The 39th International Symposium on Lattice Field Theory

I=1/2 and 3/2 $K\pi$ Scattering Length at Physical Pion Mass

using domain wall fermions with all-to-all propagators

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Overview

Aim: compute $\Delta E_{K\pi}$ from lattice quantities

- Kaon-pion scattering amplitude
  - Check chiral perturbation theory with strange quark
  - Study CKM matrix elements
  - Towards full phase shift calculation
  - $B \to K^*$ (main decay channel of K^* is $K\pi$)
- Previous scattering length computations
  - Wilson *et al*, 2019
  - Sasaki *et al*, 2014
  - Helmes *et al* (ETMC), 2018
- ✦ Physical pion mass

$$T_{nr} = -\frac{8\pi}{m}\sum_{l=0}^{\infty}(2l+1)P_l(cos\theta)t_l$$

$$t_l = \frac{1}{2ip}\left(e^{2i\delta_l} - 1\right)$$

$$a_0 = \lim_{p\to 0}\frac{1}{2ip}\left(e^{2i\delta_0} - 1\right)$$

$$\Delta E_{K\pi} = -\frac{2\pi(m_\pi + m_K)}{m_\pi m_K L^3}a_0\left(1 + c_1\frac{a_0}{L} + c_2\frac{(a_0)^2}{L^2}\right) + \mathcal{O}(L^{-6}) \quad [1]$$

$c_1 = -2.837297, \quad c_2 = 6.375183$

[1] M Lüscher. Nucl. Phys. B, 354, 1991

Simulation details

- all-to-all propagators

$$S_{x,y} = D^{-1}_{x,y} = \boxed{\sum_i^{N_{ev}} \frac{1}{\lambda^{(i)}} e^{(i)}_x e^{(i)\dagger}_y} + \boxed{\sum_j^{N_\eta} D^{-1}_{x,y} \eta^{(j)}_x \eta^{(j)\dagger}_y} =: \mathbf{v}_x \mathbf{w}^\dagger_y$$

low modes, spectral decomposition [2]

high modes, stochastic sampling

- meson fields

$$M^\Gamma_x = \mathbf{w}^\dagger_x \Gamma \mathbf{v}_x$$

$$\mathbf{v}^i_x = \begin{cases} \frac{1}{\lambda^{(i)}} e^{(i)}_x & i = 1, \ldots, N_{ev} \\ D^{-1}_{x,z} (P\eta^{(i)})_z & i = N_{ev}+1, \ldots, N_{ev}+N_\eta \end{cases}$$

- correlations on each configuration

$$c(x, y) = \mathrm{Tr}[M^{\gamma_5}_x M^{\gamma_5}_y]$$

$$\mathbf{w}^i_x = \begin{cases} \frac{1}{e^{(i)}_x} & i = 1, \ldots, N_{ev} \\ \eta^{(i)} & i = N_{ev}+1, \ldots, N_{ev}+N_\eta \end{cases}$$

- smearing
  - kaon source smearing
  - suppress excited state contributions

[2] J Foley *et al.* Comput.Phys.Commun.172:145-162, 2005

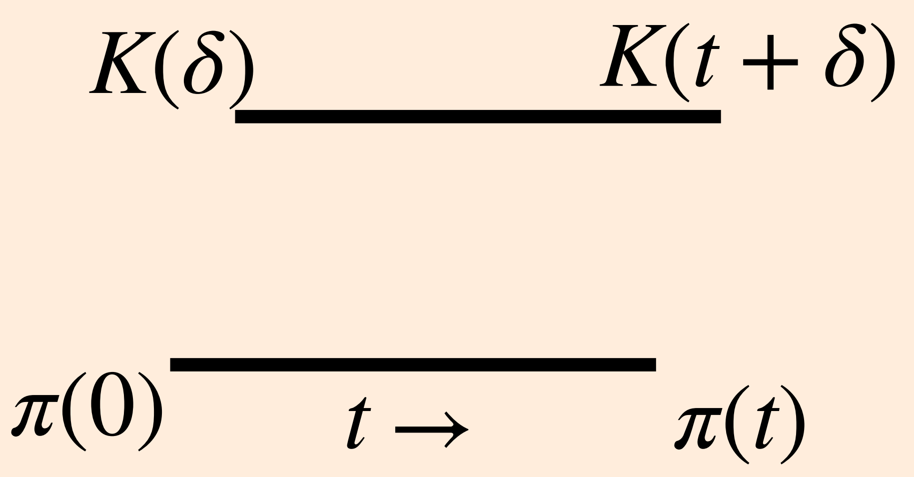
Strategy to determine $\Delta E_{K\pi}$

- look at $K\pi$ two-point function

$$C_{K\pi}(t) = \langle \mathcal{O}_{K\pi}(t)\mathcal{O}^\dagger_{K\pi}(0)\rangle$$

$$\mathcal{O}_{K\pi}(t) = K(t+\delta)\pi(t)$$

δ=1 shift to avoid contributions from correlated noise sources on the same time slice



- in the continuum

$$C_{K\pi}(t) = \langle K(t+\delta)\pi(t)(K(\delta)\pi(0))^\dagger\rangle$$
$$\sim |\langle K\pi|K(\delta)\pi(0)|0\rangle|^2 e^{-E_{K\pi}t} + \ldots$$

contains $E_{K\pi} = m_K + m_\pi + \Delta E_{K\pi}$

- on the periodic lattice

$$C_{K\pi}(t) = |\langle K\pi|K(\delta)\pi(0)|0\rangle|^2\left(e^{-E_{K\pi}t} + e^{-E_{K\pi}(T-t)}\right)$$ cosh-like term — **contains signal!**

$$+|\langle \pi|K(\delta)\pi(0)|K\rangle|^2 e^{-m_K t}e^{-m_\pi(T-t)}$$
$$+|\langle K|K(\delta)\pi(0)|\pi\rangle|^2 e^{-m_\pi t}e^{-m_K(T-t)}$$ 'around-the-world' terms — **need to remove**

$$+\ldots$$

- for noise cancellations

$$R_{K\pi}(t) = \frac{C_{K\pi}(t)}{C_K(t)C_\pi(t)} = \frac{\langle K(t+\delta)K(\delta)\pi(t)\pi(0)\rangle}{\langle K(t+\delta)K(\delta)\rangle\langle\pi(t)\pi(0)\rangle}$$

$$= R^{cosh}_{K\pi}(t) + R^{ATW}_{K\pi}(t)$$

Strategy to determine $\Delta E_{K\pi}$

- cosh-like part

$$R^{cosh}_{K\pi}(t) = A\frac{\left(e^{-(m_\pi+m_K+\Delta E_{K\pi})t} + e^{-(m_\pi+m_K+\Delta E_{K\pi})(T-t)}\right)}{\left(e^{-m_\pi t} + e^{-m_\pi(T-t)}\right)\left(e^{-m_K t} + e^{-m_K(T-t)}\right)}$$

- around-the-world part

$$R^{ATW}_{K\pi}(t) = \left|\frac{\langle\pi|K(\delta)\pi(0)|K\rangle}{\langle 0|K|K\rangle\langle\pi|\pi|0\rangle}\right|^2 \frac{e^{-m_K t}e^{-m_\pi(T-t)}}{\left(e^{-m_\pi t} + e^{-m_\pi(T-t)}\right)\left(e^{-m_K t} + e^{-m_K(T-t)}\right)}$$
$$+\left|\frac{\langle K|K(\delta)\pi(0)|\pi\rangle}{\langle 0|K|K\rangle\langle\pi|\pi|0\rangle}\right|^2 \frac{e^{-m_\pi t}e^{-m_K(T-t)}}{\left(e^{-m_\pi t} + e^{-m_\pi(T-t)}\right)\left(e^{-m_K t} + e^{-m_K(T-t)}\right)}$$

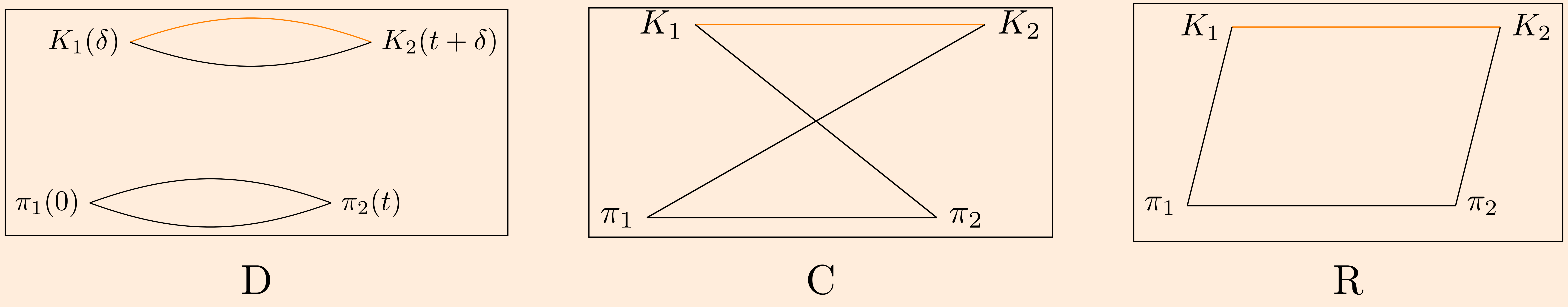
Checklist:

✓ pion two-point function

✓ kaon two-point function

✓ additional correlation functions $C_{\pi K\pi K}$ and $C_{KK\pi\pi}$

Isospin channels



Diagrammatic contributions to $C_{K\pi}$ corresponding to different Wick contractions.

