Lattice results for hybrid static potentials at short quark-antiquark separations and their parametrization

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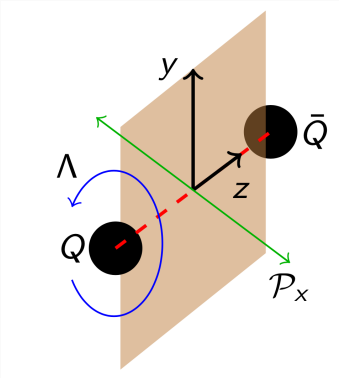
Gluonic excitations

Hybrid static potentials

= gluonic energy between a static quark and antiquark in a distance r

- Quantum numbers $\Lambda_\eta^\epsilon = \Sigma_g^+, \Pi_u, \Sigma_u^-, \ldots$
- excited gluon field $\rightarrow$ exotic quantum numbers J^{PC} are possible for *hybrid mesons*



active field of research [1] [2], both experimentally (GlueX, PANDA) and theoretically (lattice gauge theory)

[1] S. L. Olsen, T. Skwarnicki,D. Zieminska, Rev. Mod. Phys. 90 (2018) no.1, 015003 [arXiv:1708.04012 [hep-ph]]

[2] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo, C. Z. Yuan, Phys. Rept. 873 (2020), 1-154 [arXiv:1907.07583 [hep-ex]]

SU(3) lattice Yang-Mills data for hybrid static potentials

Main goal

- investigation of the small-r region of the Π_u and Σ_u^- hybrid static potentials [3] [4] [5] [6]
- precise parametrizations consistent with the continuum limit by
  - combining several small lattice spacings $0.040\,\text{fm}, \ldots, 0.093\,\text{fm}$
  - removing leading lattice discretization errors

$\rightarrow$ to be used to predict the spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons in Born-Oppenheimer approximations (coupled channels, heavy quark spin effects) [7] [8] [9] [10] [11] [12]

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[3] S. Perantonis and C. Michael, Nuclear Physics B 347 no. 3, (1990) 854 – 868

[4] K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90, 161601 (2003) [hep-lat/0207004]

[5] G. S. Bali et al. [SESAM and TχL Collaborations], Phys. Rev. D 62, 054503 (2000) [hep-lat/0003012]

[6] G. S. Bali, A. Pineda, Phys. Rev. D 69, 094001 (2004) [hep-ph/0310130]

[7] S. Perantonis and C. Michael, Nuclear Physics B 347 no. 3, (1990) 854 – 868

[8] K. J. Juge, J. Kuti and C. J. Morningstar, Nucl. Phys. Proc. Suppl. 63, 326 (1998) [hep-lat/9709131]

[9] P. Guo, A. P. Szczepaniak, G. Galata, A. Vassallo, and E. Santopinto, Phys. Rev. D 78 (2008) 056003, arXiv:0807.2721 [hep-ph]

[10] E. Braaten, C. Langmack, and D. H. Smith, Phys. Rev. D 90 no. 1, (2014) 014044, arXiv:1402.0438 [hep-ph]

