

# PRECISE DETERMINATION OF DECAY RATES FOR

## $J/\Psi \rightarrow \gamma\eta_c$ AND $\eta_c \rightarrow \gamma\gamma$

---

Brian Colquhoun

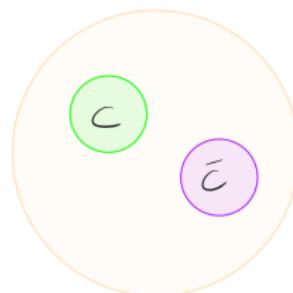
w/ Laurence Cooper, Christine Davies,  
G. Peter Lepage  
HPQCD Collaboration



University  
of Glasgow

12 August 2022

- ★ Decays with photons can be used as tests of our understanding of internal structure of mesons from strong interaction physics
- ★  $J/\Psi \rightarrow \gamma\eta_c$ : Some tension between branching fractions from lattice QCD and experimental result
- ★  $\eta_c \rightarrow \gamma\gamma$  less clear:
  - ▶ 40% uncertainty from average in PDG; 7% from fit
  - ▶ Lattice results exist for quenched & 2 sea quark flavours: low decay rate & tension with PDG fit
  - ▶ Recent  $N_f = 2$  result ([Lui et al \[2004.03907\]](#)) better, but still 14% error
- ★ This work:
  - ▶ Precise calculation by using Highly Improved Staggered Quark (HISQ) action
  - ▶ Calculate these decays with realistic sea
    - Effect of 2+1+1 quarks



- ★  $2 + 1 + 1$  HISQ gauge ensembles provided by MILC Collaboration
- ★ Lattice spacings from  $\approx 0.15$  fm down to  $\approx 0.06$  fm
- ★ Combination of  $m_s/m_l = 5$  and physical  $m_l$
- ★ Valence charm quarks also use HISQ formalism

| set | $a$ [fm]    | $N_x^3 \times N_t$ | $am_l^{\text{sea}}$ | $am_s^{\text{sea}}$ | $am_c^{\text{sea}}$ | $am_c^{\text{val}}$ |
|-----|-------------|--------------------|---------------------|---------------------|---------------------|---------------------|
| 1   | 0.15424(82) | $16^3 \times 48$   | 0.013               | 0.065               | 0.838               | 0.888               |
| 2   | 0.15088(79) | $32^3 \times 48$   | 0.00235             | 0.064               | 0.828               | 0.873               |
| 3   | 0.12404(66) | $24^3 \times 64$   | 0.0102              | 0.0509              | 0.635               | 0.664               |
| 3B  | 0.12404(66) | $24^3 \times 64$   | 0.0102              | 0.0509              | 0.635               | 0.654               |
| 4   | 0.12121(64) | $48^3 \times 64$   | 0.00184             | 0.0507              | 0.628               | 0.643               |
| 5   | 0.09023(48) | $32^3 \times 96$   | 0.0074              | 0.037               | 0.440               | 0.450               |
| 6   | 0.05926(33) | $48^3 \times 144$  | 0.0240              | 0.0240              | 0.286               | 0.274               |

$$J/\Psi \rightarrow \gamma \eta_c$$

For electromagnetic current  $j_c^\mu = \frac{2e}{3} \bar{c} \gamma^\mu c$ :

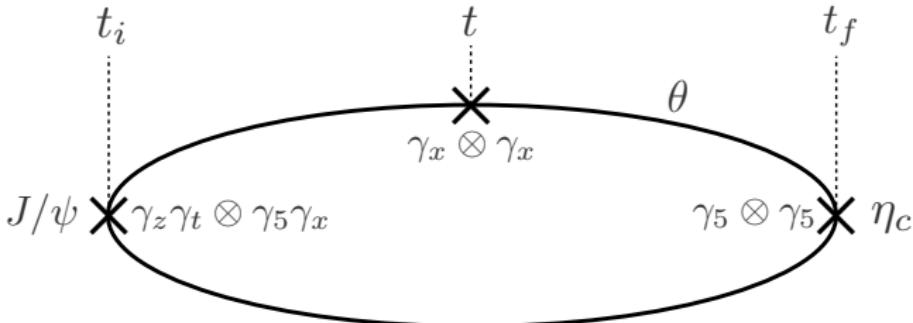
$$\langle \eta_c(p') | j_c^\mu | J/\psi(p) \rangle = \frac{V(q^2)}{M_{J/\psi} + M_{\eta_c}} \varepsilon^{\mu\alpha\beta\sigma} p'_\alpha p_\beta \epsilon_\sigma^{J/\psi}$$