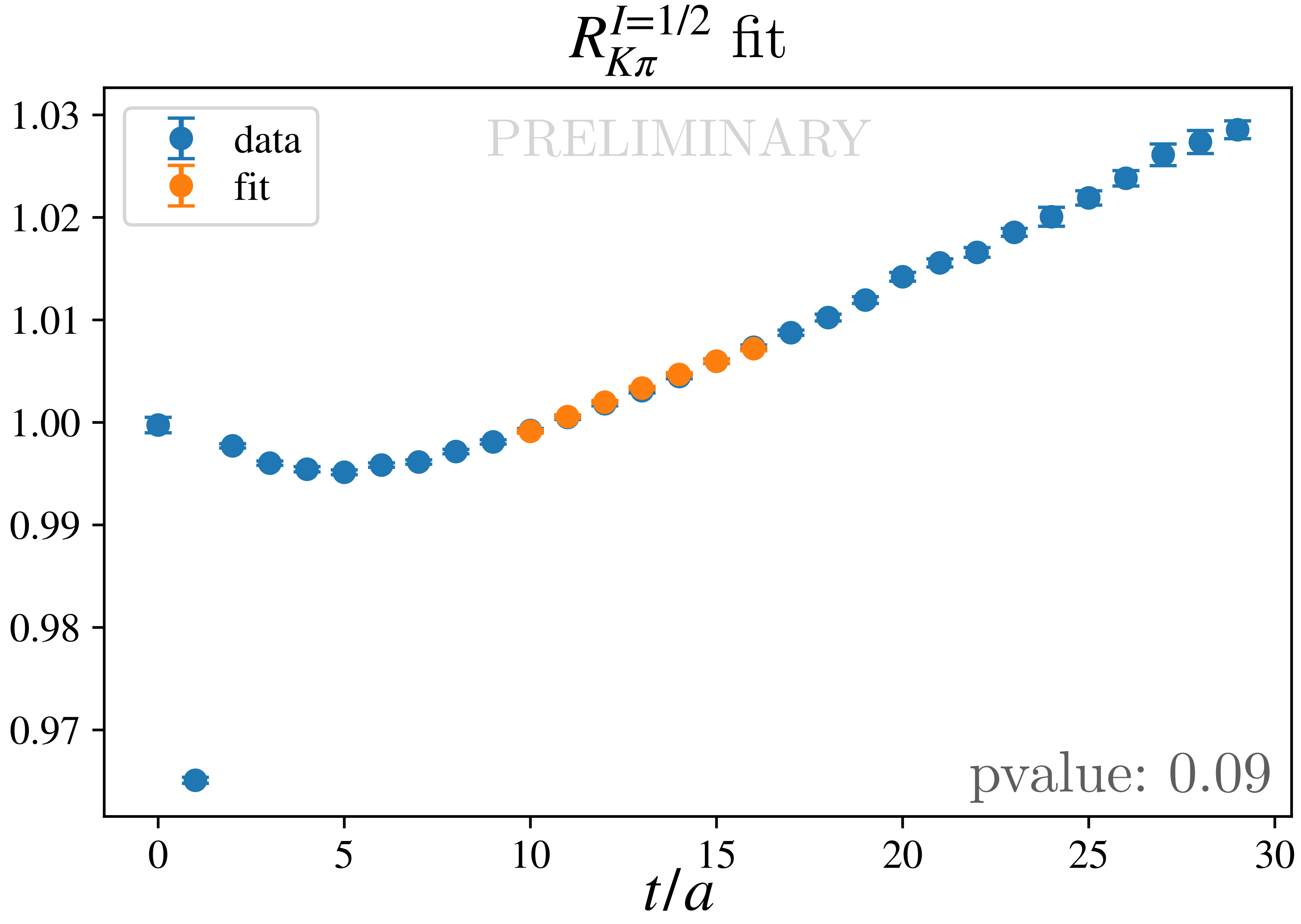
$$C_{K\pi}^{I=1/2} = D + \frac{1}{2}C - \frac{3}{2}R$$

$$C_{K\pi}^{I=3/2} = D - C$$

Fits

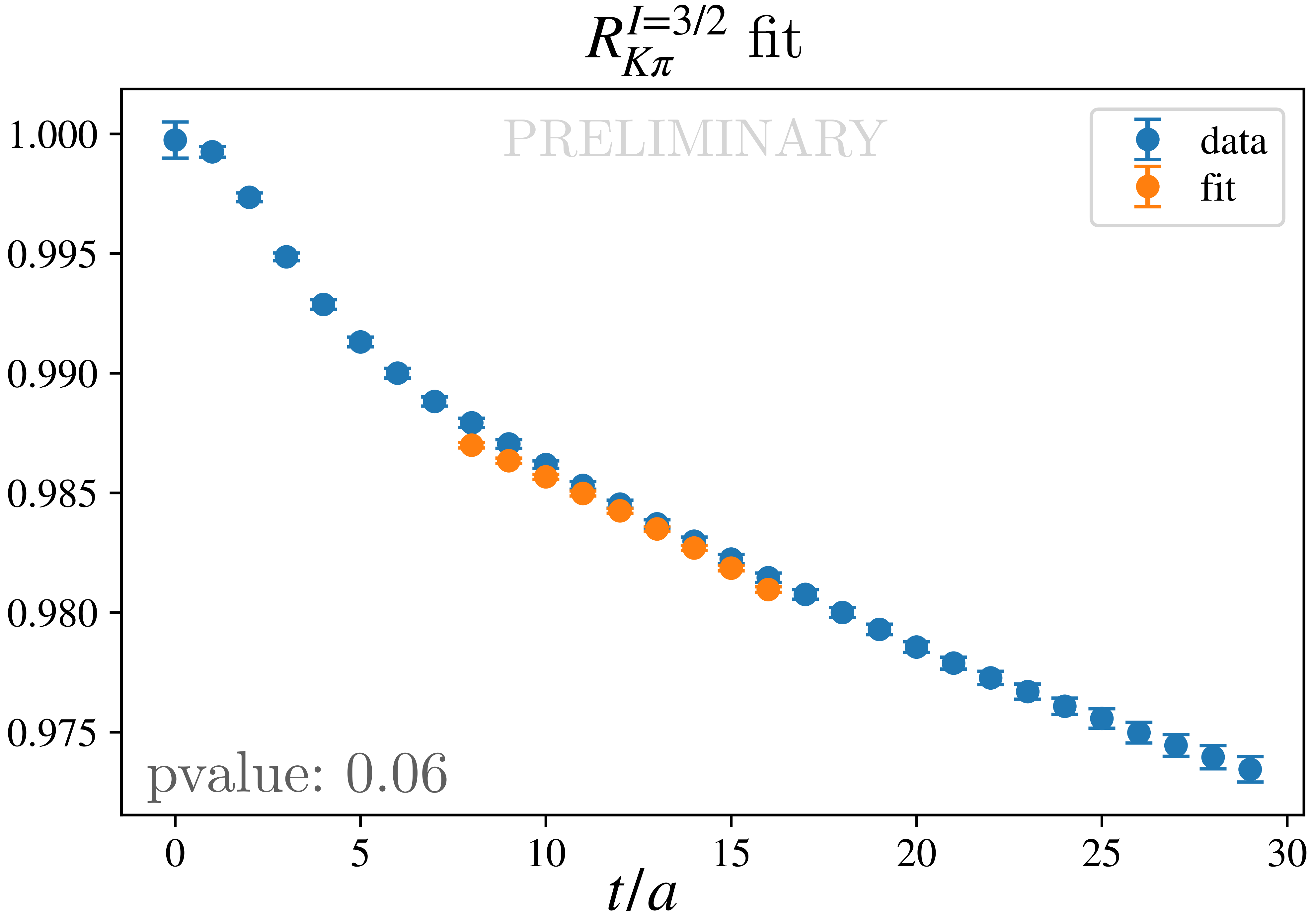
Simulation details:

- $V = 48^3 \times 96$
- $a = 1.73\ \text{GeV}^{-1}$
- $m_\pi^{lat} \approx 139$ MeV



