Doubly charm tetraquark and its quark mass dependence

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M. Padmanath

Sara Collins

Lattice 2022, Bonn, August 2022

Outline:

Doubly charm tetraquark $cc\bar{d}\bar{u}$

Padmanath, S.P.: 2202.10110, PRL
$m_{u/d}$, m_c dependence in supplemental material

Charmonium(like) resonances $\bar{c}c$, $\bar{c}q\bar{q}c$

S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02542 JHEP, 1905.03506 PRD, 2111.02934
versus experimental discoveries in 2022

both on N_f=2+1, CLS ensembles, $m_\pi \approx 280$ MeV

$cc\bar{d}\bar{u}$ = T_{cc}

Padmanath, S.P.: 2202.10110,
Phys.Rev.Lett. 129 (2022) 3, 032002
&
subsequent studies with S. Collins

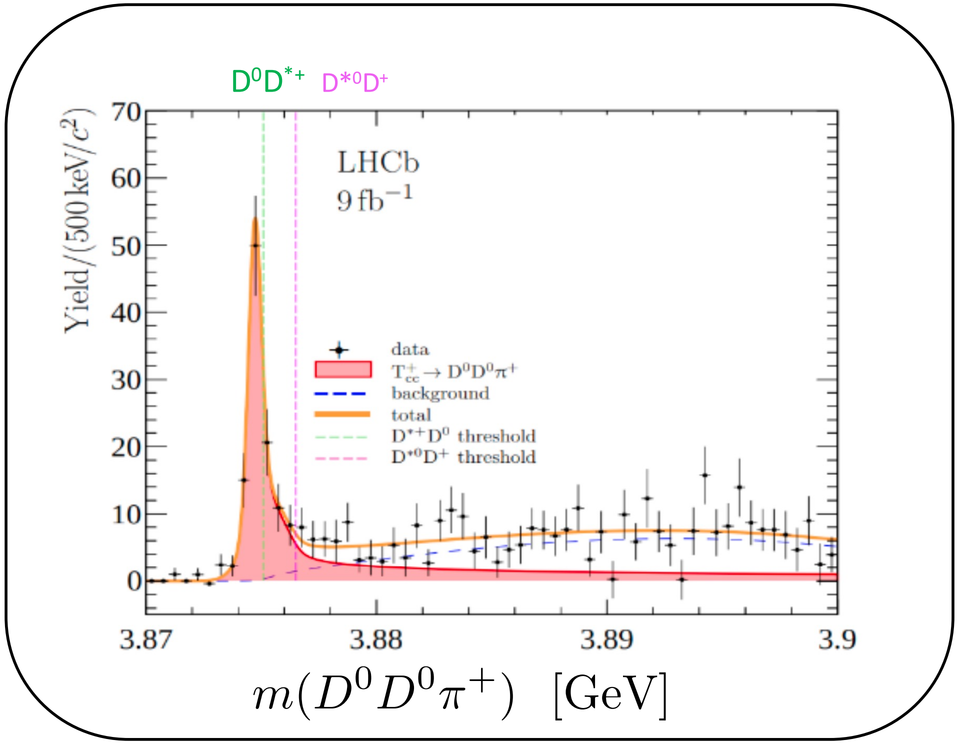
LHCb discovery of T_{cc}

$cc\bar{u}\bar{d}$

The longest lived exotic hadron ever discovered

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

I=0, $J^P=1^+$

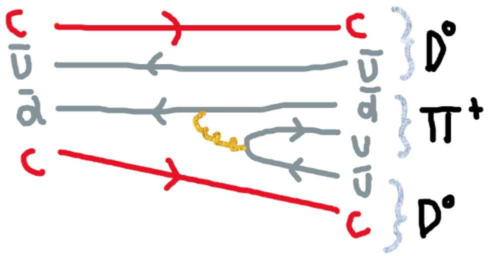


LHCb July 2021, 2109.01038, 2109.01056

The doubly charmed tetraquark T_{cc}^+, $I=0$ and favours $J^P=1^+$.

No states observed in $D^0D^+\pi^+$: eliminates possibility of $I=1$.

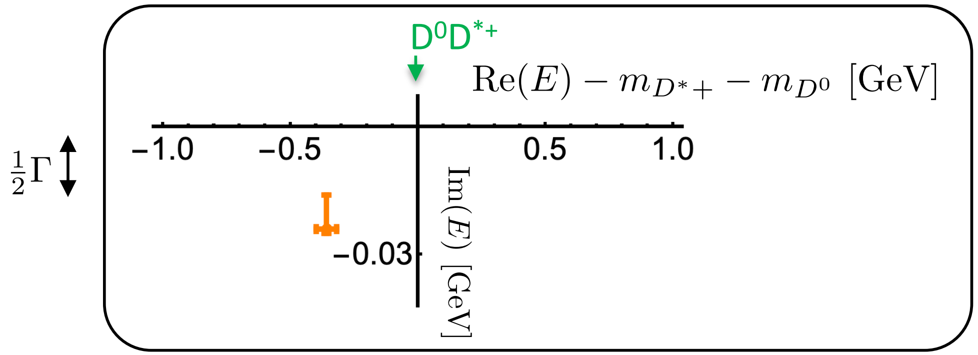
Near-threshold state: Demands pole identification to confirm existence.



Omitting $D^* \to D\pi,\ T_{cc} \to DD\pi$

T_{cc} would be a bound state

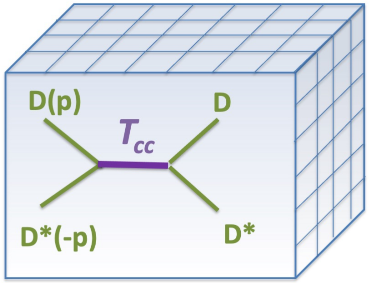
Pole in T(E) $\delta m = -0.36$ MeV



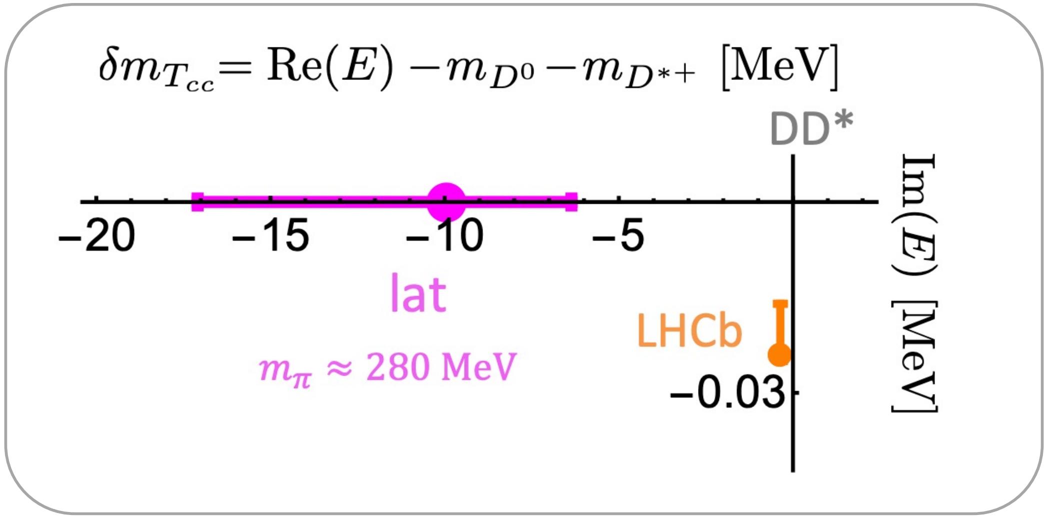
$$\delta m_{\rm pole} = -360 \pm 40\,^{+4}_{-0}\ \mathrm{keV}/c^2\,,$$
$$\Gamma_{\rm pole} = 48 \pm 2\,^{+0}_{-14}\,\mathrm{keV}\,,$$

T_{cc} on the lattice $cc\bar{d}\bar{u}$

Previous talk by Padmanath: lattice results



Pole of T(E) at $m_c^{(h)}$



	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	−0.36(4)	bound st.

closer-to physical m_c

omitting $T_{cc} \to DD\pi$, $D^* \to D\pi$

This talk: simple analytic arguments

$m_{u/d}$ increases :

$m_{u/d}^{phy} \to m_{u/d}^{lat}$

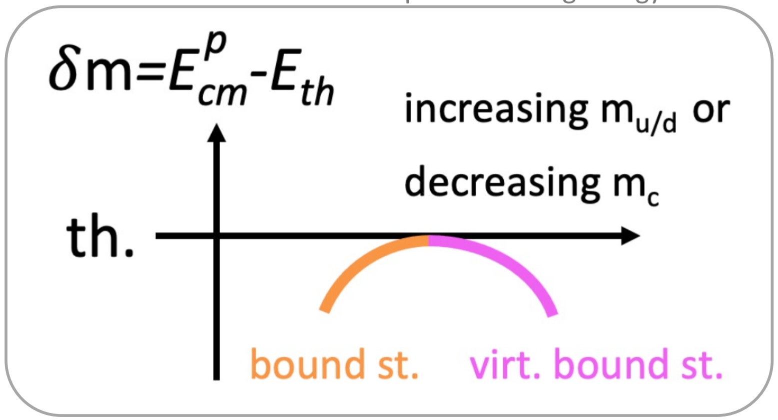
(LHCb) would-be bound st. → virtual bound st.

m_c decreases

$|\delta m_{T_{cc}}|$ increases for virtual bound st.

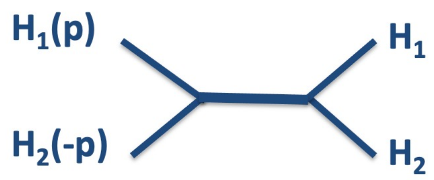
(both in agreement with the lattice result)

sketch of expected binding energy



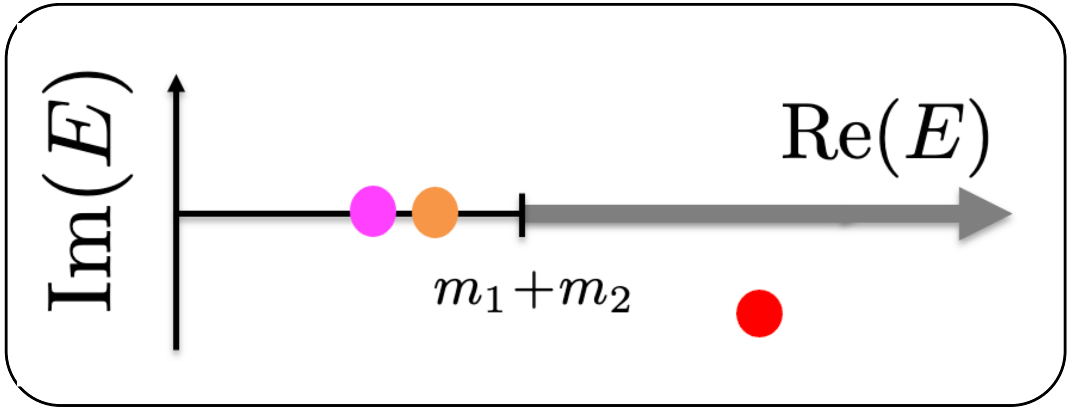
Hypothesis to be verified by future simulations

