WILL DETMOLD

Massachusetts Institute of Technology

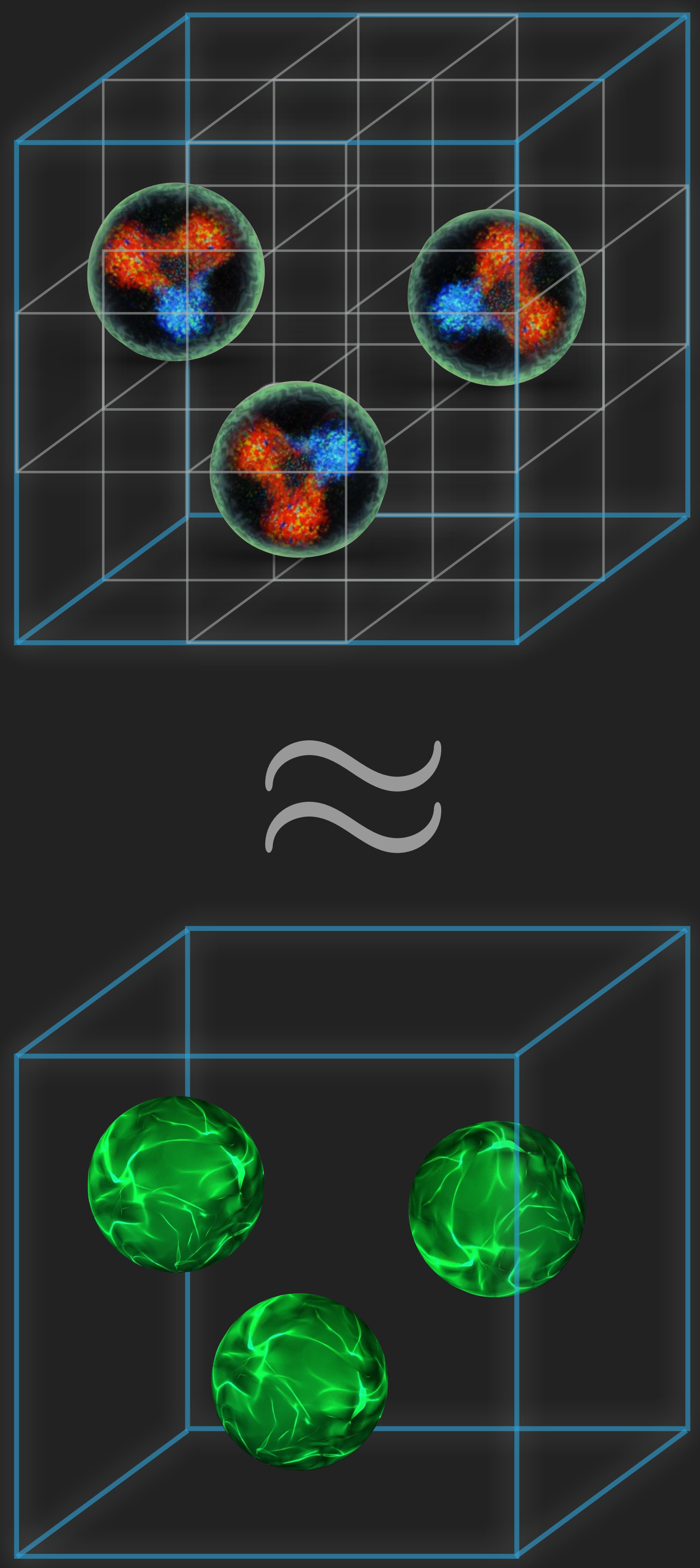
# FINITE VOLUME PIONLESS EFFECTIVE FIELD THEORY FOR NUCLEAR SYSTEMS

2102.04329 with P Shanahan
2202.03530 with X Sun, D Luo and P Shanahan
work in progress (D Luo, P Shanahan, F Romero-Lopez)

Lattice 2022, Bonn, Aug 8th, 2022

# FINITE VOLUME EFFECTS

- Major issue for LQCD calculations of nuclei is finite volume (FV) effects
- Well-known Lüscher method to understand FV effects in B=2 spectrum (and now B=3)
- Less well-developed technologies for effects on matrix elements
- Alternative: direct matching between LQCD and EFT in the same FV
  - Use EFT to extrapolate to infinite volume
  - Spectroscopy and matrix elements
  - Pionless EFT for simplicity



# PIONLESS EFT

- Leading order pionless EFT Hamiltonian

$$H = -\frac{1}{2M_N}\sum_i \nabla_i^2 + \sum_{i<j} V_2\left(\mathbf{r}_{ij}\right) + \sum_{i<j<k} V_3\left(\mathbf{r}_{ij}, \mathbf{r}_{jk}\right)$$

- Two- and three-body interactions

Three body low energy constant

$$V_2\left(\mathbf{r}_{ij}\right) = \left(\boxed{C_0} + \boxed{C_1}\sigma^{(i)} \cdot \sigma^{(j)}\right) g_\Lambda\left(\mathbf{r}_{ij}\right). \quad V_3\left(\mathbf{r}_{ij}, \mathbf{r}_{jk}\right) = \boxed{D_0}\sum_{\mathrm{cyc}} g_\Lambda\left(\mathbf{r}_{ij}\right) g_\Lambda\left(\mathbf{r}_{jk}\right)$$

Two body low energy constants

- Implement with Gaussian regulator with shifted copies to satisfy periodic boundary conditions

Regulator scale

$$g_\Lambda(\mathbf{r}, L) = \frac{\Lambda^3}{8\pi^{3/2}} \prod_{\alpha\in\{x,y,z\}} \sum_{q^{(\alpha)}=-\infty}^{\infty} \exp\left(-\boxed{\Lambda^2}\left(r^{(\alpha)} - Lq^{(\alpha)}\right)^2/4\right)$$

# VARIATIONAL METHOD

- For any wavefunction, the variational method can bound energies of eigenstates

$$E_h \leq \mathcal{E}\left[\Psi_h\right] = \frac{\int \Psi_h(\mathbf{x})^* H \Psi_h(\mathbf{x}) d\mathbf{x}}{\int \Psi_h(\mathbf{x})^* \Psi_h(\mathbf{x}) d\mathbf{x}}$$

- Find wave functions that provide the most stringent bounds
  - Some increasingly large set of basis functions
  - Wavefunction as a neural network (ongoing)
- Correlated Gaussians in many-particle coordinate space
  - Computationally efficient (integrals analytic) and expressive

# CORRELATED GAUSSIANS

- 3n spatial coordinates: $\mathbf{x} = (\mathbf{r}_1, \ldots, \mathbf{r}_n)$ with $\mathbf{x}^{(\alpha)} = (\mathbf{r}_{1,\alpha}, \ldots, \mathbf{r}_{n,\alpha})$
- Wavefunction built from correlated shifted Gaussians in each Cartesian direction $\alpha \in \{x, y, z\}$

Vector

$$\Psi_\infty^{(\alpha)}\left(A^{(\alpha)}, B^{(\alpha)}, \mathbf{d}^{(\alpha)}; \mathbf{x}^{(\alpha)}\right) = \exp\left[-\frac{1}{2}\mathbf{x}^{(\alpha)T}\boxed{A^{(\alpha)}}\mathbf{x}^{(\alpha)} - \frac{1}{2}\left(\mathbf{x}^{(\alpha)} - \boxed{\mathbf{d}^{(\alpha)}}\right)^T \boxed{B^{(\alpha)}}\left(\mathbf{x}^{(\alpha)} - \boxed{\mathbf{d}^{(\alpha)}}\right)\right]$$

Symmetric matrix    Diagonal matrix

Wavefunction parameters

- Impose periodicity:

$$\Psi_L^{(\alpha)}\left(A^{(\alpha)}, B^{(\alpha)}, \mathbf{d}^{(\alpha)}; \mathbf{x}^{(\alpha)}\right) = \sum_{\mathbf{b}^{(\alpha)}} \Psi_\infty^{(\alpha)}\left(A^{(\alpha)}, B^{(\alpha)}, \mathbf{d}^{(\alpha)}; \mathbf{x}^{(\alpha)} - \mathbf{b}^{(\alpha)}L\right)$$

$$\Psi_L(A, B, \mathbf{d}; \mathbf{x}) = \prod_{\alpha\in\{x,y,z\}} \Psi_L^{(\alpha)}\left(A^{(\alpha)}, B^{(\alpha)}, \mathbf{d}^{(\alpha)}; \mathbf{x}^{(\alpha)}\right)$$

- Symmetrise spatial wavefunction under particle interchange

$$\Psi_L^{\text{sym}}(A, B, \mathbf{d}; \mathbf{x}) = \sum_{\mathcal{P}} \Psi_L\left(A_{\mathcal{P}}, B_{\mathcal{P}}, \mathbf{d}_{\mathcal{P}}; \mathbf{x}\right)$$

X. Y. Yin and D. Blume, Phys. Rev. A 87, 063609 (2013)

# PIONLESS EFT

- Focus on nuclei up to $^4$He in s-wave
- Spatially symmetric, antisymmetric in spin-flavour

$$\Psi_h^{(N)}(\mathbf{x}) = \sum_{j=1}^{N} c_j \Psi_L^{\mathrm{sym}}\left(A_j, B_j, \mathbf{d}_j; \mathbf{x}\right) |\chi_h\rangle$$

where eg

$$\left|\chi^3\mathrm{H}, j_z = 1/2\right\rangle = \frac{1}{\sqrt{6}}\left[\left|n^\uparrow p^\uparrow n^\downarrow\right\rangle - \left|n^\downarrow p^\uparrow n^\uparrow\right\rangle - \left|p^\uparrow n^\uparrow n^\downarrow\right\rangle + \left|p^\uparrow n^\downarrow n^\uparrow\right\rangle - \left|n^\uparrow n^\downarrow p^\uparrow\right\rangle + \left|n^\downarrow n^\uparrow p^\uparrow\right\rangle\right]$$

- General ansatz with parameters: $\theta = \{\mathbf{c}, A, B, \mathbf{d}\}$

# STOCHASTIC VARIATIONAL METHOD

- Wavefunction ansatz contains linear and nonlinear parameters
- Linear parameters can be optimised via an eigenvalue problem
- Nonlinear parameters optimised stochastically [Varga & Suzuki 1995]

1. Propose a new Gaussian term with stochastically chosen parameters
2. Calculate matrix elements of Hamiltonian between terms in set

$$[\mathbb{H}]_{ij} \equiv \int \Psi_i(\mathbf{x})^* \langle \chi_h | H | \chi_h \rangle \Psi_j(\mathbf{x}) d\mathbf{x} \quad [\mathbb{N}]_{ij} \equiv \int \Psi_i(\mathbf{x})^* \Psi_j(\mathbf{x}) d\mathbf{x}$$

3. Solve generalised eigenvalue problem to determine coefficient

$$\mathbb{H}\mathbf{c} = \lambda \mathbb{N}\mathbf{c}$$

# STOCHASTIC VARIATIONAL METHOD

- Energy bound

$$E_h \leq \mathcal{E}\left[\Psi_h\right] = \frac{\int \Psi_h(\mathbf{x})^* H \Psi_h(\mathbf{x}) d\mathbf{x}}{\int \Psi_h(\mathbf{x})^* \Psi_h(\mathbf{x}) d\mathbf{x}}$$

$$[\mathbb{H}]_{ij} \equiv \int \Psi_i(\mathbf{x})^* \left\langle \chi_h | H | \chi_h \right\rangle \Psi_j(\mathbf{x}) d\mathbf{x} \quad \mathbb{H} = \mathbb{K} + \mathbb{V}_2 + \mathbb{V}_3$$