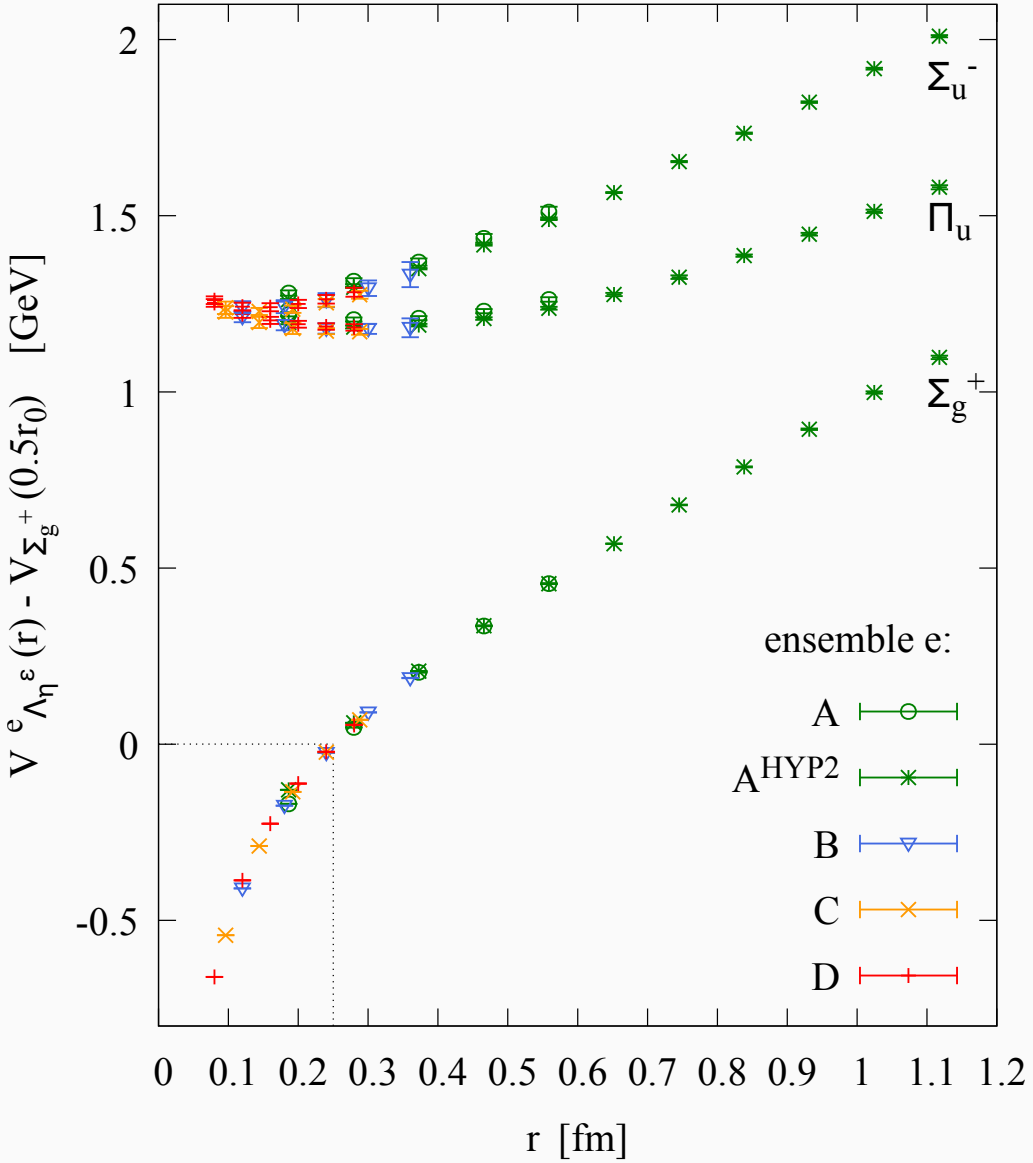
[11] R. Oncala and J. Soto, Phys. Rev. D96 no. 1, (2017) 014004, arXiv:1702.03900 [hep-ph]

[12] N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà, A. Vairo, Phys. Rev. D 99 no. 1, (2019) 014017, arXiv:1805.07713 [hep-ph]

(Hybrid) static potentials $\Lambda_\eta^\epsilon = \Sigma_g^+, \Pi_u, \Sigma_u^-$ from fine lattices

SU(3) ensemble	β	a in fm
A	6.000	0.093
B	6.284	0.060
C	6.451	0.048
D	6.594	0.040
A^{HYP2} [13]	6.000	0.093

- Optimized hybrid static potential creation operators[13]
- optimized *APE*-smearing
- Multilevel algorithm [14]