Definitions: bound state, virtual bound state & resonance



$$T(E) \propto \frac{1}{E^2 - m^2} \qquad T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$

Poles of *T(E)*, $E=E_{cm}$



Virtual bound st.	Bound st.	Resonance
p = - i |p|	p = i |p|	
example:	example:	
di-neutron	deuteron	

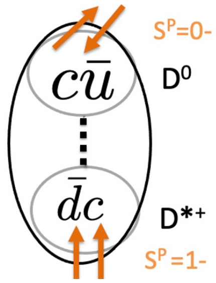
$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + (-p)^2} \; < \; m_1 + m_2$$

Possible binding mechanisms of T_{cc}

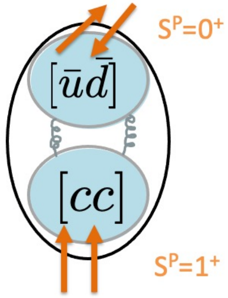
molecular

likely dominant

[e.g. Janc, Rosina 2003]



"molecular"



Molecular component: dependence on $m_{u/d}$

exchanged particles:

light mesons $\pi, \rho, ..$

increasing $m_{u/d}$

increasing m_{ex}

decreasing R or

decreasing attraction |V|

Yukava-like potential

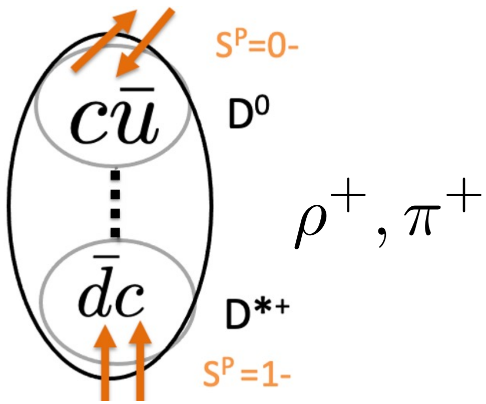
$$V(r) \propto -\frac{e^{-m_{ex} r}}{r}$$

analogous conclusion for any

fully attractive

$$V(r) = -V_0 \; f(r/R)$$

$$f = e^{-r/R}, e^{-r^2/R^2}, \theta(R-r), ...$$



subsequent lattice study:

CLQCD, Chen et al. 2206.06185

comparison of I=0,1 :

attraction in I=0 channel arises

mainly from ϱ exchange

Simplest Example: scattering in square-well potential in QM

$\delta = \arctan[\tan(qR)\frac{p}{q}] - pR$

partial wave l=0

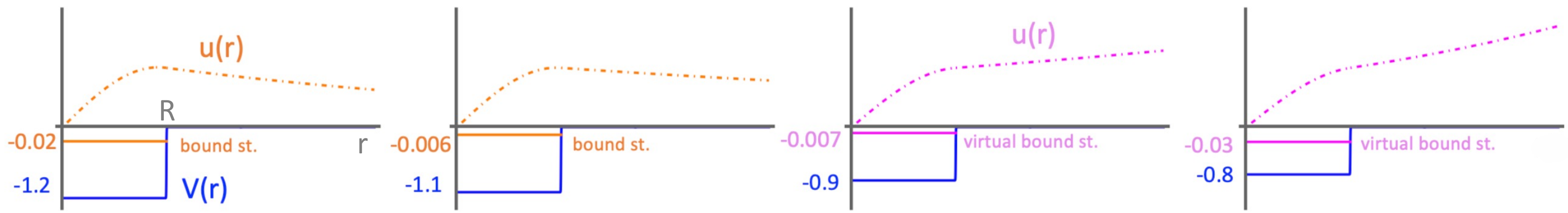
$t \propto (p\cot\delta - ip)^{-1}$

$R=1,\ m_r = \pi^2/8$

$u(r) = A\sin(qr)$ $\quad u(r) = B\sin(pr+\delta)$

p=i|p| $\quad e^{ipr} = e^{-|p|r}$

p=-i|p| $\quad e^{ipr} = e^{|p|r}$



increasing $m_{u/d}$, decreasing attraction V_0 (or decreasing R)

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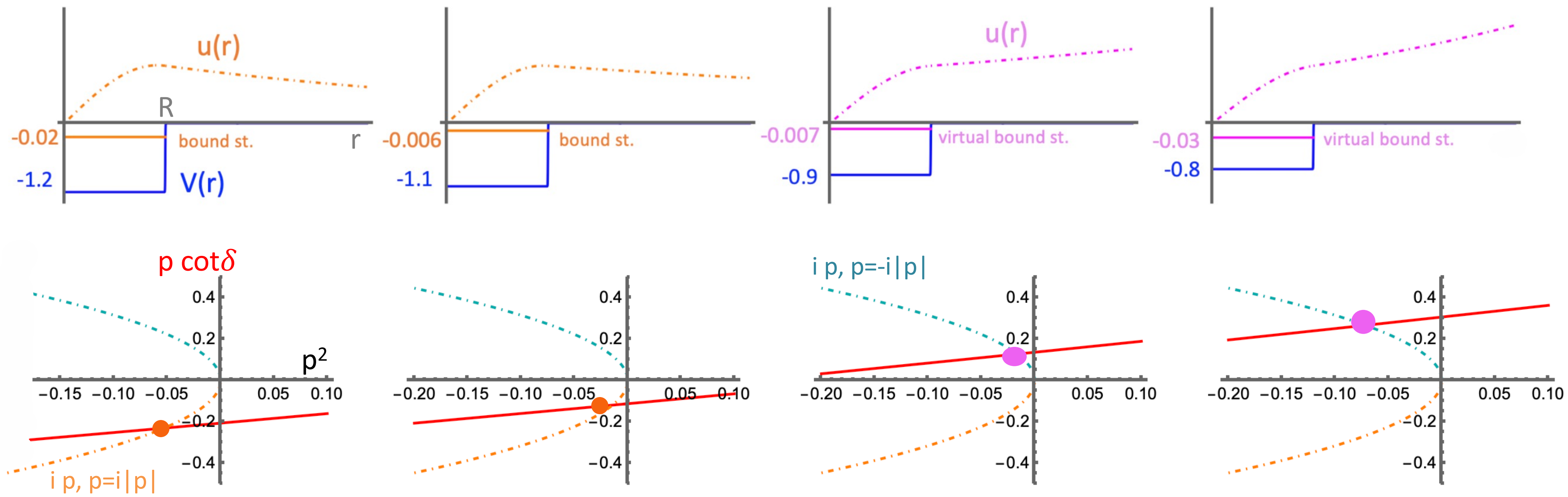

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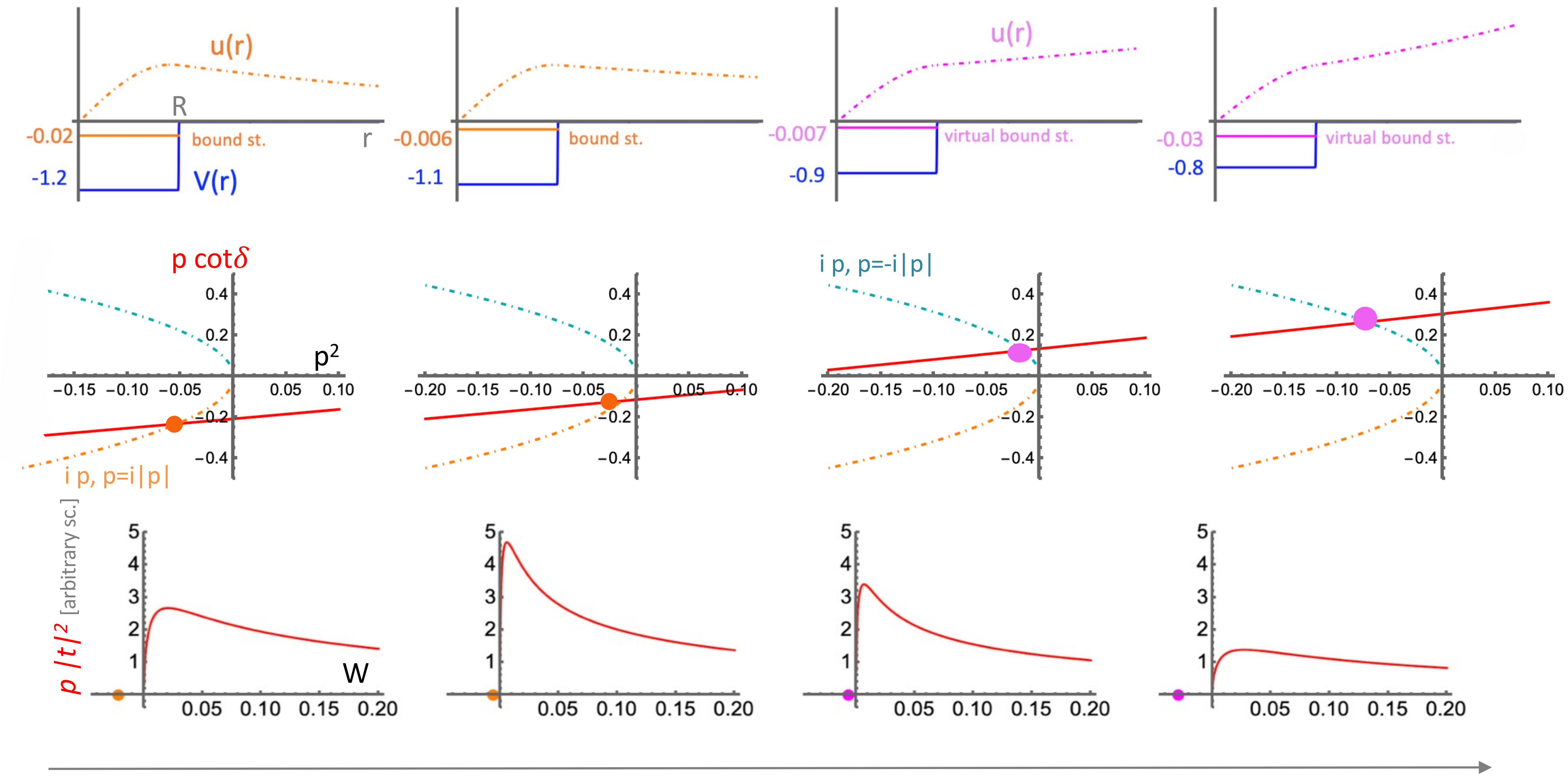
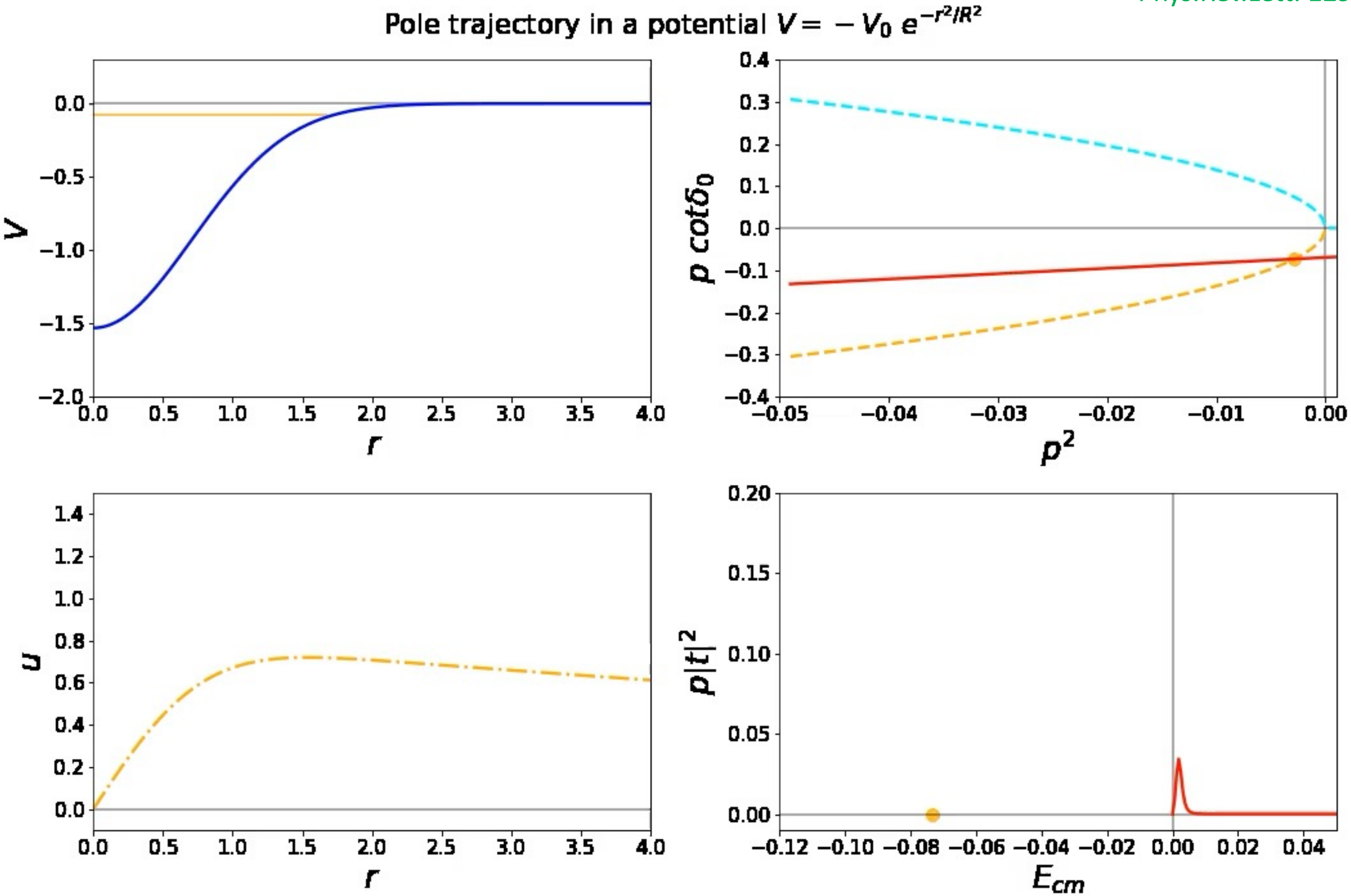