- Integrals can be performed analytically

$$[\mathbb{N}]_{ij} \equiv \int \Psi_L^{\mathrm{sym}}\left(A_i, B_i, \mathbf{d}_i; \mathbf{x}\right)^* \Psi_L^{\mathrm{sym}}\left(A_j, B_j, \mathbf{d}_j; \mathbf{x}\right) d\mathbf{x}$$

$$= \sum_{\mathcal{P},\mathcal{P}'} \prod_{\alpha \in \{x,y,z\}} \sqrt{\frac{(2\pi)^n}{\mathrm{Det}\left[C^{(\alpha)}_{i\mathcal{P};j\mathcal{P}'}\right]}} \sum_{\mathbf{b}^{(\alpha)}}^{(\alpha)} |\leq \tilde{b} \exp\left[-\frac{1}{2}\Omega^{(\alpha)}_{i\mathcal{P};\mathcal{P}'}\right]$$

Objects built from the wavefunction parameters

# STOCHASTIC VARIATIONAL METHOD

- Energy bound

$$E_h \leq \mathcal{E}\left[\Psi_h\right] = \frac{\int \Psi_h(\mathbf{x})^* H \Psi_h(\mathbf{x}) d\mathbf{x}}{\int \Psi_h(\mathbf{x})^* \Psi_h(\mathbf{x}) d\mathbf{x}}$$

$$[\mathbb{H}]_{ij} \equiv \int \Psi_i(\mathbf{x})^* \langle \chi_h | H | \chi_h \rangle \Psi_j(\mathbf{x}) d\mathbf{x} \quad \mathbb{H} = \mathbb{K} + \mathbb{V}_2 + \mathbb{V}_3$$

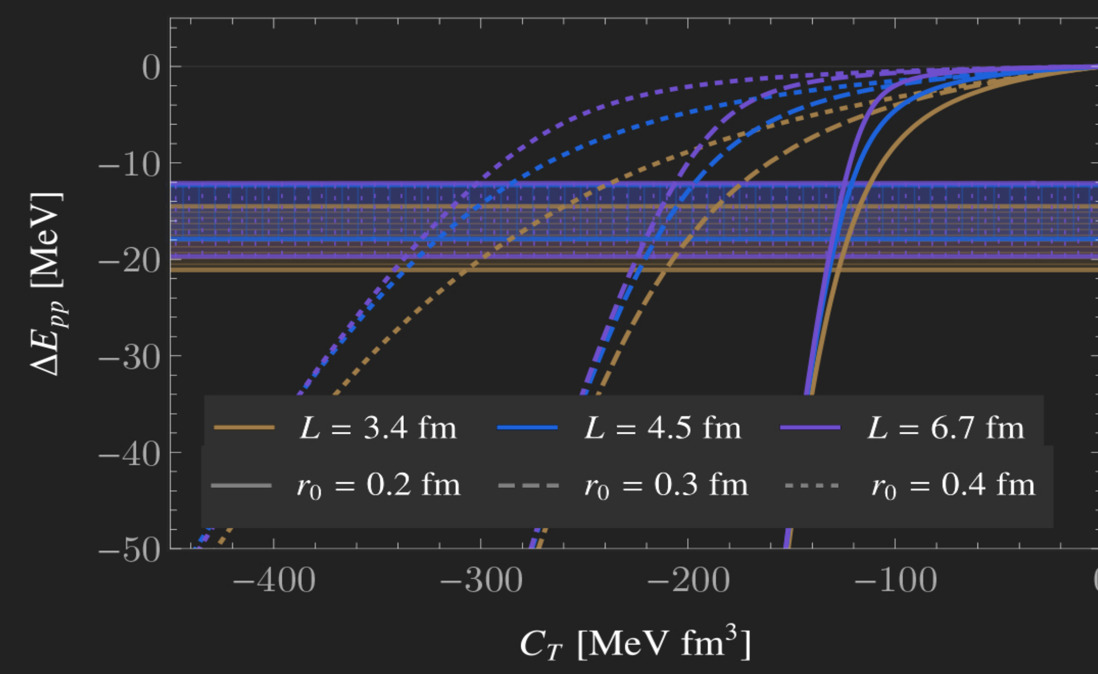
- Integrals can be performed analytically

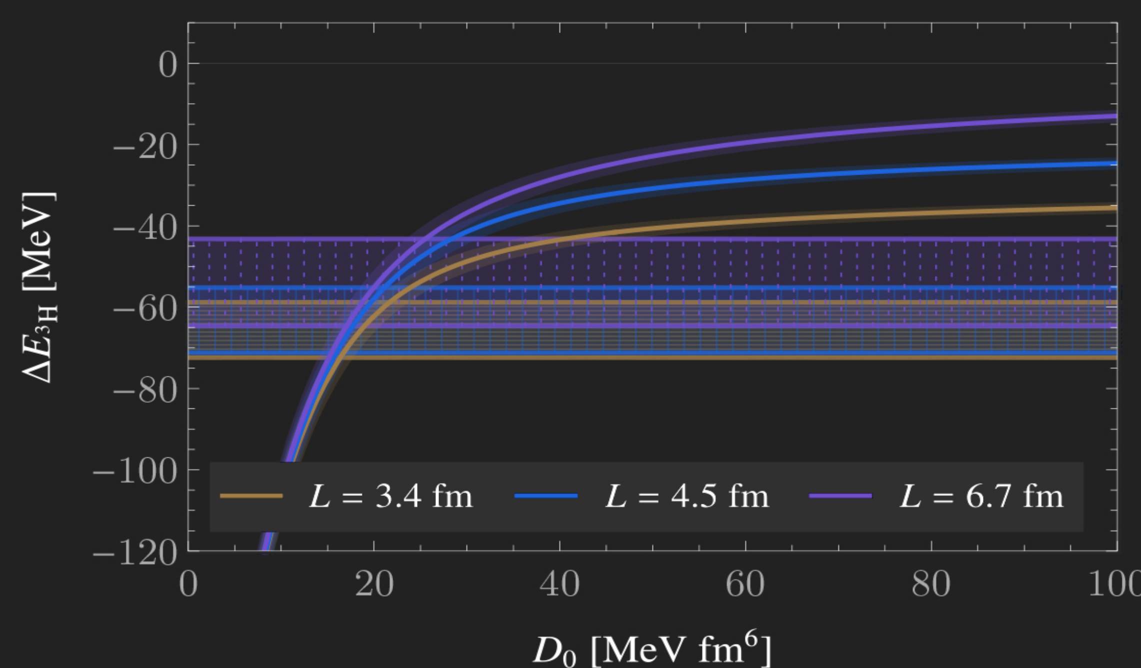
$$\begin{aligned}
[\mathbb{V}_3]_{ij} &\equiv D_0 \sum_{a\neq b\neq c}^{\text{cyc}} \int \Psi_L^{\text{sym}}\left(A_i, B_i, \mathbf{d}_i; \mathbf{x}\right)^* g_\Lambda\left(\mathbf{x}_a - \mathbf{x}_b, L\right) g_\Lambda\left(\mathbf{x}_b - \mathbf{x}_c, L\right) \Psi_L^{\text{sym}}\left(A_j, B_j, \mathbf{d}_j; \mathbf{x}\right) d\mathbf{x} \\
&= D_0 \left(\frac{\Lambda^3}{8\pi^{3/2}}\right)^2 \sum_{\mathcal{P},\mathcal{P}'} \sum_{a\neq b\neq c}^{\text{cyc}} \prod_{\alpha\in\{x,y,z\}} \sqrt{\frac{(2\pi)^n}{\text{Det}\left[\widehat{C}_{i\mathcal{P};;\mathcal{P}'}\right]}} \exp\left[-\frac{1}{2}\left(\mathbf{d}_{i\mathcal{P}}^{(\alpha)} \cdot B_{i\mathcal{P}}^{(\alpha)} \cdot \mathbf{d}_{i\mathcal{P}}^{(\alpha)} + \mathbf{d}_{j'}^{(\alpha)} \cdot B_{j\mathcal{P}'}^{(\alpha)} \cdot \mathbf{d}_{j'}^{(\alpha)}\right)\right] \\
&\times \sum_{\mathbf{b}^{(\alpha)}}^{\left|\mathbf{b}^{(\alpha)}\right|\leq\tilde{b}} \exp\left[-\frac{1}{2}\left(\left(L\mathbf{b}^{(\alpha)}\right) \cdot \left(A_{i\mathcal{P}}^{(\alpha)} + B_{i\mathcal{P}}^{(\alpha)}\right) \cdot \left(L\mathbf{b}^{(\alpha)}\right) + 2\mathbf{d}_{i\mathcal{P}}^{(\alpha)} \cdot B_{i\mathcal{P}}^{(\alpha)} \cdot \left(L\mathbf{b}^{(\alpha)}\right) - \Xi^{(\alpha)} \cdot \left[\widehat{C}_{i\mathcal{P};j\mathcal{P}'}^{(\alpha)}\right]^{-1} \cdot \vdots\right.\right. \\
&\times \sum_{q^{(\alpha)}=-\tilde{q}}^{\tilde{q}} \exp\left[-\frac{L^2}{r_0^2} q^{(\alpha)2} + \frac{q^{(\alpha)2}L^2}{2r_0^4} \mathfrak{P}_v^{[a,b]} \cdot \left[\widehat{C}_{i\mathcal{P};j\mathcal{P}\mathcal{P}}^{(\alpha)}\right]^{-1} \cdot \mathfrak{P}_v^{[a,b]} + \frac{q^{(\alpha)}L}{r_0^2} \Xi^{(\alpha)} \cdot \left[\widehat{C}_{i\mathcal{P};j\mathcal{P}\mathcal{P}}^{(\alpha)}\right]^{-1} \cdot \mathfrak{P}_v^{[a,b]}\right] \\
&\times \sum_{t^{(\alpha)}=-\tilde{q}}^{\tilde{q}} \exp\left[-\frac{L^2}{r_0^2} t^{(\alpha)2} + \frac{t^{(\alpha)2}L^2}{2r_0^4} \mathfrak{P}_v^{[b,c]} \cdot \left[\widehat{C}_{i\mathcal{P};j\mathcal{P}\mathcal{P}'}^{(\alpha)}\right]^{-1} \cdot \mathfrak{P}_v^{[b,c]} + \frac{t^{(\alpha)}L}{r_0^2} \Xi^{(\alpha)} \cdot \left[\widehat{C}_{i\mathcal{P};j\mathcal{P}'}^{(\alpha)}\right]^{-1} \cdot \mathfrak{P}_v^{[b,c]}\right] \\
&\times \exp\left[\frac{t^{(\alpha)}q^{(\alpha)}L^2}{r_0^4} \mathfrak{P}_v^{[b,c]} \cdot \left[\widehat{C}_{i\mathcal{P};j\mathcal{P}'}^{(\alpha)}\right]^{-1} \cdot \mathfrak{P}_v^{[a,b]}\right)\right]
\end{aligned}$$

Objects built from the wf parameters

# STOCHASTIC VARIATIONAL METHOD

- Match onto LQCD FV energies to determine nuclear wave functions
  - NPLQCD 2012, 2017, $m_\pi$=806 MeV
  - 2 and 3-body energies fix NN and NNN contact interactions
  - $^4$He has large uncertainties so does not improve constraints
- Determines infinite volume bindings
- NB: all LQCD calculations here at unphysically large quark masses