$$\Gamma(J/\psi \rightarrow \gamma \eta_c) = \frac{1}{4\pi} \frac{4}{3} \frac{|\vec{q}|^3}{(M_{\eta_c} + M_{J/\psi})^2} |V(0)|^2$$

$$|\vec{q}| = \frac{(M_{\eta_c} + M_{J/\psi})(M_{J/\psi} - M_{\eta_c})}{2M_{J/\psi}}$$

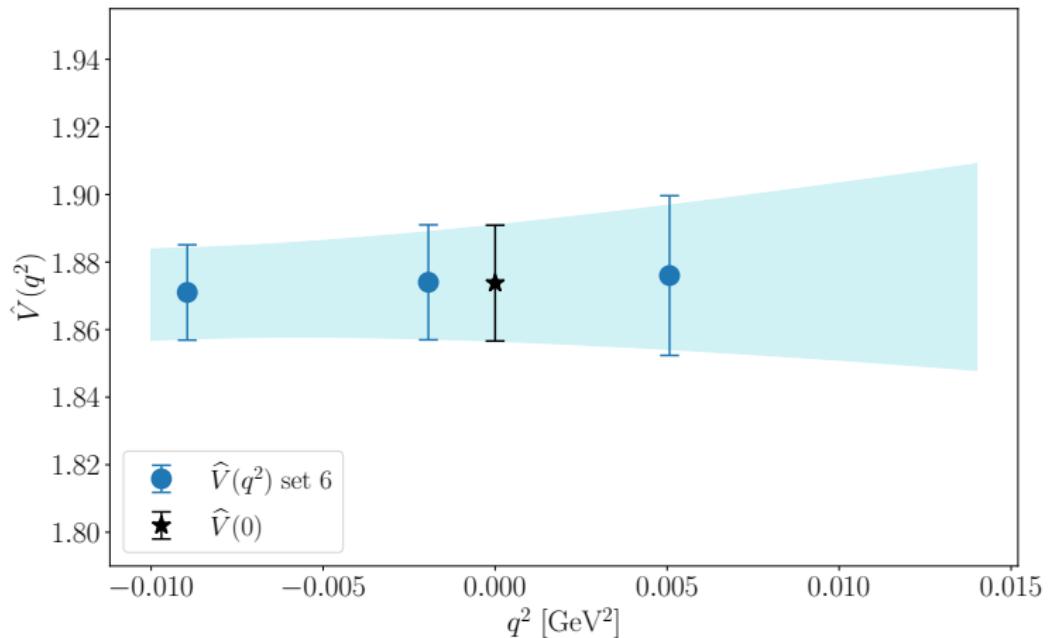
For our lattice calculation it is useful to define  $\hat{V}$ :