$$a\Delta E_{K\pi}^{I=1/2} = -0.00161(3)$$

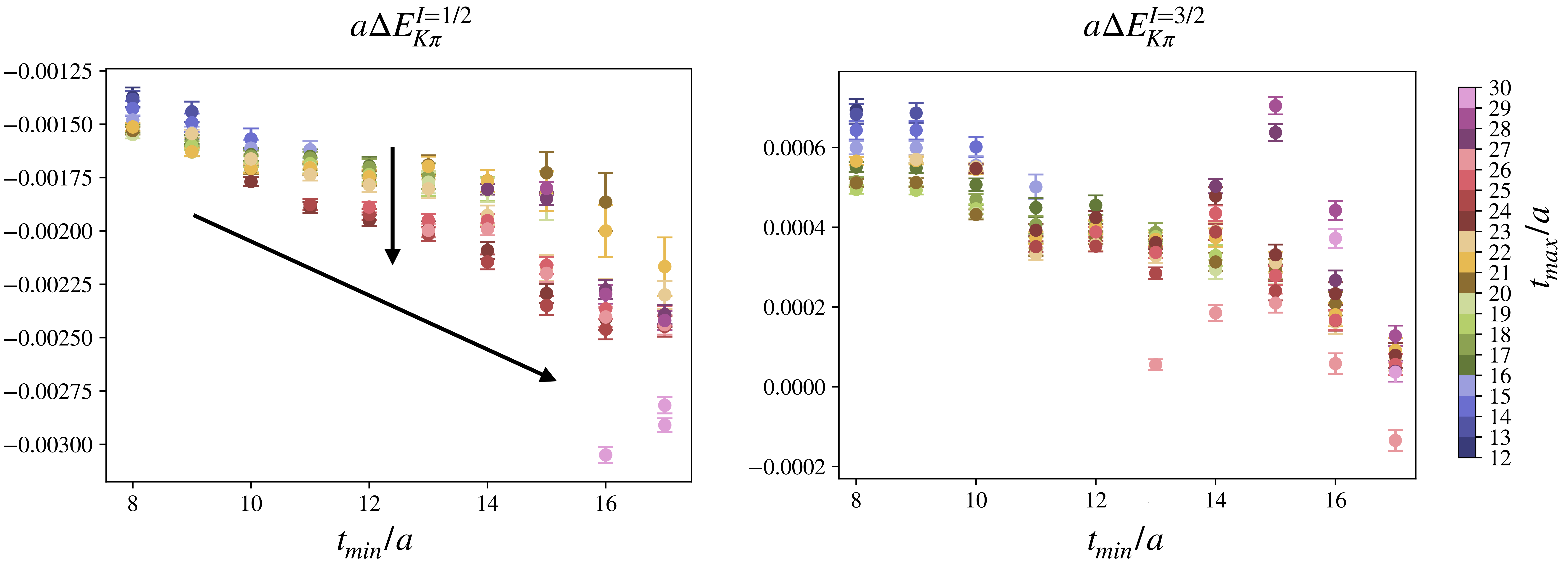
$$m_\pi a_0^{I=1/2} = 0.160(3)$$



$$a\Delta E_{K\pi}^{I=3/2} = 0.00055(1)$$

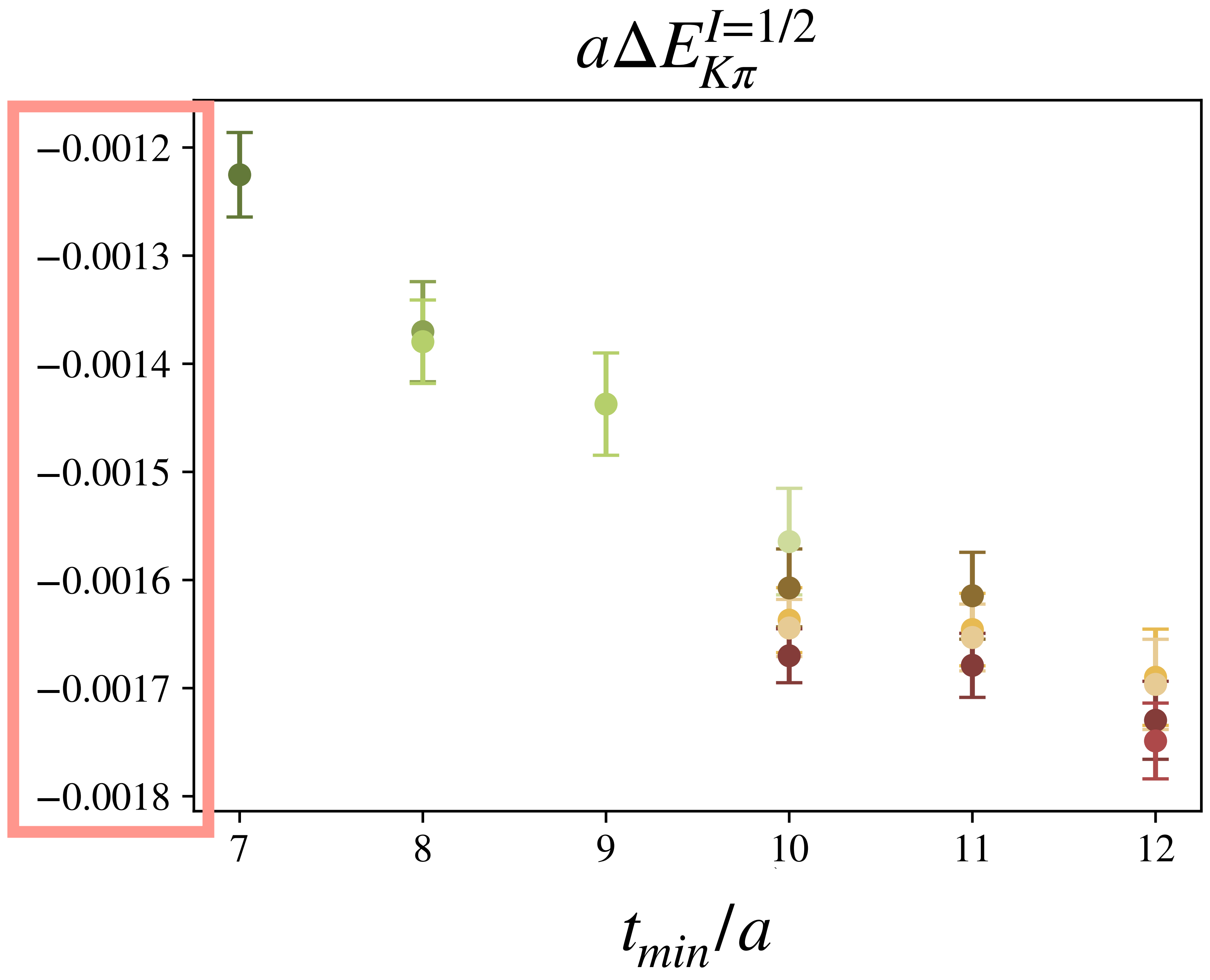
$$m_\pi a_0^{I=3/2} = -0.0471(9)$$

Fit variations

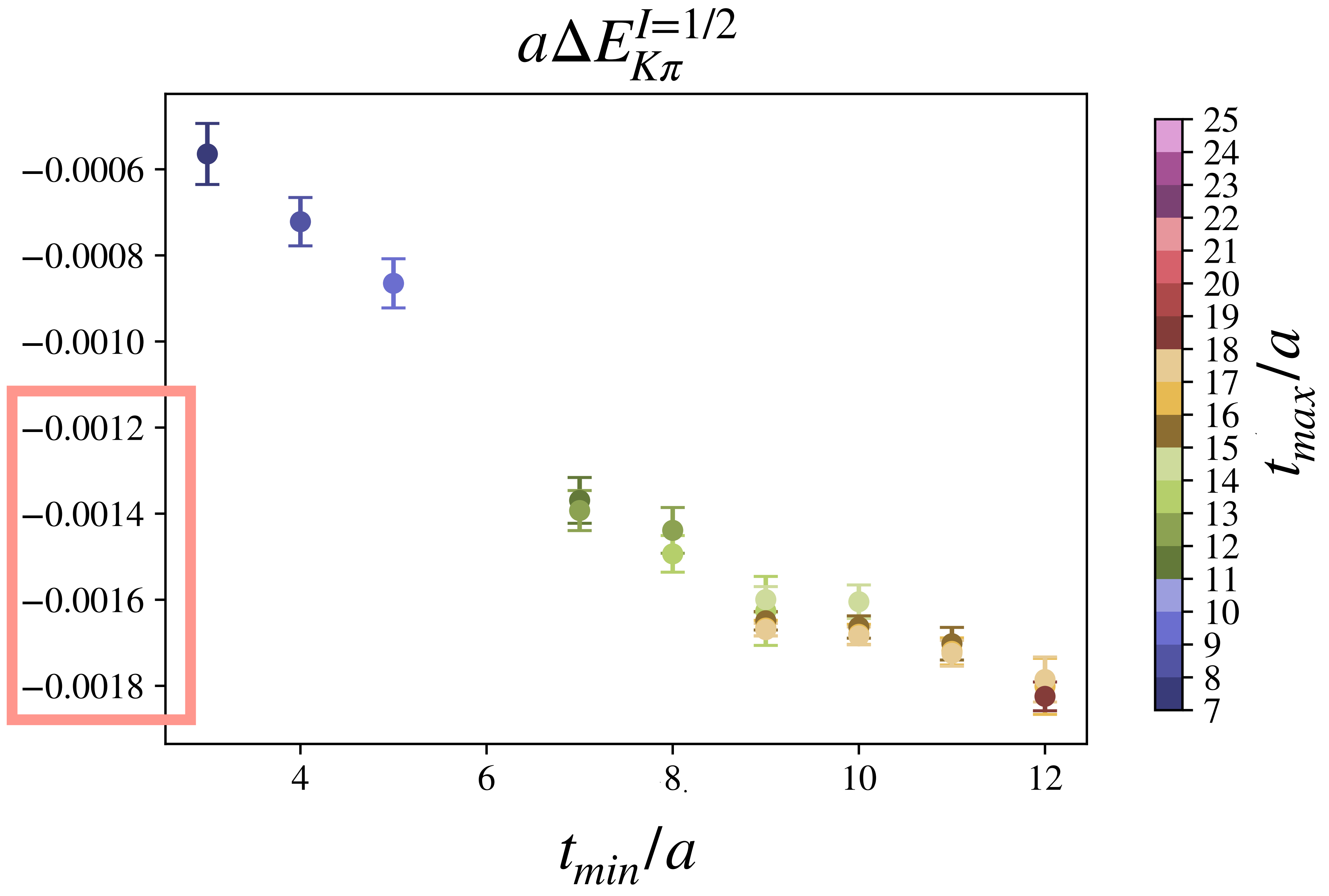


Variation in the value of $\Delta E_{K\pi}$ with fit intervals, extracted from $R_{K\pi}$ for each isospin channel using point source data.

Closer look



Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ using **point** source data.