[WD, P Shanahan. 2102.04329; Eliyahu, Bazak, Barnea *Phys.Rev.C* 102 (2020), 044003 ]

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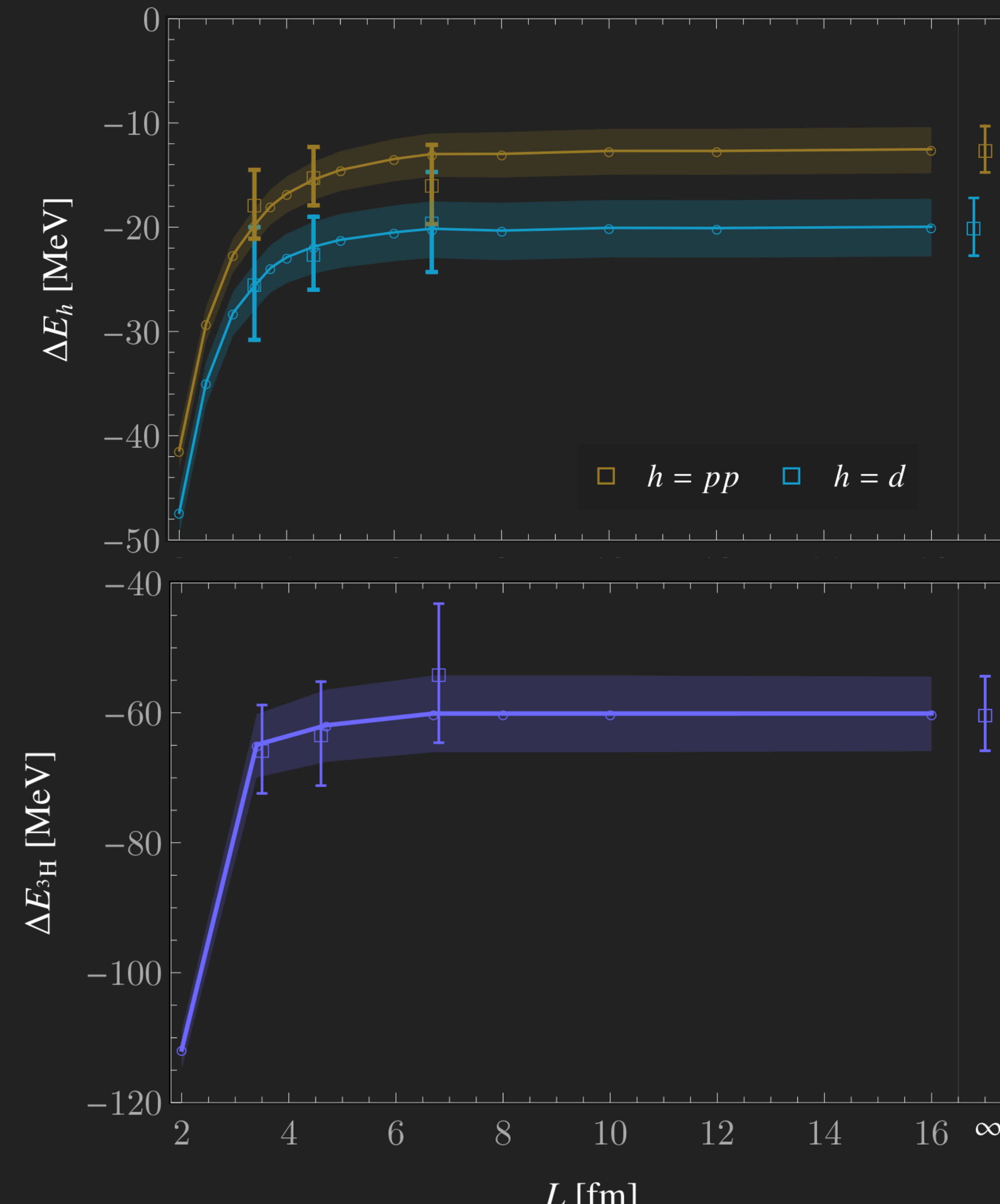
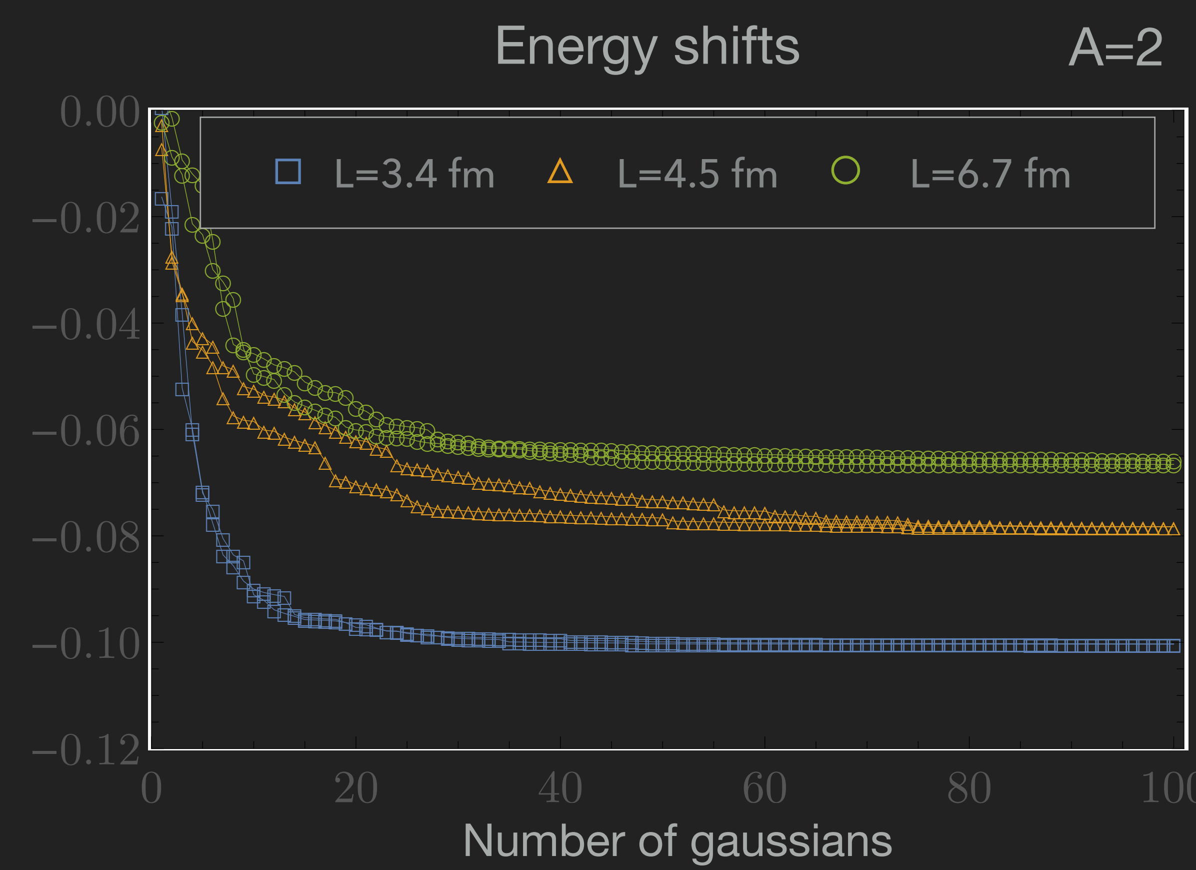
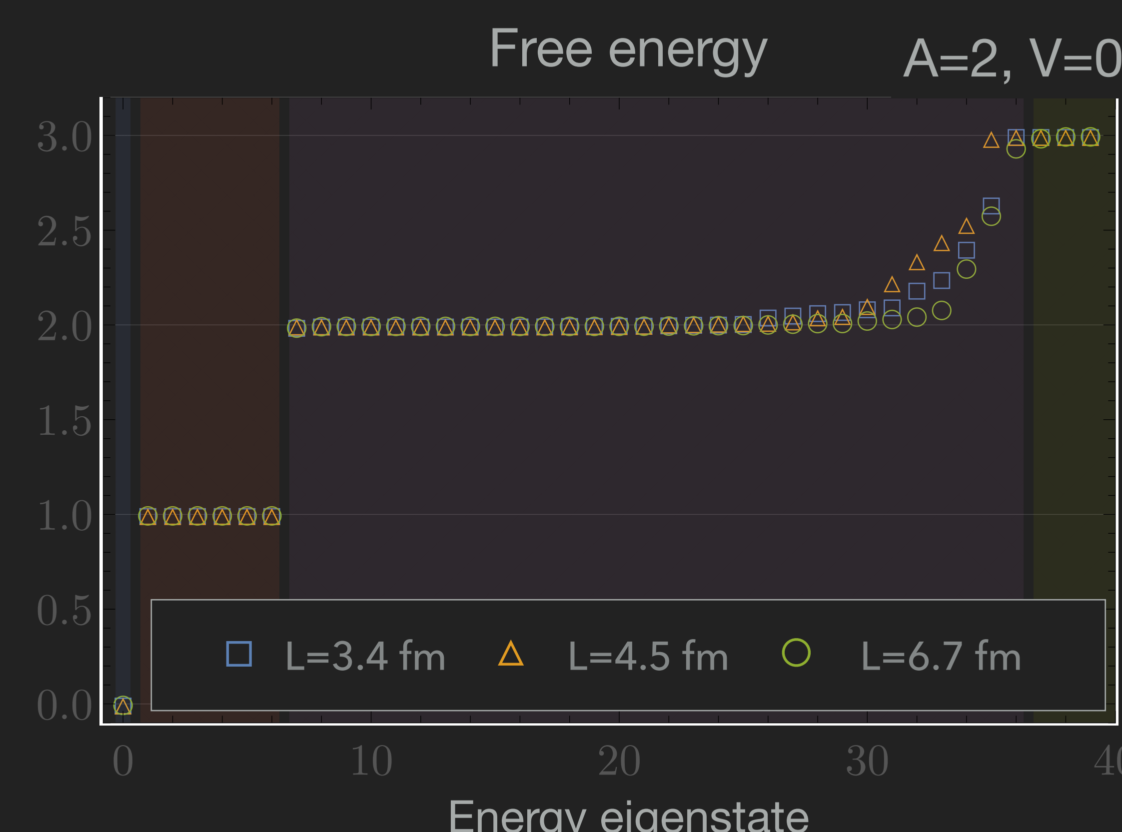



[WD, P Shanahan. 2102.04329; Eliyahu, Bazak, Barnea *Phys.Rev.C* 102 (2020), 044003 ]

# SVM

- SVM works but needs 100s of Gaussians to converge to FV ground state
  [M. Eliyahu, B. Bazak, and N. Barnea, PRC 2020]
  - Able to represent bound states
  - Also able to represent scattering states
- Cubic boundary conditions increase the cost significantly
- Going beyond A=3 is very challenging in FV