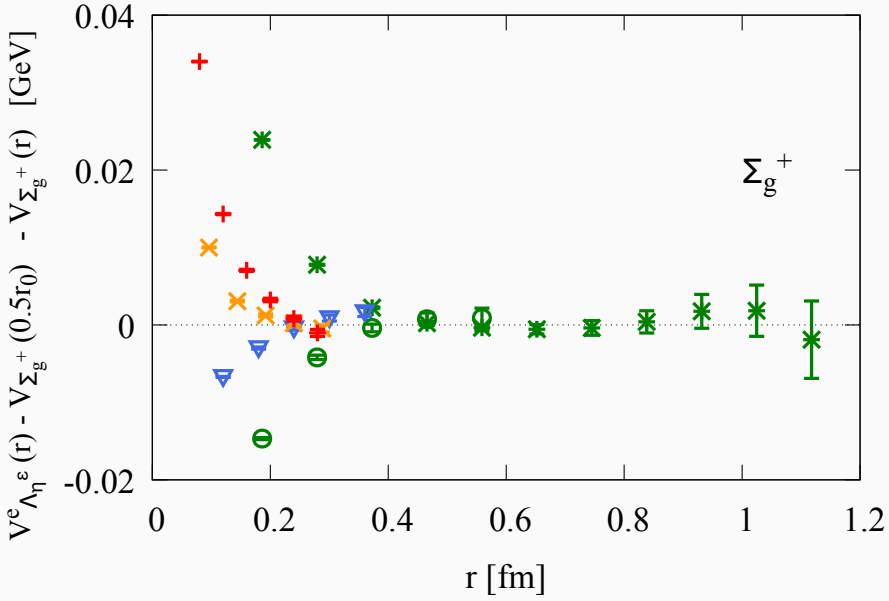


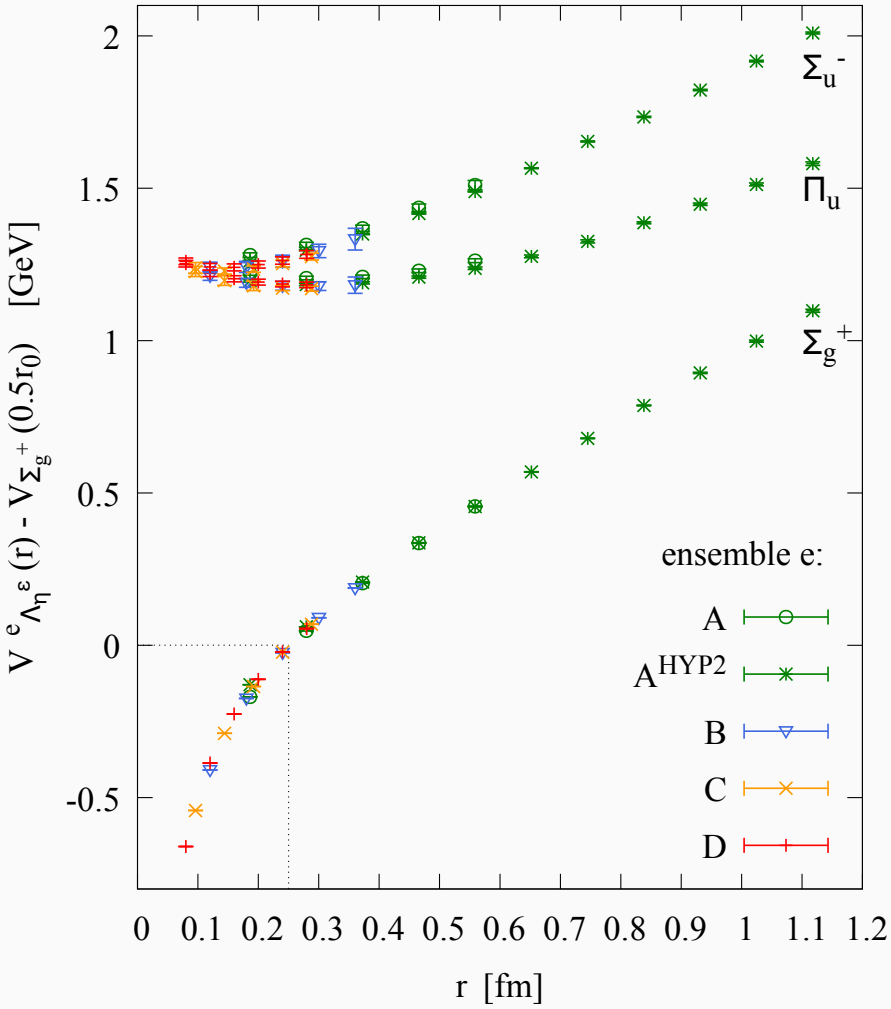
[13] S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