increasing $m_{u/d}$, decreasing attraction V_0 (or decreasing R)

All fully attractive potentials lead to analogous conclusions

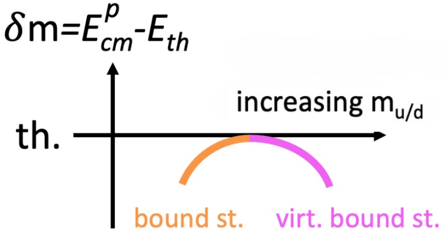
video: courtesy M. Padmanath
supplemental material of
Phys.Rev.Lett. 129 (2022) 3, 032002

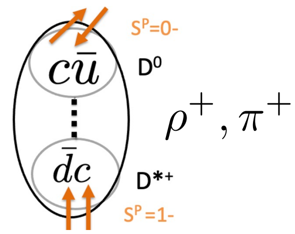


$m_{u/d}$ increases :

$m_{u/d}^{phy} \to m_{u/d}^{lat}$

(LHCb) would-be bound st. → virtual bound st.

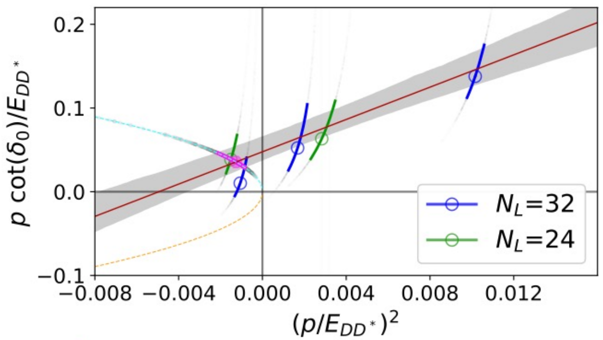


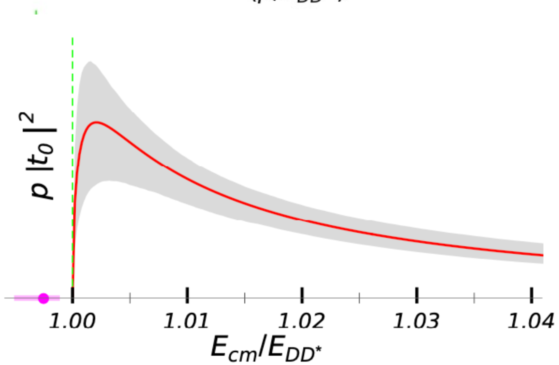


Shallow virtual bound state

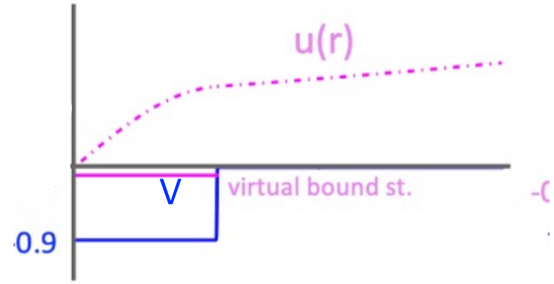
lattice results

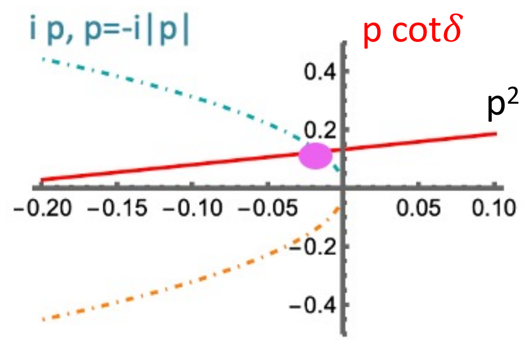
at $m_\pi \approx 280$ MeV and $m_c^{(h)}$

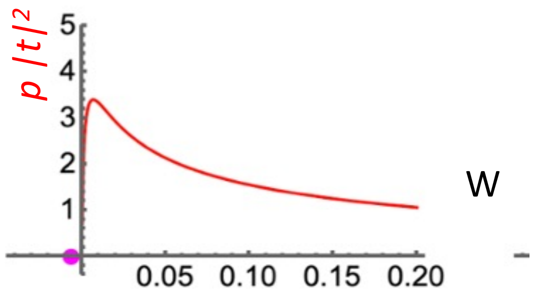




toy model



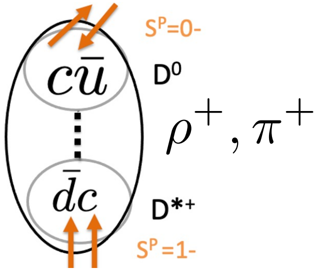




Molecular component: dependence on m_c

V(r) independent on m_c ,

m_c decreases : reduced mass m_r of D,D* system decreases



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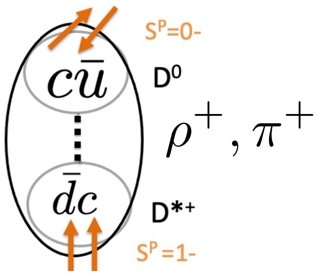
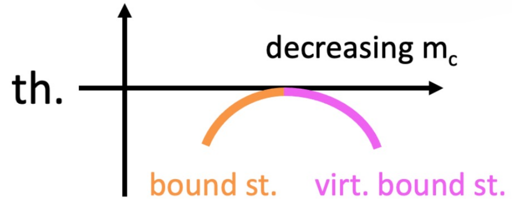
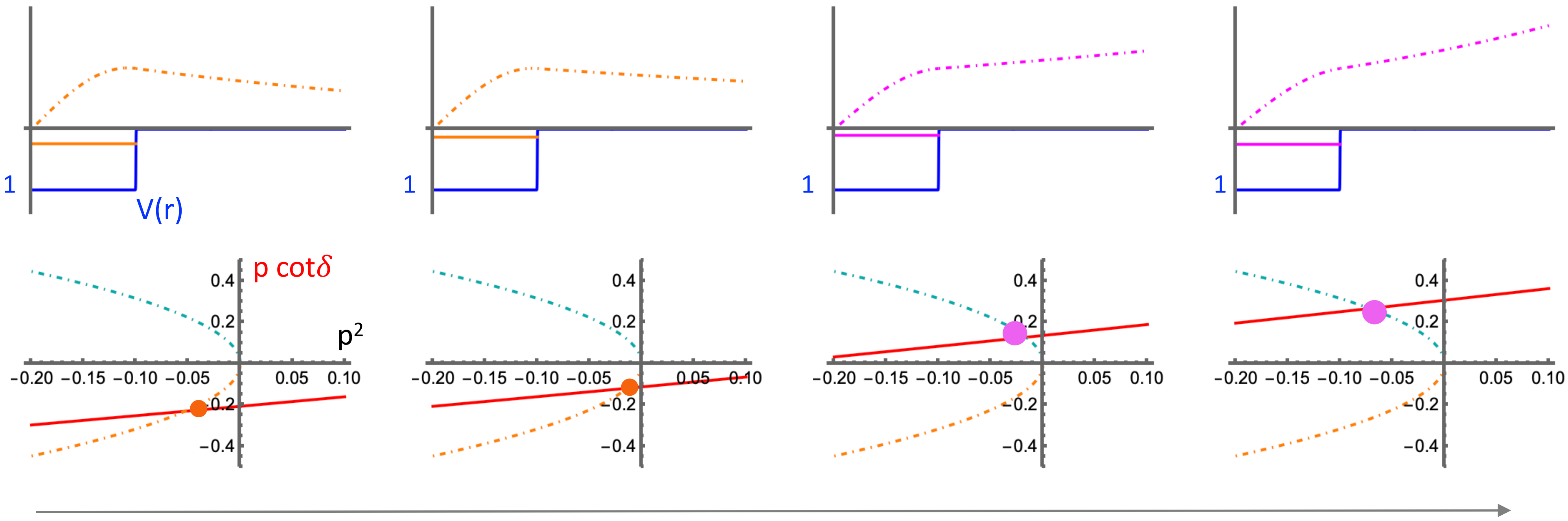


$\delta m = E^p_{cm} - E_{th}$



Square well potential (analogous conclusion for other shapes)



decreasing m_c and m_r

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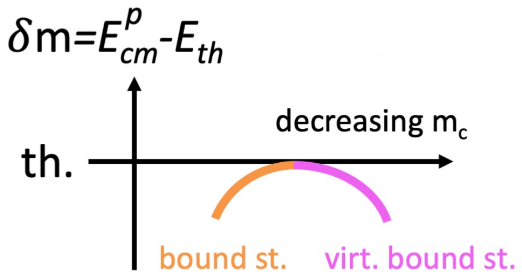