# DIFFERENTIABLE PROGRAMMING

- Obviously better to optimise all parameters in wavefunction ansatz but how?
  - Need gradients of objective function (energy bound) w.r.t parameters

$$\mathcal{E}\left[\Psi_h^{(N)}(\theta)\right] = \frac{\mathbf{c}\cdot(\mathbb{K}+\mathbb{V}_2+\mathbb{V}_3)\cdot\mathbf{c}}{\mathbf{c}\cdot\mathbb{N}\cdot\mathbf{c}}$$

$$\nabla_\theta\mathcal{E} = -\frac{\mathbf{c}\cdot(\mathbb{K}+\mathbb{V}_2+\mathbb{V}_3)\cdot\mathbf{c}}{(\mathbf{c}\cdot\mathbb{N}\cdot\mathbf{c})^2}\nabla_\theta(\mathbf{c}\cdot\mathbb{N}\cdot\mathbf{c}) + \frac{1}{\mathbf{c}\cdot\mathbb{N}\cdot\mathbf{c}}\left(\nabla_\theta(\mathbf{c}\cdot\mathbb{K}\cdot\mathbf{c}) + \nabla_\theta\left(\mathbf{c}\cdot\mathbb{V}_2\cdot\mathbf{c}\right) + \nabla_\theta\left(\mathbf{c}\cdot\mathbb{V}_3\cdot\mathbf{c}\right)\right)$$

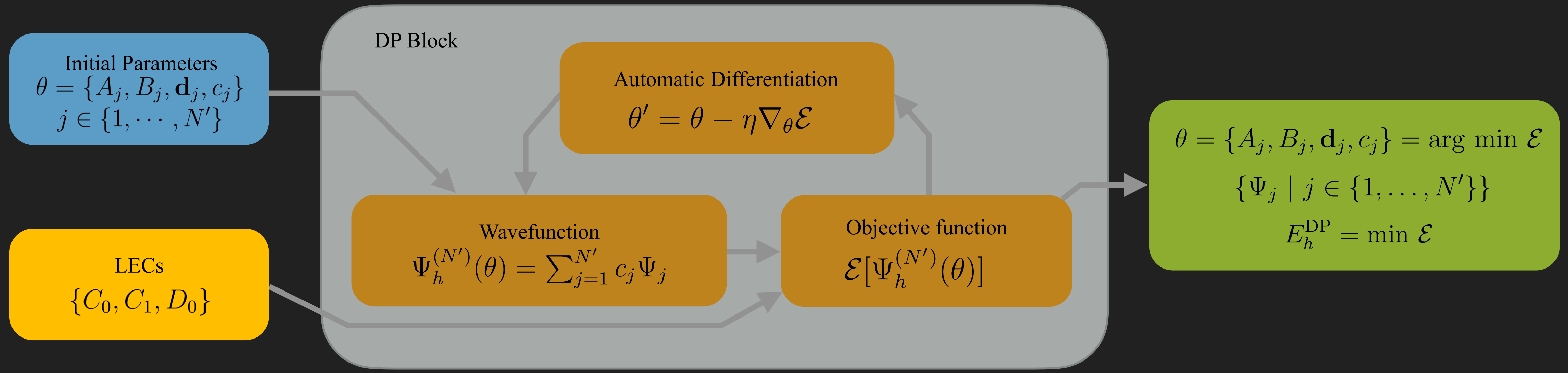
$$\nabla_\theta(\mathbf{c}\cdot\mathbb{X}\cdot\mathbf{c}) = \sum_{i,j}\left(\nabla_\theta\left(c_i c_j\right)[\mathbb{X}]_{ij} + c_i c_j \nabla_\theta[\mathbb{X}]_{ij}\right)$$

- Very large chain rule expressions can be computed using automatic differentiation
  - Automatic differentiation (AD) development driven by ML: at the heart of backpropagation for training of neural networks
  - Very efficient and easy too use implementations in ML frameworks

Sun et al., 2202.03530

# DIFFERENTIABLE PROGRAMMING

- Implement in two stages
  - Stage 1: DP to optimise a set of N′ Gaussians given input LECs
  - Stage 2: combine sets of optimised Gaussians using GEVP



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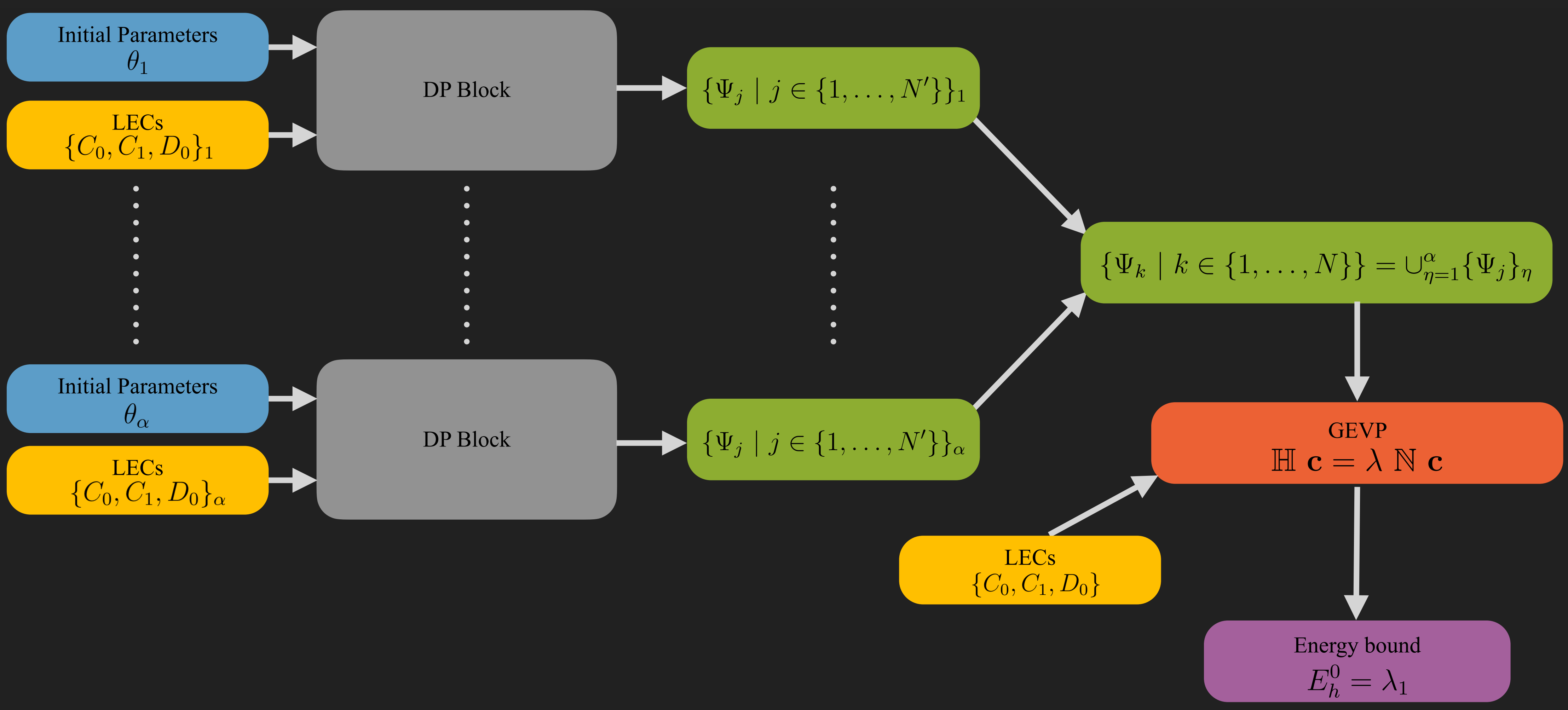
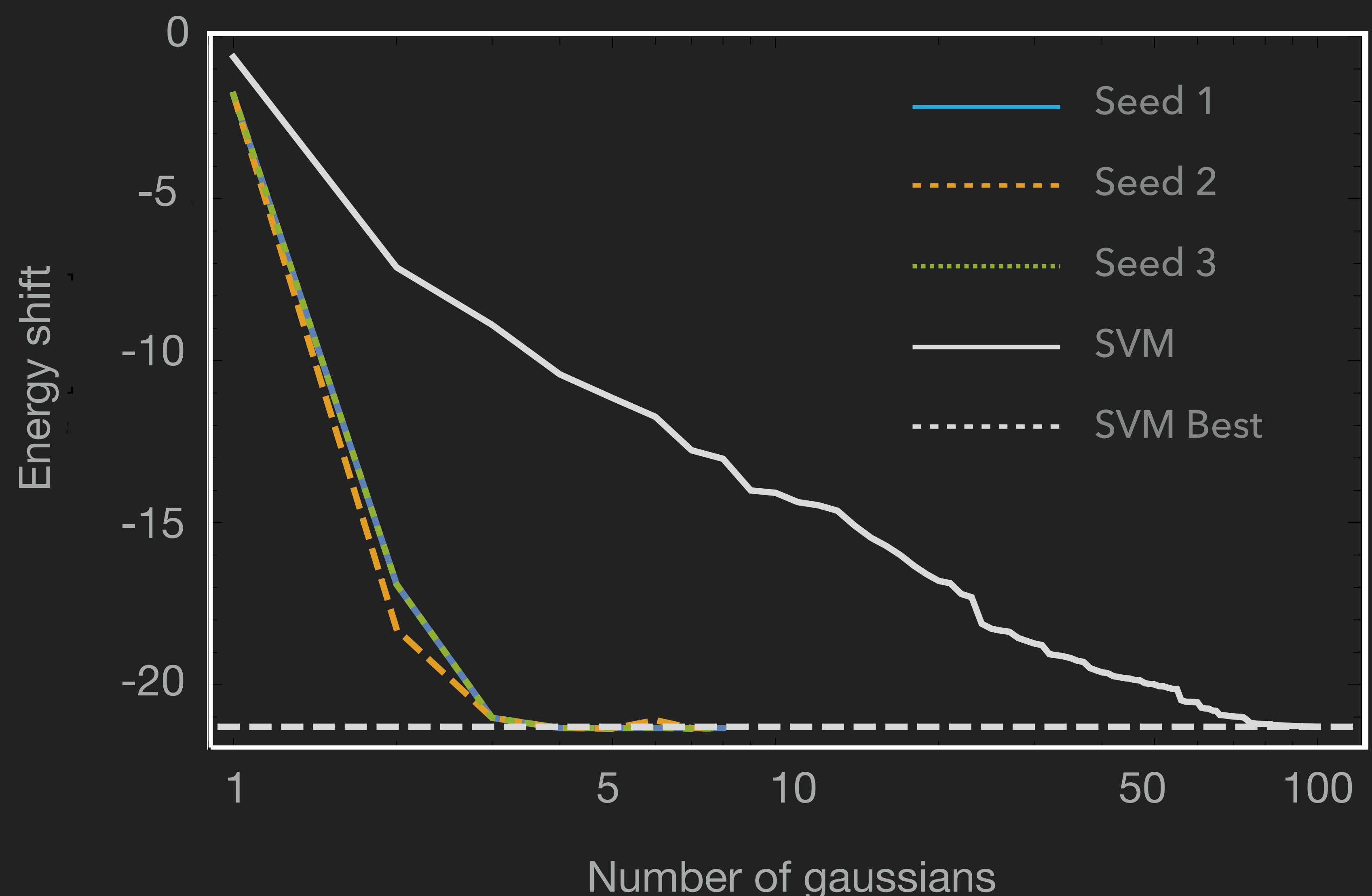


# DIFFERENTIABLE PROGRAMMING

- Batched AD controls memory use (necessary for GPU)
  - Computational graph of the program computed in chunks and discarded after gradient computed
- First order gradient descent based on AD gradients
  - Self-adaptive step sizes
  - Step clipping
- Optimisation cost for $n$-body $N$-term wavefunction: $O\left(N^2 n! n^3\right)$
  - Sequential construction for $N$=$\alpha$×$N$'-term: $O\left(\alpha N'^2 n! n^3\right)$