[14] M. Lüscher and P. Weisz, JHEP 09 (2001), 010 [arXiv:hep-lat/0108014 [hep-lat]]

(Hybrid) static potentials $\Lambda_\eta^\epsilon = \Sigma_g^+$, Π_u, Σ_u^- from fine lattices

Visualization of lattice discretization errors





Reducing lattice discretization errors

Static potential at tree level of perturbation theory:

- $V^{\text{continuum}}(r) \propto \frac{1}{r}$
- $V^{\text{lattice}}(r) \propto G(\mathbf{r})$
  $G(\mathbf{r})$ = tree-level lattice gluon propagator

Methods of tree-level improvement:

(1) Improving the separation: $r \to r_{\textbf{impr}}$

- initially used for static force [R. Sommer, Nucl. Phys. B 411, 839 (1994) [arXiv:hep-lat/9310022v1]]
- r_{impr} determined from $\frac{1}{4\pi r_{\textbf{impr}}} = G(\mathbf{r})$

(2) Correcting the potential value: $\tilde{V}(r) = V(r) - \Delta V^{\textbf{lat}}(r)$

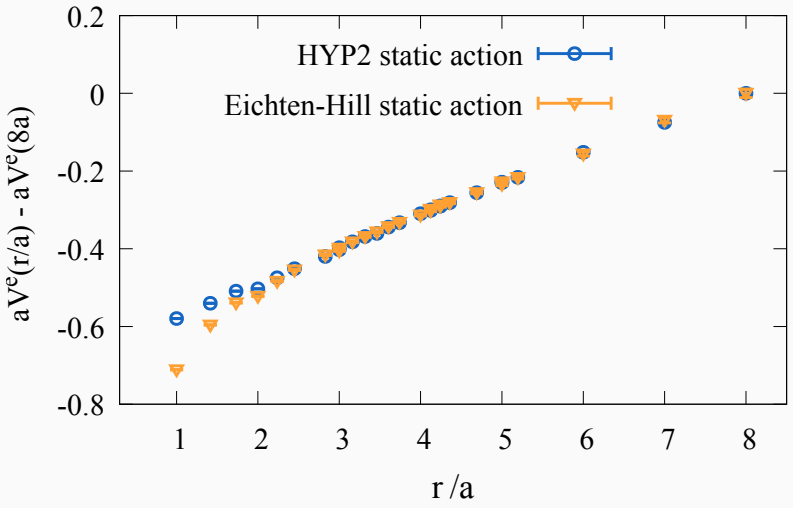
- $\Delta V^{\text{lat}}(r) = \alpha' \left(\frac{1}{r} - \frac{G(\mathbf{r})}{a} \right)$
- α' determined from a fit of lattice data

[C. Michael, Phys. Lett. B 283, 103 (1992) [arXiv:hep-lat/9205010v1]]

[A. Hasenfratz, R. Hoffmann, and F. Knechtli, Nucl. Phys. Proc. Suppl. 106, 418 (2002) [arXiv:hep-lat/0110168v1]]

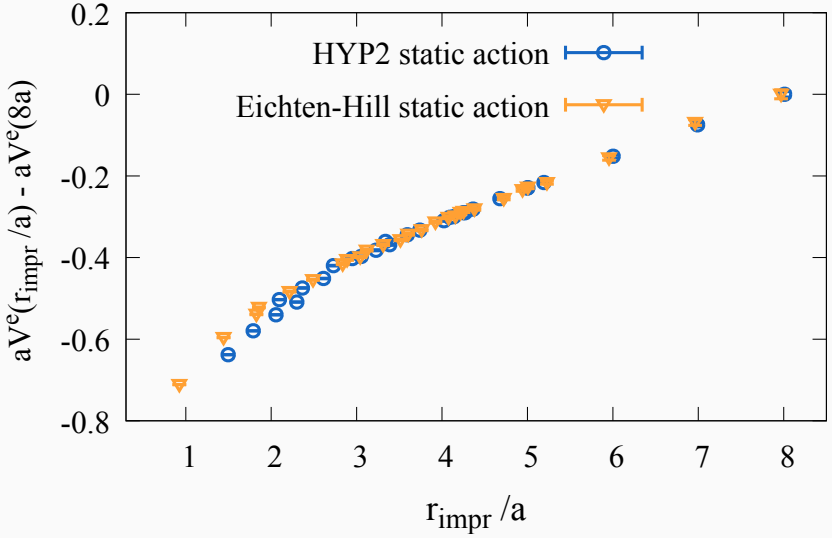
Methods of tree-level improvement for the static potential

