)	0.7331(21)
w_0	0.8798(24)	0.8798(24)	0.8798(24)	0.8798(24)	0.8798(24)	0.8798(24)
M_{ss}^2	0.4772(07)		0.4772(07)		0.4772(08)	
f_K/f_π	1.1948(22)	1.1948(22)	1.1949(22)	1.1949(22)	1.1949(22)	1.1949(22)
a^{-1}|48I_M	1.7283(31)	1.7283(31)	1.7283(31)	1.7283(31)	1.7285(31)	1.7285(31)
a^{-1}|64I_M	2.3515(32)	2.3517(32)	2.3519(32)	2.3518(32)	2.3519(32)	2.3520(32)
a^{-1}|96I_M	2.6874(42)	2.6872(42)	2.6870(42)	2.6870(42)	2.6872(42)	2.6870(42)
	without χ^2 weight on physical ensembles					
	A + 32ID		A		B	
f_π	0.12929(59)	0.12924(60)	0.12929(60)	0.12920(60)	0.12982(44)	0.12978(44)
f_K	0.15451(63)	0.15446(63)	0.15466(62)	0.15458(62)	0.15504(42)	0.15500(42)
$t_0^{1/2}$	0.7372(30)	0.7375(30)	0.7363(28)	0.7368(28)	0.7324(22)	0.7326(21)
w_0	0.8854(34)	0.8858(34)	0.8841(31)	0.8847(31)	0.8793(24)	0.8795(24)
M_{ss}^2	0.4775(12)		0.4773(11)		0.4772(07)	
f_K/f_π	1.1951(29)	1.1952(29)	1.1962(29)	1.1964(28)	1.1942(22)	1.1943(22)
a^{-1}|48I_M	1.7308(41)	1.7311(40)	1.7305(39)	1.7310(38)	1.7284(32)	1.7285(31)
a^{-1}|64I_M	2.3483(38)	2.3476(38)	2.3492(35)	2.3484(35)	2.3530(33)	2.3527(32)
a^{-1}|96I_M	2.6794(54)	2.6787(54)	2.6808(49)	2.6800(49)	2.6886(43)	2.6881(42)