# DIFFERENTIABLE PROGRAMMING

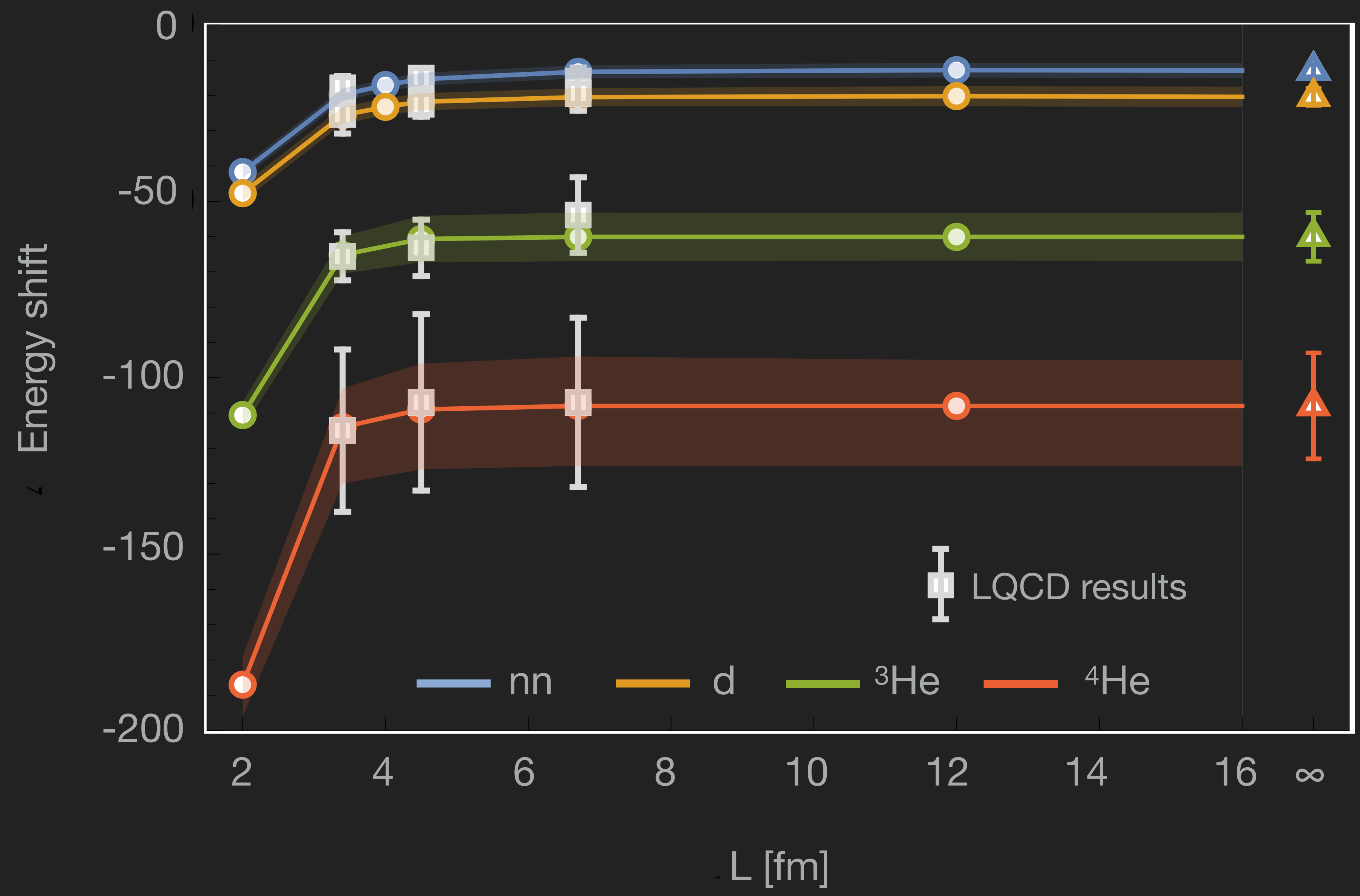
- DP-GEVP converges MUCH faster than SVM and finds slightly lower minimum




Sun et al., 2202.03530

# FVEFT SPECTRA USING DIFFERENTIABLE PROGRAMMING

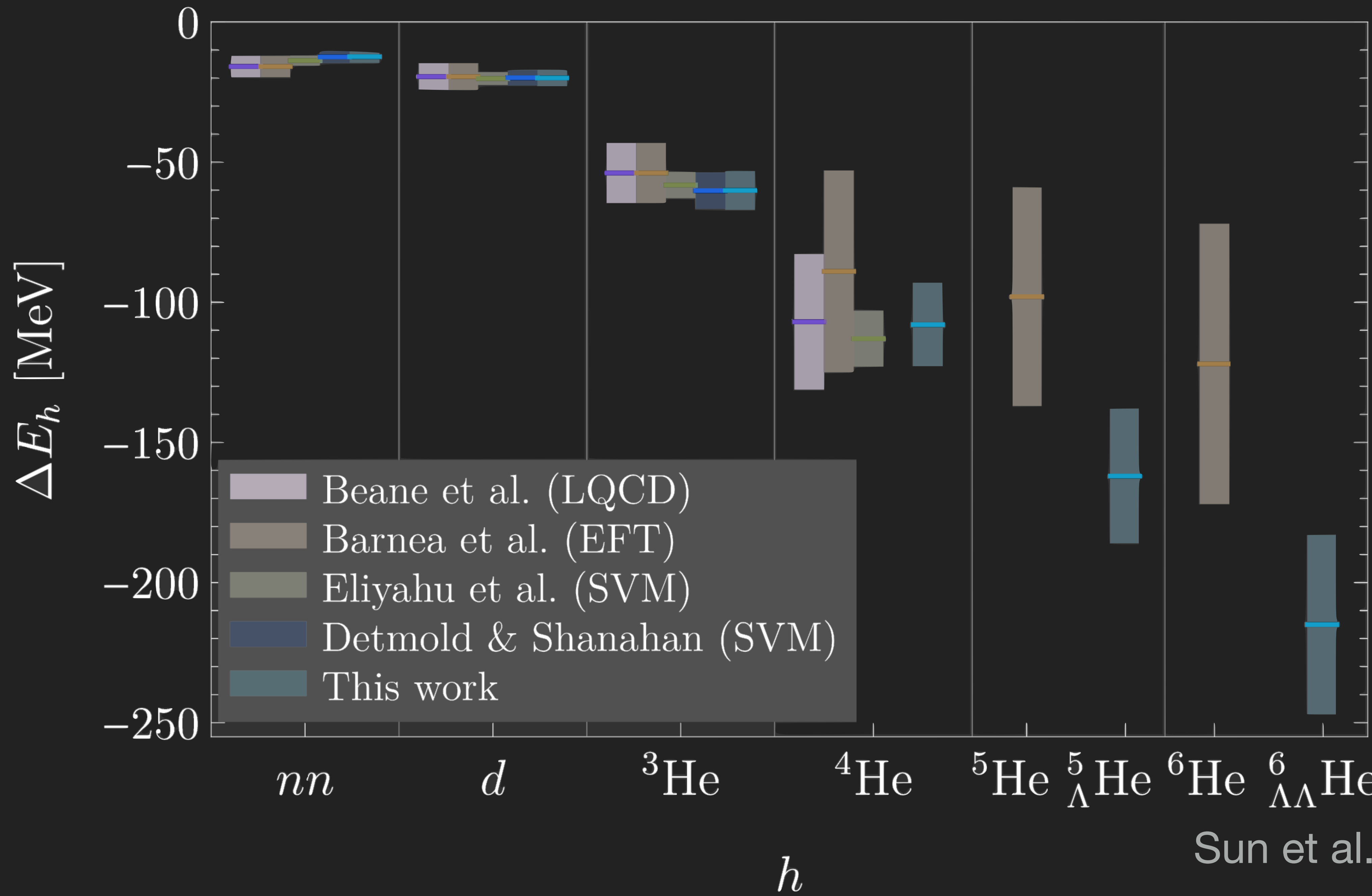
- Once LECs determined can study volume dependence




Sun et al., 2202.03530

# INFINITE VOLUME SPECTRA USING DIFFERENTIABLE PROGRAMMING

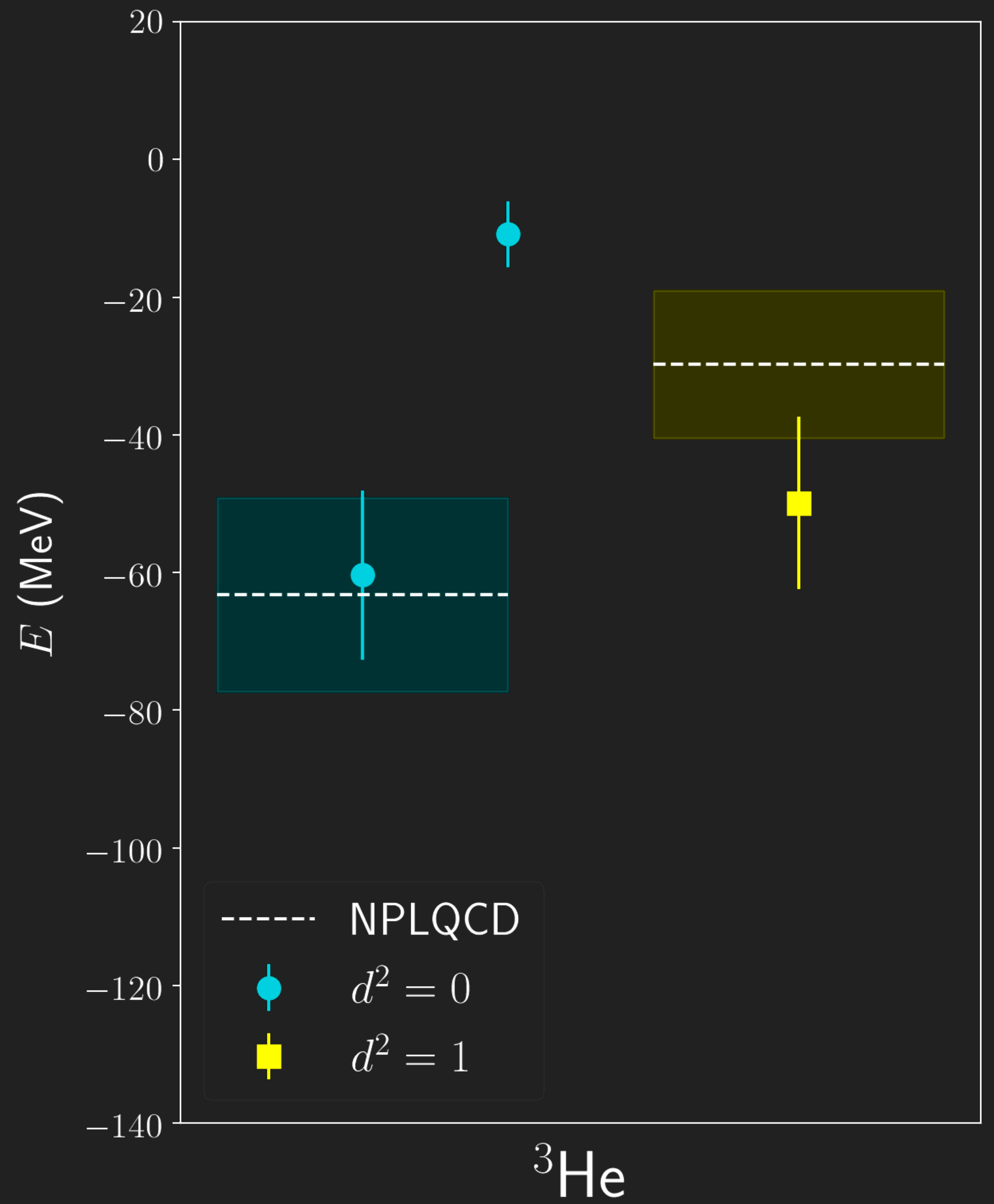
- Diff programming offers better way to optimise wavefunction




