Lattice study on a tetra-quark state T_{bb} in the HAL QCD method

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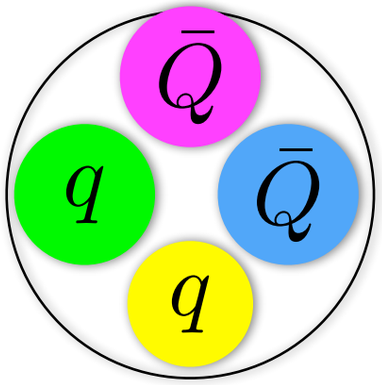
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Lattice2022@Bonn, Aug.8-13, 2022

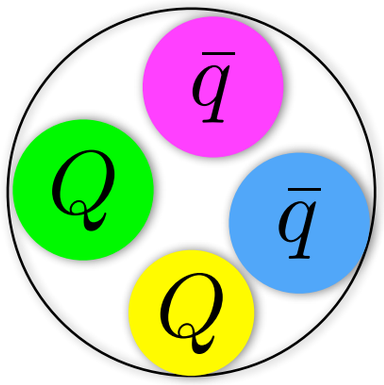
I. Introduction

Heavy tetra-quark states T_{QQ}



$\bar{Q}$: heavy anti-quark

q : light quark



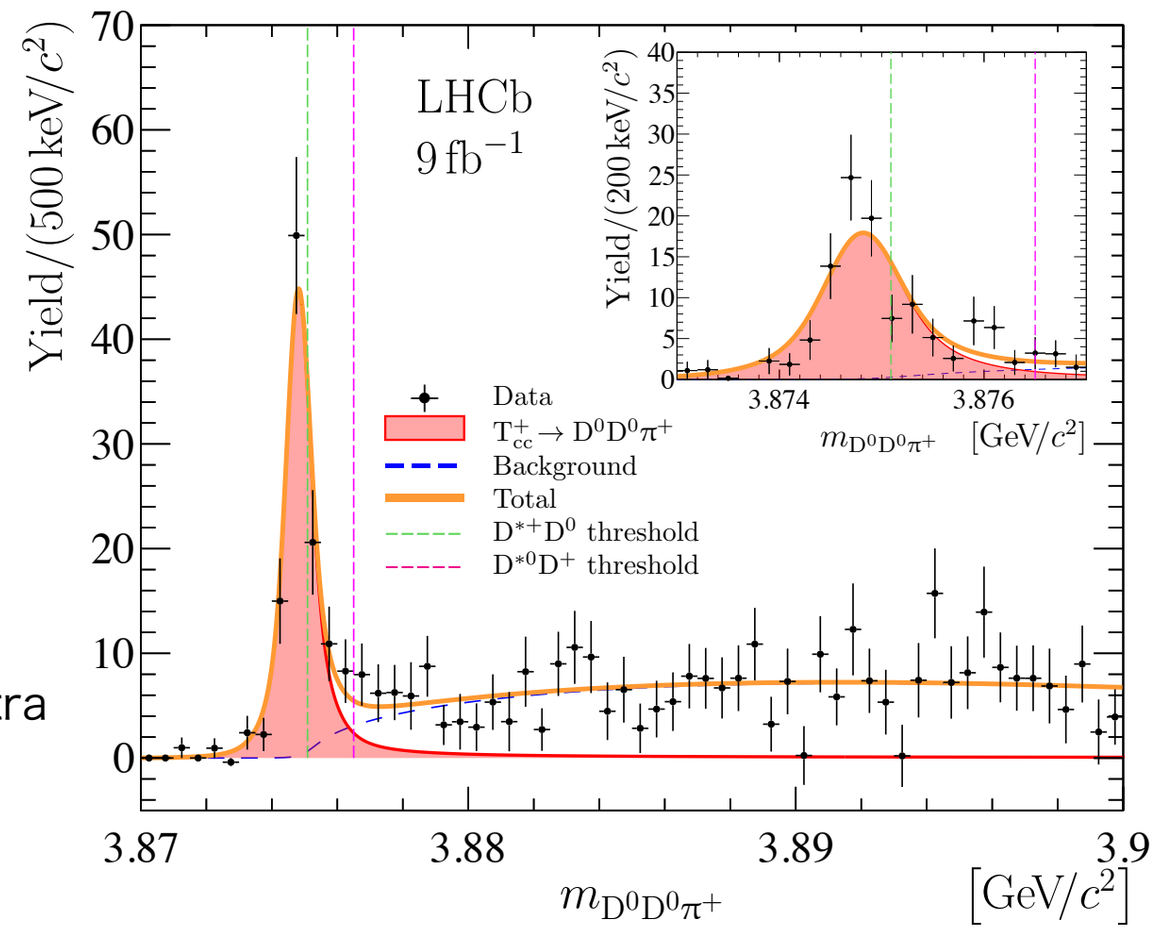
Q : heavy quark

$\bar{q}$: light anti-quark

genuine tetra-quark states

$T_{cc}(cc\bar{u}\bar{d})$ observation by LHCb.

Aaij et al. (LHCb Collaboration) , Nature Phys. (2022)



No definite confirmation by previous Lattice QCD calculations.

Guerrieri et al., PoS LATTICE 2014(2014) 106: hadron spectra

Ikeda et al., PLB729(2014) 85: potential method

The latest lattice study suggest a virtual state.

Padmanath-Prelovsek, PRL129(2022)3,032002: hadron spectra

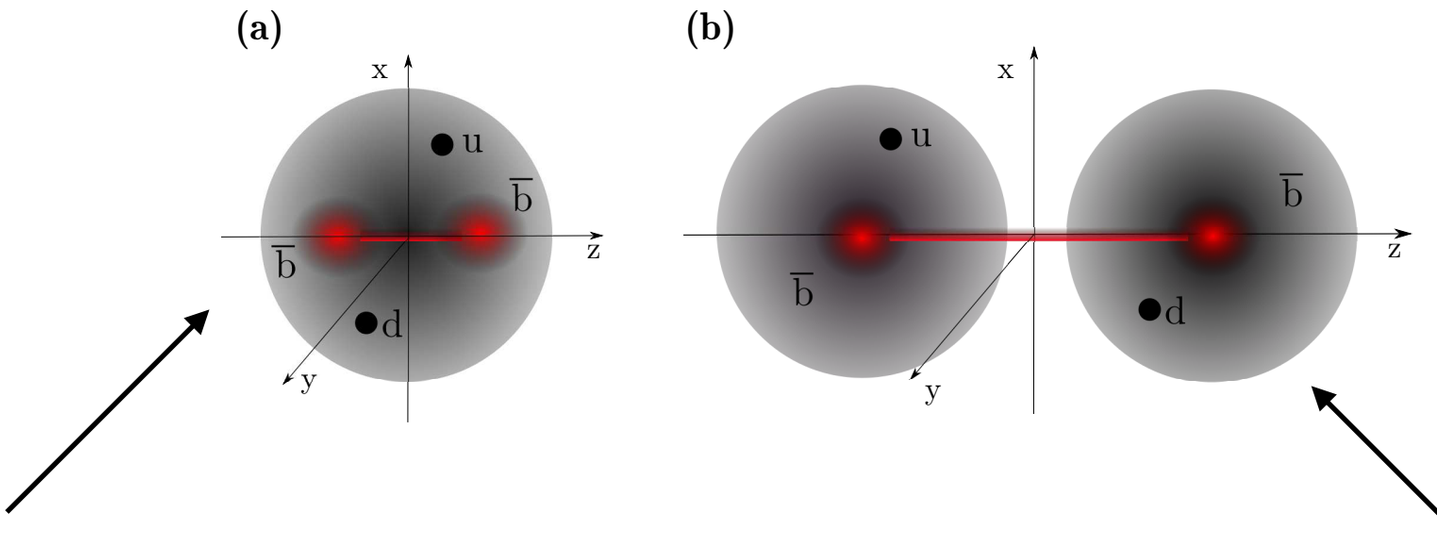
talks@lattice2022

$$T_{bb}(\bar{b}\bar{b}ud)$$

not observed yet, but is more likely to exist as a bound state than charmed tetra-quark states.