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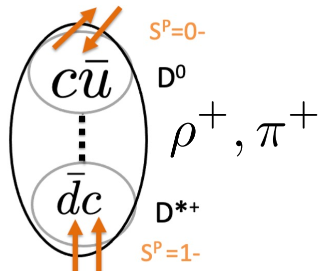

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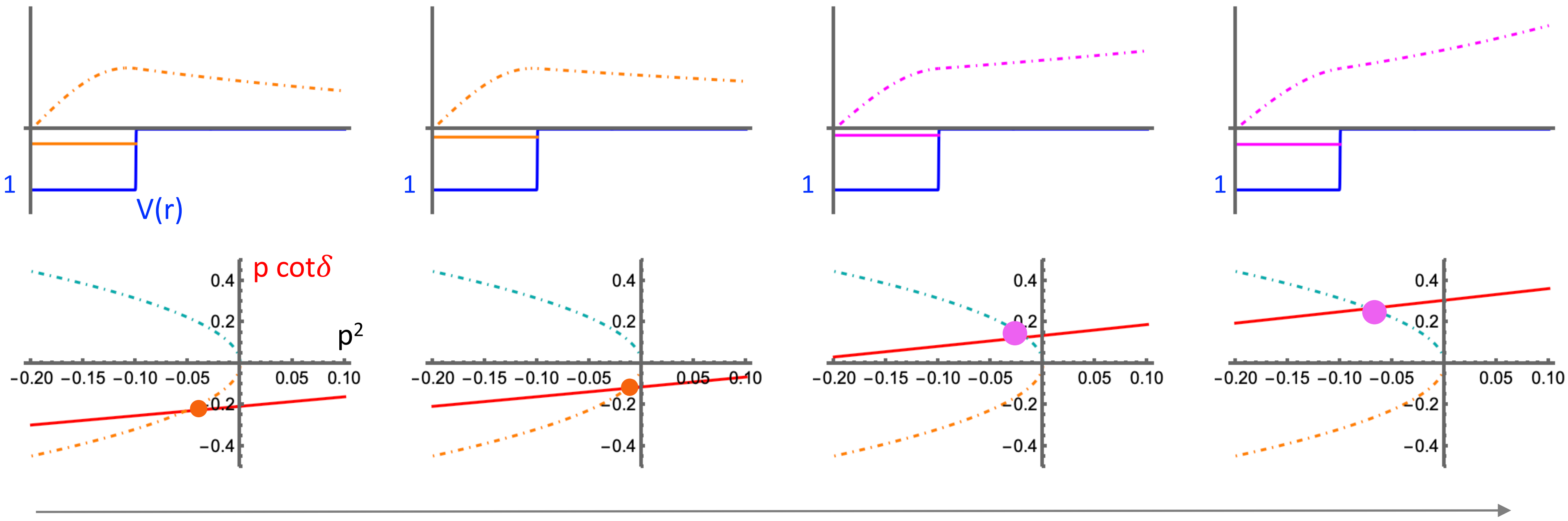

decreasing m_c and m_r

	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
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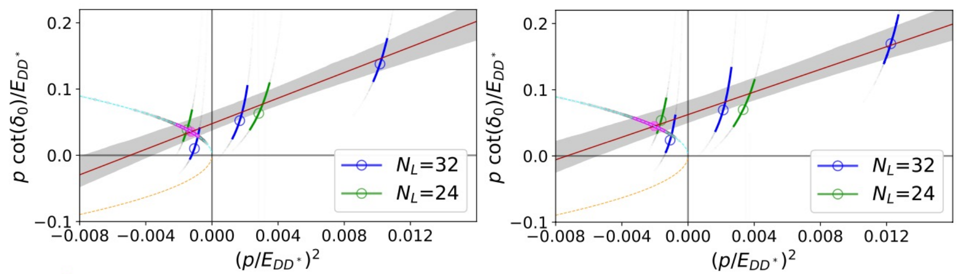


lattice results

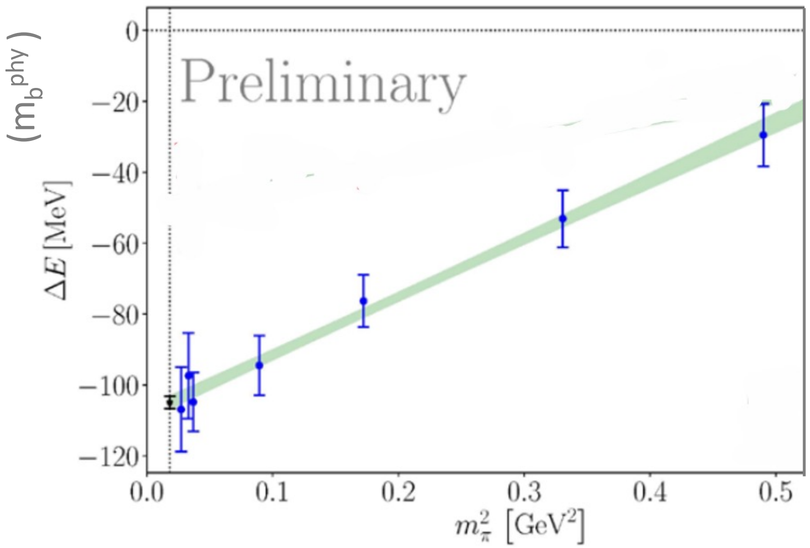
[QQ][ud] dominated state: dependence on m_Q and $m_{u,d}$
known only for a bound state well bellow threshold

$bb\bar{d}\bar{u}$ $\quad I=0,\ J^P=1^+$

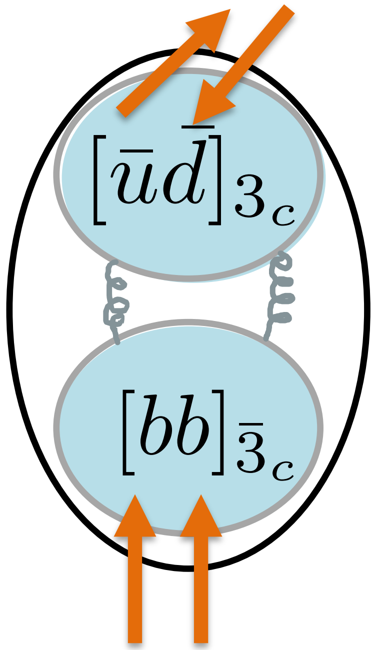
Colquhoun, Francis, Hudspith, Maltman, Lewis
1810.10550 , *PoS* LATTICE2021 (2022) 144
supports internal structure below
see also next talk by S. Aoki

good and bad diquark properties:

Francis et al, 2201.03332



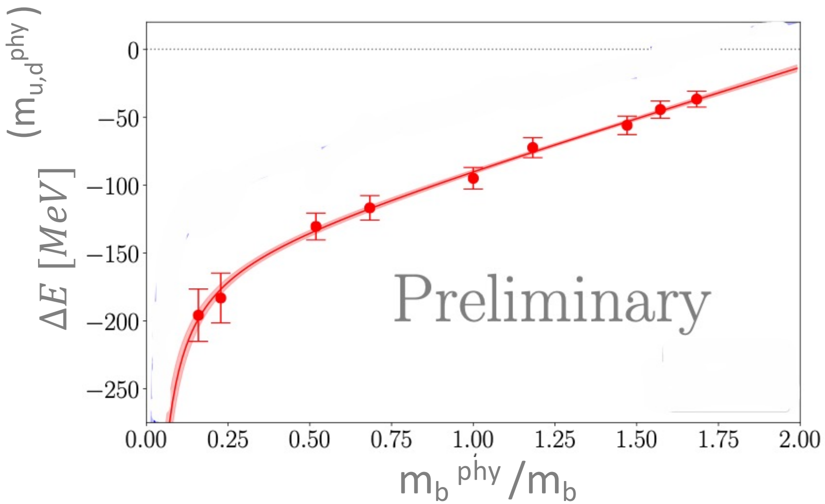
$m_{u,d}$ increases →



$S^P = 0^+$
good anti-diquark

color Coulomb
strongly binds bb
←

$S^P = 1^+$



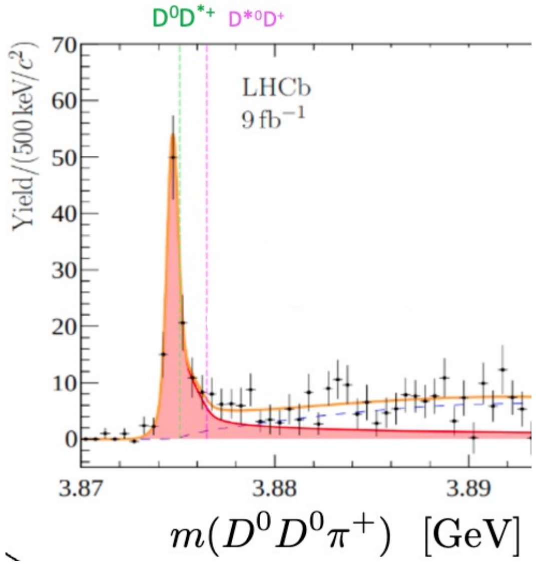
m_b decreases →

As [QQ][ud] dominated state reaches threshold:
virtual bound st. or resonance ?
not known yet (from rigorous studies)

Conclusion on the doubly charm tetraquark

$cc\bar{u}\bar{d}$

I=0, $J^P=1^+$



- The longest lived exotic hadron ever discovered
- It lies very close to DD* threshold: t(E) has to be extracted
- virtual bound state pole slightly below DD* at $m_{u/d}>m_{u/d}^{phy}$
  virtual bound state pole further below th. as m_c is decreased:

  consistent with expectations from dominant molecular Fock comp.
  (this alone does not rule out the presence of other Fock components or other binding mechanisms)

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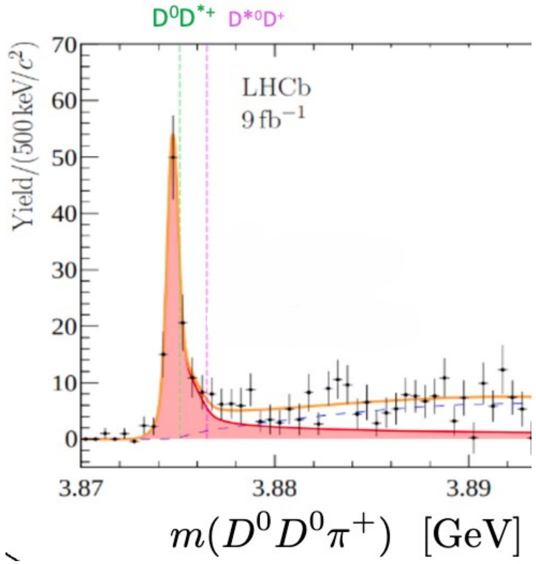


- current study: $m_\pi \simeq 280$ MeV :
  $D^* \not\to D\pi,\ T_{cc} \not\to DD\pi$
  $DD\pi$ above analyzed region