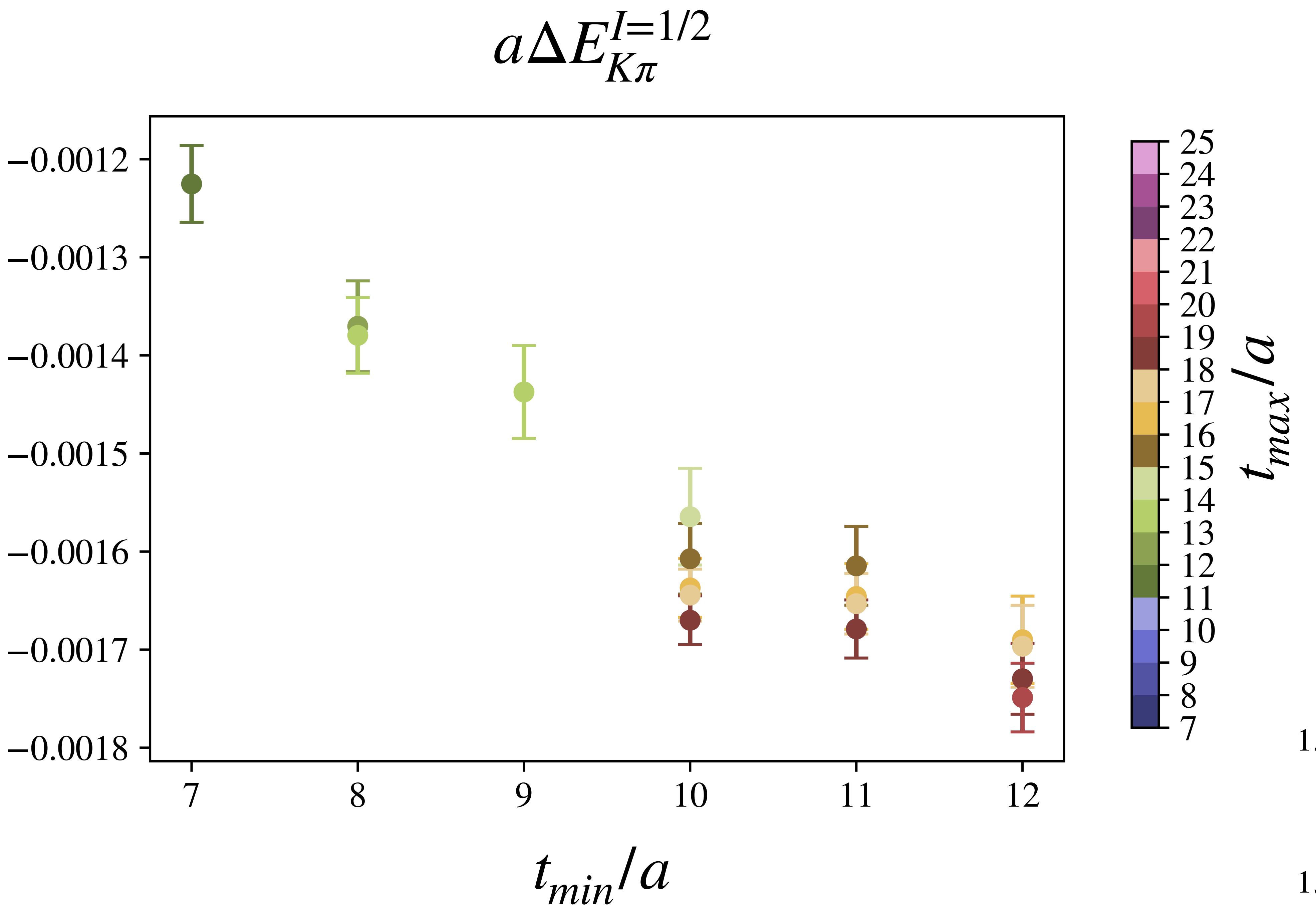
Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ using **smeared** source data.

Explanations

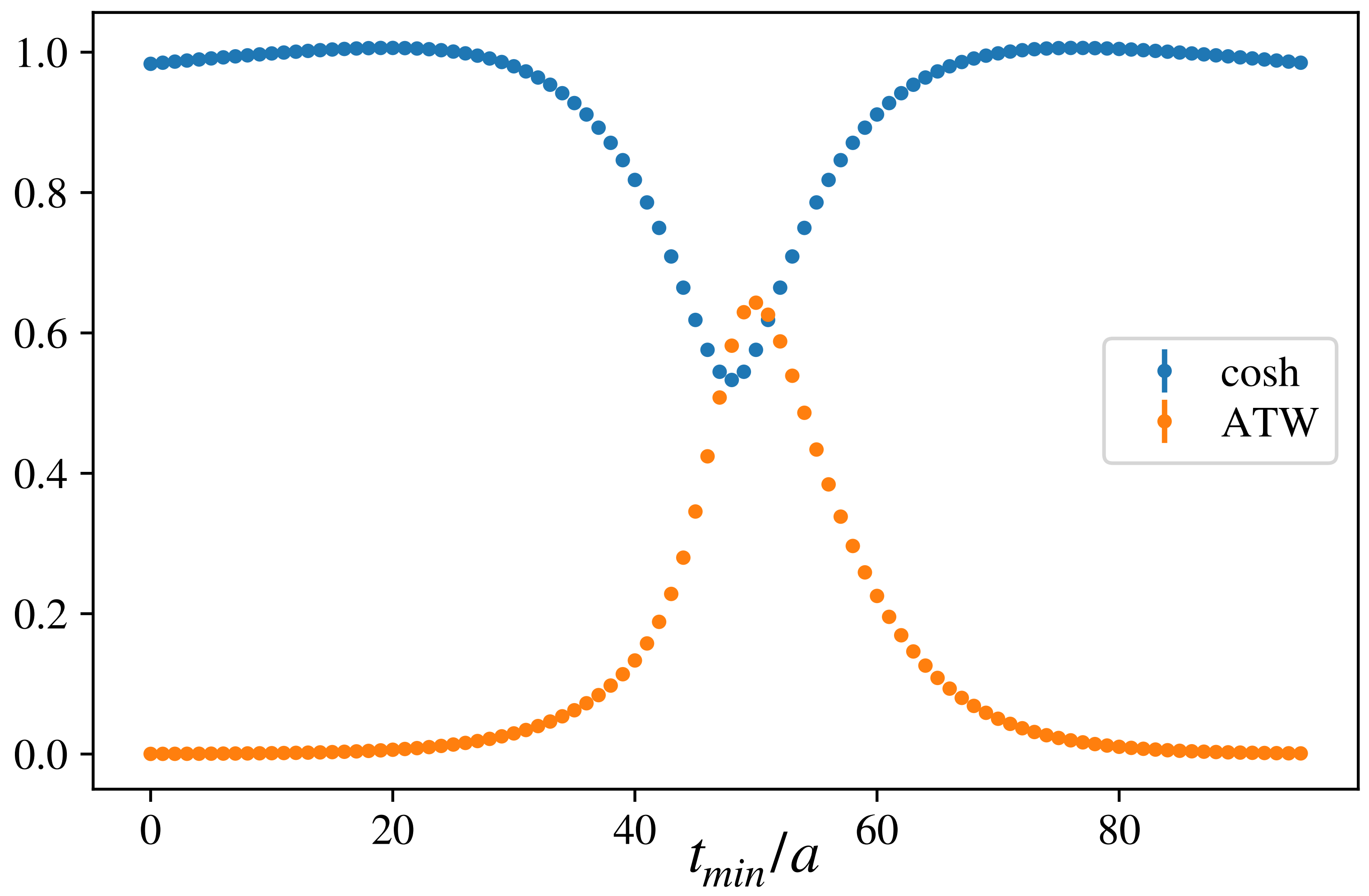
1. excited states
   - distillation? See next talk by N. Lachini

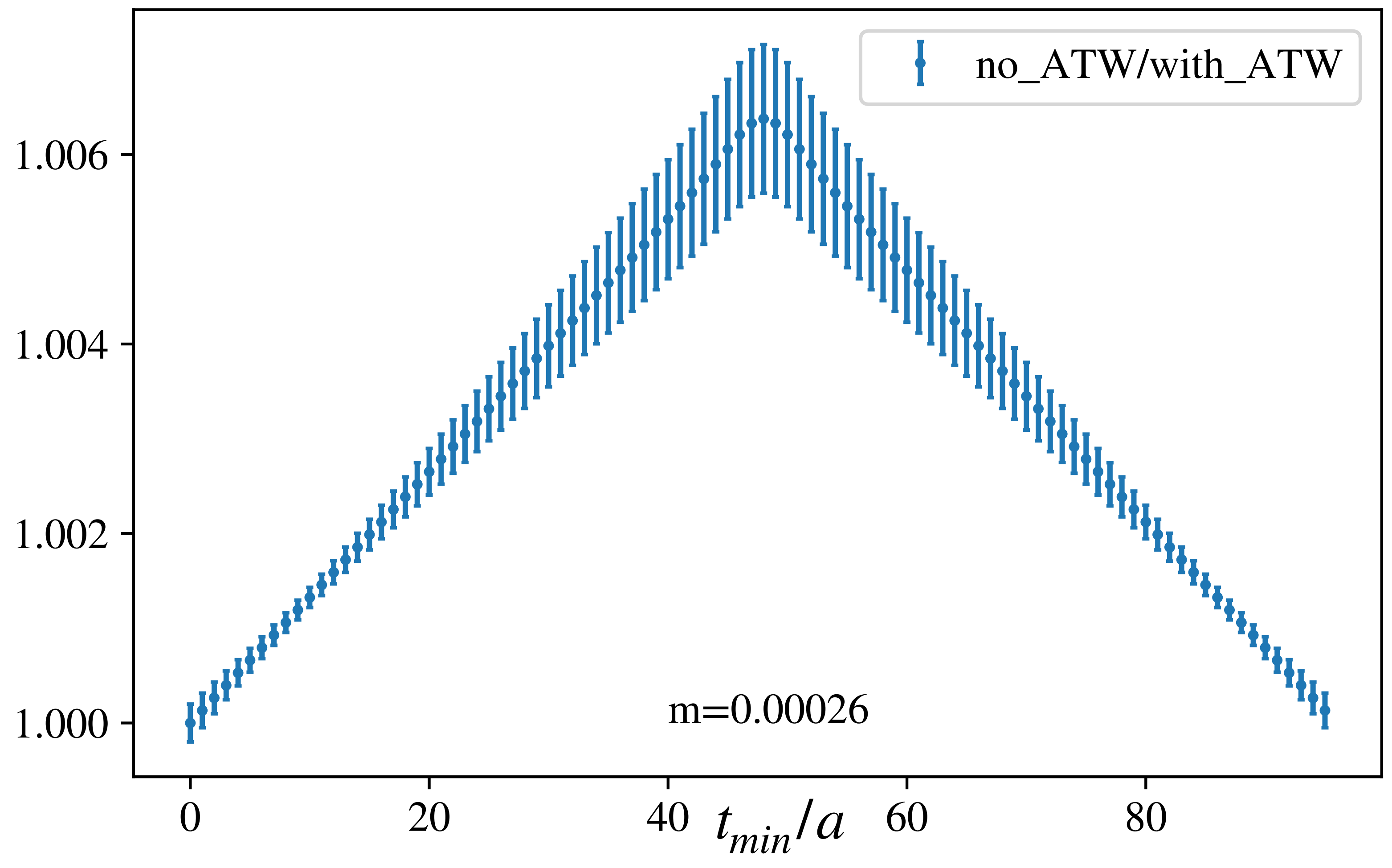
Excited states contributions are present but are expected to decay with t_{min}.