Sun et al., 2202.03530

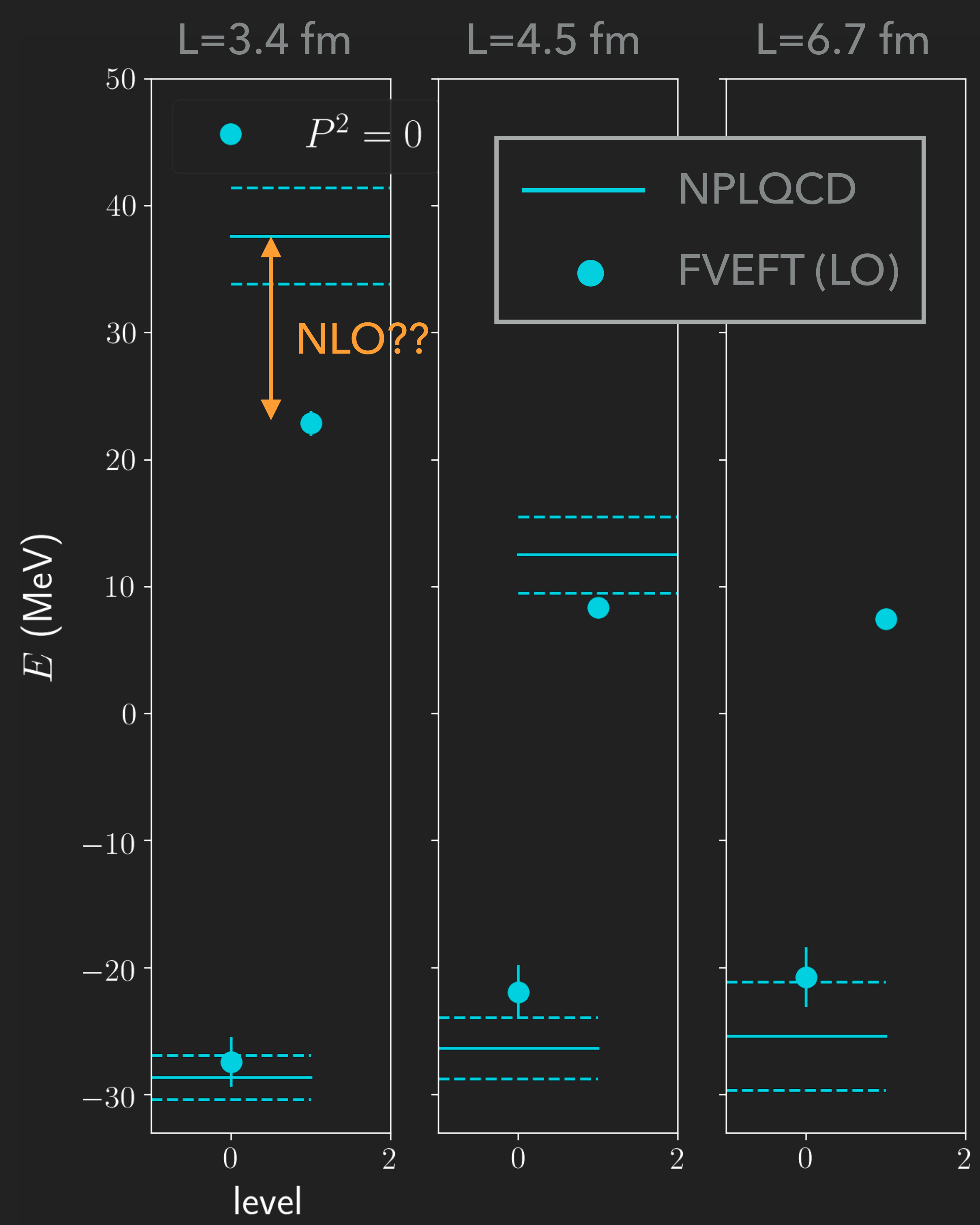
# BOOSTED SYSTEMS

- Gaussian WFs do not have definite total momentum P but FV eigenstates do
- Can measure $P^2$ on optimised states
  - Find values very close to integers up to $P^2=3$
  - Better optimisation required to go higher
- NPLQCD 2012 results for A=2 for $P^2=0,1,2$ and A=3 for $P^2=0,1$
  - EFT matched only to $P^2=0$
- In principle can also project to given total momentum by adding constraint term to Hamiltonian



# BEYOND LEADING ORDER - EXCITED STATES

- Variational method produces more than ground state
- LQCD not restricted to ground states
  - Excited FV NN scattering states from NPLQCD 2012 ($A_1^+$ cubic rep)
  - Many more NN states from recent variational study
- Figure shows FVEFT with only LO
- Currently implementing terms from NLO Lagrangian
  - Will match to excited states to determine NLO counterterms



# PIONLESS EFT

- LQCD-EFT matching is a powerful tool to extrapolate to infinite  volume
  - Spectroscopy and matrix elements
- Differential programming approach vastly improves on SVM
  - More efficient representation of states
  - Able to go to larger nuclei
- Extensions
  - Hypernuclei and excited states
  - NLO pionless EFT, Pionful EFT

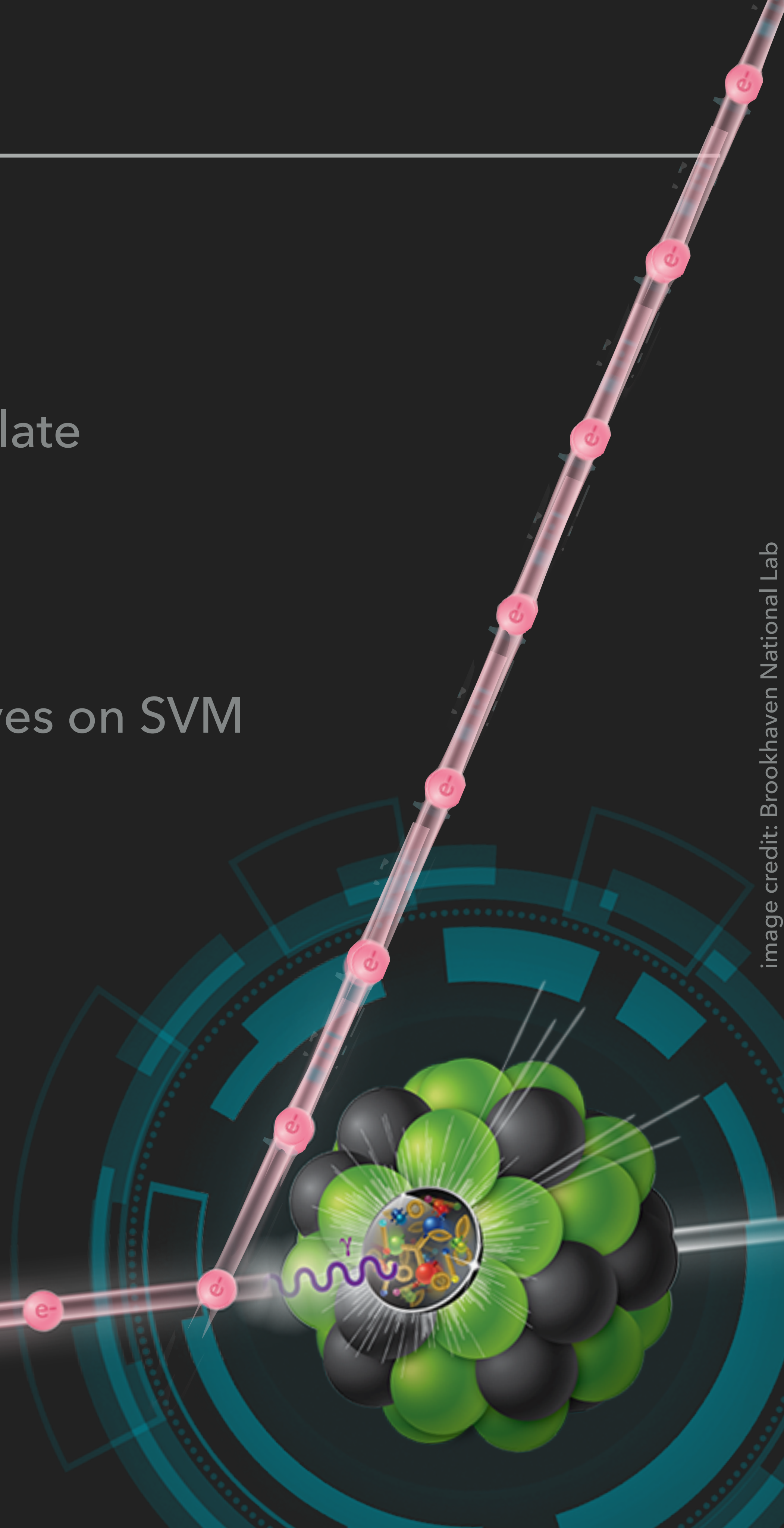
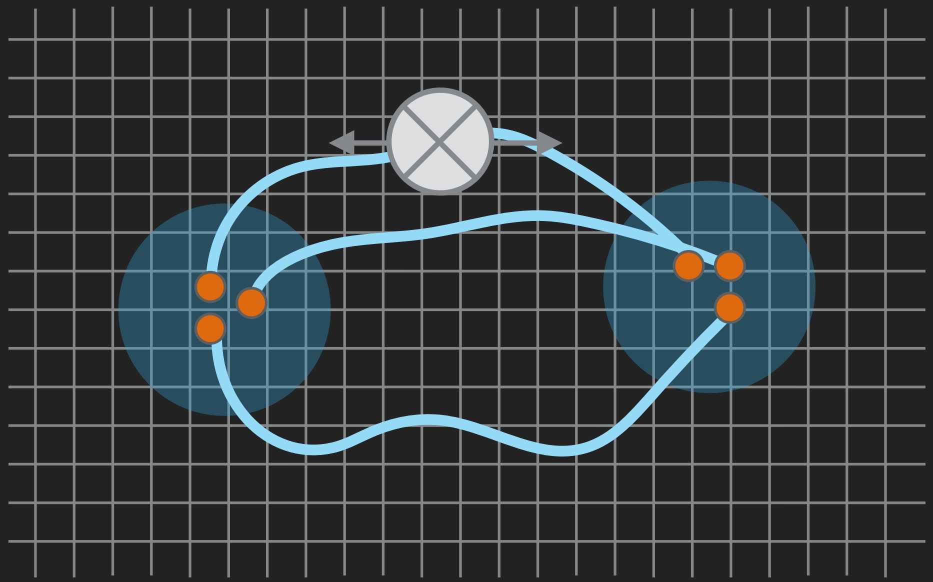