PRELIMINARY - blocking studies for autocorrelations to be done

Fitting with Different Physics Inputs

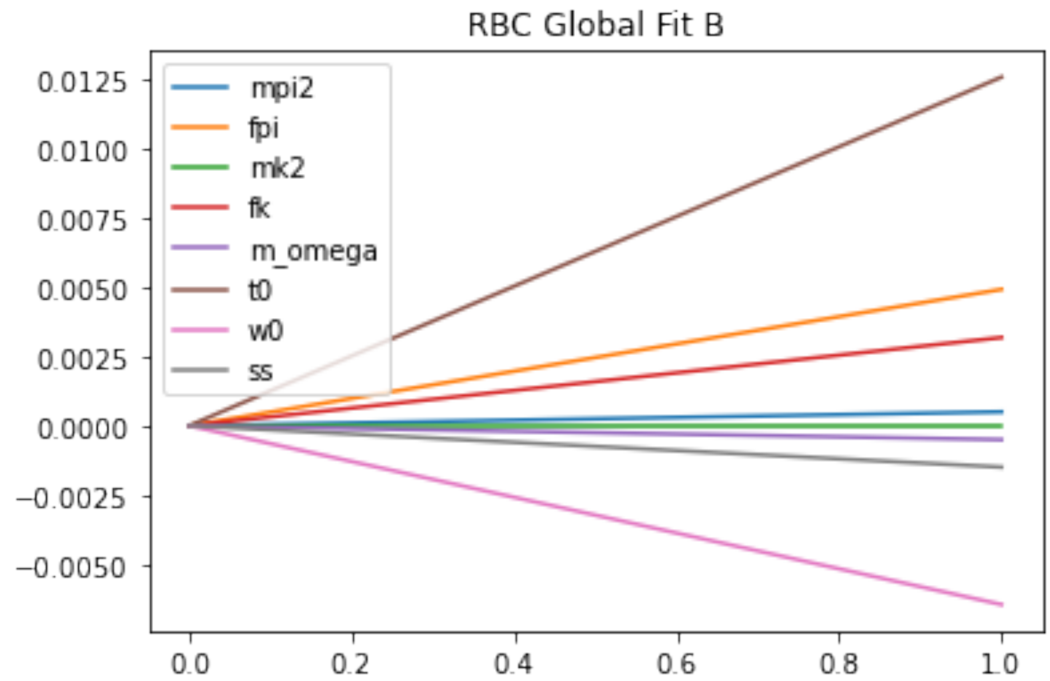
w_0, M_{ss}

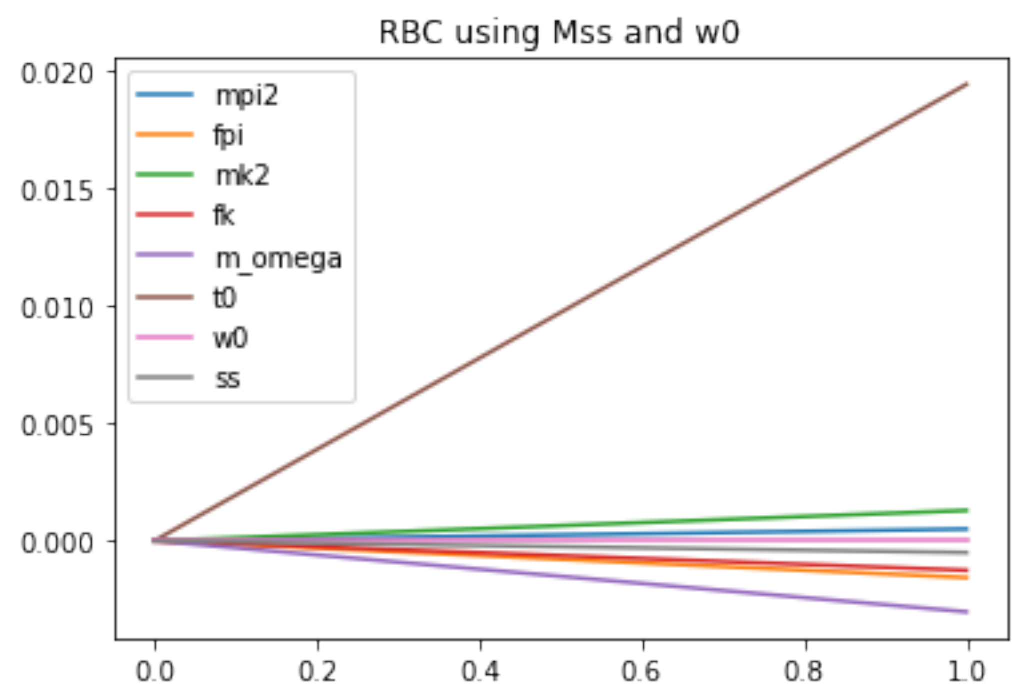
	BMW		RBC
w_0	0.87346	0.8798	
M_{ss}	0.68989	0.6908	
	with χ^2 weight on physical ensembles B		
f_π	0.13061(26)	0.12968(26)	0.12969(44)
f_K	0.15579(28)	0.15495(26)	0.15496(42)
$t_0^{1/2}$	0.72747(36)	0.73332(42)	0.7331(21)
M_K^2	0.24516(33)	0.24573(35)	0.24572
M_Ω	1.6790(27)	1.6723(26)	1.67245
f_K/f_π	1.1928(21)	1.1949(22)	1.1949(22)
a^{-1}|48I_M	1.7189(08)	1.7044(08)	1.7285(31)
a^{-1}|64I_M	2.3539(15)	2.3343(16)	2.3519(32)
a^{-1}|96I_M	2.6942(11)	2.6715(12)	2.6872(42)

- From RBC fit, produce w_0 and M_{ss}.
- Feed the central values for w_0 and M_{ss}, along with m_π into a second global fit and check the result.
- This second fit is done with no $O(a^2)$ errors for m_π, w_0 and M_{ss}.

Fitting with Different Physics Inputs

- Plot of a^2 dependence of various observables
- Top fit has no a^2 dependence in m_π, m_K, and m_Ω
- Lower fit has no a^2 dependence in m_π, w_0, and M_{ss}





Summary

- Essentially physical point MDWF+I ensembles for 3 lattice spacings
  - * Ensembles away from physical point allow for ~5% adjustments in quark masses to reach truly physical results.
  - * For HVP project, additional "nearby" ensembles have recently been generated (Lehner). These give consistent results with those shown here.
  - * May be included in the future into a common fit.
- Same results to a few parts in 10^4 for different pion mass cuts
  - * Indicates systematic effects from ChPT expansion are small
- Inclusion of coarse MDWF+ID ensembles shows need for a^4 terms in t_0.