Closer look



Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$.

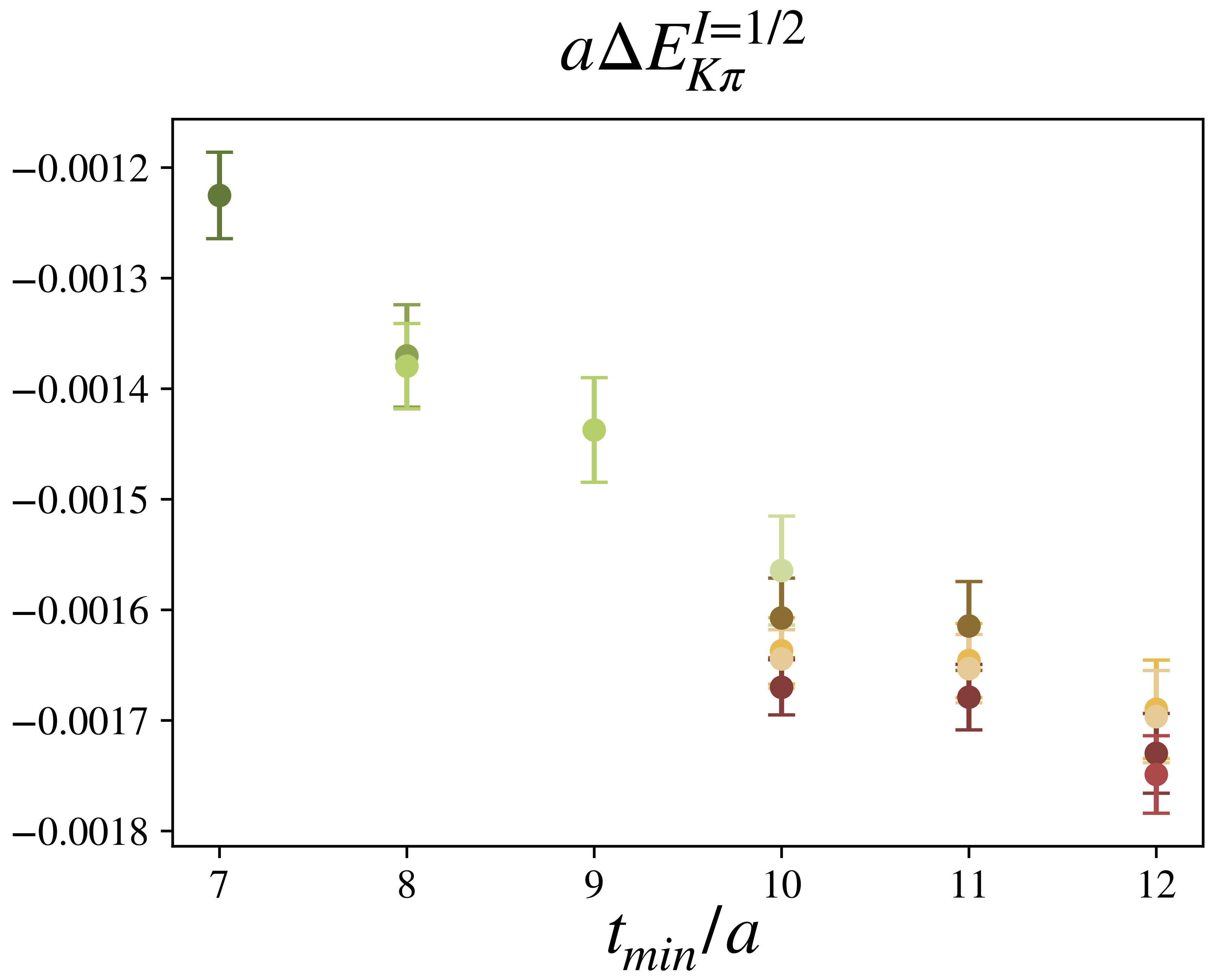




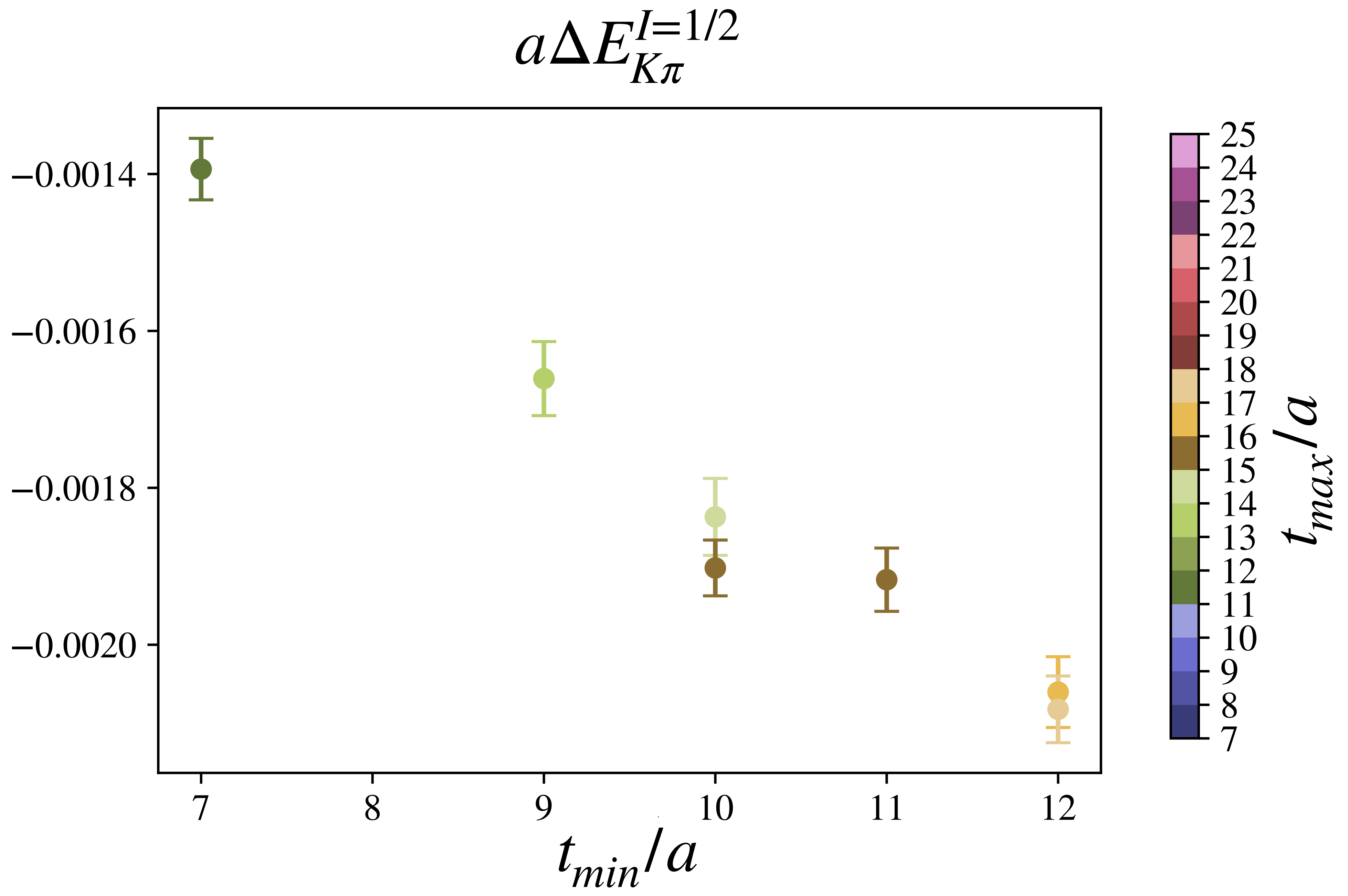
Explanations

1. excited states
   - distillation? See next talk by N. Lachini
2. around-the-world terms

Closer look



Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ **with** around-the-world terms accounted for.