image credit: Brookhaven National Lab

FIN

# LQCD MOMENTUM FRACTIONS

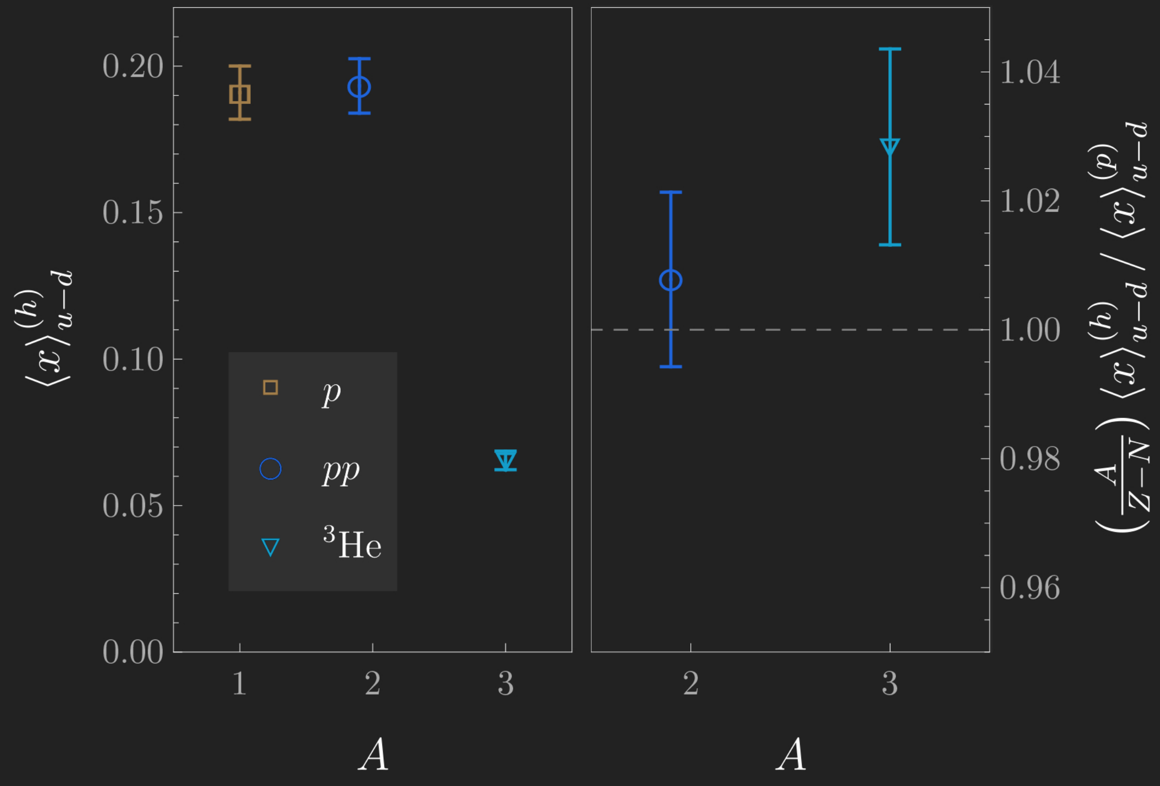


- NPLQCD calculation of various matrix elements in nuclei: focus on momentum fraction
  - Local twist-2 operator matrix element

$$\bar{q}\gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}} q$$

  - Unphysical quark masses for which pion mass is 806 MeV
  - Single fixed volume L~4.5 fm
  - *pp* and $^3$*He* systems

[WD et al. PRL 2021]




| | $p$ | $pp$ | $^3$He |
|---|---|---|---|
| $\langle x \rangle^{(h)}_{u-d}$ | 0.191(1)(9) | 0.194(2)(9) | 0.066(1)(3) |
| $\left(\frac{A}{Z-N}\right) \frac{\langle x \rangle^{(h)}_{u-d}}{\langle x \rangle^{(p)}_{u-d}}$ | – | 1.007(14) | 1.028(15) |

See Phiala Shanahan's talk EG.00002 @ 11:57 today

# TWIST-2 OPERATORS

- EFT: match QCD operators to all possible hadronic operators with same symmetries
- Used in pion and N sectors to connect lattice PDF moments to experiment [Arndt & Savage; Chen & Ji; Detmold et al.,…]
- Isoscalar, spin independent operator matching:

$$\bar{q}\gamma^{\{\mu_1}D^{\mu_2}\dots D^{\mu_n\}}q \longrightarrow + c_n N^\dagger \mathcal{V}^{\mu_1\cdots\mu_n}N + c_n' N^\dagger S^{\{\mu_1}A^{\mu_2}\mathcal{V}^{\mu_3\cdots\mu_n\}}N + \dots$$
$$+\boxed{\alpha_n} N^\dagger \mathcal{V}^{\mu_1\cdots\mu_n}NN^\dagger N + \boxed{\beta_n} N^\dagger \mathcal{V}^{\mu_1\cdots\mu_n}\tau_j^{\xi+}NN^\dagger\tau_j^{\xi+}N + \dots$$

Two body counterterms

- where

$$\mathcal{V}^{\mu_1\cdots\mu_n} = \left(v + i\frac{D}{M}\right)^{\mu_1}\dots\left(v + i\frac{D}{M}\right)^{\mu_n}$$

$$\tau_j^{\xi\pm} = \frac{1}{2}\left(\xi^\dagger\tau_j\xi \pm \xi\tau_j\xi^\dagger\right)$$

[J W Chen, WD PLB 2005]

# NUCLEAR PDF MOMENTS

- Nucleon matrix elements (includes pion loop effects)

$$v_{\mu_1}\ldots v_{\mu_n}\langle N\,|\mathcal{O}^{\mu_1\cdots\mu_n}|\,N\rangle = \langle x^n\rangle_q$$

- Nuclear matrix elements

$$\begin{aligned}\langle x^n\rangle_{q|A} &\equiv v_{\mu_1}\ldots v_{\mu_n}\langle A\,|\mathcal{O}^{\mu_1\cdots\mu_n}|\,A\rangle \\ &= \langle x^n\rangle_q\left[A+\underbrace{\boxed{\alpha_n\left\langle A\left|\left(N^\dagger N\right)^2\right|A\right\rangle}}_{\text{Dominant term}}+\beta_n\left\langle A\left|\left(N^\dagger\tau N\right)^2\right|A\right\rangle\right]+\ldots\end{aligned}$$

- $\beta_n$ term suppressed by $N_c^2$ [Kaplan & Savage 96; K & Manohar 97]

- Ellipsis includes higher-body operators, terms with derivatives: higher-order in power-counting

[J W Chen, WD PLB 2005]

# MATRIX ELEMENTS

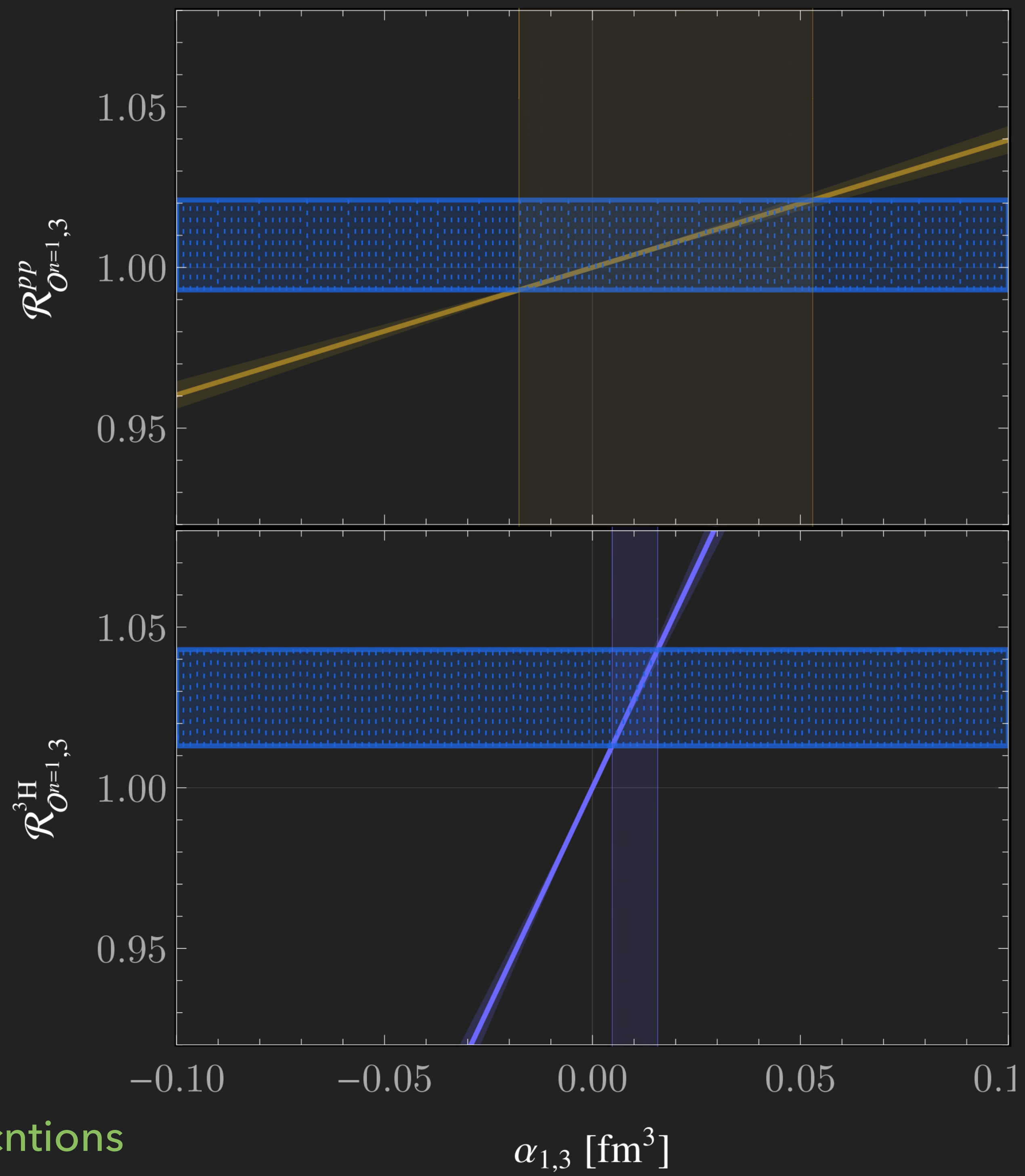
- Given EFT wavefunctions, matrix elements are easily computed
- LQCD matching determines EFT counterterms
  - Enables infinite volume prediction for matrix elements
  - Example: isovector momentum fraction

$$\mathcal{R}^h_{\mathcal{O}^n,3} \equiv \frac{A^h}{(Z^h - N^h)\langle x^n\rangle_3} \frac{\left\langle \Psi_h \middle| \mathcal{O}_3^{(n)} \middle| \Psi_h \right\rangle}{\langle \Psi_h \mid \Psi_h \rangle}$$

$$= \left(1 + \frac{\boxed{\alpha_{n,3}}}{(Z^h - N^h)\langle x^n\rangle_3} \boxed{h_h(\Lambda, L)}\right)$$