- one of the future challenges m_π^{phy} :
  $D^* \to D\pi,\ T_{cc} \to DD\pi$

formalisms developed by three groups, particularly suitable for DDπ : [Blanton, Sharpe, Lopez, 2105.12094, 2111.12734, talk by S. Sharpe]

Three-particle finite-volume formalism for $\pi^+\pi^+K^+$ and related systems

Implementing the three-particle quantization condition for $\pi^+\pi^+K^+$ and related systems

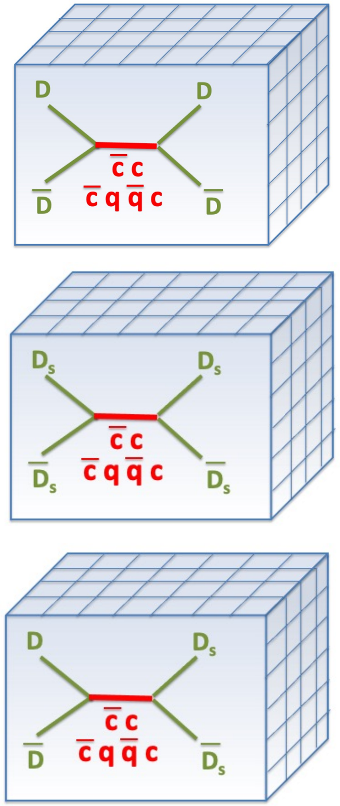
$\bar{c}c$, $\bar{c}q\bar{q}c$ I=0

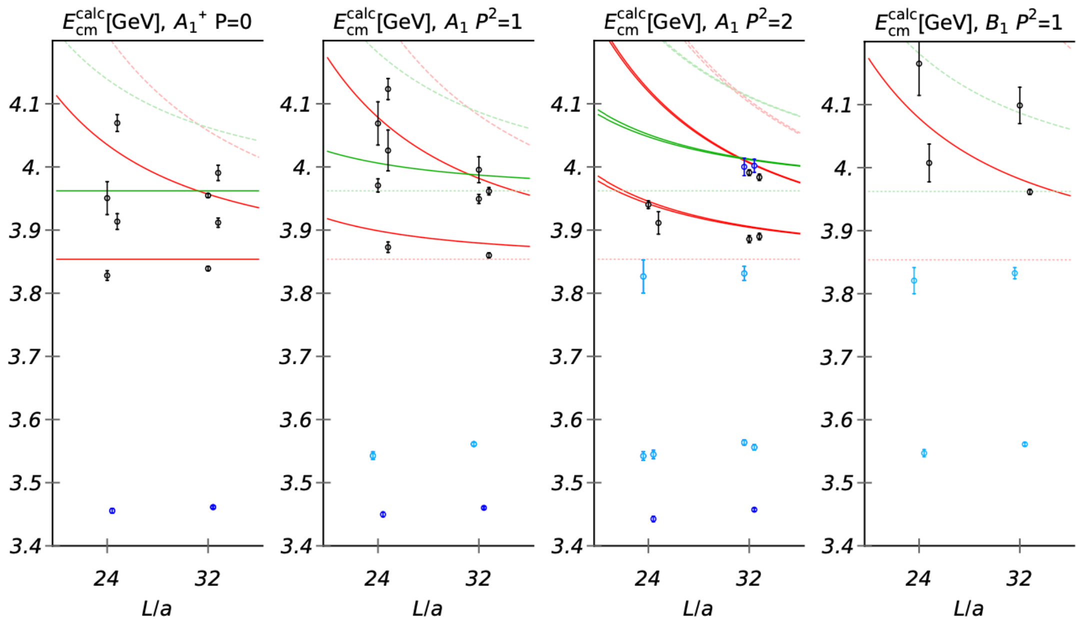
S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02542 JHEP, 1905.03506 PRD, 2111.02934

Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

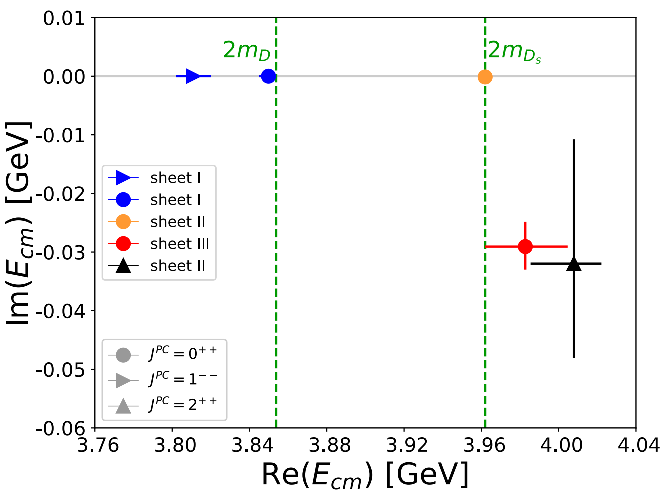
$D\bar{D} - D_s\bar{D}_s$





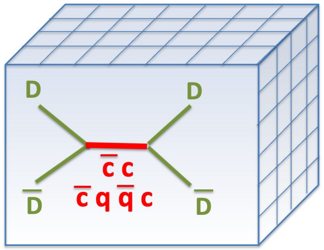
Luscher formalism

$t_{ij}(E)$



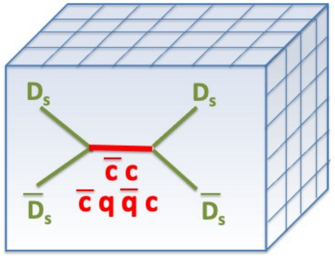
Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$



$\bar{D}_s D_s$ J^P=0$^+$ state

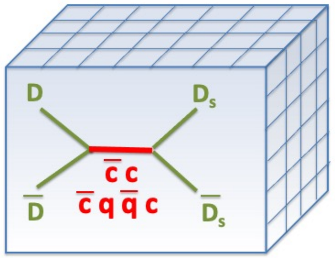
likely related to X(3915) / χ_{c0}(3930) / X(3960)
[BaBar, LHCb 2009.00026, LHCb 2022 indico..../1176505/]
explaining why it has narrow width to $\underline{D}$D.
Supported by some pheno studies:
Lebed, Polosa 1602.08421, Oset et al . 2207.08490, Guo et al, 2101.01021, ….



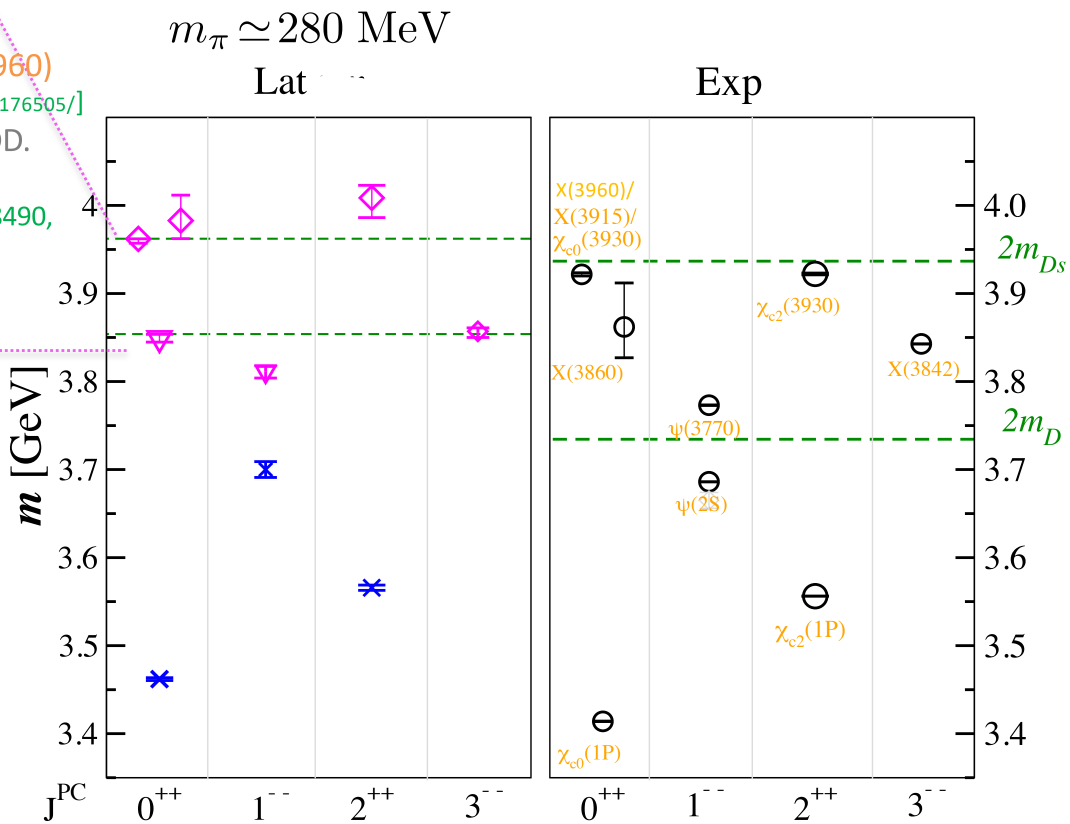
$\bar{D}D$ J^P=0$^+$ state

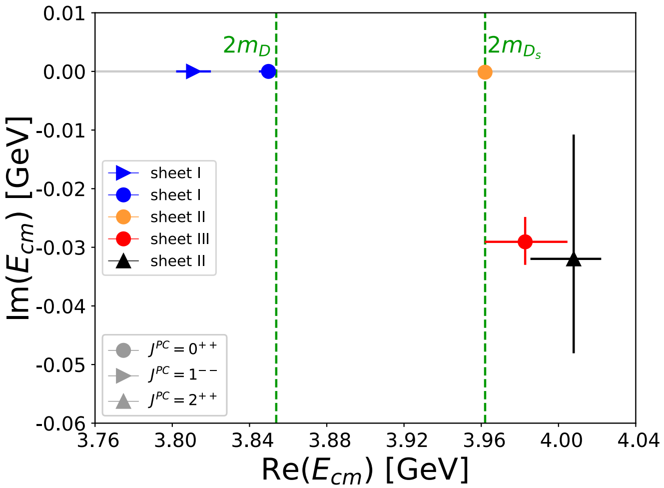
predicted in models [Oset et al, 0612179 PRD, Hildago Duque et al 1305.4487, Baru et al 1605.09649 PLB]

seen in dispersive re-analysis of exp. [Danilkin et al 2111.15033]



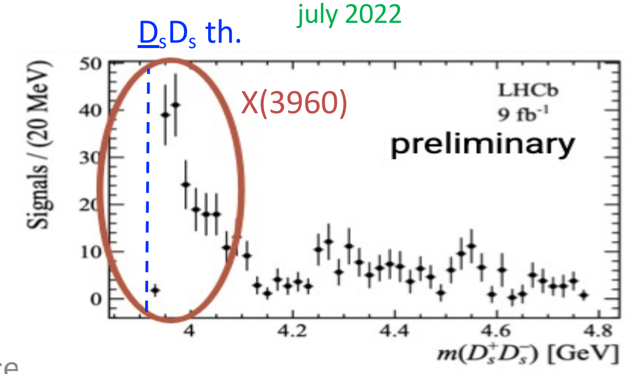
+ expected conventional charmonia





X(3915) / χ_{c0}(3930) / X(3960)
likely the same state
currently named χ_{c0}(3914) in PDG

indico.cern.ch/event/1176505/
july 2022



Backup

Lattice details

CLS ensembles with u/d, s dynamical quarks

$a \simeq 0.086$ fm

$N_L = 24, 32$

lat exp

$m_{u/d} > m_{u/d}^{exp}$

$m_s < m_s^{exp}$

$m_u + m_d + m_s = m_u^{exp} + m_d^{exp} + m_s^{exp}$

$m_c \gtrsim m_c^{exp}$

m [MeV]	lat	exp
m_π	280(3)	137
m_D	1927(2)	1867
m_{Ds}	1981(1)	1968
M_{av}	3103(3)	3068