Unimproved data



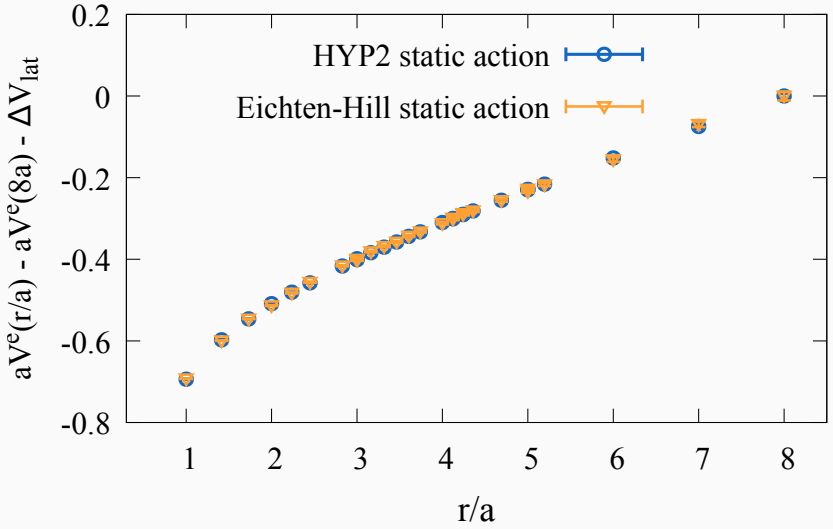
(1) Improving the separation:

$$r \longrightarrow r_{\mathbf{impr}}$$



(2) Correcting the potential value:

$$\tilde{V}(r) = V(r) - \Delta V^{\mathbf{lat}}(r)$$



Parametrization of the ordinary static potential Σ_g^+

$\rightarrow$ subtraction of a-dependent self-energy C^e

$\rightarrow$ determination and subtraction of lattice discretization errors $\Delta V^{\text{lat},e}(r)$

8-parameter fit of lattice data from all ensembles:

$$V_{\Sigma_g^+}^{\text{fit},e}(r) = V_{\Sigma_g^+}(r) + C^e + \Delta V_{\Sigma_g^+}^{\text{lat},e}(r) \tag{1}$$

- $e \in \{A, B, C, D, A^{\text{HYP2}}\}$
- C^e: a-dependent self energy
- $\Delta V_{\Sigma_g^+}^{\text{lat},e}(r) = \alpha' \left(\frac{1}{r} - \frac{G^e(r/a)}{a}\right)$: lattice discretization error at tree-level
- $V_{\Sigma_g^+}(r) = -\frac{\alpha}{r} + \sigma r$ parametrization of ordinary static potential Σ_g^+

Parametrization of the hybrid static potentials Π_u and Σ_u^-

10-parameter fit of lattice data for $\Lambda_\eta^\epsilon = \Pi_u$ and Σ_u^-:

$$V_{\Lambda_\eta^\epsilon}^{\text{fit},e}(r) = V_{\Lambda_\eta^\epsilon}(r) + C^e + \Delta V_{\text{hybrid}}^{\text{lat},e}(r) + A_{2,\Lambda_\eta^\epsilon}'^e a^2 \tag{2}$$

- $\Delta V_{\text{hybrid}}^{\text{lat},e}(r) = -\frac{1}{8}\Delta V_{\Sigma_g^+}^{\text{lat},e}(r)$: lattice discretization error at tree-level