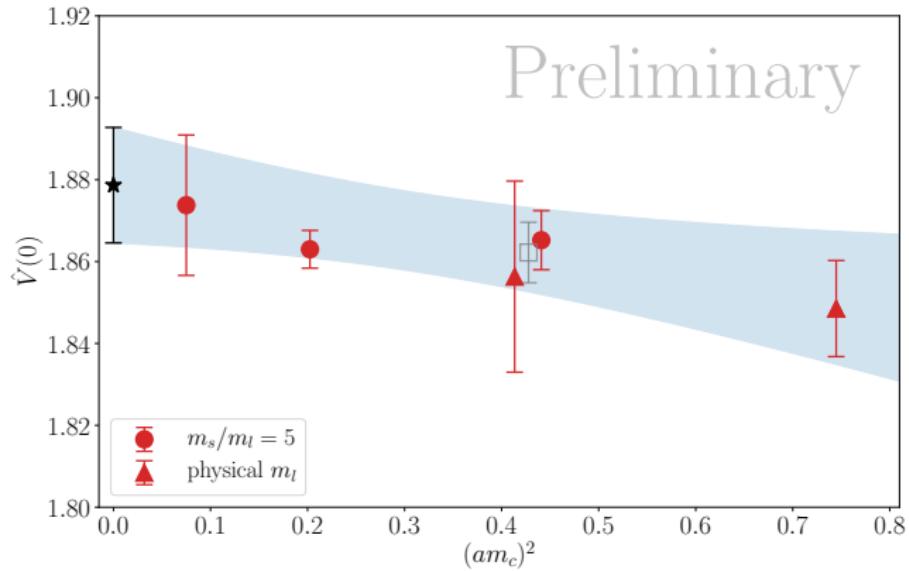
$$V(q^2) = 2 \times \frac{2e}{3} \times \hat{V}(q^2)$$



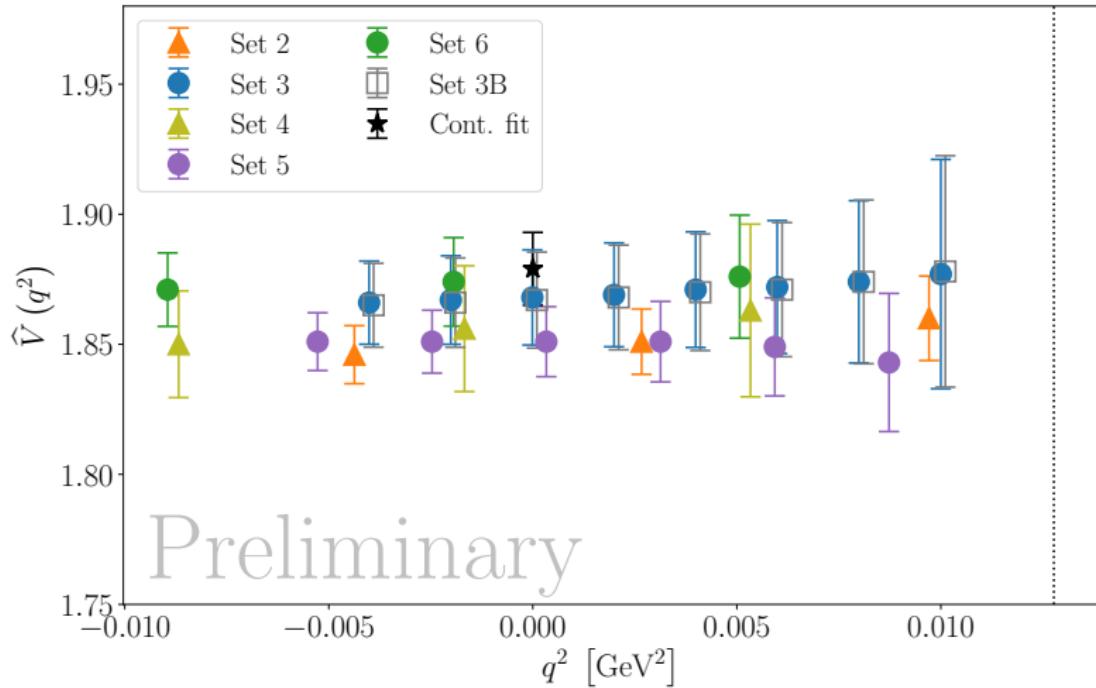
| Set | $\theta$  | $T = t_f - t_i$        |
|-----|---|------------------------|
| 1   | 0.1750, 0.3031, 0.3914, 0.4631                                    | 12, 13, 14, 15, 16, 17 |
| 2   | 0.6406, 0.9060, 1.1097  | 13, 14, 15, 16         |
| 3   | 0.2111, 0.2986, 0.3657, 0.4223,<br>0.4721, 0.5172, 0.5586, 0.5972 | 16, 17, 18, 19, 20, 21 |
| 4   | 0.7719, 1.0918, 1.3373  | 17, 18, 19, 20         |
| 5   | 0.2423, 0.3427, 0.4197,<br>0.4846, 0.5418, 0.5936                 | 21, 24, 27, 30         |
| 6   | 0.3774, 0.5338, 0.6538  | 33, 36, 39, 42         |