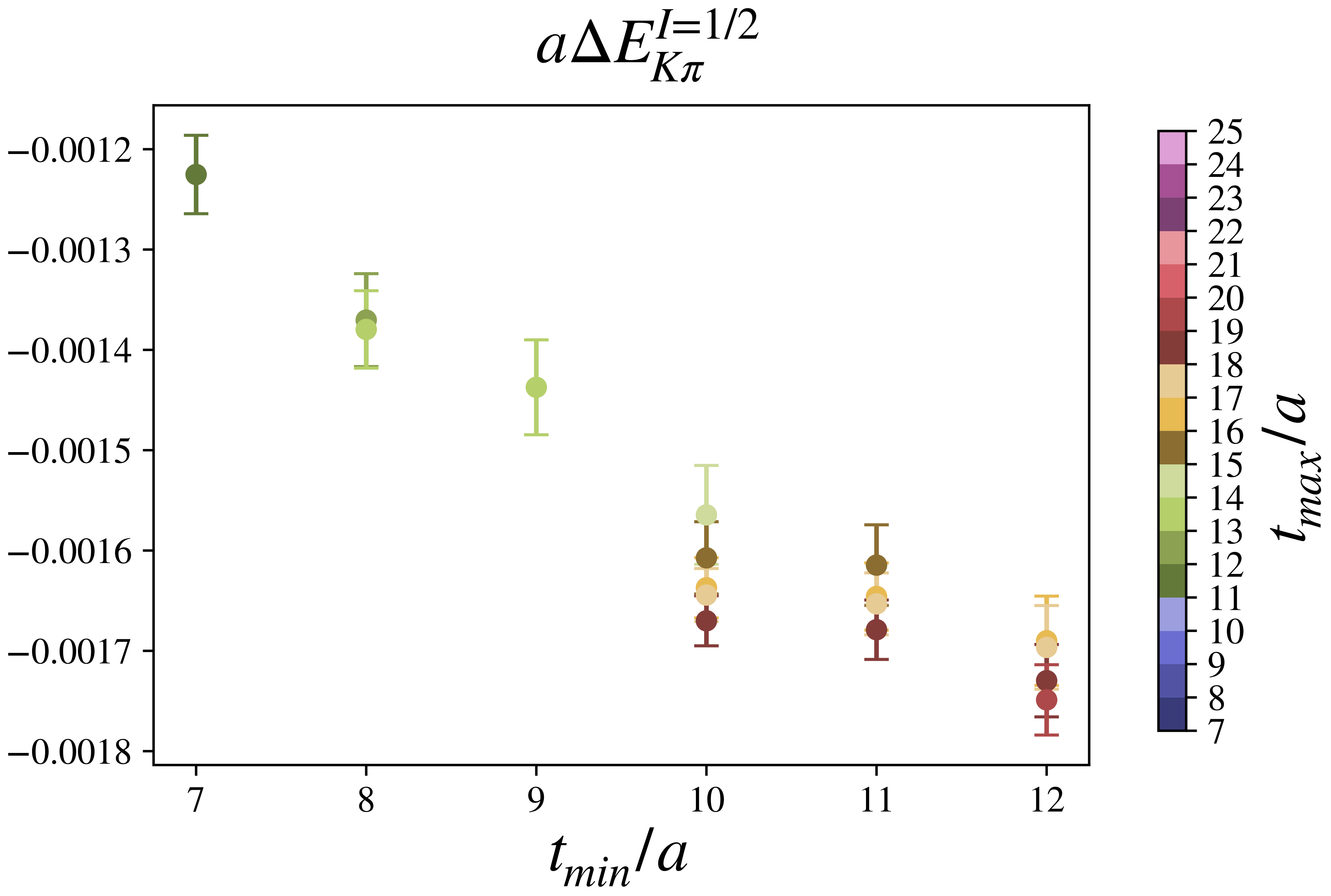
Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ **without** around-the-world terms accounted for.

Explanations

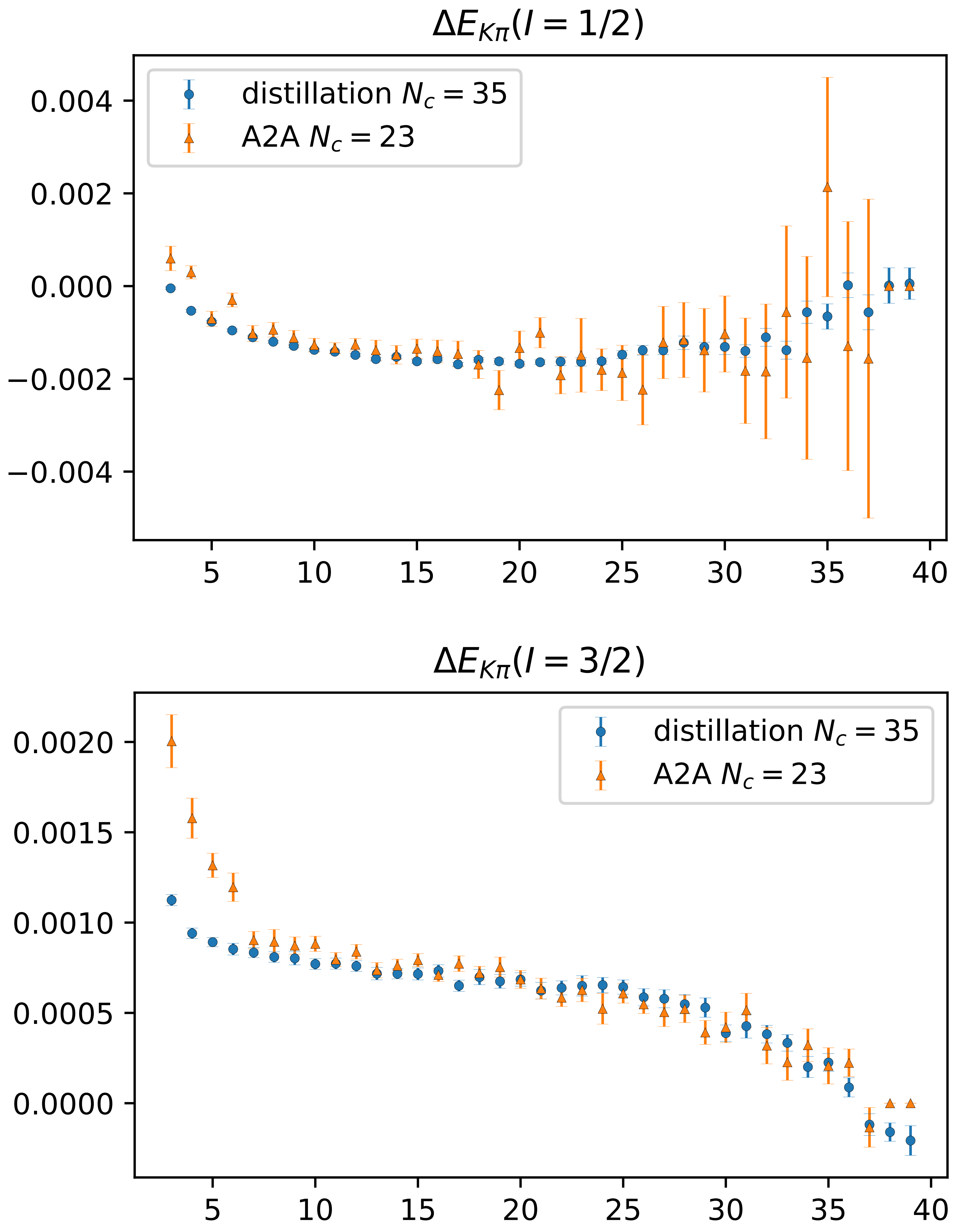
1. excited states
   - distillation? See next talk by N. Lachini
2. around-the-world terms

ATW contributions cannot be ignored but are unlikely to be the the source of the overall trend with t_{min}.

Closer look



Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ using all-to-all data.

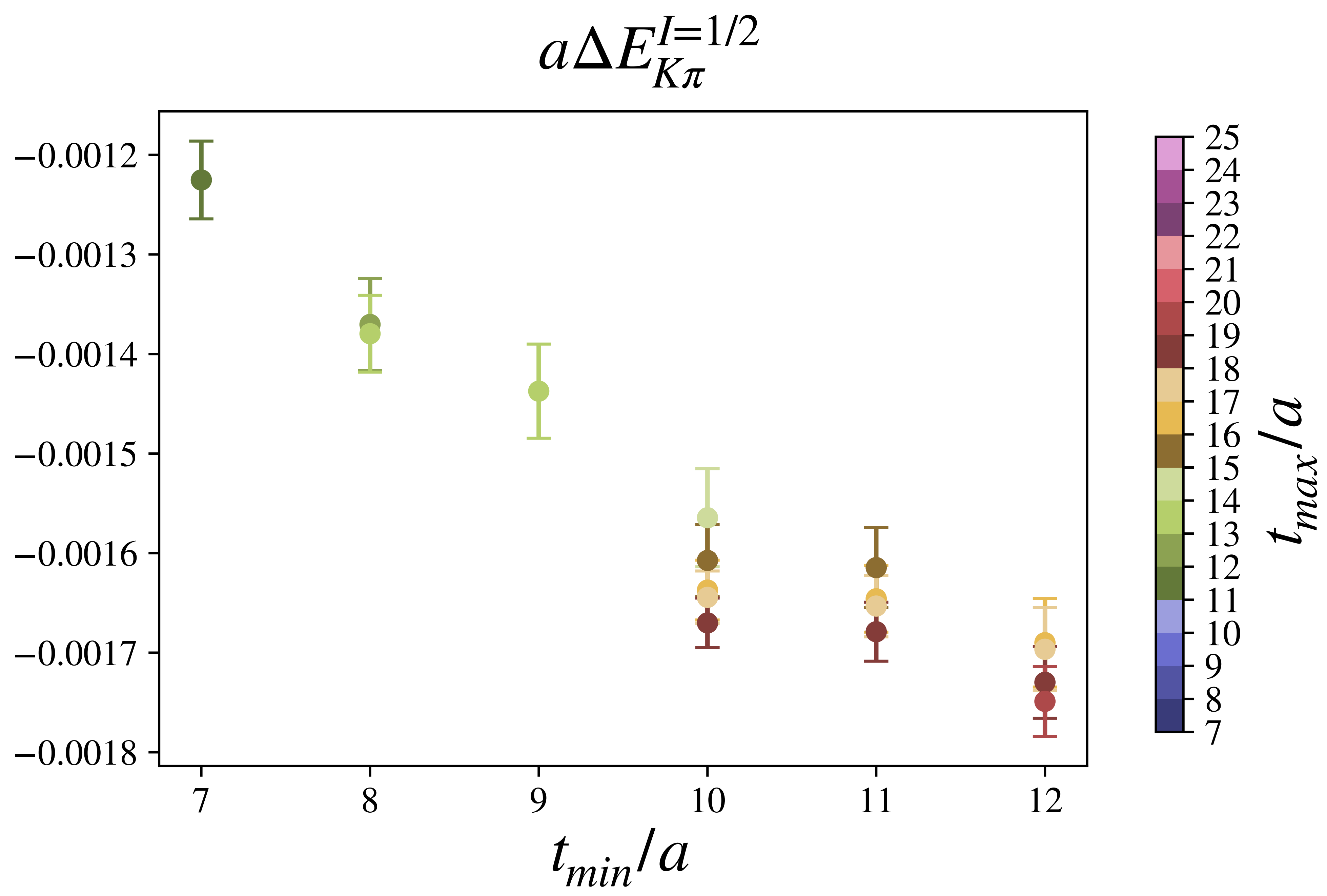


Plots for distillation vs all-to-all data courtesy of F. Erben and N. Lachini

<u>Explanations</u>

1. excited states
   - distillation? See next talk by N. Lachini
2. around-the-world terms
3. statistics
   - number of configs available: 23
4. other physics

Summary



Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ using all-to-all data.