Expected mechanism

picture from Bicudo et al., PRD93(2016) 034501



one-gluon-exchange (short distance)

screening by light quarks (long distance)

screens Coulomb potential

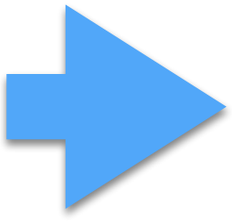
bound state

if $\bar{b}\bar{b}$ are heavy enough

latest lattice QCD calculations

1. single channel potential between $\bar{B}$ and $\bar{B}^*$, static b quark

Bicudo et al., PRD93(2016) 034501



one bound state in $I(J^P) = 0(1^+)$

$E_B = 90^{+43}_{-36}$ MeV, extrapolated to physical pion mass

2. coupled channel potential for $\bar{B}\bar{B}^*$ and $\bar{B}^*\bar{B}^*$, static b quark

Bicudo et al., PRD95(2017) 034502

binding energy is reduced to $E_B = 59^{30}_{-38}$ MeV

3. spectra with NRQCD b quark, a bound state in $0(1^+)$.

$E_B = 165.0(32.5)$ MeV $(a \to 0)$ Junnarkar et al., PRD99(2019) 034507

$E_B = 120(24)(10)$ MeV Leskovec et al., PRD100(2019) 014503

$E_B = 167(19)$ MeV Mohanta-Basak, PRD102(2020) 094516

moving b quark $E_B \uparrow$ coupled channel $E_B \downarrow$

Aims of our study

1. confirm an existence of a bound T_{bb} in the HAL QCD (potential) method

2. investigate properties of T_{bb} in the HAL QCD method

3. perform a coupled channel analysis with moving b quarks by NRQCD

Takafumi Aoki mainly worked on this project and got his master degree this March. Unfortunately, he left physics and is now working in a private company.

Plan of my talk

I. Introduction (4 pages)

II. Methodology (4 pages)

1. HAL QCD method

2. NRQCD for heavy quarks

3. Operators

4. Lattice QCD configurations

III. Numerical results (8 pages)

1. Leading order potential (3 pages)

2. Scattering analysis (5 pages)

IV. Conclusion (1 pages)

II. Methodology

1. HAL QCD method

Since the $\mathcal{B}^* \equiv \bar{B}^*\bar{B}^*$ threshold is about 45 MeV above $\mathcal{B} \equiv \bar{B}\bar{B}^*$ threshold, we consider the coupled channel analysis in our study.

We employ the time-dependent coupled channel HAL QCD method at the leading order in the derivative expansion.

Ishii et al. (HAL QCD), PLB712(2012)437

Aoki et al. (HAL QCD), Proc. Japan Acad. B87 (2011) 509

Ishii, Aoki and Hatsuda, PRL99(2007)022001

time-dependent 2×2 coupled channel equation $\quad \alpha, \beta = 0(\mathcal{B}), 1(\mathcal{B}^*)$

$$\left(\frac{\nabla^2}{2\mu^\alpha} - \frac{\partial}{\partial t} + \frac{1+3\delta^{\alpha 2}}{8\mu^\alpha}\frac{\partial^2}{\partial t^2}\right) R_\xi^\alpha(\mathbf{r},t) \simeq \sum_\beta \tilde{\Delta}^{\alpha\beta}(t) \int \mathrm{d}^3\mathbf{r}'\, U^{\alpha\beta}(\mathbf{r},\mathbf{r}') R_\xi^\beta(\mathbf{r}',t)$$

$$\equiv \mathcal{K}_\xi^\alpha$$

$$\tilde{\Delta}^{\alpha\beta}(t) = \sqrt{\frac{Z_1^\beta Z_2^\beta}{Z_1^\alpha Z_2^\alpha}} \frac{\mathrm{e}^{-(m_1^\beta + m_2^\beta)t}}{\mathrm{e}^{-(m_1^\alpha + m_2^\alpha)t}}$$ correction factor

Leading order in the derivative expansion

$$\begin{pmatrix} V^{00}(\mathbf{r}) & \tilde{\Delta}^{01}(t)V^{01}(\mathbf{r}) \\ \tilde{\Delta}^{10}(t)V^{10}(\mathbf{r}) & V^{11}(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathcal{K}_0^0(\mathbf{r},t) & \mathcal{K}_1^0(\mathbf{r},t) \\ \mathcal{K}_0^1(\mathbf{r},t) & \mathcal{K}_1^1(\mathbf{r},t) \end{pmatrix} \begin{pmatrix} R_0^0(\mathbf{r},t) & R_1^0(\mathbf{r},t) \\ R_0^1(\mathbf{r},t) & R_1^1(\mathbf{r},t) \end{pmatrix}^{-1}$$

2. Non-Relativistic QCD (NRQCD)

Since b quark is too heavy to treat relativistically on current lattice spacings, we employ NRQCD action for b quarks.

$O(v^4)$ Hamiltonian

$$\mathcal{H}_\psi = \mathcal{H}_0 + \sum_i c_i \delta\mathcal{H}^{(i)}, \quad \mathcal{H}_0 = -\frac{1}{2M}\Delta^{(2)},$$

$$\delta\mathcal{H}^{(1)} = -\frac{1}{2M}\sigma\cdot\mathbf{B}, \quad \delta\mathcal{H}^{(2)} = \frac{i}{8M^2}(\Delta\cdot\mathbf{E} - \mathbf{E}\cdot\Delta), \quad \delta\mathcal{H}^{(3)} = -\frac{1}{8M}\sigma\cdot(\Delta\times\mathbf{E} - \mathbf{E}\times\Delta),$$

$$\delta\mathcal{H}^{(4)} = -\frac{1}{8M^3}(\Delta^{(2)})^2, \quad \delta\mathcal{H}^{(5)} = \frac{1}{24M}\Delta^{(4)}, \quad \delta\mathcal{H}^{(6)} = -\frac{1}{16nM^2}(\Delta^{(2)})^2, \tag{32}$$

$c_i = 1$ values at tree level with tadpole improvement $U_\mu \to U_\mu/u_0$

Mass of heavy-light meson in NRQCD

kinetic energy $E_X(\mathbf{p}) = \sqrt{\mathbf{p}^2 + (M_X^{\rm kin})^2} - \delta$ ⟵ additive mass renormalization

⇒ kinetic mass $M_X^{\rm kin} = \dfrac{\mathbf{p}^2 - (E_X(\mathbf{p}) - E_X(\mathbf{0}))^2}{2(E_X(\mathbf{p}) - E_X(\mathbf{0}))}.$

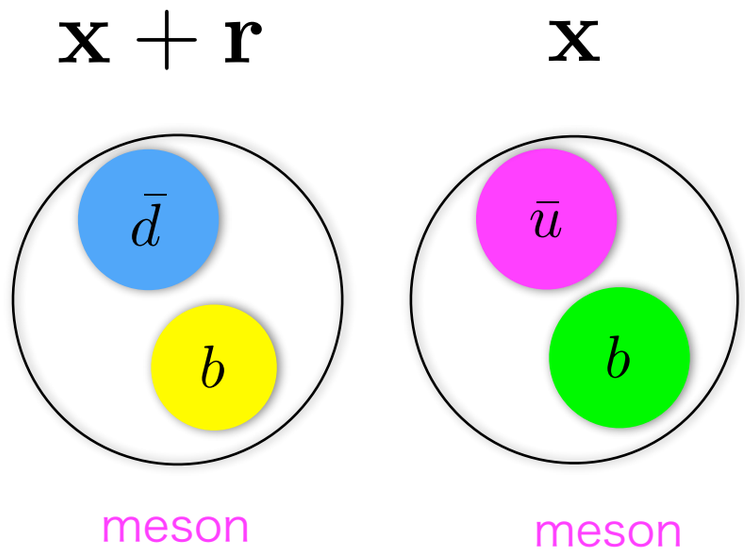
non-perturbative subtraction but noisier.

3. Operators

Sink operators (2 types of meson-meson operators)

$$\mathcal{B}_j \equiv \sum_{\mathbf{x}} \underbrace{(\bar{u}(\mathbf{y})\gamma_5 b(\mathbf{y}))}_{\bar{B}(\mathbf{y})} \underbrace{(\bar{d}(\mathbf{x})\gamma_j b(\mathbf{x}))}_{\bar{B}^*(\mathbf{x})} - [u \leftrightarrow d], \quad \mathbf{y} \equiv \mathbf{x} + \mathbf{r},$$

$$\mathcal{B}^*_j \equiv \epsilon_{jkl} \sum_{\mathbf{x}} (\bar{u}(\mathbf{y})\gamma_k b(\mathbf{y}))\big(\bar{d}(\mathbf{x})\gamma_l b(\mathbf{x})\big) - [u \leftrightarrow d],$$



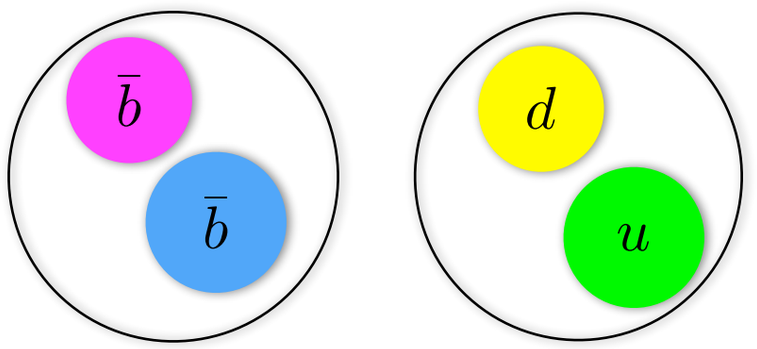
Since both $\mathcal{B}$ and $\mathcal{B}^*$ source operators create similar combinations of states, we introduce a diquark source operator.