★ Twist,  $\theta$ , relates to spatial momenta through:  $q^y = \theta\pi/aN_x$

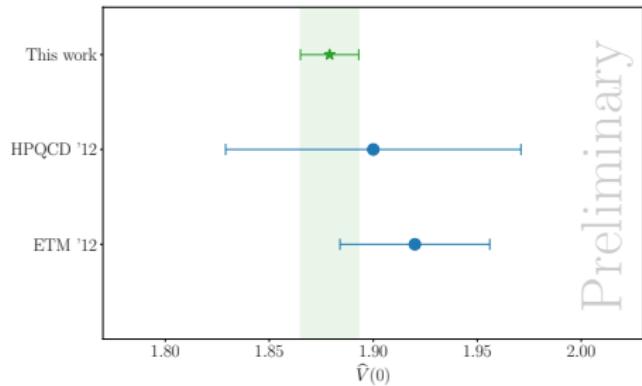




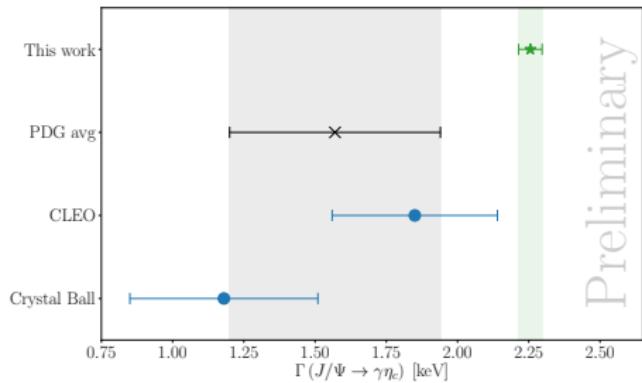
$$\begin{aligned} \widehat{V}_{\text{latt}}(0) = A \times & \left[ 1 + \sum_{i=1}^3 \kappa_{amc}^{(i)} \left( \frac{am_c}{\pi} \right)^{2i} + \kappa_{\text{sea},c} \delta^{\text{sea},c} + \kappa_{\text{val},c} \delta^{\text{val},c} \right. \\ & \left. + \kappa_{\text{sea},uds}^{(0)} \delta^{\text{sea},uds} \left\{ 1 + \kappa_{\text{sea},uds}^{(1)} (\Lambda a)^2 + \kappa_{\text{sea},uds}^{(2)} (\Lambda a)^4 \right\} \right] \end{aligned}$$



# Summary results $J/\Psi$ decays



Preliminary



Preliminary

$$\hat{V}(0) = 1.879(14)$$

Fitting uncertainty only

$$\Gamma(J/\Psi \rightarrow \gamma\eta_c) = 2.256(41) \text{ keV}$$

HPQCD '12 [1208.2855]  
ETM '12 [1206.1445]  
CLEO [0805.0252]  
Crystal Ball Phys. Rev. D 34, 711

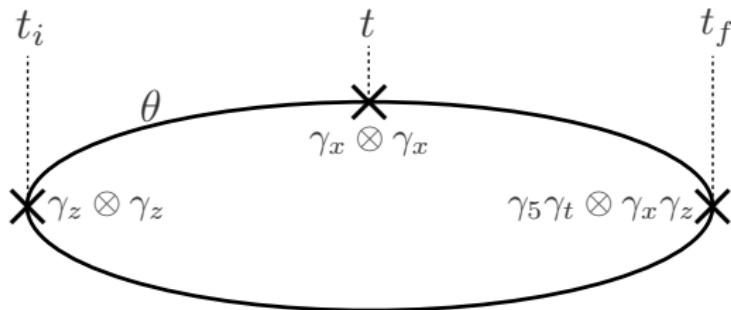
$$\eta_c \rightarrow \gamma\gamma$$

For form factor  $F(Q_1^2, Q_2^2)$ :

$$\mathcal{M}_{\mu\nu} = 2 \left( \frac{2}{3} e \right)^2 F(Q_1^2, Q_2^2) \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma.$$