- $K\pi$ scattering length study at physical pion mass
  $\rightarrow$ new features and new issues
- **interesting**: around-the-world contributions are time-dependent and present at all times - cannot be ignored in studies at physical point
- **unexplained**: time-dependent trend in the value of $\Delta E_{K\pi}$ with variation in fit interval
- **outlook**: distillation analysis, other analyses at physical point

Explanations

1. excited states
   - distillation? See next talk by N. Lachini
2. around-the-world terms
3. statistics
   - number of configs available: 23
4. other physics

Thank you.
Questions/comments?

Simulation details:
- $V = 48^3 \times 96$
- $a = 1.73\ \mathrm{GeV}^{-1}$
- $m_\pi^{lat} \approx 139$ MeV

Extra: fits

	m_π/MeV	$m_\pi a_0^{I=1/2}$	$m_\pi a_0^{I=3/2}$
Wilson, 2019	239	0.46(3)	
	284	0.79(13)	
Sasaki, 2014	extrap	0.142(14)(27)	-0.0469(24)(20)
	166	0.158(36)	-0.108(12)
ETMC	phys/extrap	0.163(3)	-0.059(2)
Our results (preliminary)	139 (phys)	0.160(3)	-0.0471(9)

Extra: 'around-the-world' matrix elements

$\langle \pi | K(\delta)\pi(0) | K \rangle$

$$C_{\pi K\pi K} = \langle \pi(\Delta)K(t+\delta)\pi(t)K(0)\rangle$$
$$= \langle 0|\pi|\pi\rangle\langle \pi|K(\delta)\pi(0)|K\rangle\langle K|K|0\rangle e^{-m_\pi(\Delta-t)}e^{-m_K t}$$
$$+ \text{ other terms}\ldots$$

$$\frac{C_{\pi K\pi K}}{C_\pi(\Delta-t)C_K(t+\delta)} = \text{ cons } + \text{other time-dep terms}$$

$$\text{cons} = \frac{\langle \pi|K(\delta)\pi(0)|K\rangle}{\langle 0|K|K\rangle\langle \pi|\pi|0\rangle}e^{m_K\delta}$$

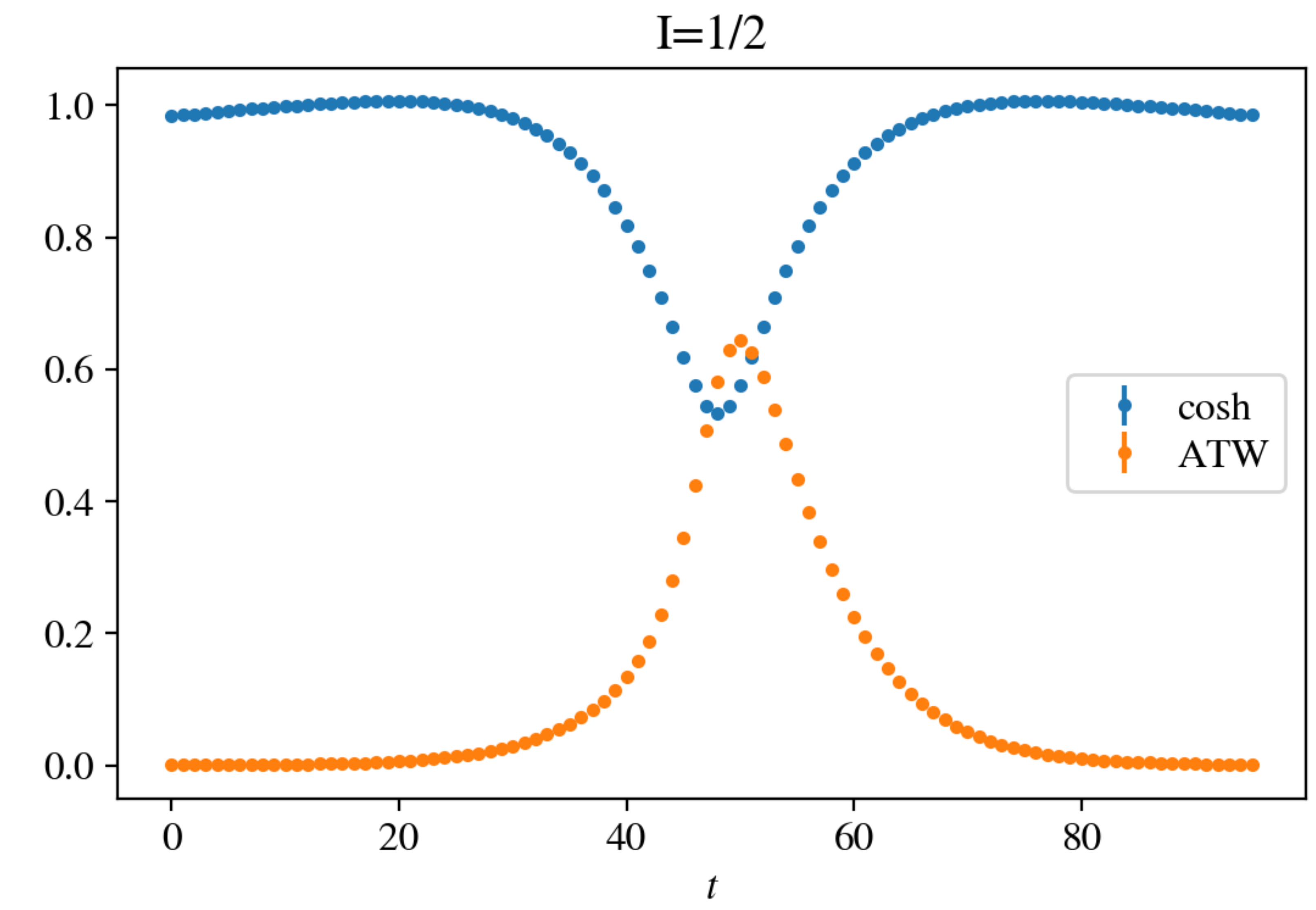
$\langle K | K(\delta)\pi(0) | \pi \rangle$

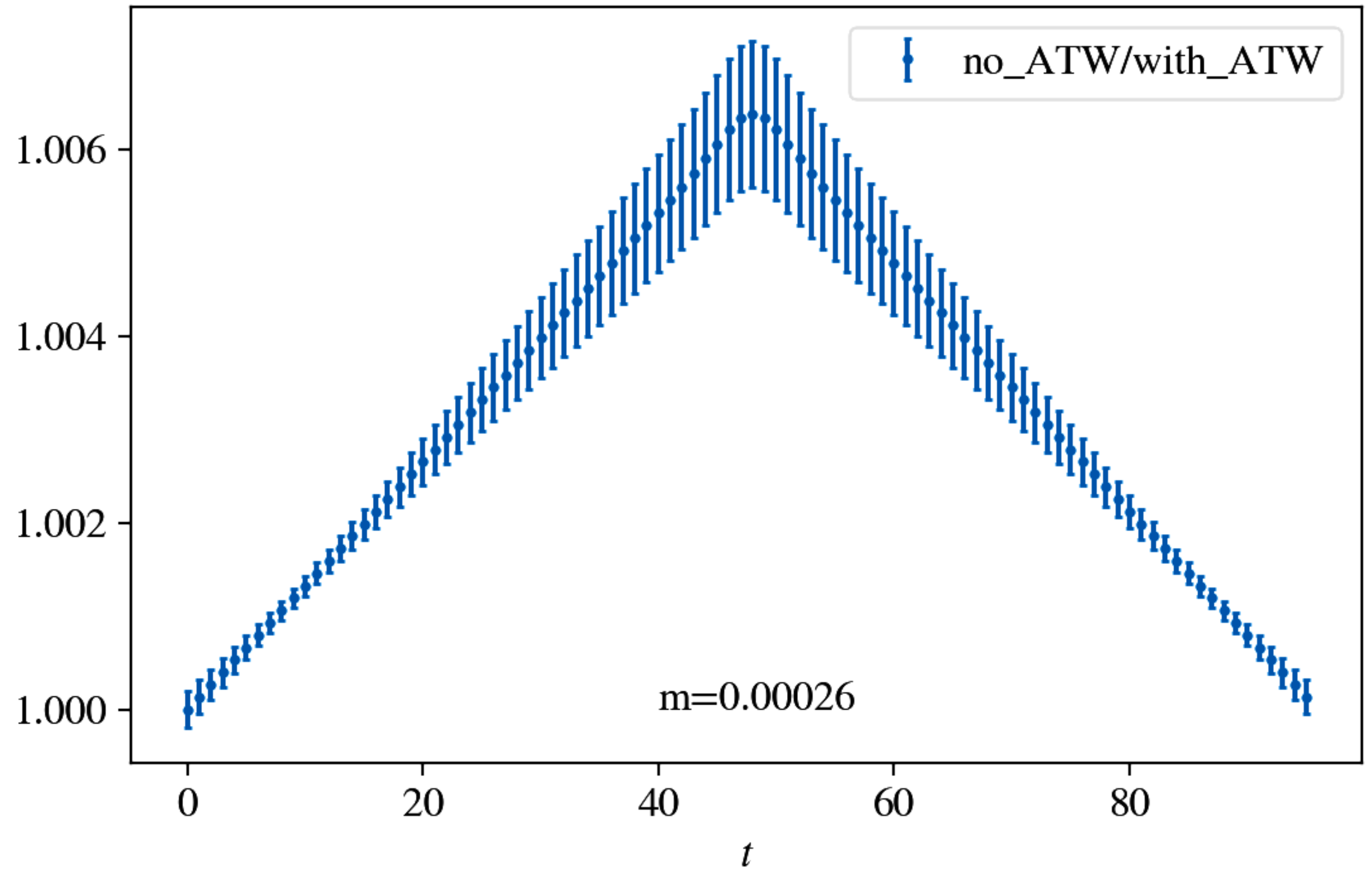
$$C_{KK\pi\pi} = \langle K(\Delta)K(t+\delta)\pi(t)\pi(0)\rangle$$
$$= \langle 0|K|K\rangle\langle K|K(\delta)\pi(0)|\pi\rangle\langle \pi|\pi|0\rangle e^{-m_K(\Delta-t)}e^{-m_\pi t}$$
$$+ \text{ other terms}\ldots$$

$$\frac{C_{KK\pi\pi}}{C_\pi(t)C_K(\Delta-t-\delta)} = \text{ cons } + \text{other time-dep terms}$$

$$\text{cons} = \frac{\langle K|K(\delta)\pi(0)|\pi\rangle}{\langle \pi|\pi|0\rangle\langle 0|K|K\rangle}e^{-m_K\delta}$$

