

Towards the finite-volume spectrum of the Roper resonance

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Introduction and Motivation

■ The Roper Resonance $N^*(1440)$

- Discovered 1964: Partial wave analysis of $N\pi$ -scattering [Roper 1964]
- First excited nucleon state: $I(J^P) = 1/2(1/2^+)$
- Mass: $m_R = 1.365 \text{ GeV}$ (B.-W.: $m_R = 1.440 \text{ GeV}$) [PDG]
- Decays: $R \rightarrow N\pi$, $R \rightarrow N\pi\pi$, ...

■ Remarkable features of Roper:

- m_R is light, but too light?
 - disagrees with quark models! (2nd radial excitation of N)
- $\Gamma(R \rightarrow N\pi) \simeq \Gamma(R \rightarrow N\pi\pi)$
 - 2 and 3 particle final states equally likely!

Introduction and Motivation

■ How to tackle the problem?

- Lattice QCD
[Lang *et al.* (2017), Liu *et al.* (2017), ...]
- Effective field theory
[Borasoy *et al.* (2006), Djukanovic *et al.* (2010), van Kolck *et al.* (2011), Gegelia *et al.* (2016), ...]
- Why not both? → **Finite-volume (FV) formalism**
[Lüscher (1985), Wiese (1989), ...]

■ In this talk: FV formalism for the Roper resonance

DS and Meißner, *Commun. Theor. Phys.* **72** (2020) 075201

DS, Mai and Meißner (in progress)

Finite volume formalism

- Finite volume (FV) formalism for the Roper resonance
 - Idea (continuous k_0): Place system in a cubic box of length L
 - discretized spatial momenta \vec{k}
 - replace Euclidean loop integral with sum:

$$\int \frac{d^4 k_E}{(2\pi)^4} (\dots) \mapsto \int_{-\infty}^{\infty} \frac{dk_4}{2\pi} \frac{1}{L^3} \sum_{\vec{k}} (\dots)$$



Advertisement:

Bethe Forum "Multihadron
Dynamics in a Box"
→ next week!

Credit: Mai

Summary of DS and Mei^ßner (2020)

- Effective Lagrangian for the Roper resonance in ChPT (flavor $SU(2)$ and isospin-limit)
- Degrees of freedom: Roper R , Nucleon N , Delta-resonance Δ and Pions π

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{\pi R} + \mathcal{L}_{\pi\Delta} + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi NR} + \mathcal{L}_{\pi\Delta R}$$

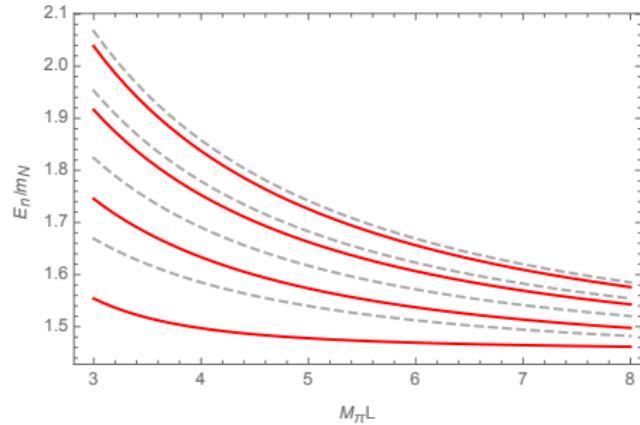
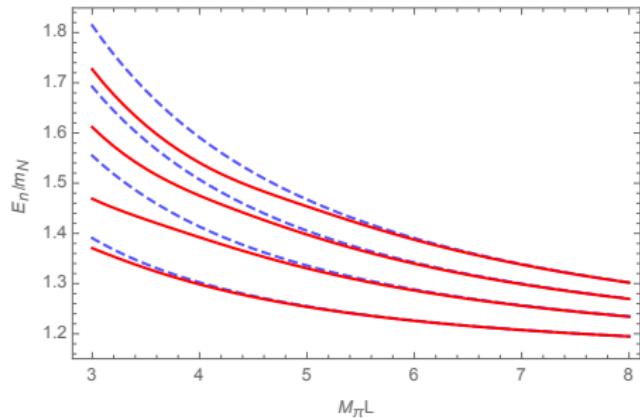
- Calculate the self-energy of the Roper:



- Calculate the energy levels of the Roper in FV

DS and Mei^ßner, *Commun. Theor. Phys.* **72** (2020) 075201

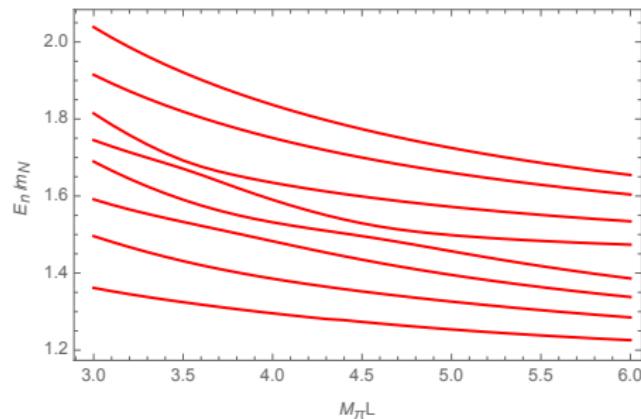
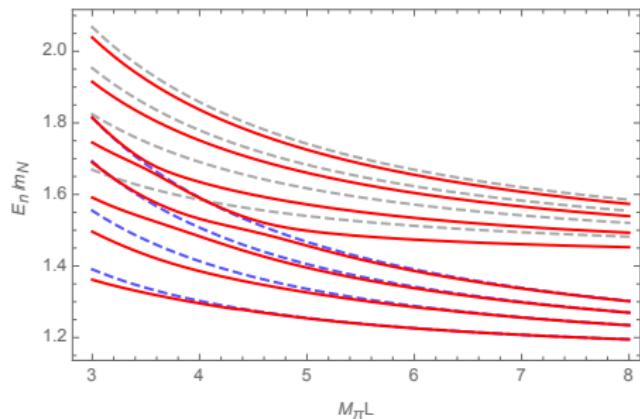
Finite volume energy levels from ChPT