Form factor  $F(0,0)$  then relates to the width:

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = \pi \alpha_{\text{em}}^2 \left( \frac{2}{3} \right)^4 M_{\eta_c}^3 |F(0,0)|^2.$$



| set | $\theta$ |
|-----|----------|
| 1   | 5.928    |
| 2   | 11.598   |
| 3   | 7.151    |
| 4   | 13.976   |
| 5   | 6.936    |
| 6   | 6.833    |

$$C_{\eta_c \rightarrow \gamma\gamma}(t, t_i) = \left(\frac{2}{3}e\right)^2 \lim_{t_f - t \rightarrow \infty} \int dt_i e^{-\omega_1(t_i - t)} C(t_i, t, t_f)$$

Ji et al. [[hep-lat/0101014](#)] & [[hep-lat/0103007](#)]

For on-shell photons:

$$\omega_1 = |\vec{q}_1| = |\vec{q}_2| = \frac{M_{\eta_c}}{2}$$

- ★ Twists  $\theta$  are chosen to give on-shell photons:  $\theta = aN_x M_{\eta_c} / 2\pi$ 
  - ▶  $M_{\eta_c} = 2.9783$  GeV

# Fitting correlators

Fit two sets of correlators:

$$C_{2\text{pt}}(t) = \sum_i^{N_n} (a_{n,i})^2 f(E_{n,i}, t) - \sum_i^{N_o} (a_{o,i})^2 (-1)^t f(E_{o,i}, t)$$

with

$$f(E, t) = e^{-Et} + e^{-E(N_t - t)}.$$

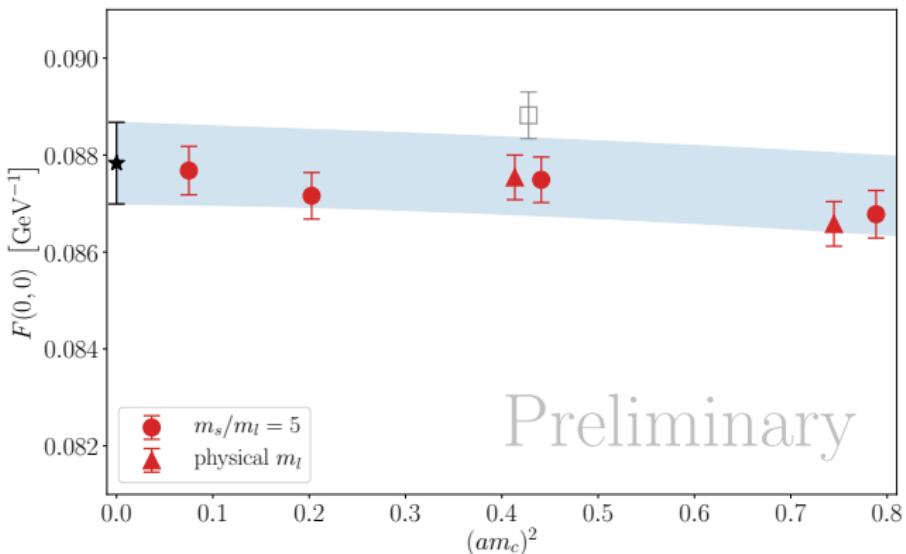
and

$$C_{\eta_c \rightarrow \gamma\gamma}(t) = \sum_i^{N_n} a_{n,i} b_{n,i} \exp(-E_{n,i} t) - \sum_i^{N_o} a_{o,i} b_{o,i} (-1)^t \exp(-E_{o,i} t)$$