Source operators (wall quark source with gauge fixing)

$$\mathcal{J}_{\mathcal{B}^\dagger_j} = (\bar{b}\gamma_5 u)(\bar{b}\gamma_j d) - [u \leftrightarrow d]$$ meson-meson

$$\mathcal{J}_{\mathcal{D}^\dagger_j} = (\epsilon^{abc}\bar{b}^b\gamma_j C\bar{b}^c)(\epsilon^{ade}d^d C\gamma_5 u^e) - [u \leftrightarrow d]$$

diquark operator



4. Lattice QCD configurations

$2+1$ flavor full QCD configurations generated by the PACS-CS Collaboration with the Iwasaki gauge action and the Wilson-Clover light quark action

400 configurations

Configuration	$V_{\rm lat} = L_s^3 \times L_t$	a [fm]	L_s [fm]	κ_{ud}	κ_s	$c_{\rm sw}$	M_b	u_0
PACS-CS-A	$32^3 \times 64$	0.0907(13)	2.902(42)	0.13700	0.13640	1.715	1.919	0.868558(42)
PACS-CS-B	$32^3 \times 64$	0.0907(13)	2.902(42)	0.13727	0.13640	1.715	1.919	0.868793(43)
PACS-CS-C	$32^3 \times 64$	0.0907(13)	2.902(42)	0.13754	0.13640	1.715	1.919	0.869005(44)

M_b is taken to give $M_{bb}^{\rm spin-avg.} \simeq 9450$ MeV

Configuration	m_π [MeV]	m_ρ [MeV]	$M_{\bar{B}}^{\rm spinavg}$ [MeV]	$\Delta E_{\bar{B}\bar{B}^*}$ [MeV]
PACS-CS-A	701(1)	1102(1)	5440(174)	49.4(2.6)
PACS-CS-B	571(0)	1011(1)	5382(269)	44.9(1.6)
PACS-CS-C	416(1)	920(3)	5332(220)	42.7(3.9)

decreasing

$$M_{\bar{B}}^{\rm spin-avg.} = \frac{1}{4} M_{\bar{B}} + \frac{3}{4} M_{\bar{B}^*} \qquad \Delta E_{\bar{B}\bar{B}^*} = E_{\bar{B}^*}(\mathbf{0}) - E_{\bar{B}}(\mathbf{0})$$

$M_{\bar{B}}^{\rm spin-avg.} \simeq 5313$ MeV within about 5% statistical errors

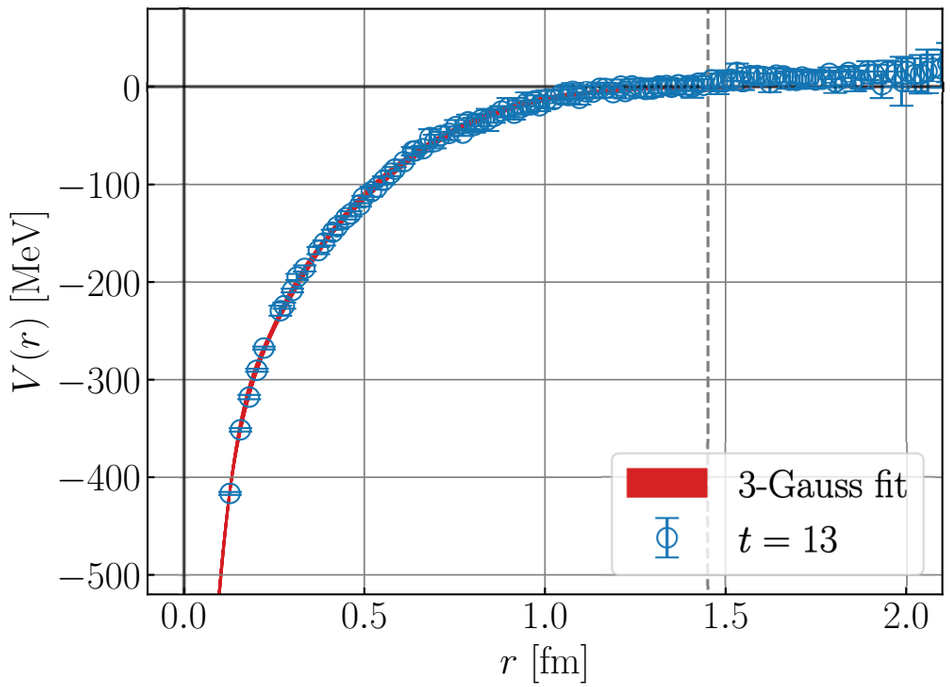
III. Numerical Results

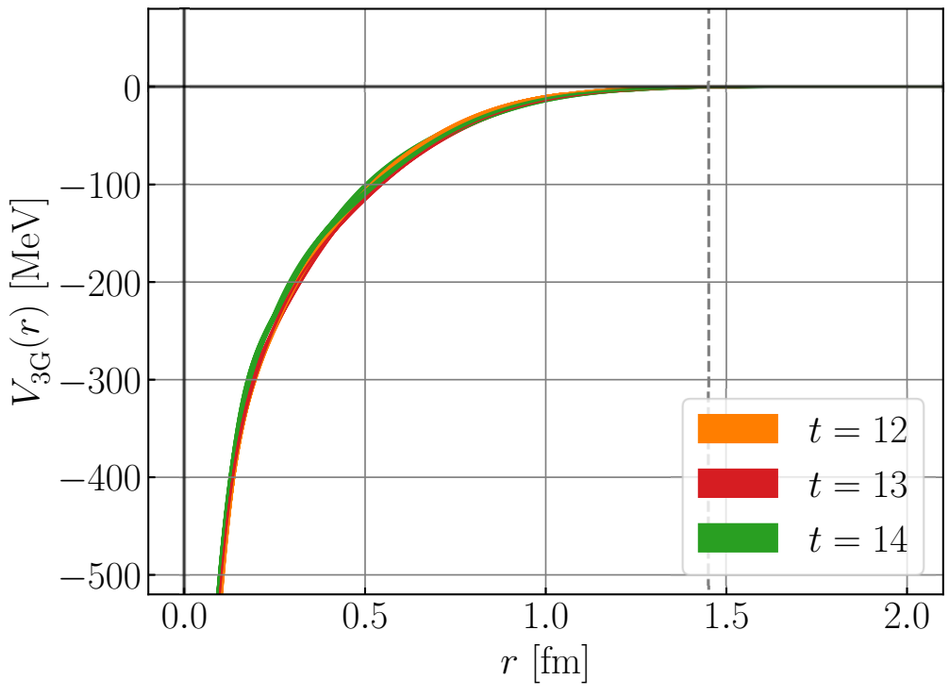
Takafumi Aoki and Sinya Aoki, in preparation.