■ Energy levels for πN and $\pi\Delta$ system:

- red lines: (interacting) energy levels,
- blue-dashed: non-interacting (free) πN levels,
- grey-dashed: non-interacting (free) $\pi\Delta$ levels
- avoided level crossing in πN case around $m_R/m_N \approx 1.45$

Finite volume energy levels from ChPT



- Energy levels for the coupled-channel ($\pi N/\pi \Delta$):
 - avoided level crossing seen around $m_R/m_N \approx 1.45$ and where free energy levels overlap
 - Problem: Δ -resonance is **not** a final state, it decays!
→ Possible Solution: Particle-Dimer-Formalism

[Bedaque, Hammer, van Kolck (1999)]

The Dimer formalism

- Consider the Lagrangian [Pang *et al.* (2017)]

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 - \frac{1}{2m} \vec{\nabla}^2 \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 - \frac{D_0}{6} (\psi^\dagger \psi)^3$$

- ψ : non-relativistic field with mass m
- C_0, D_0 : 2- and 3-body contact interactions

- Now consider

$$\begin{aligned}\mathcal{L}' = & \psi^\dagger \left(i\partial_0 - \frac{1}{2m} \vec{\nabla}^2 \right) \psi + \Delta T^\dagger T - \frac{g}{\sqrt{2}} (T^\dagger \psi \psi + \text{h.c.}) \\ & + h T^\dagger T \psi^\dagger \psi\end{aligned}$$

- T : non-dynamical auxiliary field → "the dimer field"

The Dimer formalism

- Integrate out T :

$$\begin{aligned}\mathcal{L}' &= \psi^\dagger \left(i\partial_0 - \frac{1}{2m} \vec{\nabla}^2 \right) \psi - \frac{g^2(\psi^\dagger \psi)^2}{2(\Delta + h\psi^\dagger \psi)} \\ &= \psi^\dagger \left(i\partial_0 - \frac{1}{2m} \vec{\nabla}^2 \right) \psi - \frac{g^2}{2\Delta} (\psi^\dagger \psi)^2 + \frac{g^2 h}{2\Delta^2} (\psi^\dagger \psi)^3 + \dots\end{aligned}$$

- $\mathcal{L}' = \mathcal{L}$, if $C_0 = \frac{g^2}{\Delta}$ and $D_0 = -\frac{3g^2 h}{\Delta^2}$
- We reformulated a 3-body problem as a 2-body problem!

Covariant particle-dimer approach for the Roper

- Roper resonance appears in system with pions and nucleons → $N\pi\pi$ -system
- Consider Lagrangian

$$\begin{aligned}\mathcal{L}_{\pi N} = & \mathcal{L}_{dyn.} + c_1(\phi^\dagger\phi)^2 + c_2\psi^\dagger\phi^\dagger\phi\psi \\ & + c_3\psi^\dagger\phi^\dagger(\phi + \phi^\dagger)\phi\psi + c_4\psi^\dagger\phi^\dagger\phi^\dagger\phi\phi\psi + \dots\end{aligned}$$

- ϕ : non-relativistic pion field, ψ : non-relativistic nucleon field
- $c_{1,2,3,4}$: LECs

- Where is the "covariant" part? → $\mathcal{L}_{dyn.}$

Covariant particle-dimer approach for the Roper

- $\mathcal{L}_{dyn.}$: [Colangelo *et al.* (2006), Bernard, UGM *et al.* (2008)]

$$\mathcal{L}_{dyn.} = \phi^\dagger 2W_\pi (i\partial_t - W_\pi) \phi + \psi^\dagger 2W_N (i\partial_t - W_N) \psi ,$$

$$W_\pi = \left[M_\pi^2 - \vec{\nabla}^2 \right]^{1/2}, \quad W_N = \left[m_N^2 - \vec{\nabla}^2 \right]^{1/2}$$

- Note: pion and nucleon field have same dimension!
- Propagators for pion and nucleon:

$$S_N(p_0, \vec{p}) = \frac{1}{2\omega_N(\vec{p}) [\omega_N(\vec{p}) - p_0 - i\epsilon]}, \quad \omega_N(\vec{p}) = \sqrt{|\vec{p}|^2 + m_N^2}$$

$$S_\pi(p_0, \vec{p}) = \frac{1}{2\omega_\pi(\vec{p}) [\omega_\pi(\vec{p}) - p_0 - i\epsilon]}, \quad \omega_\pi(\vec{p}) = \sqrt{|\vec{p}|^2 + M_\pi^2}$$