$M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$

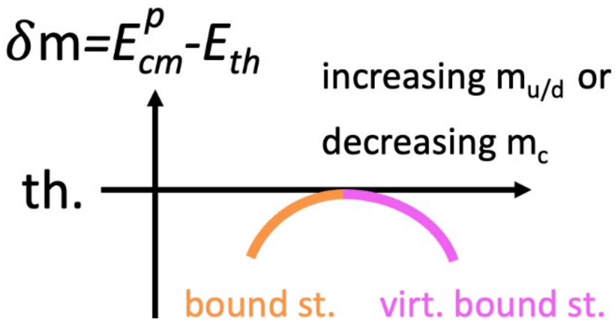
separation between D$\underline{\text{D}}$ and Ds$\underline{\text{D}}$s threshols smaller than in exp

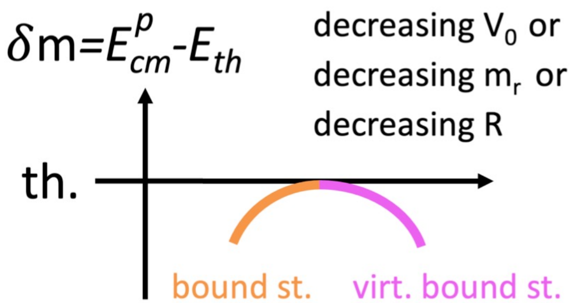
Wick contractions evaluated with distillation or stochastic distillation method.

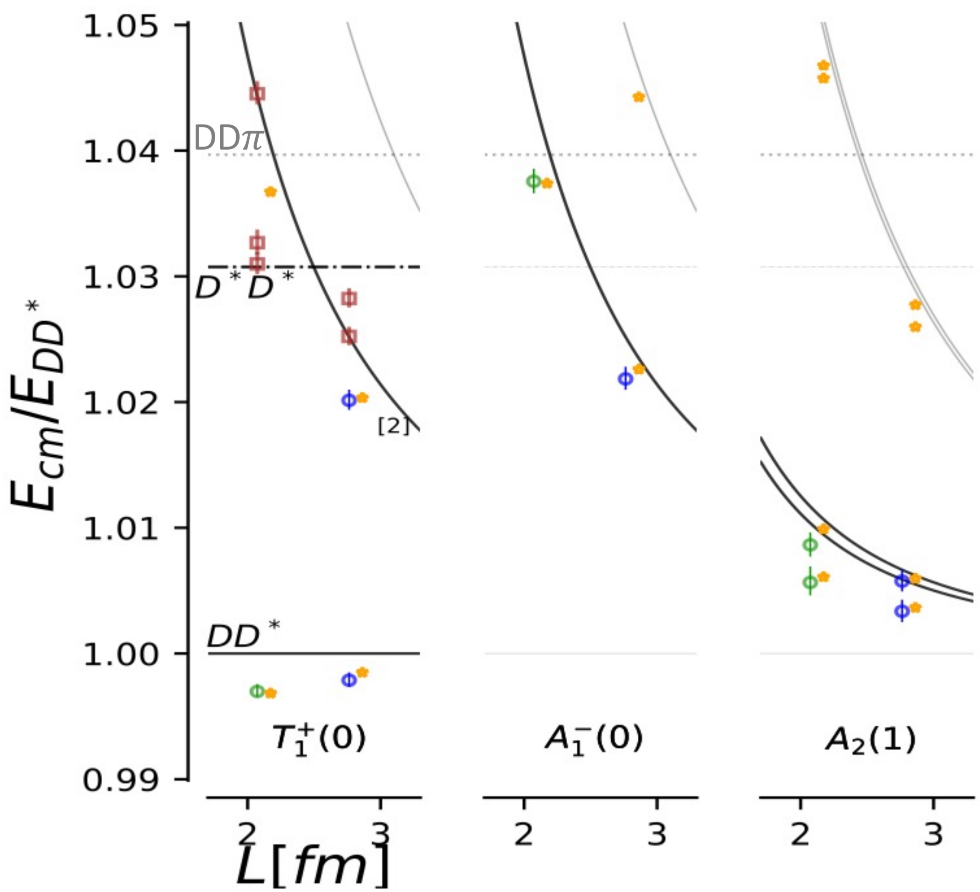
Lattice results

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9),0]	$-0.36(4)$	bound st.

$$V(r) = -V_0 \; f(r/R)$$







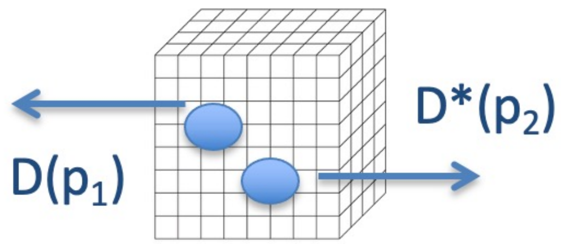
Interpolators

Example: P=0

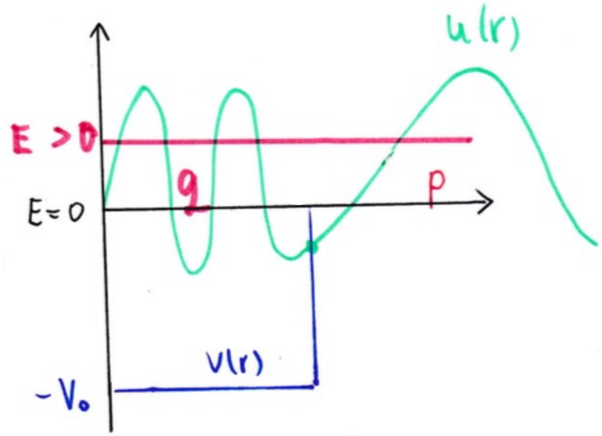
$J^P=1^+$ -> cubic irrep T_1^+

P=D, V=D*

$$
\begin{aligned}
O^{l=0} =& P(\{0,0,0\})V_z(\{0,0,0\}) \\
O^{l=0} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&+P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=2} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&-2[P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=0} =& V_{1x}[0,0,0]V_{2_y}[0,0,0] - V_{1y}[0,0,0]V_{2x}[0,0,0]
\end{aligned}
$$



s-wave scattering on spherical potential well



$$A \sin qr \qquad B \sin(pr + \delta_0)$$

$$u(R) = A \sin qR = B \sin(pR + \delta)$$
$$u'(R) = q\, A \cos qR = p\, B \cos(pR + \delta)$$

dividing both eqs $$\frac{1}{q} \tan qR = \frac{1}{p} \tan(pR + \delta)$$

$$\delta_0(p) = \arctan\left(\frac{p}{q}\tan(qR)\right) - pR + n\pi$$

$$q = \sqrt{2\mu(V_0 + E)} = \sqrt{2\mu V_0 + p^2}$$

$$\chi^2(\{a\}) = \sum_L \sum_{\vec{P}\Lambda n} \sum_{\vec{P}'\Lambda' n'} dE_{cm}(L, \vec{P}\Lambda n; \{a\}) \qquad (1)$$
$$\mathcal{C}^{-1}(L; \vec{P}\Lambda n; \vec{P}'\Lambda' n') dE_{cm}(L, \vec{P}'\Lambda' n'; \{a\}) \ .$$

Here

$$dE_{cm}(L, \vec{P}\Lambda n; \{a\}) = E_{cm}(L, \vec{P}\Lambda n) - E_{cm}^{an.}(L, \vec{P}\Lambda n; \{a\})$$

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i \frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)} \qquad (5)$$

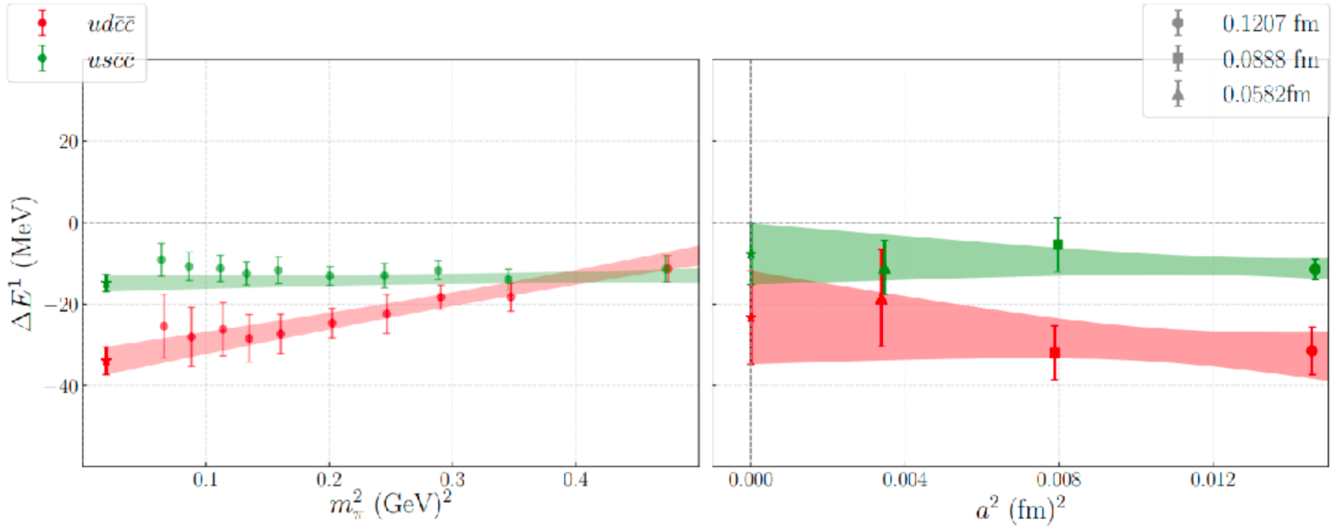
We parametrize it with the effective range expansion

$$\tilde{K}^{-1} = \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} . \qquad (6)$$

other lattice studies of Tcc

Previous lattice QCD study of T_{cc} channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



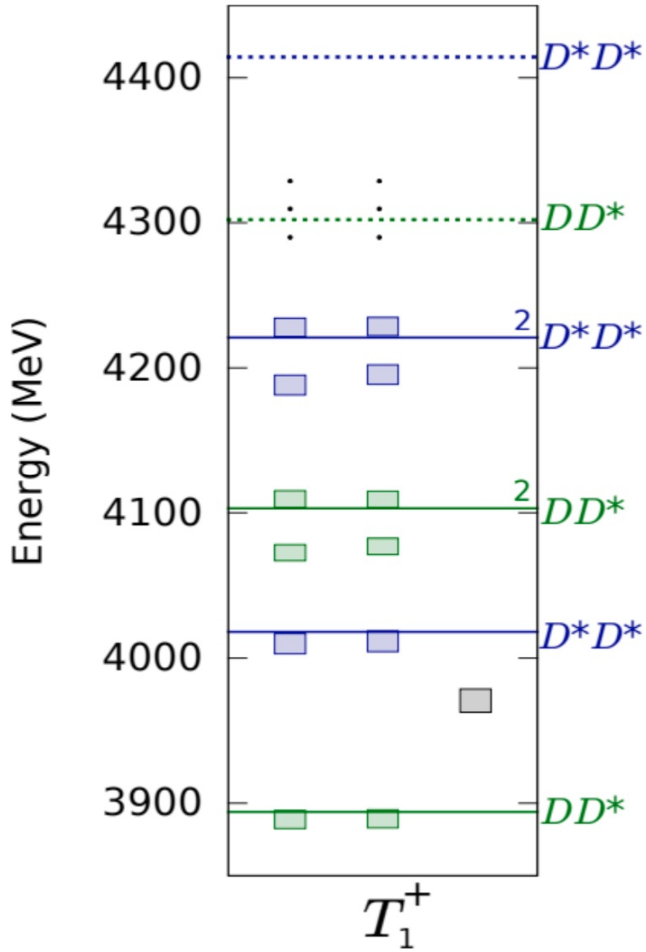
lowest finite-volume eigen-energy for P=0, $J^P=1^+$, I=0

- Study performed on LQCD ensembles with different lattice spacings. Single volume and only rest frame finite-volume irreps considered.
- Including a meson-meson and diquark-antidiquark interpolator. Diquark-antidiquark interpolators do not influence the low energy spectrum.
- The ground state energy subjected to chiral and continuum extrapolations.
- A finite-volume energy level 23(11) MeV below DD^* threshold. No rigorous scattering analysis and no pole structure determined.