1. Leading order potentials

1-1. Single channel LO potential

$m_\pi \simeq 700$ MeV





1. attractive at distances smaller than 1.0 fm

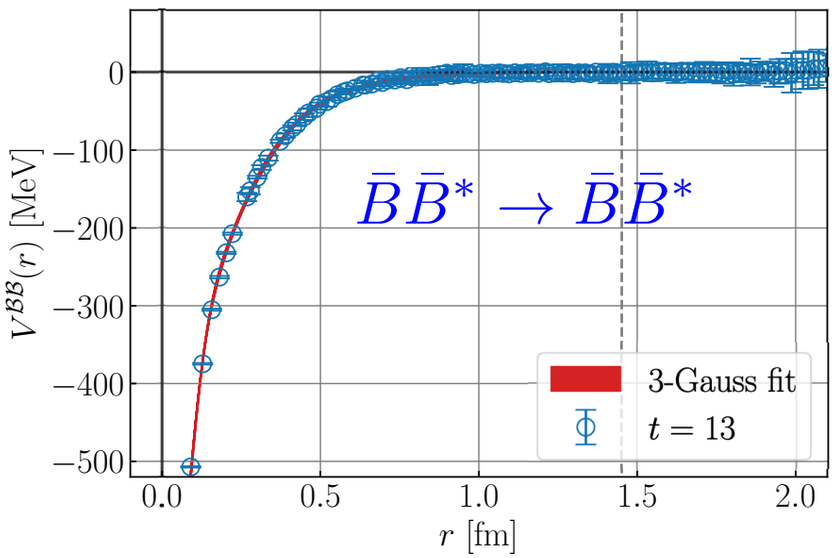
2. 3-Gaussian fit

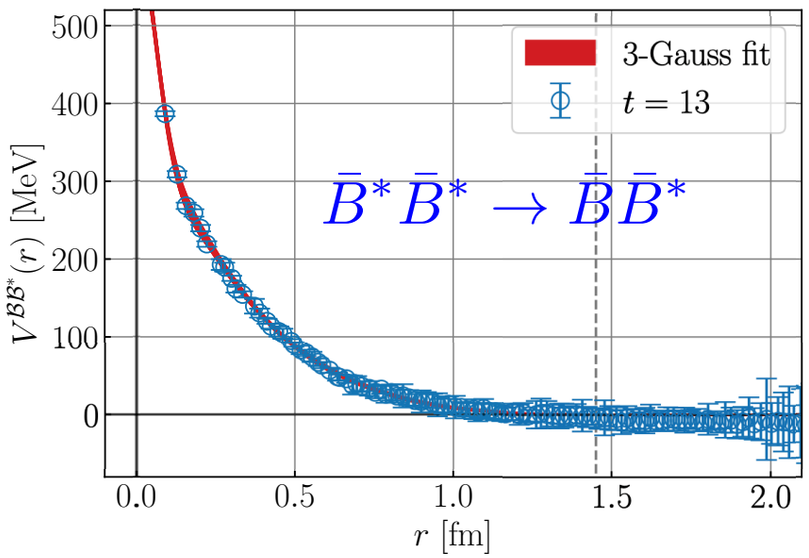
$$V_{3\mathrm{G}}(r) = \mathcal{V}_0 e^{-r^2/\rho_0^2} + \mathcal{V}_1 e^{-r^2/\rho_1^2} + \mathcal{V}_2 e^{-r^2/\rho_2^2}$$

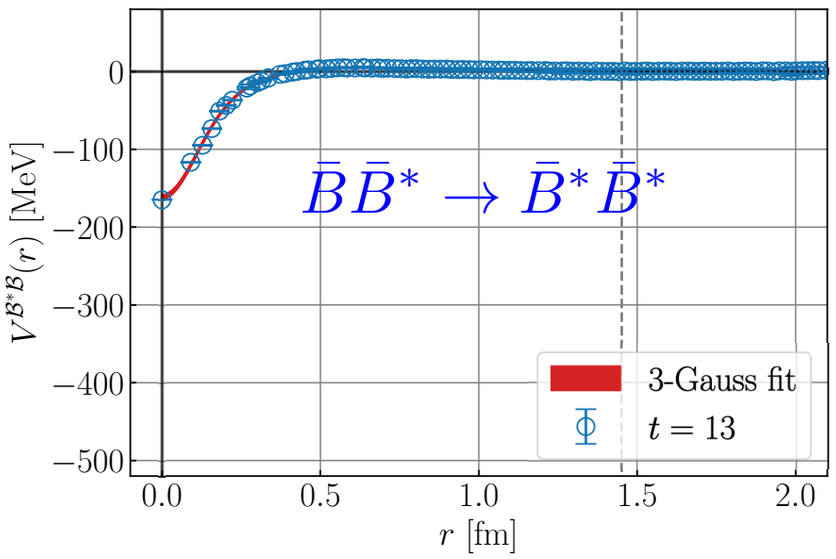
3. small time dependence suggests that contributions from inelastic states are well under control.

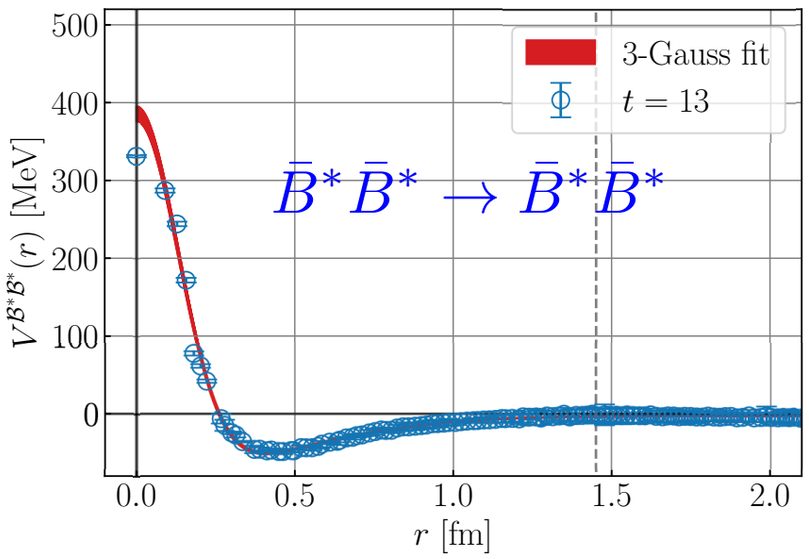
1-2. Coupled channel LO potential

$m_\pi \simeq 700$ MeV









Hermiticity is badly broken.

shorter than a single channel

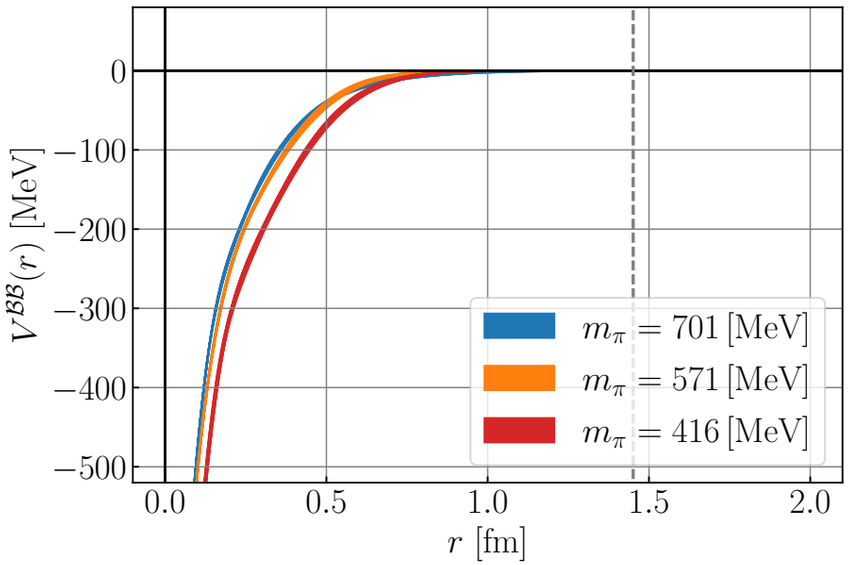
A diagonal potential $V^{\mathcal{B}\mathcal{B}}$ is attractive at distances smaller than 0.8 fm.

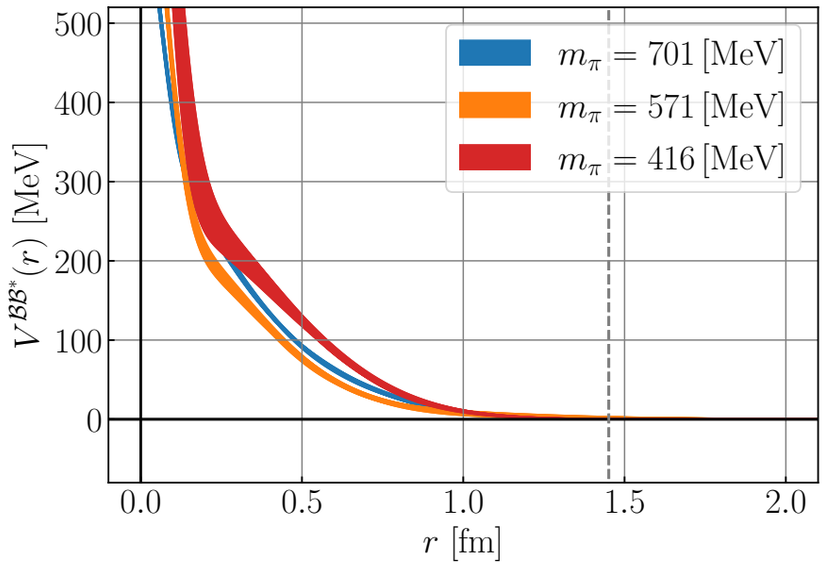
Other one $V^{\mathcal{B}^*\mathcal{B}^*}$ has a repulsive core surrounded by an attractive pocket at $r \simeq 0.4$ fm.