Parametrization of the hybrid static potentials Π_u and Σ_u^-

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- $\Delta V_{\text{hybrid}}^{\text{lat},e}(r) = -\frac{1}{8}\Delta V_{\Sigma_g^+}^{\text{lat},e}(r)$: lattice discretization error at tree-level
- hybrid static potential parametrization $V_{\Lambda_\eta^\epsilon}(r)$ (based on pNRQCD[15]):

$$V_{\Pi_u}(r) = \frac{A_1}{r} + A_2 + A_3 r^2 \tag{3}$$

$$V_{\Sigma_u^-}(r) = \frac{A_1}{r} + A_2 + A_3 r^2 + \frac{B_1 r^2}{1 + B_2 r + B_3 r^2}, \tag{4}$$

[15] M. Berwein, N. Brambilla, J. Tarrús Castellá, and A. Vairo, Phys. Rev. D 92 no. 11, (2015) 114019, arXiv:1510.04299 [hep-ph]

Parametrization of the hybrid static potentials Π_u and Σ_u^-

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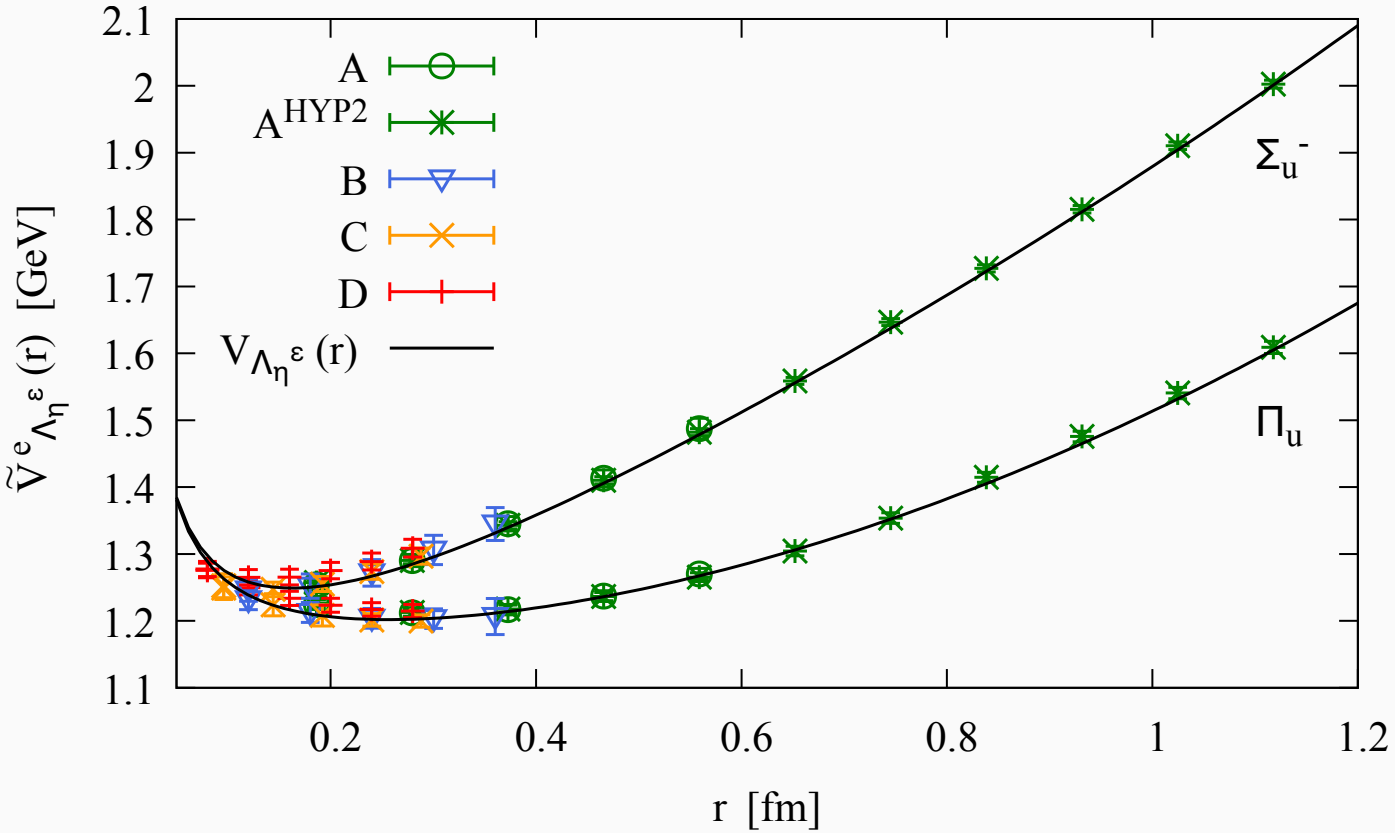
- $A_{2,\Lambda_\eta^\epsilon}'^e$: leading order discretization error in the difference to the ordinary static potential Σ_g^+ ($\propto a^2$)