Previous lattice QCD study of T_{cc} channel

Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

finite-volume
eigen-energies for
P=0, $J^P=1^+$, I=0

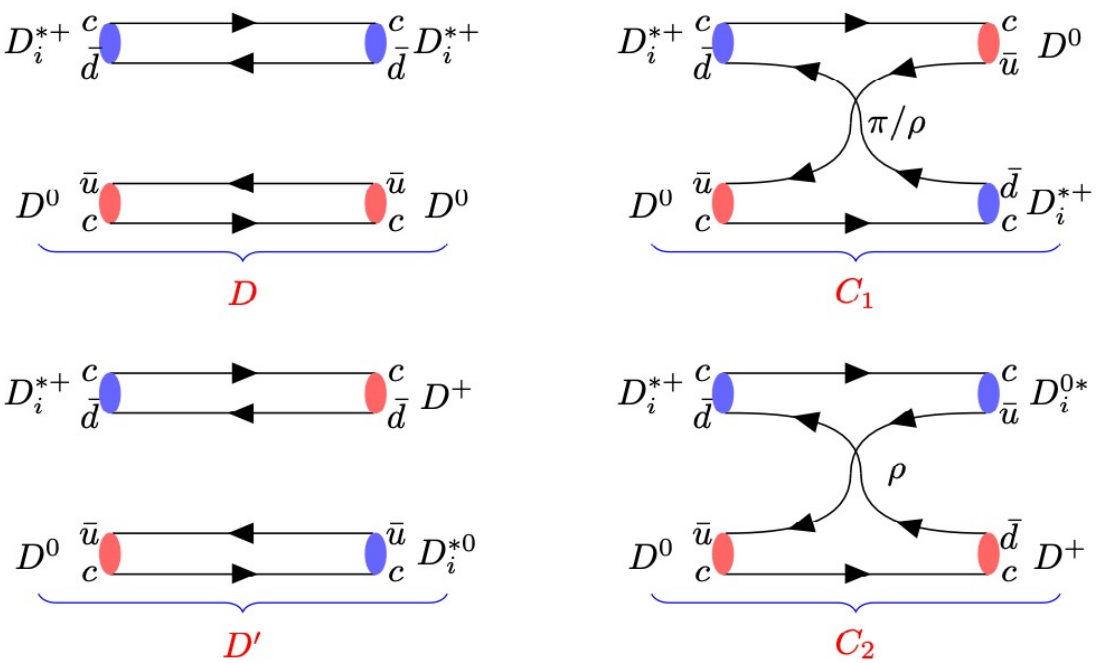


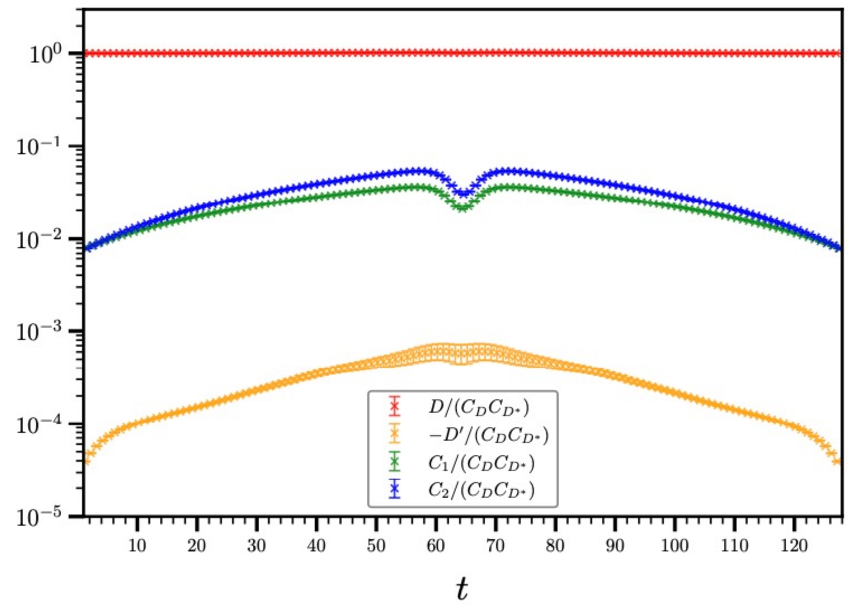
- Single volume rest frame study on a relatively coarse lattice ($a_s \sim 0.12$ fm).
- Large basis of meson-meson and diquark-antidiquark interpolators.
- Diquark-antidiquark interpolators do not influence the low energy spectrum.
- No statistically significant energy shifts observed near DD^* threshold.
  $\Rightarrow$ No scattering amplitude extraction.

Subsequent lattice QCD study of T_{cc} channel

CLQCD, Chen et al. 2206.06185

comparison of I=0,1 :
attraction in I=0 channel arises
mainly from ϱ exchange





$$C^{(I)}(p,t) = D - C_1(\pi/\rho) + (-)^{I+1}\left(D' - C_2(\rho)\right)$$

Phenomenological theoretical predictions

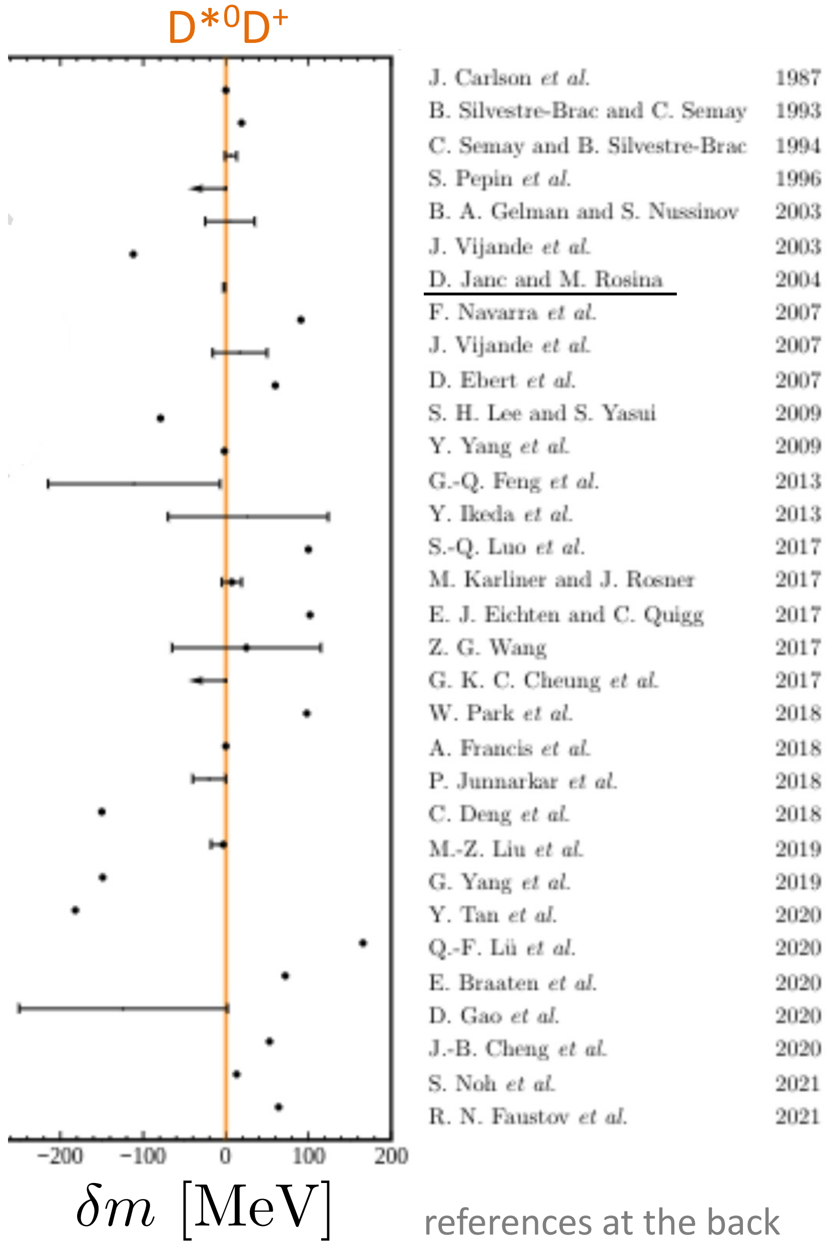
❖ **Phenomenological approaches** →

* Janc & Rosina , Few Body Syst. 35, 175 (2004), hep-ph/0405208

one of the most sophisticated quark model predictions:

V_{ij} between all pairs of quarks, ground state energy of four-body problem

$$\delta m = -1.6 \pm 1.0 \text{ MeV}$$



references at the back

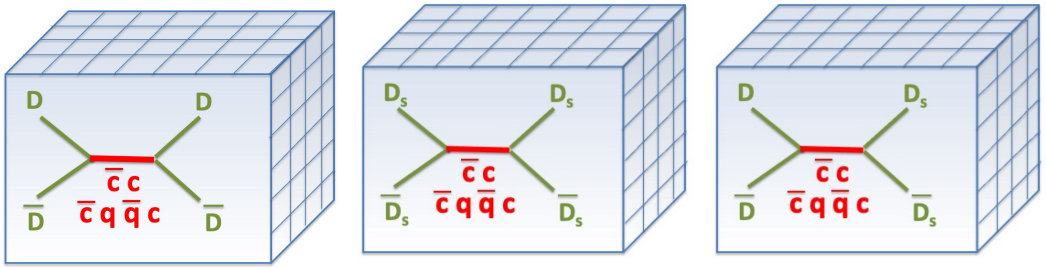
Theoretical PREdictions

courtesy: Ivan Polyakov, EPS-HEP 2021

(references at the back)