Constructing the particle-dimer EFT

- Now: Build the dimer Lagrangian
- What are the dimer fields?
 - Roper R : quantum numbers $J^P = 1/2^+$, mass m_R
 - Delta Δ : quantum numbers $J^P = 3/2^+$, Δ -resonance
 - Dimer for $N\pi$ -interactions
 - Sigma σ : quantum numbers $J^P = 0^+$, σ -resonance ($f_0(500)$)
 - Dimer for $\pi\pi$ -interactions

■ Full Lagrangian:

$$\mathcal{L}_{Dimer} = \mathcal{L}_{dyn.} + \mathcal{L}_T$$

- $\mathcal{L}_{dyn.}$ as before
- \mathcal{L}_T contains the dimer fields and their interactions

Constructing the particle-dimer EFT

- The full dimer Lagrangian:

$$\begin{aligned}\mathcal{L}_T = & R^\dagger 2W_R (i\partial_t - W_R) R + f_1 R^\dagger \phi^\dagger \phi R - f_2 [R^\dagger \phi \psi + R \phi^\dagger \psi^\dagger] \\ & - f_3 [R^\dagger \phi \Delta + \Delta^\dagger \phi^\dagger R] - f_4 [R^\dagger \sigma \psi + \psi^\dagger \sigma^\dagger R] \\ & + \alpha_\Delta m_\Delta^2 \Delta^\dagger \Delta + g_1 \Delta^\dagger \phi^\dagger \phi \Delta - g_2 [\Delta^\dagger \phi \psi + \Delta \phi^\dagger \psi^\dagger] \\ & + \alpha_\sigma M_\sigma^2 \sigma^\dagger \sigma + h_1 \psi^\dagger \sigma^\dagger \sigma \psi - h_2 [\sigma^\dagger \phi \phi + \sigma \phi^\dagger \phi^\dagger] \\ & - G_{R\sigma} [R^\dagger \phi^\dagger \sigma \psi + \psi^\dagger \sigma^\dagger \phi R] - G_{R\Delta} [R^\dagger \phi^\dagger \phi \Delta + \Delta^\dagger \phi^\dagger \phi R] \\ & - G_{\Delta\sigma} [\Delta^\dagger \phi^\dagger \sigma \psi + \psi^\dagger \sigma^\dagger \phi \Delta]\end{aligned}$$

Constructing the particle-dimer EFT

■ Things to note from \mathcal{L}_T :

- Proper dimer field R is dynamical: $W_R = [m_R^2 - \vec{\nabla}^2]^{1/2}$,
 Δ and σ dimer fields are not
- Dimer fields can interact among themselves
- Integrating out Δ and σ , we obtain $R \rightarrow N\pi$ and $R \rightarrow N\pi\pi$ interactions

■ Bare dimer propagators:

$$S_R(p_0, \vec{p}) = \frac{1}{2\omega_R(\vec{p}) [\omega_R(\vec{p}) - p_0 - i\epsilon]} , \quad \omega_R(\vec{p}) = \sqrt{|\vec{p}|^2 + m_R^2}$$

$$D_\Delta^0(p_0, \vec{p}) = -\frac{1}{\alpha_\Delta m_\Delta^2} , \quad D_\sigma^0(p_0, \vec{p}) = -\frac{1}{\alpha_\sigma M_\sigma^2}$$

Self-energy of the Roper resonance

- Dressed Roper-dimer propagator:

$$S_R^d(p_0, \vec{p}) = \frac{1}{2\omega_R(\vec{p}) [\omega_R(\vec{p}) - p_0 - i\epsilon] - \Sigma_R(p_0, \vec{p})}$$

- Poles:

$$2\omega_R(\vec{p}) [\omega_R(\vec{p}) - p_0] - \Sigma_R(p_0, \vec{p}) \stackrel{!}{=} 0$$

- Self-energy contributions:



Finite volume energy levels

- Poles of the Roper dimer propagator in the finite volume:

$$2\omega_R(\vec{p}) [\omega_R(\vec{p}) - p_0] - \Sigma_R^L(p_0, \vec{p}) = 0$$

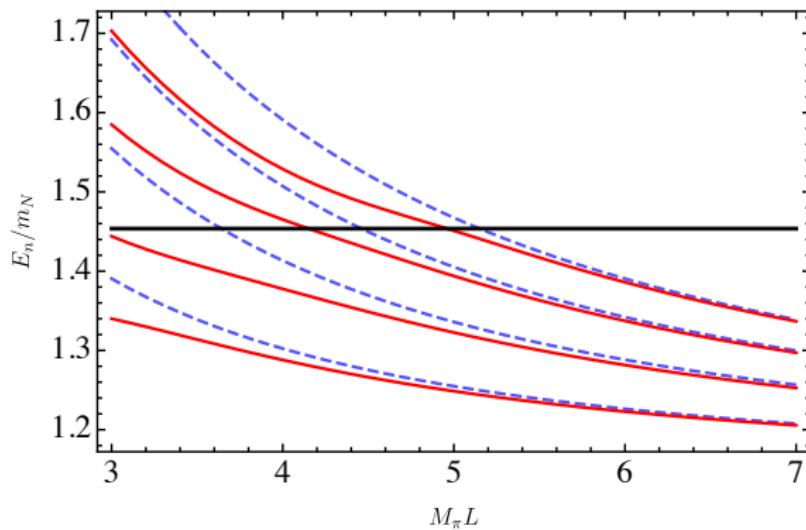
- Go to rest frame: $p_0 = E$, $\vec{p} = 0$

$$\begin{aligned} 0 &= 2m_R [m_R - E] - \Sigma_R^L(E) \\ \Rightarrow m_R - E &= \frac{1}{2m_R} \Sigma_R^L(E) \end{aligned}$$

- Solve numerically!