Extra: 'around-the-world' effects





$$R_{K\pi}^{cosh}(t) = A\frac{\left(e^{-(m_\pi+m_K+\Delta E_{K\pi})t} + e^{-(m_\pi+m_K+\Delta E_{K\pi})(T-t)}\right)}{\left(e^{-m_\pi t} + e^{-m_\pi(T-t)}\right)\left(e^{-m_K t} + e^{-m_K(T-t)}\right)}$$

$$\xrightarrow{t\to 0} A(1 - \Delta E_{K\pi}t)$$

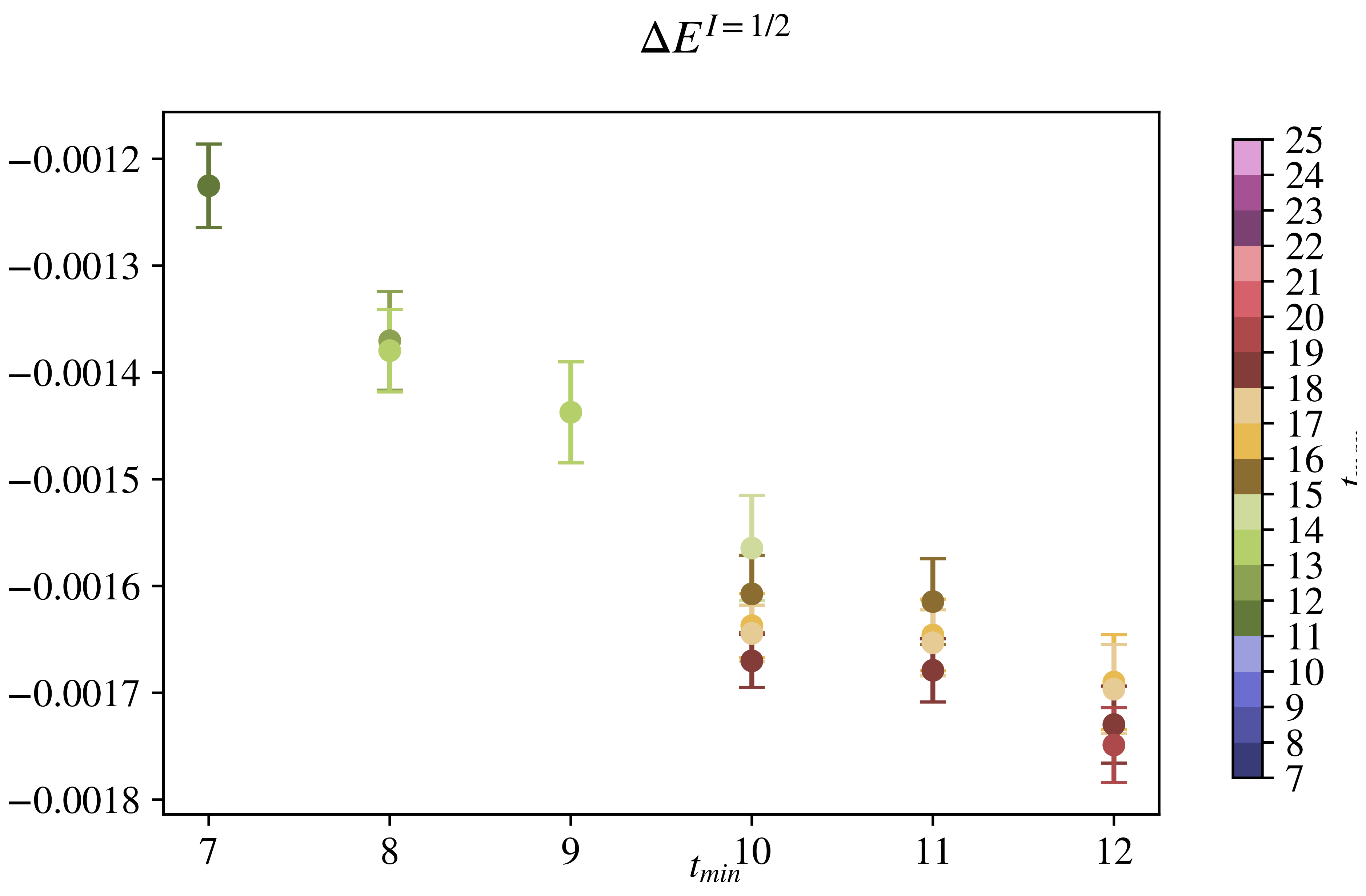
$$R_{K\pi}^{ATW}(t) = A_{\pi K\pi K}\frac{e^{-m_K t}e^{-m_\pi(T-t)}}{\left(e^{-m_\pi t} + e^{-m_\pi(T-t)}\right)\left(e^{-m_K t} + e^{-m_K(T-t)}\right)}$$

$$+A_{KK\pi\pi}\frac{e^{-m_\pi t}e^{-m_K(T-t)}}{\left(e^{-m_\pi t} + e^{-m_\pi(T-t)}\right)\left(e^{-m_K t} + e^{-m_K(T-t)}\right)}$$

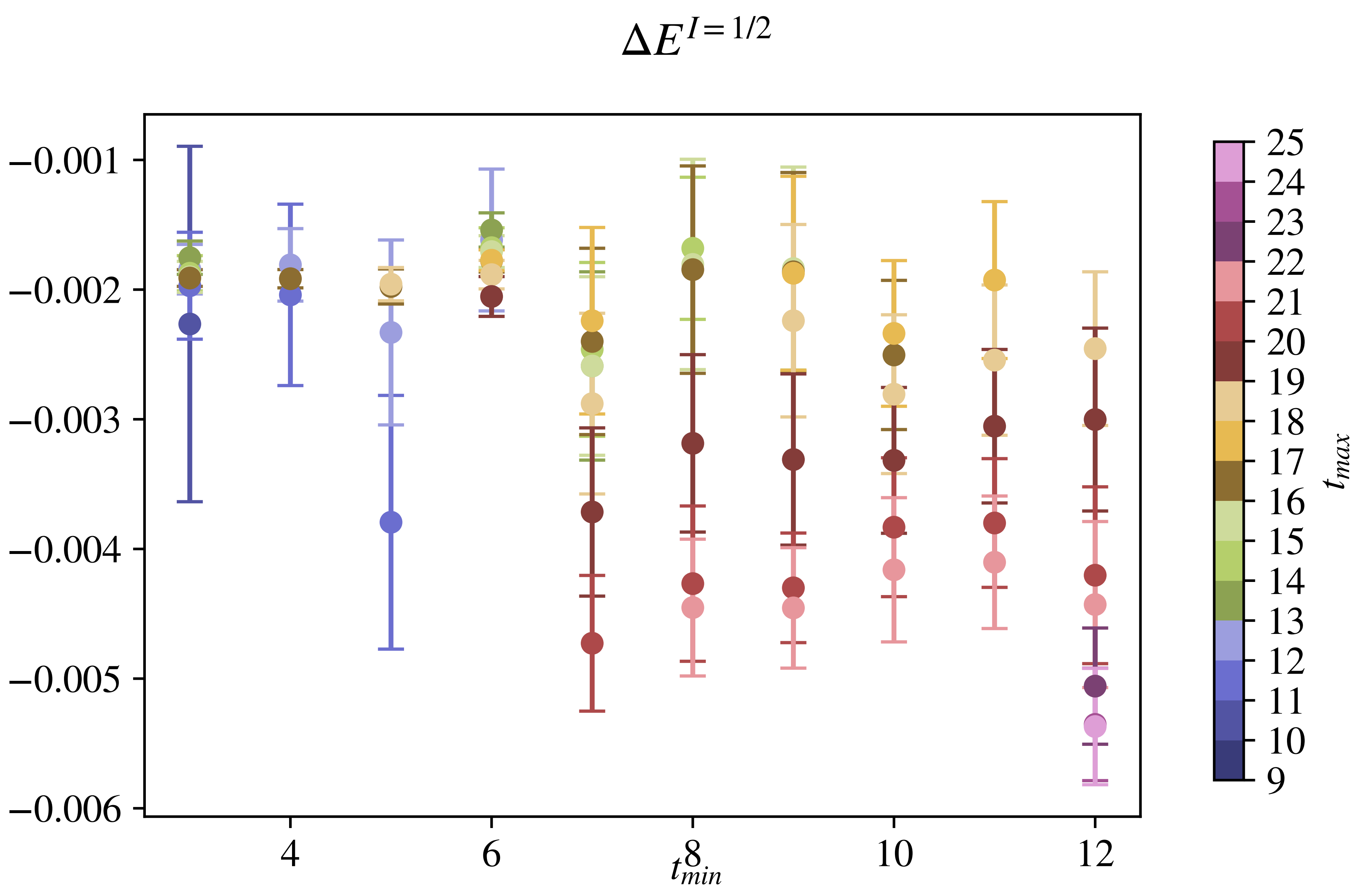
$$\xrightarrow{t\to 0} A_{\pi K\pi K}(1 + m_\pi t)) + A_{KK\pi\pi}(1 + m_K t)$$

extra: excited state fits

$m_\pi a = 0.0803(2), m_K = 0.2884(2)$



Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ using **ground state** ansatz.



Variation in fit values of $\Delta E_{K\pi}^{I=1/2}$ using **excited state** ansatz.

<u>Explanations</u>

1. excited states
   - distillation? See next talk by N. Lachini