Hybrid static potentials Π_u and Σ_u^-

$\Rightarrow$ **Improved data points** are obtained by subtracting the a-dependent self-energy and the discretization errors:

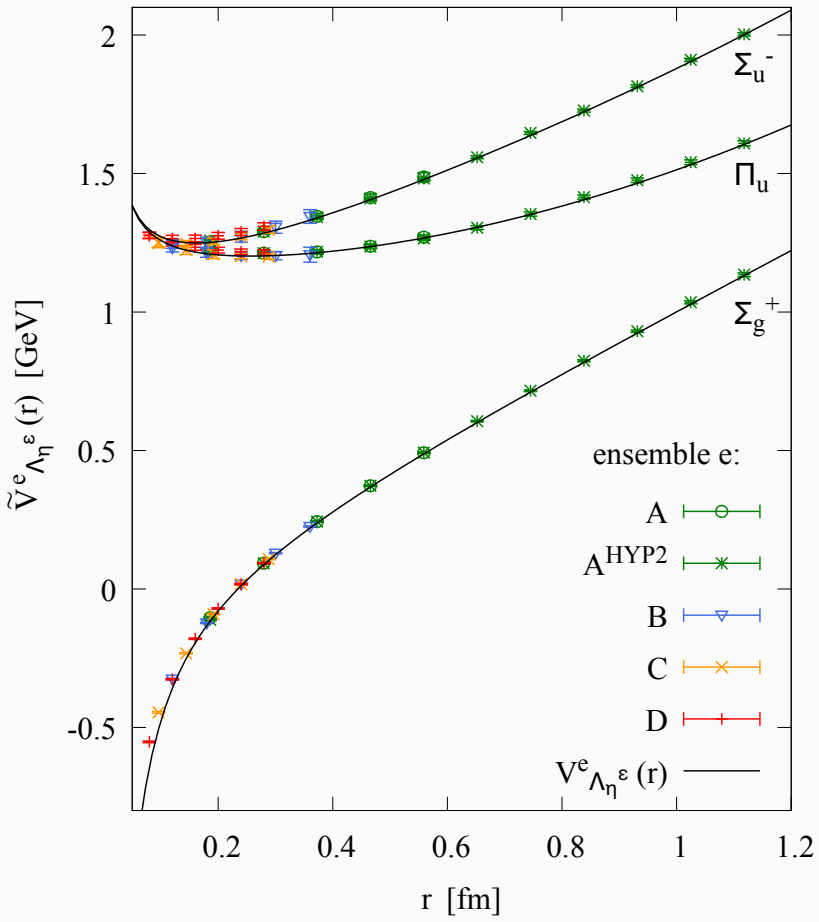
$$\tilde{V}^e_{\Lambda^\epsilon_\eta}(r) = V^e_{\Lambda^\epsilon_\eta}(r) - C^e - \Delta V^{\mathsf{lat},e}_{\mathsf{hybrid}}(r) - A'^e_{2,\Lambda^\epsilon_\eta} a^2 \tag{5}$$