Finite volume energy levels

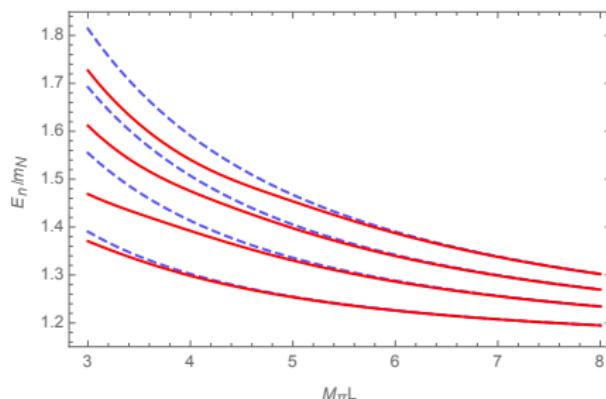
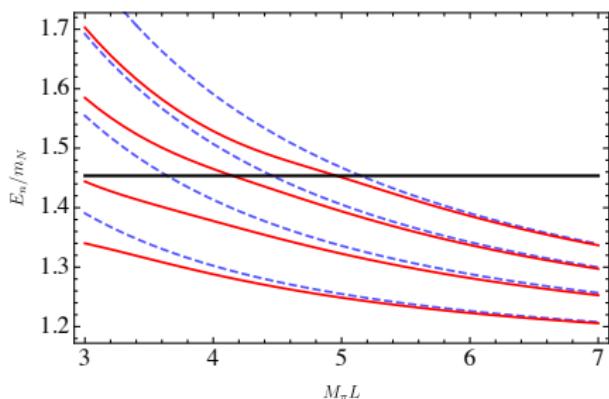
- First consider: $\Sigma_R^L(E) = \Sigma_{N\pi}^L(E)$
- Energy level spectrum: (preliminary)



- red lines: (interacting) energy levels, blue-dashed: free πN levels, black line: $m_R/m_N \approx 1.45$

Finite volume energy levels

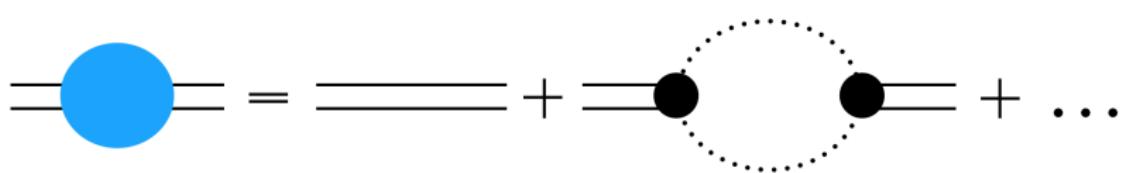
- Compare our result from the non-relativistic EFT with the ChPT result: [DS and Mei  ner (2020)]



- Very good agreement between the spectra!
- Much simpler EFT leads to this almost identical result!
- Now: Dimer fields...

Self-energy of the dimer fields

- The σ -dimer propagator:



$$D_\sigma(p) = -\frac{1}{\alpha_\sigma M_\sigma^2 + \Sigma_\sigma(p)}, \text{ with}$$

$$\Sigma_\sigma(p) = \frac{1}{2}(2h_2)^2 \int \frac{d^4 k}{(2\pi)^4 i} S_\pi(p - k) S_\pi(k)$$

- simple case due to two equal masses
- concentrate for now only on σ -dimer

Self-energy of the Roper: The $N\sigma$ -case

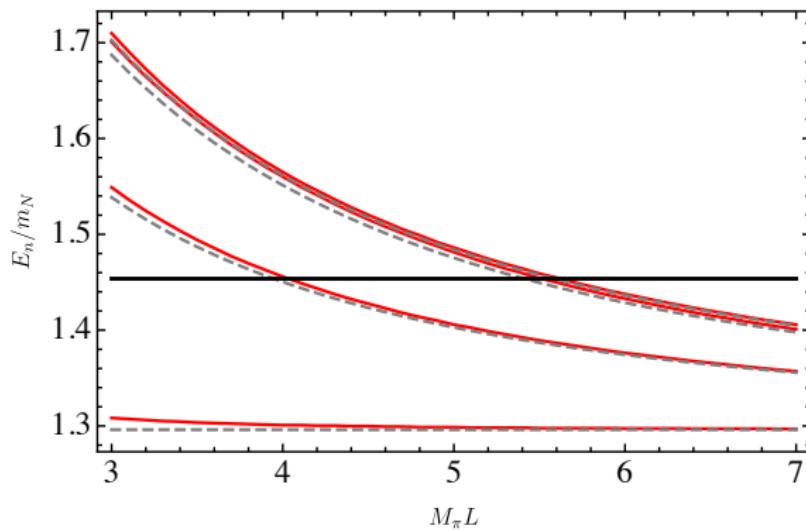
- Self-energy contribution in the finite volume:

$$\begin{aligned}\Sigma_{N\sigma}^L(E) = & -f_4^2 \frac{1}{L^3} \sum_{\vec{k}} \frac{1}{2\omega_N(\vec{k})} \left\{ \alpha_\sigma M_\sigma^2 \right. \\ & + 2h_2^2 \frac{1}{L^3} \sum_{\vec{l}} \frac{1}{4\omega_\pi(\vec{k}-\vec{l})\omega_\pi(\vec{l})} \\ & \cdot \left. \frac{1}{[\omega_\pi(\vec{k}-\vec{l}) + \omega_\pi(\vec{l}) + \omega_N(\vec{k}) - E]} \right\}^{-1}\end{aligned}$$