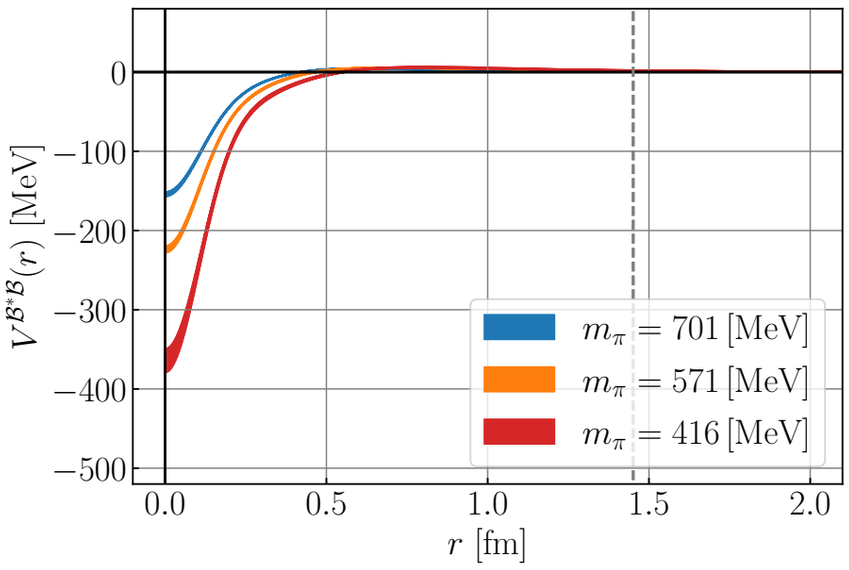
Off-diagonal interactions between $\mathcal{B}$ and $\mathcal{B}^*$ are comparable to diagonal ones

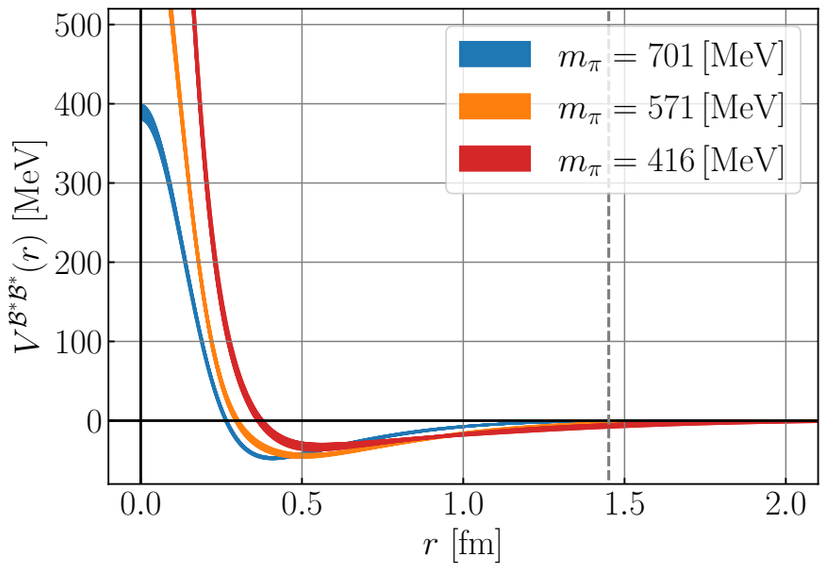
→ Coupled channel analysis is important. (Single channel one may be insufficient.)

1-3. Pion mass dependence









Both diagonal and off-diagonal potentials become stronger and more long-ranged, as the pion mass decreases.

A mixing effect increases toward the physical pion mass.

2. Scattering analysis

Inclusion of virtual $\mathcal{B}^*$ effects

Hermiticity is badly broken, probably due to the LO approximation.

We consider scattering only in the $\mathcal{B}$ channel below $\mathcal{B}^*$ threshold,

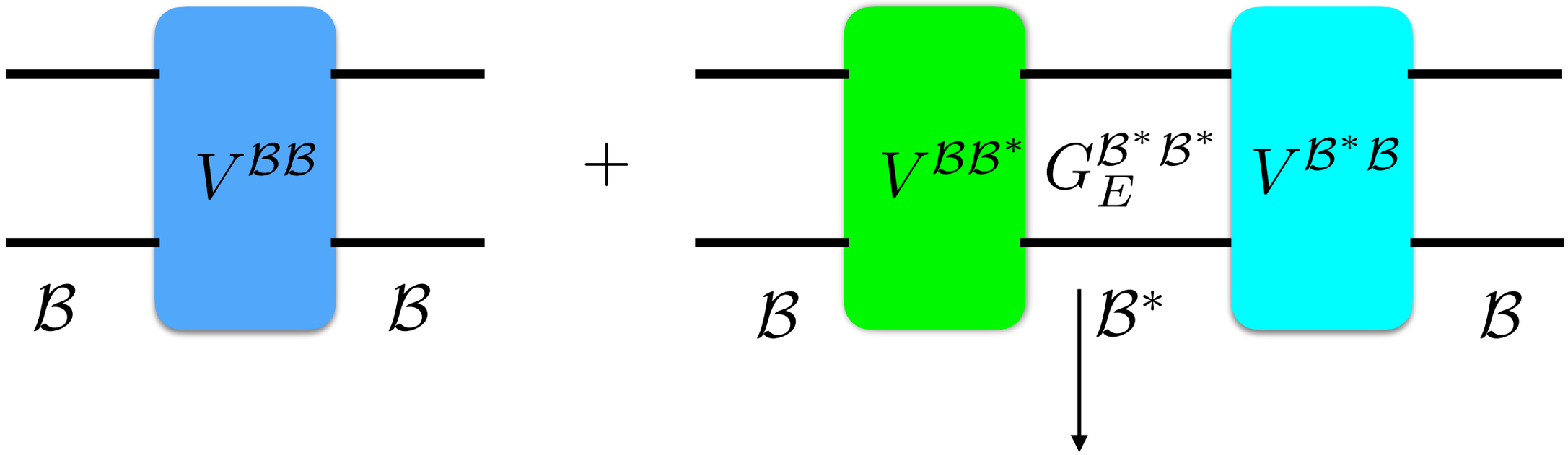
while employing 2×2 coupled channel potentials to include virtual $\mathcal{B}^*$ states.