From wavefucntions

Two body counterterm

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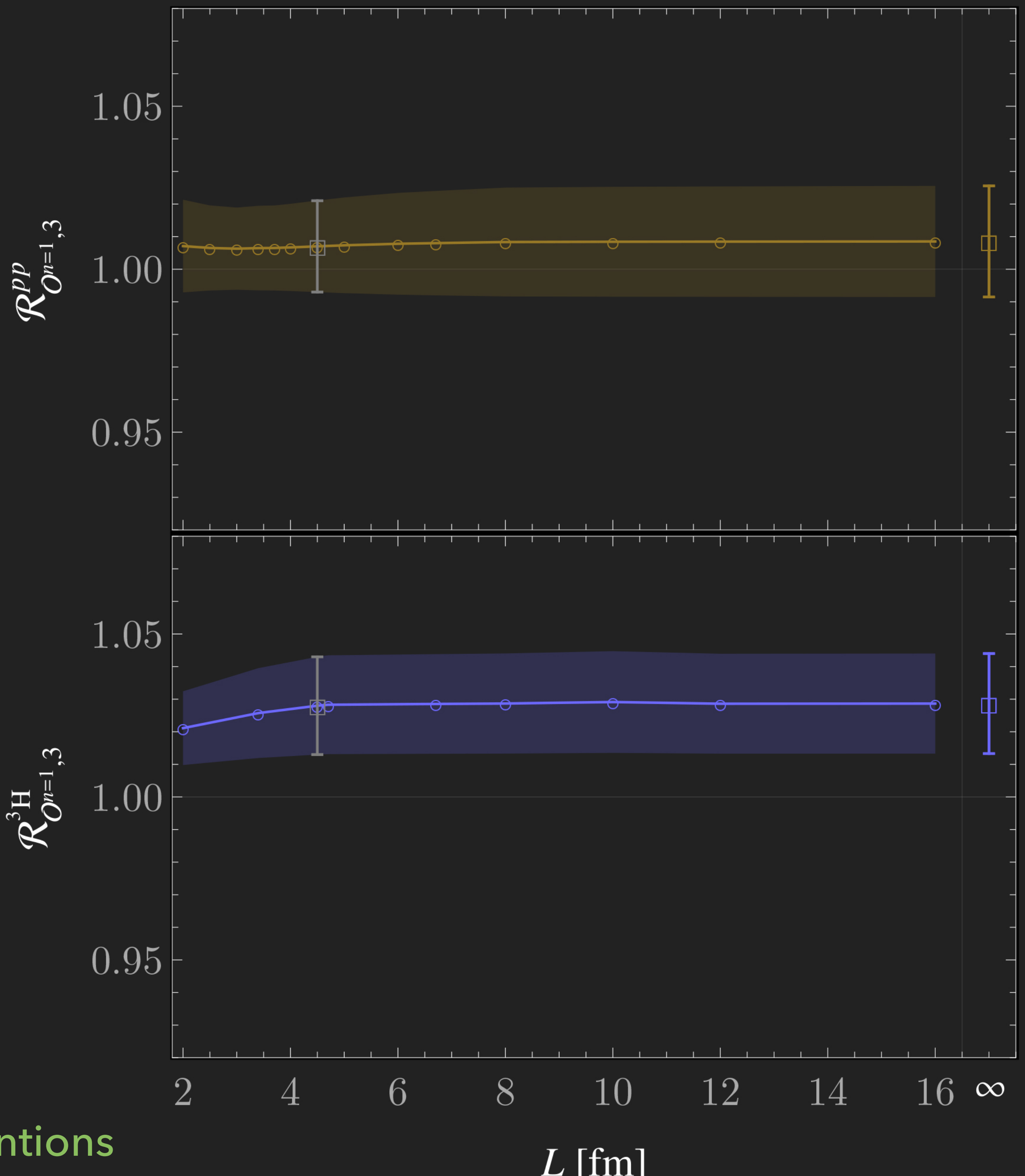
From wavefucntions

Two body counterterm

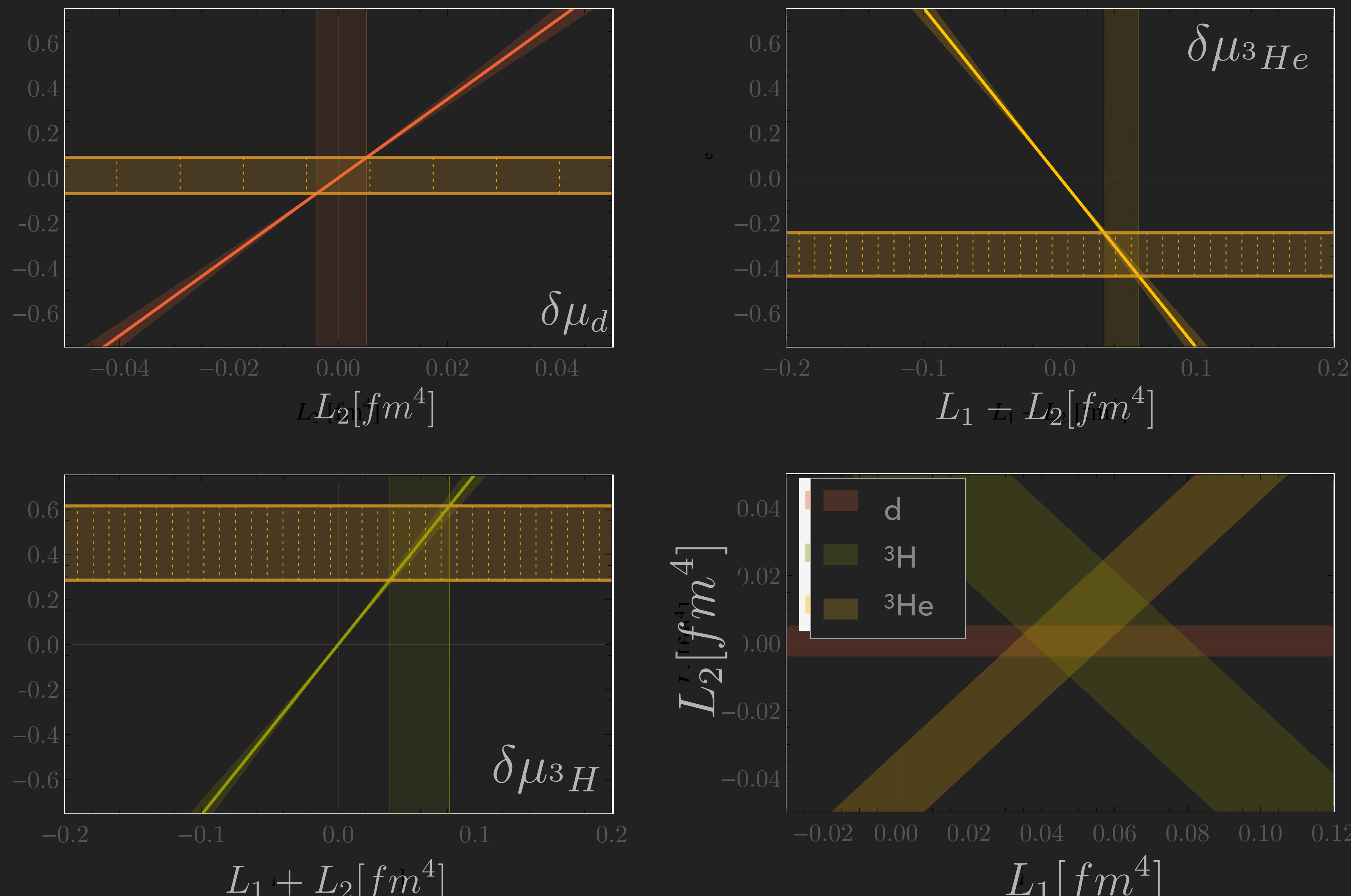



[WD, P Shanahan. 2102.04329]

# MAGNETIC MOMENTS

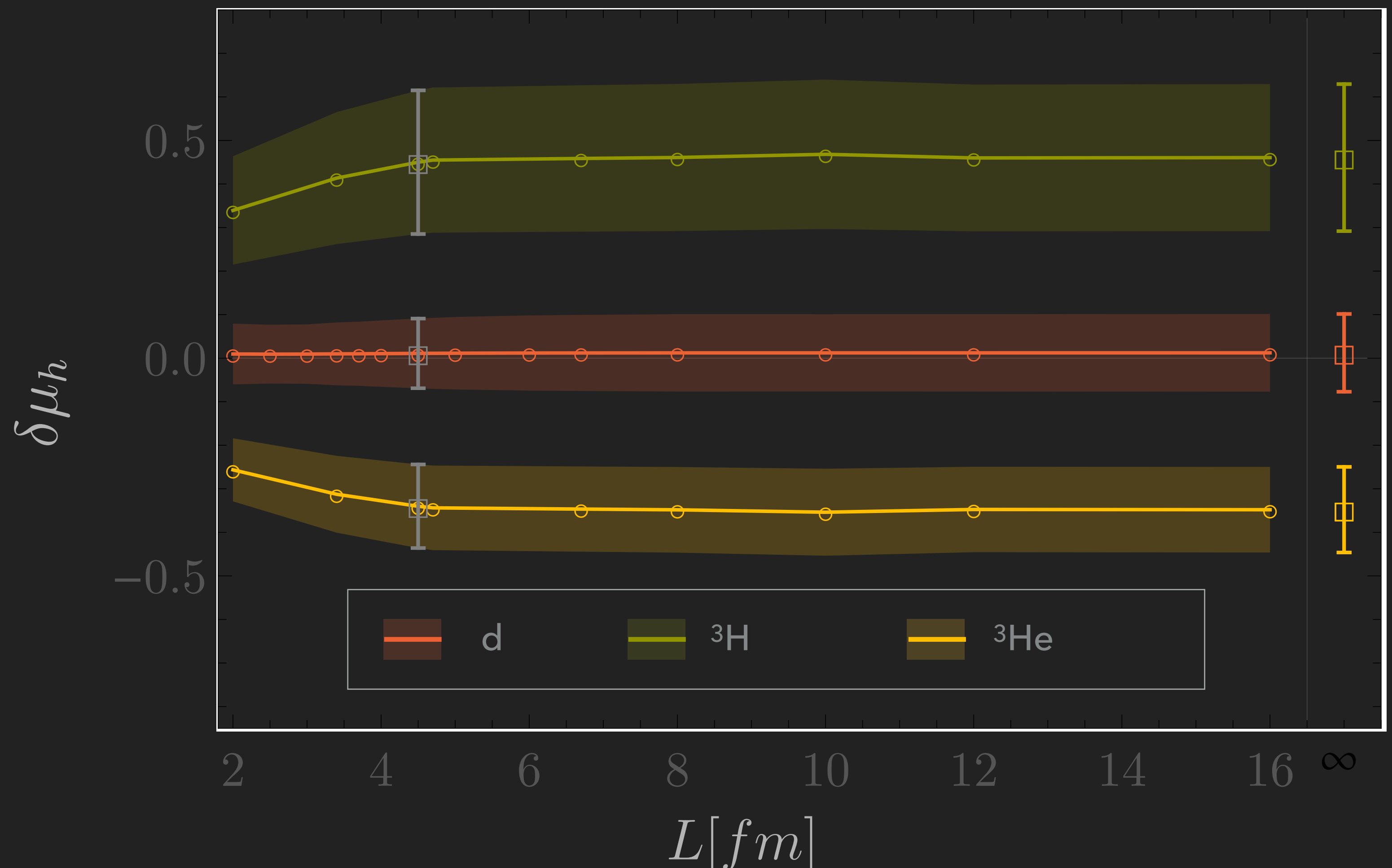
- Eg: magnetic moments

$$J_i^{EM} = \frac{e}{2M_N} N^\dagger \left(\kappa_0 + \tau_3\kappa_1\right)\sigma_i N \quad - eL_2 i\epsilon_{ijk}\left(N^T P_k N\right)^\dagger \left(N^T P_j N\right) + eL_1 \left(N^T P_i N\right)^\dagger \left(N^T \bar{P}_3 N\right) + \text{ h.c.}$$



# MAGNETIC MOMENTS

- LQCD-EFT matching can be extended to matrix elements
- Eg: magnetic moments




W. Detmold and P. E. Shanahan, Phys. Rev. D 103, 074503 (2021)