- implement the double sum and find energy levels!
- corresponding equation in infinite volume can be expressed as a parameter integral (preliminary result)

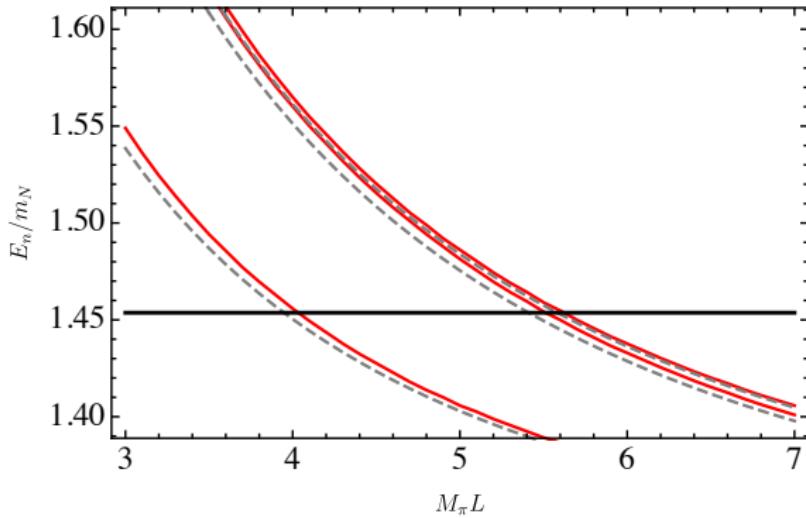
Finite volume energy levels

- Now consider: $\Sigma_R^L(E) = \Sigma_{N\sigma}^L(E)$
- Energy level spectrum: (preliminary)



- red lines: (interacting) energy levels, gray-dashed: free $\pi\pi N$ levels, black line: $m_R/m_N \approx 1.45$

Finite volume energy levels



- lowest lying non-interacting energy levels $N(0)\pi(0)\pi(0)$, $N(1)\pi(-1)\pi(0)$, $N(2)\pi(-2)\pi(0)$, and $N(0)\pi(1)\pi(-1)$
- avoided level crossing not visible (small scattering length $a_{\pi\pi}^{I=0}$) → more investigation needed!

Summary

- The particle-dimer formalism is a useful tool to investigate 3-body problems
- A covariant particle-dimer framework was introduced for the Roper resonance
- A self-energy approach was used to obtain the finite-volume energy spectrum
 - the non-relativistic $N\pi$ -system is in good agreement with the result from ChPT
 - three particle $N\pi\pi$ -states have been observed in the system including the σ -dimer field

Outlook

- Further investigation of the nucleon σ -dimer spectrum
- What about the Δ -dimer field?
- What can be said about the full system?
 - Limitations to the self-energy approach? (E.g. unitarity?)
- Can we apply the self-energy approach to different hadronic systems, e.g. $a_1(1260)$?

...exciting things are in front of us!

Thank you for your attention!



My card



DS and Meißner (2020)

Spares:

The Roper resonance in ChPT

■ Pions, Nucleons and the Roper resonance:

- Pions included in unitary matrix U
- Nucleon and Roper spin-1/2 fields

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4} \text{Tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) + \frac{F^2}{4} \text{Tr} \left(U \chi^\dagger + \chi U^\dagger \right)$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left(i \not{D} - m_{N0} + \frac{1}{2} g_A \not{\epsilon} \gamma_5 \right) \Psi_N$$

$$\mathcal{L}_{\pi R}^{(1)} = \bar{\Psi}_R \left(i \not{D} - m_{R0} + \frac{1}{2} g_R \not{\epsilon} \gamma_5 \right) \Psi_R$$

$$\mathcal{L}_{\pi R}^{(2)} = c_1^R \bar{\Psi}_R \text{Tr} (\chi_+) \Psi_R$$

The Roper resonance in ChPT

■ The Δ Resonance:

- Spin-3/2 Rarita-Schwinger fields

$$\begin{aligned}\mathcal{L}_{\pi\Delta}^{(1)} = & -\bar{\Psi}_\mu^i \xi_{ij}^{3/2} \left\{ \left(iD^{jk} - m_{\Delta 0} \delta^{jk} \right) g^{\mu\nu} \right. \\ & -i \left(\gamma^\mu D^{\nu,jk} + \gamma^\nu D^{\mu,jk} \right) + i\gamma^\mu D^{jk} \gamma^\nu + m_{\Delta 0} \delta^{jk} \gamma^\mu \gamma^\nu \\ & + \frac{g_1}{2} \psi^{jk} \gamma_5 g^{\mu\nu} + \frac{g_2}{2} \left(\gamma^\mu u^{\nu,jk} + u^{\mu,jk} \gamma^\nu \right) \gamma_5 \\ & \left. + \frac{g_3}{2} \gamma^\mu \psi^{jk} \gamma_5 \gamma^\nu \right\} \xi_{kl}^{3/2} \Psi_\nu^l\end{aligned}$$