2×2 coupled channel potential

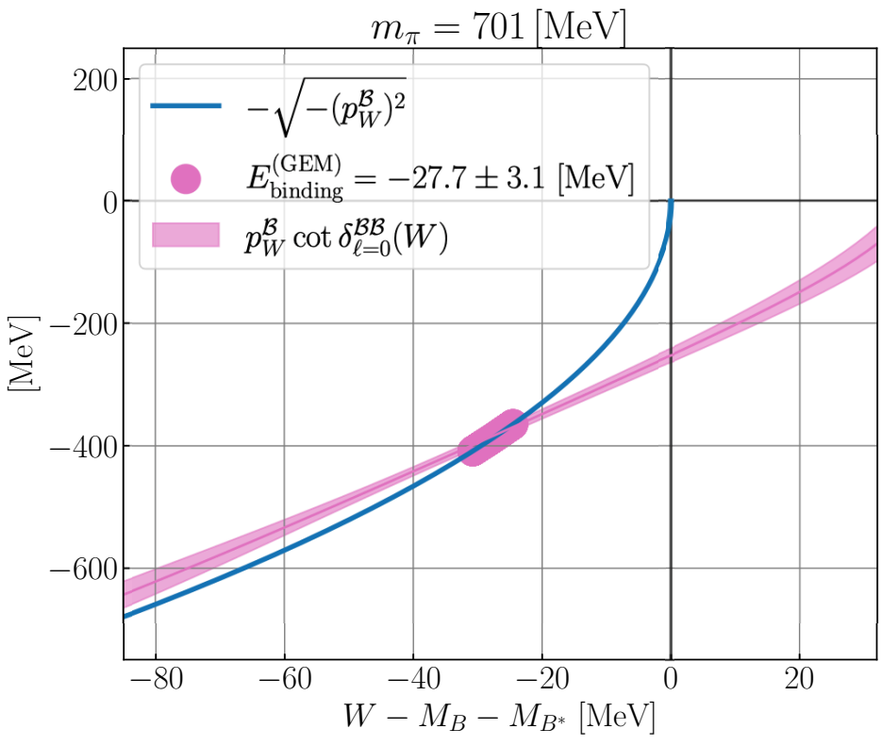
the effective single channel potential

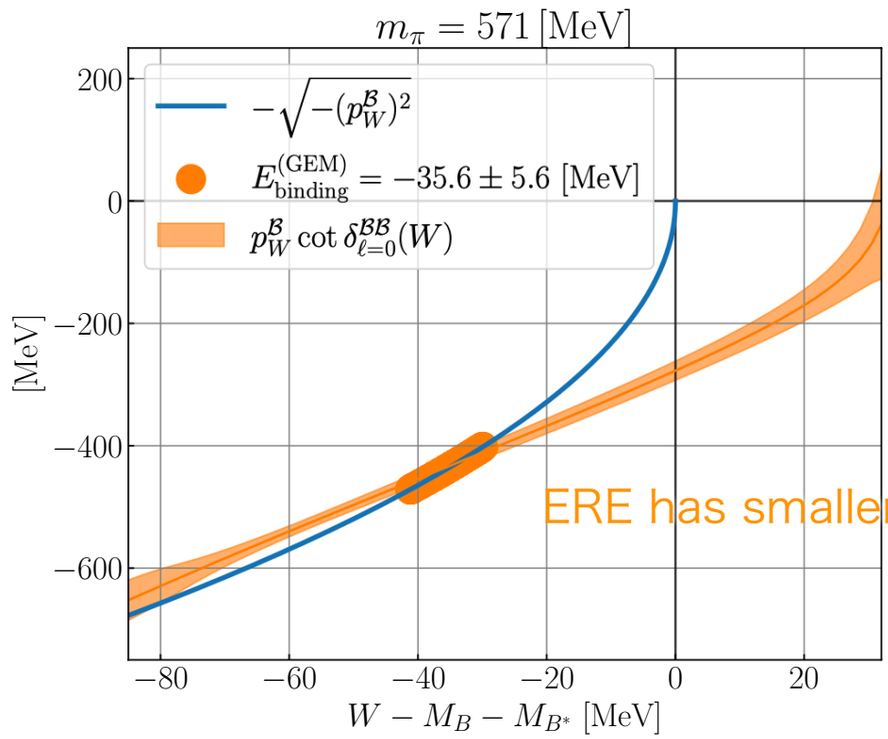
$$(H_0 + U^{\mathcal{B}\mathcal{B}}_{\text{eff},E})\Psi_B = E\Psi_B$$

$$U^{\mathcal{B}\mathcal{B}}_{\text{eff},E}(\mathbf{x},\mathbf{y}) = V^{\mathcal{B}\mathcal{B}}(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) + V^{\mathcal{B}\mathcal{B}^*}(\mathbf{x})G^{\mathcal{B}^*\mathcal{B}^*}_E(\mathbf{x},\mathbf{y})V^{\mathcal{B}^*\mathcal{B}}(\mathbf{y}),$$

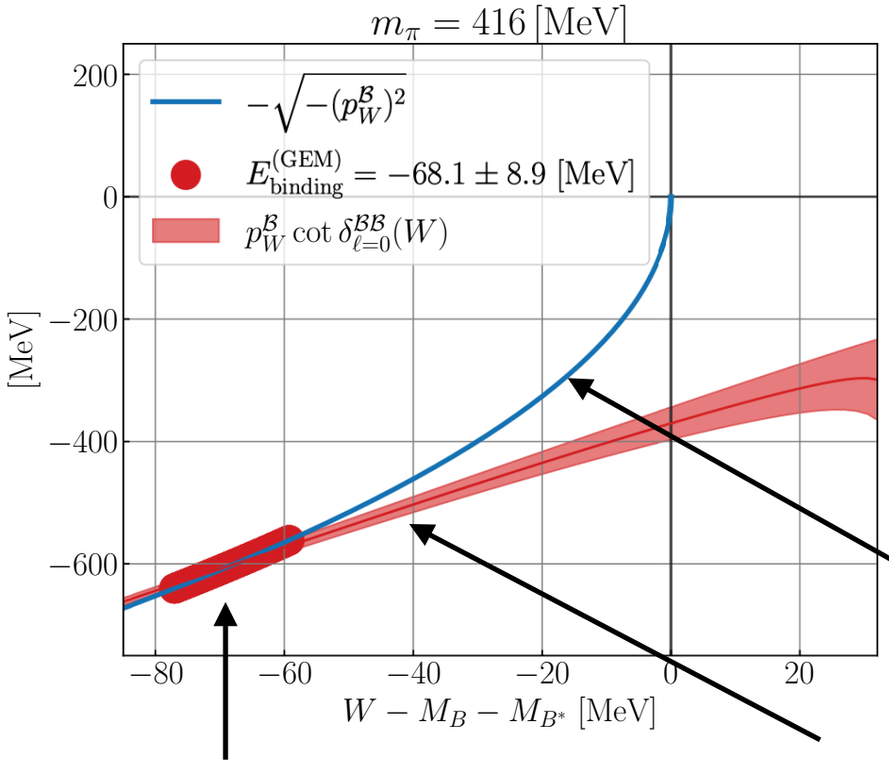


Effective Range Expansion (ERE) and a bound state





ERE has smaller errors



bound state energy by the Gaussian Expansion Method (GEM)

One bound T_{bb} at each quark mass !

ERE and GEM give consistent binding energy.

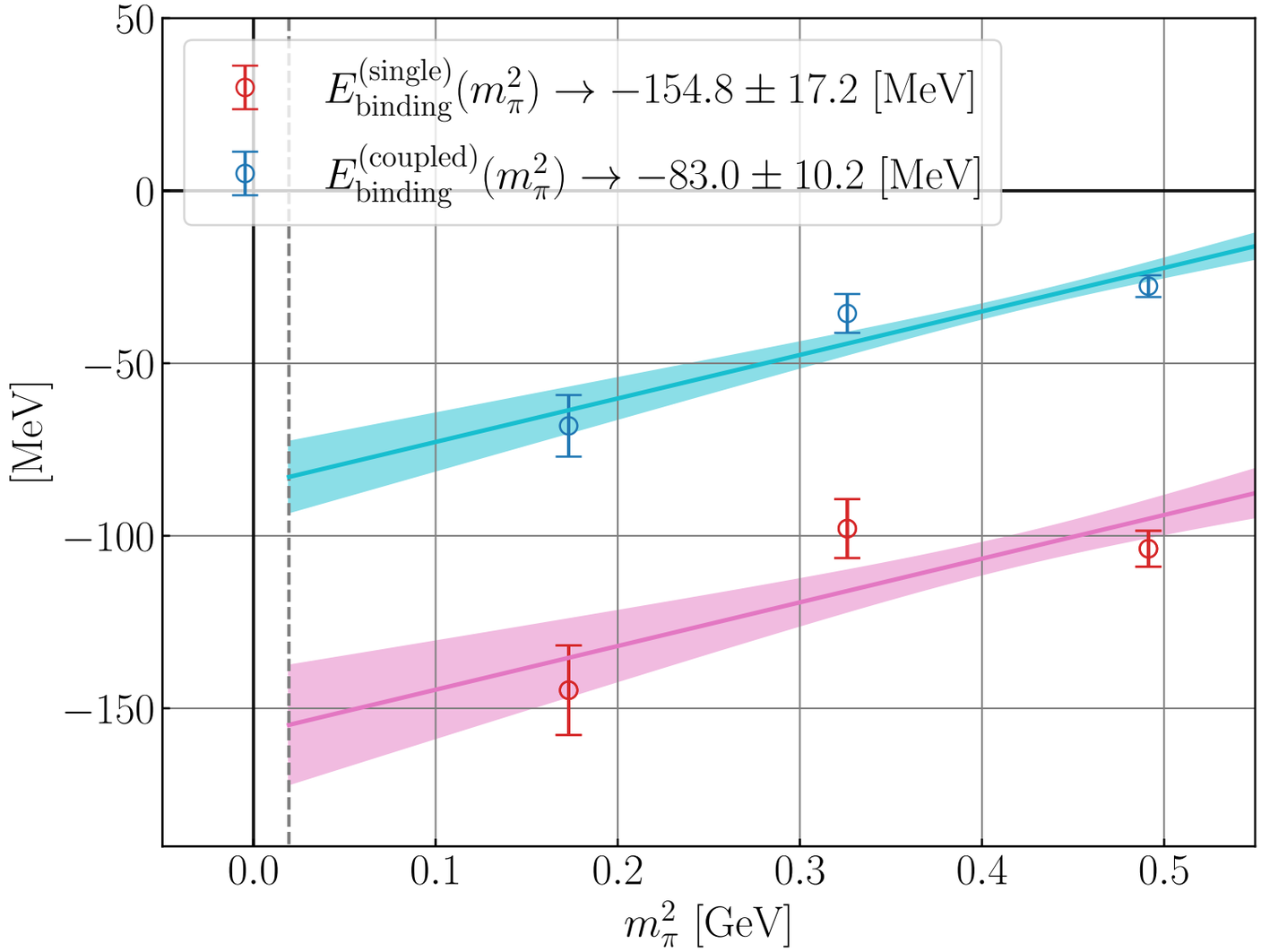
reliability of two analyses

$-\sqrt{-p^2}$

analytic continuation of ERE, physical pole condition is satisfied

$$\frac{\mathrm{d}}{\mathrm{d}p^2}\left[p\cot\delta(p) - (-\sqrt{-p^2})\right]\Big|_{p^2=-p_{\rm BS}^2} < 0\,.$$