Extract  $F_{\text{latt}}(0, 0)$  by:

$$F_{\text{latt}}(0, 0) = \frac{b_{n,0}}{\left(\frac{M_{\eta_c}^2}{2}\right)}$$

- ★  $M_{\eta_c}$  corresponds to ‘connected’  $\eta_c$  ([HPQCD '20 \[2005.01845\]](#))



$$F_{\text{latt}}(0, 0) = F(0, 0) \times \left[ 1 + \sum_{i=1}^3 \kappa_{am_c}^{(i)} \left( \frac{am_c}{\pi} \right)^{2i} + \kappa_{\text{sea},c} \delta^{\text{sea},c} + \kappa_{\text{val},c} \delta^{\text{val},c} \right. \\ \left. + \kappa_{\text{sea},uds}^{(0)} \delta^{\text{sea},uds} \left\{ 1 + \kappa_{\text{sea},uds}^{(1)} (\Lambda a)^2 + \kappa_{\text{sea},uds}^{(2)} (\Lambda a)^4 \right\} \right]$$

## $\eta_c$ Results

Continuum result gives

$$F(0,0) = 0.08783(84)$$

From which we can determine the width:

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = 6.78(13) \text{ keV}$$

Experimental average:  $6.1^{+2.2}_{-1.9}$  keV

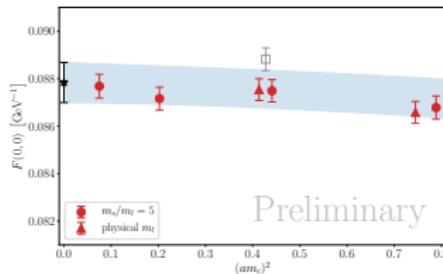
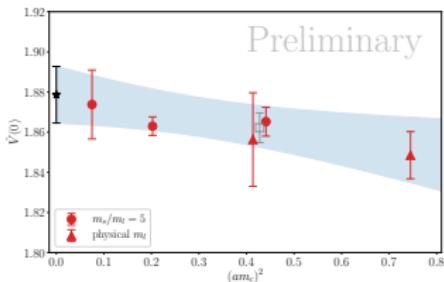
Expectation in nonrelativistic limit ([Czarnecki & Melnikov '01 \[hep-ph/0109054\]](#)):

$$\frac{\Gamma(J/\Psi \rightarrow e^+e^-)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} = \frac{1}{3Q_c^2} \times \left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{v^2}{c^2}\right)\right) \approx \frac{3}{4}$$

Using HPQCD result for  $\Gamma(J/\Psi \rightarrow e^+e^-)$  ([HPQCD '20 \[2005.01845\]](#)) we find:

$$\frac{\Gamma(J/\Psi \rightarrow e^+e^-)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} = 0.832(17)$$

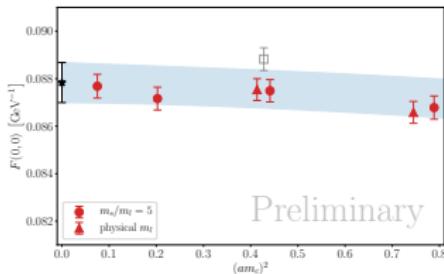
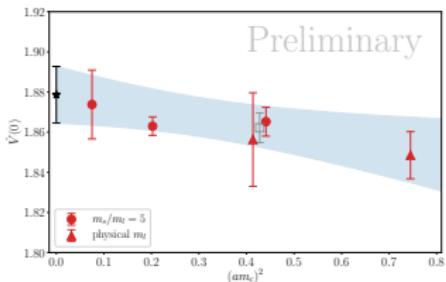
# Summary and outlook



- ★  $\Gamma(J/\Psi \rightarrow \gamma\eta_c) = 2.256(41) \text{ keV}$ 
  - ▶  $\widehat{V}(0) = 1.879(14)$
- ★  $\Gamma(\eta_c \rightarrow \gamma\gamma) = 6.78(13) \text{ keV}$ 
  - ▶  $F(0,0) = 0.08783(84)$
- ★  $\frac{\Gamma(J/\Psi \rightarrow e^+e^-)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} = 0.832(17)$ 
  - ▶ Suggests corrections to tree level ( $\approx 3/4$ ) not substantial:  $\mathcal{O}(10\%)$