The Roper resonance in ChPT

- Interactions among the fields:

$$\mathcal{L}_{\pi NR}^{(1)} = \frac{g_{\pi NR}}{2} \bar{\Psi}_R \not{\mu} \gamma_5 \Psi_N + \text{h.c.},$$

$$\mathcal{L}_{\pi N\Delta}^{(1)} = h \bar{\Psi}_\mu^i \xi_{ij}^{3/2} \Theta^{\mu\nu}(z_1) \omega_\nu^j \Psi_N + \text{h.c.},$$

$$\mathcal{L}_{\pi\Delta R}^{(1)} = h_R \bar{\Psi}_\mu^i \xi_{ij}^{3/2} \Theta^{\mu\nu}(z_2) \omega_\nu^j \Psi_R + \text{h.c.}.$$

- Low energy constants (LECs) $g_{\pi NR}$, h , h_R , ...
 - some can be fixed from experiment
 - "maximal mixing" [Beane, van Kolck (2005)]

The Roper resonance in ChPT

- Effective Lagrangian for the Roper in Chiral Perturbation Theory (ChPT):

- Several methods to describe resonances in ChPT
(e.g. unitarization method [Meißner, Oller (2000)])
- Resonance fields can be introduced explicitly
- Degrees of freedom: Pion (π), Nucleon (N),
Delta-Resonance (Δ), Roper (R)
- Use flavor $SU(2)$ and isospin-limit ($m_u = m_d$)

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{\pi R} + \mathcal{L}_{\pi\Delta} + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi NR} + \mathcal{L}_{\pi\Delta R}$$

Self-energy of the Roper resonance

- Interactions between R , N and Δ lead to self-energy corrections of the propagator:

$$iS_R(p) = \frac{i}{\not{p} - m_{R0} - \Sigma_R(\not{p})}$$

- Poles of the Roper:

$$[\not{p} - m_{R0} - \Sigma_R(\not{p})] \Big|_{\not{p}=Z} \stackrel{!}{=} 0 , \quad Z = m_R - i \frac{\Gamma_R}{2}$$

- $\Sigma_R(\not{p})$ Roper self-energy, m_R mass, Γ_R decay width

Self-energy of the Roper resonance



$$\Sigma_R(\not{p}) = \underbrace{\Sigma_R^{(2)}(\not{p})}_{\text{contact int.}} + \underbrace{\Sigma_{\pi R}^{(3)}(\not{p}) + \Sigma_{\pi N}^{(3)}(\not{p}) + \Sigma_{\pi \Delta}^{(3)}(\not{p})}_{\text{loops}} + \mathcal{O}(p^4)$$

- Evaluate loop integrals with standard methods
 - Problem: mass scales m_R , m_N , and m_Δ spoil power-counting!
 - Solution: modified EOMS scheme [Gegelia et al. (2016)]

Finite volume formalism

- Replace Euclidean loop integral with sum:

$$\int \frac{d^4 k_E}{(2\pi)^4} (\dots) \mapsto \int_{-\infty}^{\infty} \frac{dk_4}{2\pi} \frac{1}{L^3} \sum_{\vec{k}} (\dots)$$

- FV momenta are given by $\vec{k} = \frac{2\pi}{L} \vec{n}$, with $\vec{n} \in \mathbb{Z}^3$
- Poles of the Roper propagator

$$\not{p} - m_{R0} - \Sigma_R^L(\not{p}) = 0$$

- $\Sigma_R^L(\not{p})$ is the Roper self-energy in FV

Finite volume energy levels

- Using $\phi = E$ (on-shell condition) and ignoring all regular functions (exponentially suppressed) we get:

$$\begin{aligned}m_R - E &= \frac{3g_{\pi NR}^2}{128\pi^2 F_\pi^2 E} (E + m_N)^2 \\&\quad \cdot [(E - m_N)^2 - M_\pi^2] \tilde{B}_0^L(E^2, m_N^2, M_\pi^2) \\&+ \frac{h_R^2}{96\pi^2 F_\pi^2 m_\Delta^2 E} [(m_\Delta + E)^2 - M_\pi^2] \\&\quad \cdot \lambda(E^2, m_\Delta^2, M_\pi^2) \tilde{B}_0^L(E^2, m_\Delta^2, M_\pi^2)\end{aligned}$$

- Solve equation numerically for E
- Consider nucleon only ($h_R = 0$), delta only ($g_{\pi NR} = 0$) and combined system

Self-energy of the Roper resonance

- Self-energy contributions:

$$\Sigma_{\pi R}^{(3)}(\not{p}) = \frac{3g_R^2}{4F_\pi^2} \int \frac{d^4k}{(2\pi)^4} \frac{i\not{k}\gamma_5 (\not{p} - \not{k} + m_R) \not{k}\gamma_5}{[(p - k)^2 - m_R^2 + i\epsilon][k^2 - M_\pi^2 + i\epsilon]} ,$$

$$\Sigma_{\pi N}^{(3)}(\not{p}) = \frac{3g_{\pi NR}^2}{4F_\pi^2} \int \frac{d^4k}{(2\pi)^4} \frac{i\not{k}\gamma_5 (\not{p} - \not{k} + m_N) \not{k}\gamma_5}{[(p - k)^2 - m_N^2 + i\epsilon][k^2 - M_\pi^2 + i\epsilon]} ,$$

$$\Sigma_{\pi\Delta}^{(3)}(\not{p}) = \frac{2h_R^2}{F_\pi^2} \int \frac{d^4k}{(2\pi)^4} \frac{(p - k)_\mu G^{\mu\nu}(k) (p - k)_\nu}{(p - k)^2 - M_\pi^2 + i\epsilon} .$$