Chiral extrapolation of binding energy



coupled channel analysis

single channel analysis

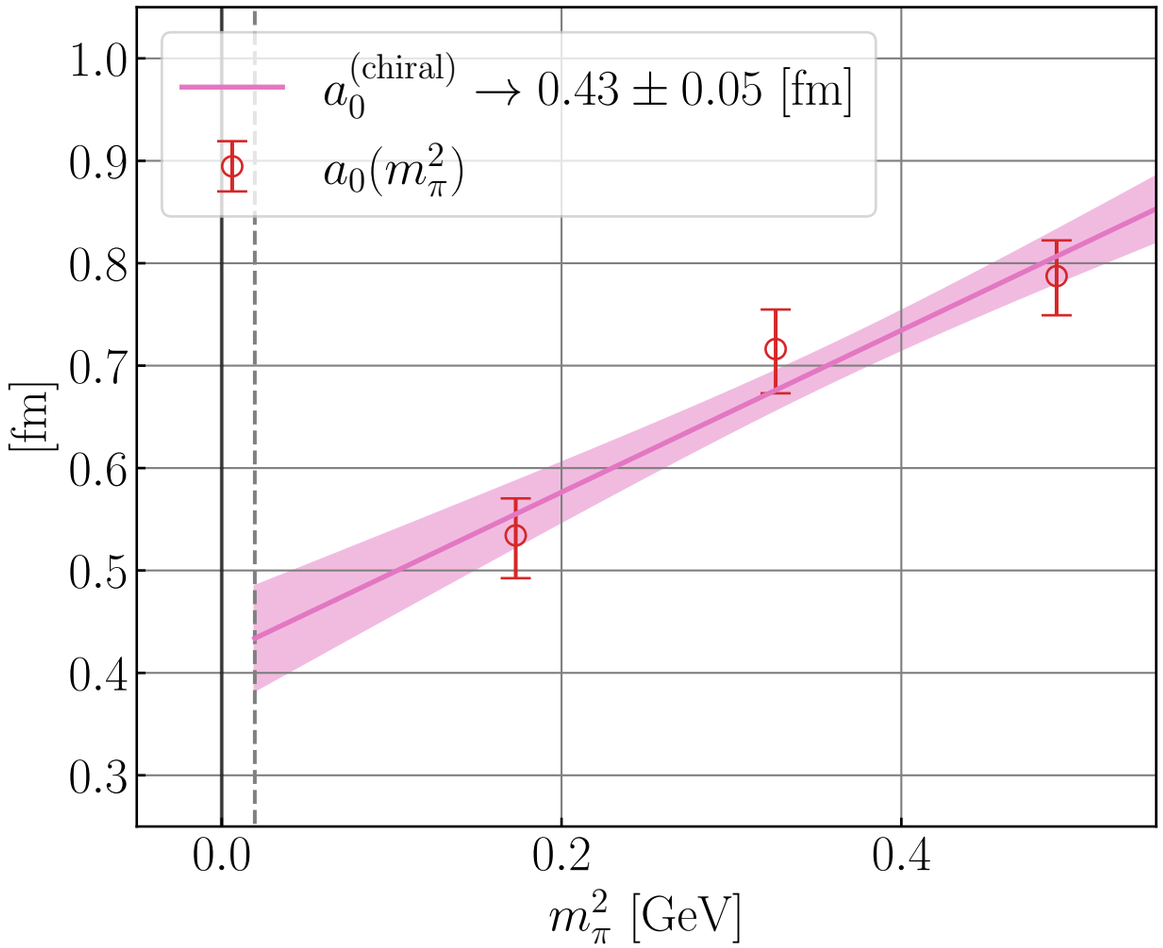
@physical pion mass $m_\pi = 140$ MeV

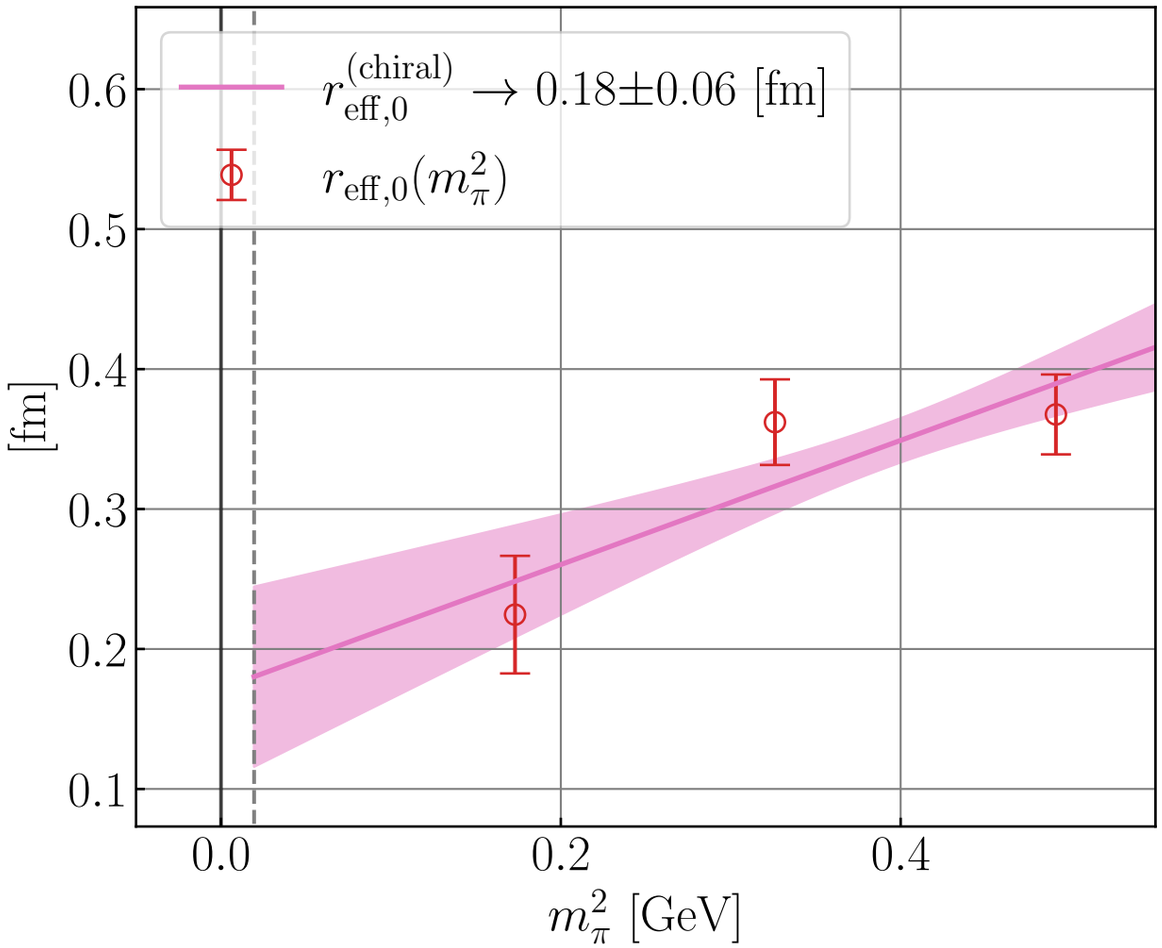
$$E^{(\mathrm{single,phys})}_{\mathrm{binding}} = -154.8 \pm 17.2 \text{ MeV}, \quad E^{(\mathrm{coupled,phys})}_{\mathrm{binding}} = -83.0 \pm 10.2 \text{ MeV},$$

effects of virtual processes such as $\mathcal{B} \to \mathcal{B}^* \to \mathcal{B}$