- ★ Results to be finalised in continuum + full error budget
- ★ From full  $J/\Psi \rightarrow \gamma\eta_c$  we will obtain  $J/\Psi \rightarrow e^+e^-$
- ★ Follow-on studies:  $\eta_b \rightarrow \gamma\gamma$  &  $\pi_0 \rightarrow \gamma\gamma$

# Summary and outlook



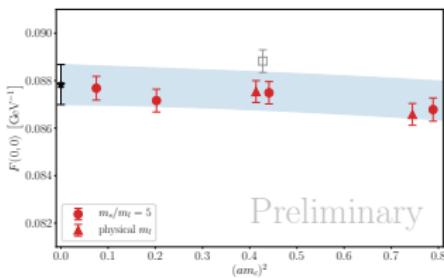
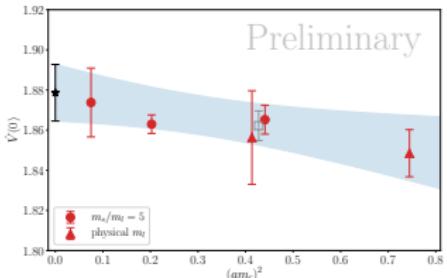
- ★  $\Gamma(J/\Psi \rightarrow \gamma\eta_c) = 2.256(41) \text{ keV}$ 
  - ▶  $\widehat{V}(0) = 1.879(14)$
- ★  $\Gamma(\eta_c \rightarrow \gamma\gamma) = 6.78(13) \text{ keV}$ 
  - ▶  $F(0,0) = 0.08783(84)$
- ★  $\frac{\Gamma(J/\Psi \rightarrow e^+e^-)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} = 0.832(17)$ 
  - ▶ Suggests corrections to tree level ( $\approx 3/4$ ) not substantial:  $\mathcal{O}(10\%)$

- ★ Results to be finalised in continuum + full error budget
- ★ From full  $J/\Psi \rightarrow \gamma\eta_c$  we will obtain  $J/\Psi \rightarrow e^+e^-$
- ★ Follow-on studies:  $\eta_b \rightarrow \gamma\gamma$  &  $\pi_0 \rightarrow \gamma\gamma$

Prediction

Experimentally measured

# Summary and outlook



- ★  $\Gamma(J/\Psi \rightarrow \gamma\eta_c) = 2.256(41) \text{ keV}$ 
  - ▶  $\hat{V}(0) = 1.879(14)$
- ★  $\Gamma(\eta_c \rightarrow \gamma\gamma) = 6.78(13) \text{ keV}$ 
  - ▶  $F(0,0) = 0.08783(84)$
- ★  $\frac{\Gamma(J/\Psi \rightarrow e^+e^-)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} = 0.832(17)$ 
  - ▶ Suggests corrections to tree level ( $\approx 3/4$ ) not substantial:  $\mathcal{O}(10\%)$

Thank you!

- ★ Results to be finalised in continuum + full error budget
- ★ From full  $J/\Psi \rightarrow \gamma\eta_c$  we will obtain  $J/\Psi \rightarrow e^+e^-$
- ★ Follow-on studies:  $\eta_b \rightarrow \gamma\gamma$  &  $\pi_0 \rightarrow \gamma\gamma$