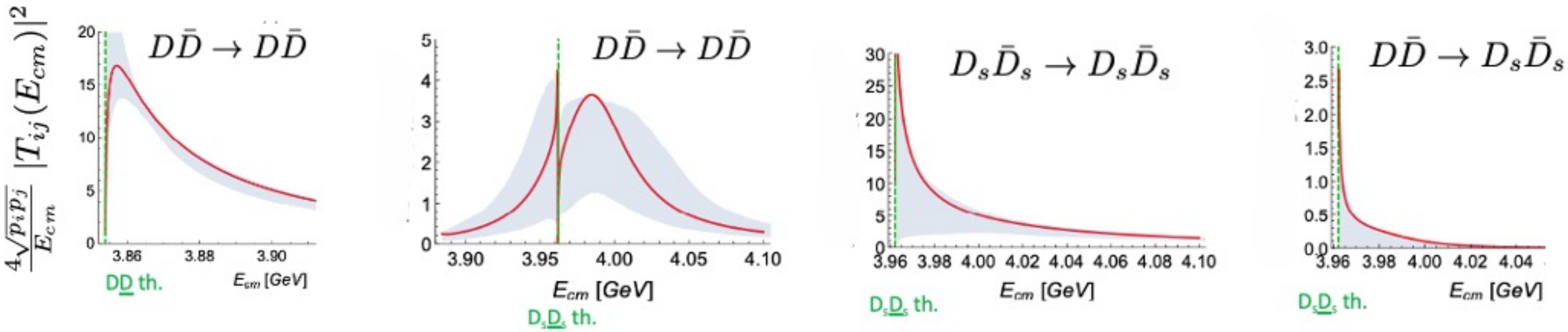
Resonances from coupled-channel scattering

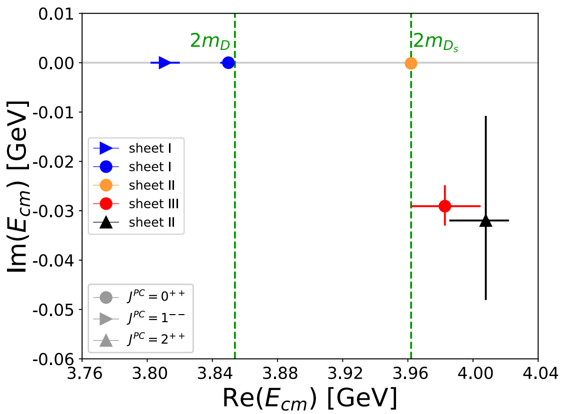
- most results by HadSpec. coll.: mostly light meson sector
- example in heavy-quark sector $\bar{c}c\,,\ \bar{c}q\bar{q}c$ q=u,d,s $I=0$



two coupled channels

$$D\bar{D} - D_s\bar{D}_s$$





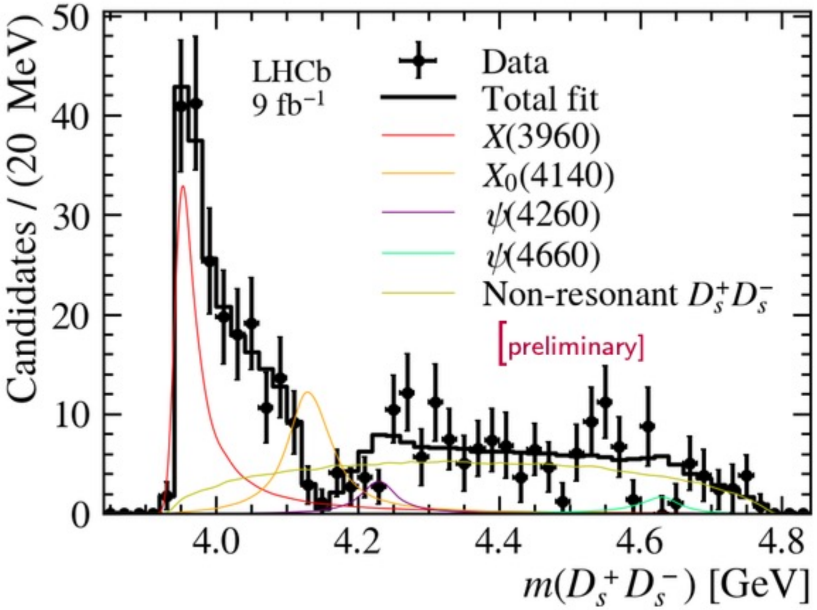
S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP
physics interpretation: two slides later

$\bar{c}c$, $\bar{c}q\bar{q}c$

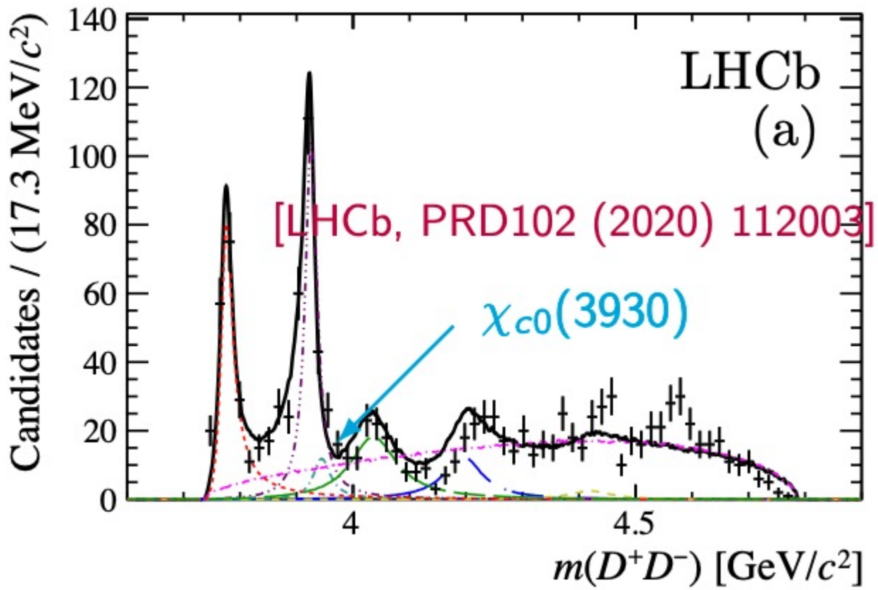
likely yes

Is $X(3960)$ the same as $\chi_{c0}(3930)$ from D^+D^-?

$B^+ \to (D_s^+ D_s^-) K^+$ by LHCb:



$B^+ \to (D^+ D^-) K^+$ by LHCb:



- Assuming to be the same, $\mathcal{B}(\chi_{c0} \to D^+D^-)/\mathcal{B}(\chi_{c0} \to D_s^+D_s^- P) \sim 0.3$
  large molecular component, or large tetraquark component, $T_{\psi\phi}$
- [JHEP 06 (2021) 035] finds a state coupled to $D_s^+D_s^-$ on the lattice

$\bar{c}c$, $\bar{c}q\bar{q}c$

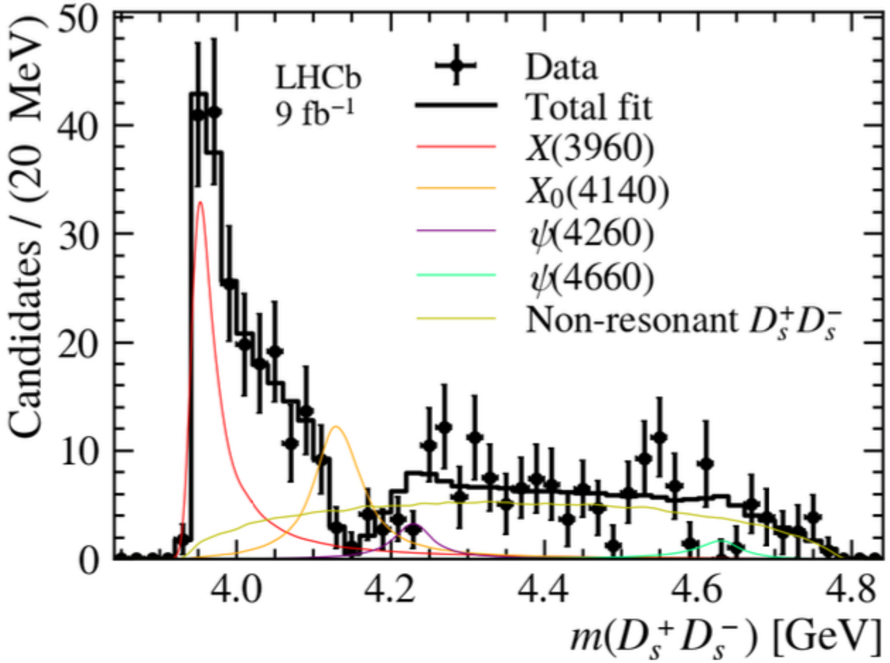
slide by Mikhashenko, Hadron Spectrum in Confinement, aug 2022

likely yes

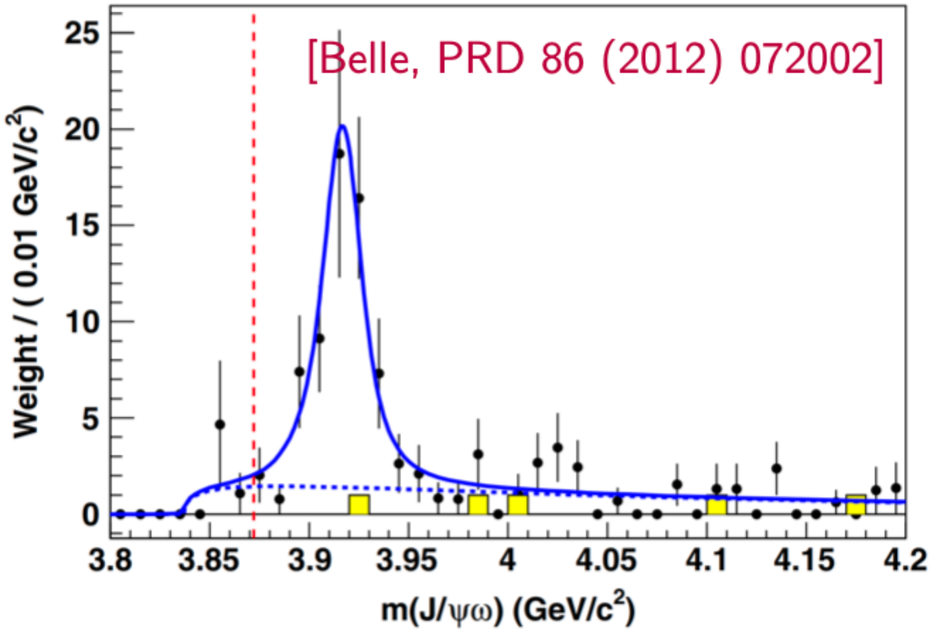
Is $X(3960)$ the same as $\chi_{c0}(3915)$?

[LHCb-PAPER-2022-018, 019 (in preparation)]

$B^+ \to (D_s^+ D_s^-) K^+$ by LHCb:



$\gamma\gamma \to J/\psi\omega$ by Belle:



- Belle sees a clean state in $J/\psi\omega$ with $J^P = 0^+$
- The $D_s^+ D_s^-$ signal might be a tail of the $\chi_{c0}(3915)$ state