⟶ 40-50 % reduction of the binding energy

Chiral extrapolation of ERE parameters

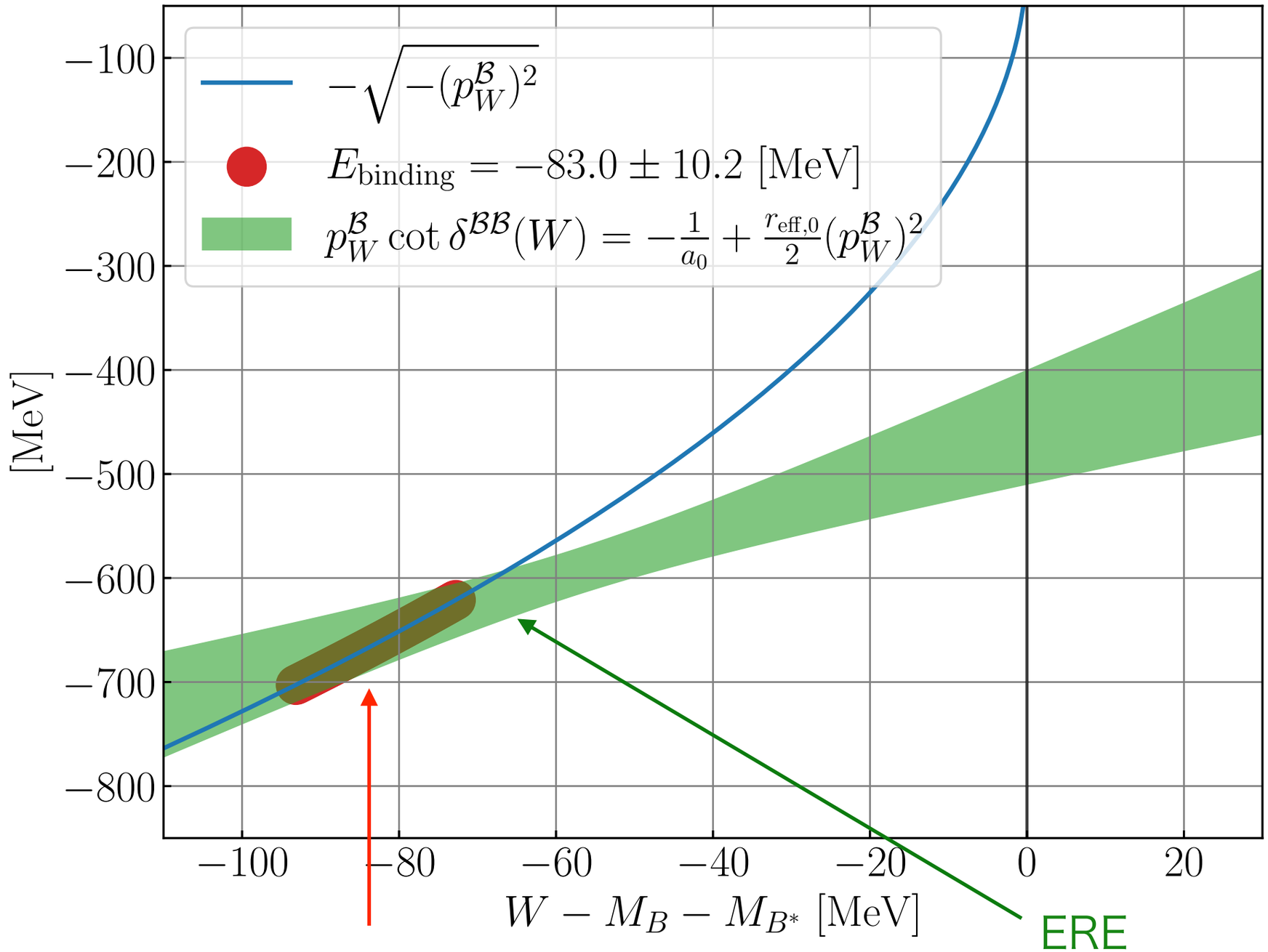




@physical pion mass $m_\pi = 140$ MeV

$$a_0^{(\mathrm{coupled,phys})} = 0.43 \pm 0.05 \text{ fm}, \quad r_{\mathrm{eff},0}^{(\mathrm{coupled,phys})} = 0.18 \pm 0.06 \text{ fm},$$

Consistency between two chiral extrapolation



Both chiral extrapolations are consistent.

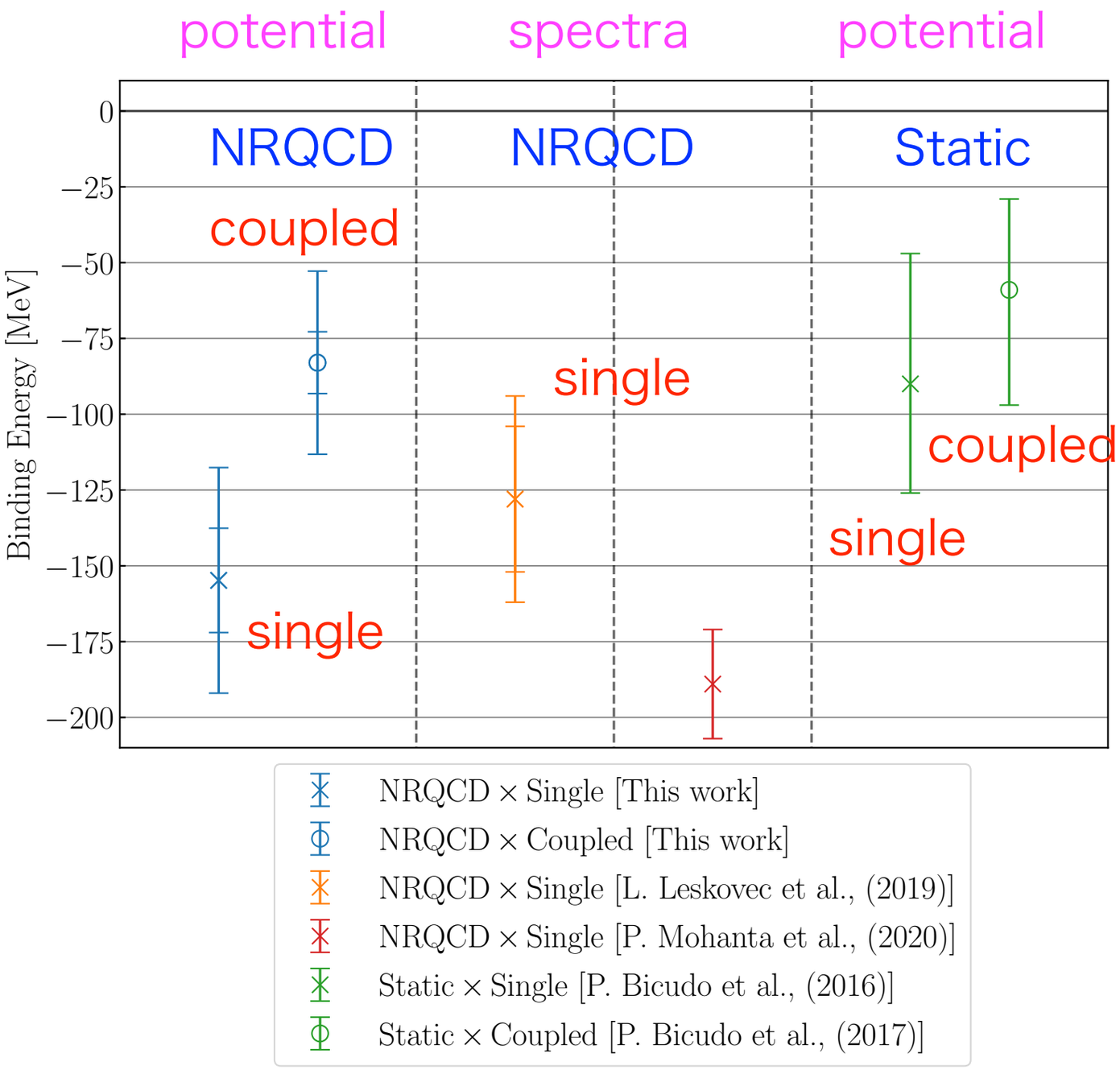
reliability of chiral extrapolations

V. Conclusions

S-wave potential between $\bar{B}$ and $\bar{B}^*$ mesons in $I(J^P) = 0(1^+)$ HAL QCD method

$$E_{\text{binding}}^{(\text{single,phys})} = -154.8 \pm 17.2 \pm 20 \text{ MeV}, \quad E_{\text{binding}}^{(\text{coupled,phys})} = -83.0 \pm 10.2 \pm 20 \text{ MeV}.$$

Systematics (empirical)



1. Consistency among NRQCD (single)

2. The binding energy increases from Static to NRQCD.

3. The binding energy decreases from single to coupled.

Thank you !