Prediction

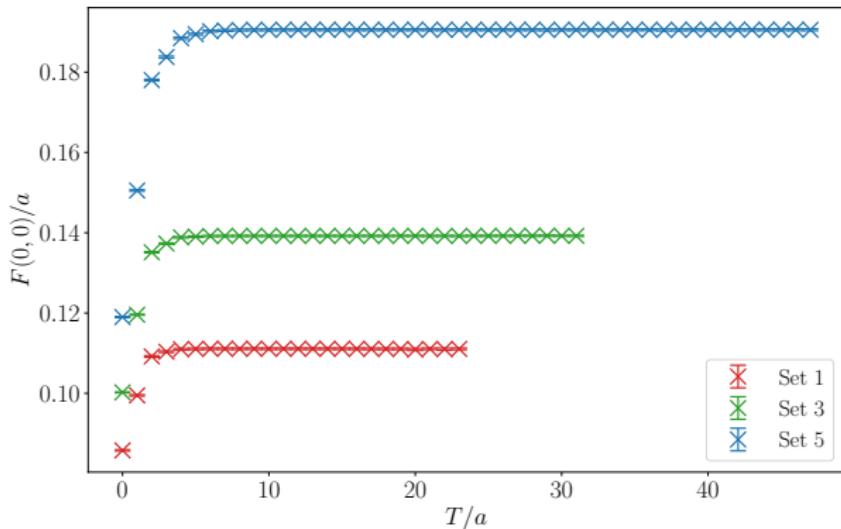
Experimentally measured

EXTRA STUFF

$$\begin{aligned}
C_{3\text{pt}}(t, T = t_f - t_i) = & \\
& \sum_{i,j}^{N_n, N_n} a_{n,i}^{\gamma_z \gamma_t \otimes \gamma_5 \gamma_x} e^{-E_{a_n,i} t} V_{nn,ij} a_{n,j}^{\gamma_5 \otimes \gamma_5} e^{-E_{a_n,j} (T-t)} \\
& - \sum_{i,j}^{N_n, N_o} (-1)^{T-t} a_{n,i}^{\gamma_z \gamma_t \otimes \gamma_5 \gamma_x} e^{-E_{a_n,i} t} V_{no,ij} a_{o,j}^{\gamma_5 \otimes \gamma_5} e^{-E_{a_o,j} (T-t)} \\
& - \sum_{i,j}^{N_o, N_n} (-1)^t a_{o,i}^{\gamma_z \gamma_t \otimes \gamma_5 \gamma_x} e^{-E_{a_o,i} t} V_{on,ij} a_{n,j}^{\gamma_5 \otimes \gamma_5} e^{-E_{a_n,j} (T-t)} \\
& + \sum_{i,j}^{N_o, N_o} (-1)^T a_{o,i}^{\gamma_z \gamma_t \otimes \gamma_5 \gamma_x} e^{-E_{a_o,i} t} V_{oo,ij} a_{o,j}^{\gamma_5 \otimes \gamma_5} e^{-E_{a_o,j} (T-t)}.
\end{aligned}$$

$$C_{2\text{pt}}(t) = \sum_i^{N_n} (a_{n,i})^2 f(E_{n,i}, t) - \sum_i^{N_o} (a_{o,i})^2 (-1)^t f(E_{o,i}, t)$$

$$C_{\eta_c \rightarrow \gamma\gamma}(t_f - t) = \sum_{t_i - t = -T}^T e^{-\omega_1(t_i - t)} C_{3\text{pt}}(t_i, t, t_f)$$



We can define the ratio:

$$R_{ee} = \frac{\mathcal{B}(J/\psi \rightarrow \eta_c e^+ e^-)}{\mathcal{B}(J/\psi \rightarrow \gamma \eta_c)}.$$

$$\begin{aligned} \frac{dR_{ee}}{dq^2} &= \frac{\alpha}{3\pi q^2} \left| \frac{V(q^2)}{V(0)} \right|^2 \left( 1 - \frac{4m_e^2}{q^2} \right)^{\frac{1}{2}} \left( 1 + \frac{2m_e^2}{q^2} \right) \\ &\times \left( \left( 1 + \frac{q^2}{M_{J/\psi}^2 - M_{\eta_c}^2} \right) - \frac{4m_{J/\psi}^2 q^2}{\left( M_{J/\psi}^2 - M_{\eta_c}^2 \right)^2} \right)^{\frac{3}{2}}. \end{aligned}$$

$R_{ee}$  can be determined by integrating this function, so as to determine  $\mathcal{B}(J/\psi \rightarrow \eta_c e^+ e^-)$ .