Finite volume formalism

- Define FV correction:

$$\tilde{\Sigma}_R^L(\not{p}) := \Sigma_R^L(\not{p}) - \text{Re}\{\Sigma_R(\not{p})\}$$

- We obtain for the poles (using $\not{p} = E$)

$$\begin{aligned} 0 &\stackrel{!}{=} E - m_{R0} - \left[\tilde{\Sigma}_R^L(E) + \text{Re}\{\Sigma_R(E)\} \right] \\ &= E - \underbrace{\left[m_{R0} + \text{Re}\{\Sigma_R(E)\} \right]}_{m_R} - \tilde{\Sigma}_R^L(E) \\ \Leftrightarrow m_R - E &= -\tilde{\Sigma}_R^L(E) \quad \rightarrow \text{energy levels!} \end{aligned}$$

Finite volume energy levels



■ Energy Levels of the Roper system:

$$\begin{aligned}m_R - E &= -\tilde{\Sigma}_R^L(E) \\&= -\left\{\tilde{\Sigma}_{\pi R}^{L,(3)}(E) + \tilde{\Sigma}_{\pi N}^{L,(3)}(E) + \tilde{\Sigma}_{\pi \Delta}^{L,(3)}(E)\right\}\end{aligned}$$

- Intermediate $R\pi$ -states do not produce pole!
→ *regular function*, only exponentially suppressed terms
- Intermediate $N\pi$ - and $\Delta\pi$ -states produce pole!
→ *irregular function*, imaginary part in infinite volume

Finite volume Formalism

- Example: Regular function → no pole on the real axis

$$A_0(m^2) = -16\pi^2 \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2} = -16\pi^2 \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2 + m^2}$$

- Finite volume:

$$A_0^L(m^2) = -16\pi^2 \int_{-\infty}^{\infty} \frac{dk_4}{2\pi} \frac{1}{L^3} \sum_{\vec{k}} \frac{1}{k_4^2 + |\vec{k}|^2 + m^2}$$

$$\begin{aligned}\Rightarrow \tilde{A}_0^L(m^2) &= -16\pi^2 \sum_{\vec{n} \neq 0} \int \frac{d^4 k_E}{(2\pi)^4} \frac{e^{iL\vec{k} \cdot \vec{n}}}{k_4^2 + |\vec{k}|^2 + m^2} \\ &= -4m^2 \sum_{|\vec{n}| \neq 0} \frac{K_1(mL|\vec{n}|)}{mL|\vec{n}|}\end{aligned}$$

Finite volume Formalism

- Example: Irregular function \rightarrow poles on the real axis

$$B_0(E^2, m_X^2, M_\pi^2) = 16\pi^2 \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{[k_E^2 + M_\pi^2][(\hat{P} - k_E)^2 + m_X^2]}$$

- Value of $\hat{P}_\mu = (iE, \vec{0})$ determines the pole
 - if $E \simeq m_X$, no pole \rightarrow Bessel-function
 - if $E > m_X$, pole \rightarrow Lüscher-function
- Lüscher-function: $B_0^L(E^2, m_X^2, M_\pi^2) \propto \frac{1}{4\pi^{3/2} E L} \mathcal{Z}_{00}(1, q^2)$

$$\mathcal{Z}_{00}(1, q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2(E)}$$

Self-energy of the Roper resonance

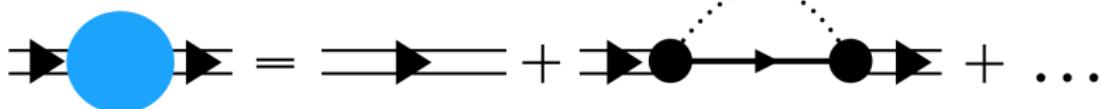


$$\Sigma_R(p_0, \vec{p}) = \Sigma_{N\pi}(p_0, \vec{p}) + \Sigma_{N\sigma}(p_0, \vec{p}) + \Sigma_{\Delta\pi}(p_0, \vec{p})$$

- $\Sigma_{N\pi}(p_0, \vec{p})$ with internal nucleon and pion
 - two stable particles
- $\Sigma_{\Delta\pi}(p_0, \vec{p})$ and $\Sigma_{N\sigma}(p_0, \vec{p})$ with internal Δ - and σ -dimer, respectively
 - two unstable particles, but dimer-propagator is a constant
 - "dress" the dimer-propagators for 3-particle dynamics!

Self-energy of the dimer fields

- The Δ -dimer propagator:



$$D_{\Delta}(p) = -\frac{1}{\alpha_{\Delta} m_{\Delta}^2 + \Sigma_{\Delta}(p)}, \text{ with}$$

$$\Sigma_{\Delta}(p) = g_2^2 \int \frac{d^4 k}{(2\pi)^4 i} S_N(p - k) S_{\pi}(k)$$

→ complicated self-energy due to two different masses

Self-energy of the Roper: The $N\pi$ -case

- Self-energy from $N\pi$ -system in the infinite volume:

$$\Sigma_{N\pi}(p) = f_2^2 \int \frac{d^4 k}{(2\pi)^4 i} S_N(p - k) S_\pi(k) := f_2^2 J_{N\pi}(p) , \text{ where}$$

$$\begin{aligned} J_{N\pi}(p) &= \int \frac{d^4 k}{(2\pi)^4 i} \frac{1}{2\omega_N(\vec{p} - \vec{k}) [\omega_N(\vec{p} - \vec{k}) - (p_0 - k_0) - i\epsilon]} \\ &\quad \cdot \frac{1}{2\omega_\pi(\vec{k}) [\omega_\pi(\vec{k}) - k_0 - i\epsilon]} \\ &= \int \frac{d^3 k}{(2\pi)^3} \frac{1}{4\omega_N(\vec{p} - \vec{k}) \omega_\pi(\vec{k}) [\omega_N(\vec{p} - \vec{k}) + \omega_\pi(\vec{k}) - p_0]} \\ &= \frac{i\lambda^{1/2}(p^2, m_N^2, M_\pi^2)}{16\pi p^2} \quad (\text{dimensional reg.}) \end{aligned}$$

Self-energy of the Roper: The $N\pi$ -case

- Self-energy from $N\pi$ -system in the finite volume:
(go to rest frame: $\rho_0 = E$, $\vec{p} = 0$)

$$\Sigma_{N\pi}(E) \rightarrow \Sigma_{N\pi}^L(E) = f_2^2 J_{N\pi}^L(E) , \text{ where}$$

$$\begin{aligned} J_{N\pi}^L(E) &= \frac{1}{L^3} \sum_{\vec{k}} \frac{1}{4\omega_N(\vec{k})\omega_\pi(\vec{k})[\omega_N(\vec{k}) + \omega_\pi(\vec{k}) - E]} \\ &= \frac{1}{4\pi^{3/2}EL} \mathcal{Z}_{00}\left(1, \tilde{q}^2(E)\right) , \end{aligned}$$

$$\text{with } \tilde{q}^2(E) = \left(\frac{L}{2\pi}\right)^2 \frac{\lambda(E^2, m_N^2, M_\pi^2)}{4E^2}$$

Self-energy of the Roper: The $N\sigma$ -case

- Self-energy from $N\sigma$ -system in the infinite volume:

$$\begin{aligned}\Sigma_{N\sigma}(p) &= f_4^2 \int \frac{d^4 k}{(2\pi)^4 i} S_N(p - k) D_\sigma(k) \\ &= f_4^2 \int \frac{d^4 k}{(2\pi)^4 i} \frac{1}{2\omega_N(\vec{p} - \vec{k}) [\omega_N(\vec{p} - \vec{k}) - (p_0 - k_0) - i\epsilon]} \\ &\quad \cdot \left\{ -\frac{1}{\alpha_\sigma M_\sigma^2 + \Sigma_\sigma(k)} \right\}, \text{ where}\end{aligned}$$

$$\Sigma_\sigma(k) = \int \frac{d^3 l}{(2\pi)^3} \frac{2h_2^2}{4\omega_\pi(\vec{k} - \vec{l})\omega_\pi(\vec{l}) [\omega_\pi(\vec{k} - \vec{l}) + \omega_\pi(\vec{l}) - k_0 - i\epsilon]}$$

Self-energy of the Roper: The $N\sigma$ -case

- First, go to rest frame ($p_0 = E$, $\vec{p} = 0$) and integrate out the time component k_0 :

$$\begin{aligned}\Sigma_{N\sigma}(E) = & -f_4^2 \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2\omega_N(\vec{k})} \left\{ \alpha_\sigma M_\sigma^2 \right. \\ & + 2h_2^2 \int \frac{d^3 l}{(2\pi)^3} \frac{1}{4\omega_\pi(\vec{k} - \vec{l})\omega_\pi(\vec{l})} \\ & \cdot \left. \frac{1}{[\omega_\pi(\vec{k} - \vec{l}) + \omega_\pi(\vec{l}) + \omega_N(\vec{k}) - E - i\epsilon]^{-1}} \right\}\end{aligned}$$

Self-energy of the Roper: The $N\sigma$ -case

- Self-energy contribution in the infinite volume:
 - replace inner integral with its result

$$\Sigma_{N\sigma}(E) = -f_4^2 \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2\omega_N(\vec{k})} \cdot \left\{ \alpha_\sigma M_\sigma^2 + \frac{i h_2^2}{8\pi} \frac{\lambda^{1/2} (\sigma(\vec{k}), M_\pi^2, M_\pi^2)}{\sigma(\vec{k})} \right\}^{-1}$$

$$\text{with } \sigma(\vec{k}) := E^2 + m_N^2 - 2E\omega_N(\vec{k})$$

- simplify structure inside the brace and integrate over \vec{k} in dimensional reg.

Self-energy of the Roper: The $N\sigma$ -case

- Self-energy contribution in the infinite volume:
(preliminary result)

$$\begin{aligned}\Sigma_{N\sigma}(E) = & (\alpha_\sigma + 1) \frac{\mu_\sigma^2 M_\sigma^2 f_4^2}{M_\sigma^4 + c^2} \frac{i\lambda^{1/2}(E^2, m_N^2, \mu_\sigma^2)}{16\pi E^2} \\ & + \frac{c f_4^2}{16\pi^2(M_\sigma^4 + c^2)E^2} \int_0^1 dx \left(\frac{(4M_\pi^2)^2 \sqrt{x(1-x)}}{4M_\pi^2(1-x) - \mu_\sigma^2} \right) \\ & \cdot \lambda^{1/2}(E^2, m_N^2, 4M_\pi^2(1-x))\end{aligned}$$

$$\text{with } c = \frac{h_2^2}{8\pi}, \text{ and } \mu_\sigma^2 = \frac{4M_\pi^2 c^2}{M_\sigma^4 + c^2}$$