Summary

- Parametrization of $SU(3)$ lattice results for hybrid static potentials Π_u and Σ_u^- at r as small as $0.08\,\mathrm{fm}$
- Elimination of leading lattice discretization errors
  - at tree level: $\Delta V^{\mathsf{lat}}(r) = \alpha' \left(\frac{1}{r} - \frac{G(\mathbf{r})}{a} \right)$
  - in the difference to Σ_g^+: $A'^{e}_{2,\Lambda_\eta^\epsilon} a^2$
- Improvements important: Born-Oppenheimer predictions of heavy hybrid meson masses change by $\mathcal{O}(10 \ldots 45\ \mathrm{MeV})$
- All fit parameters and data are provided in *C. Schlosser and M. Wagner, Phys. Rev. D 105, 054503 (2022) [arXiv:2111.00741v1 [hep-lat]]*

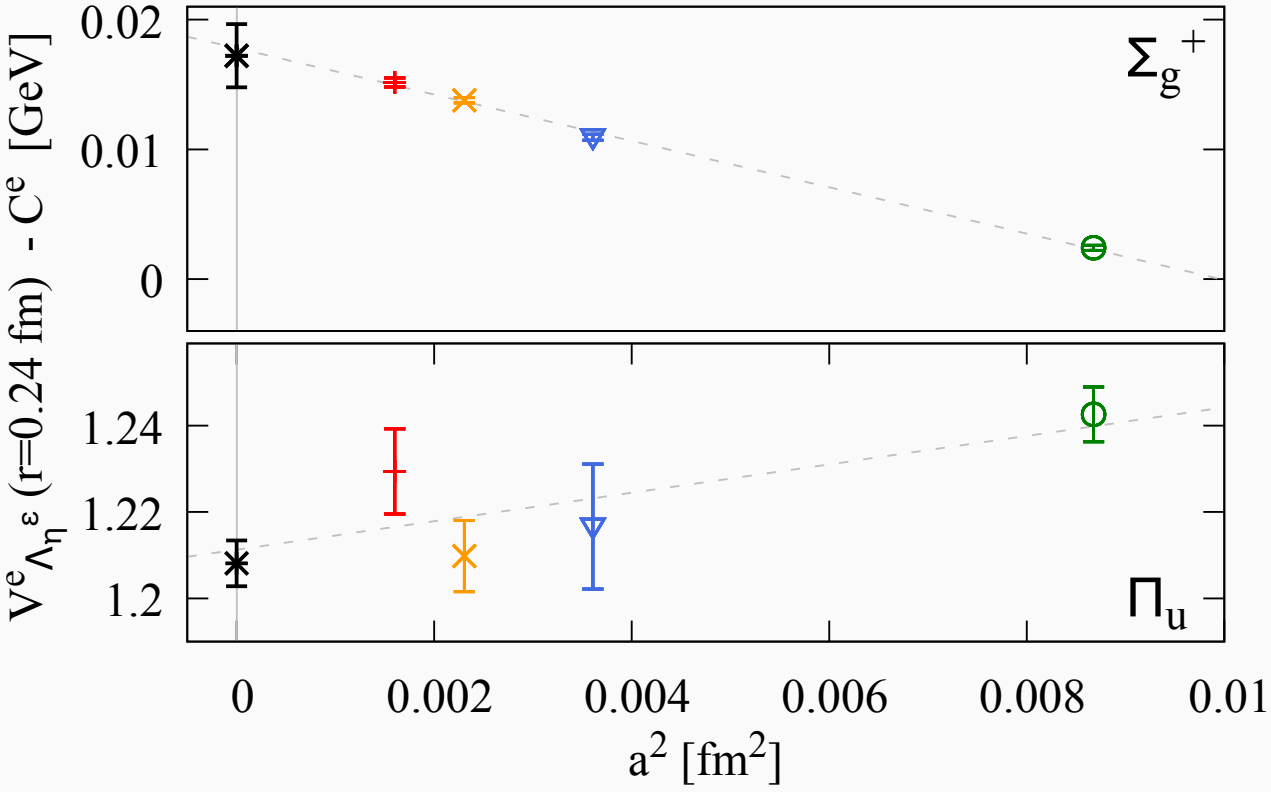
Improved lattice data:



Continuum extrapolation of lattice data at $r = 0.24\,\text{fm}$

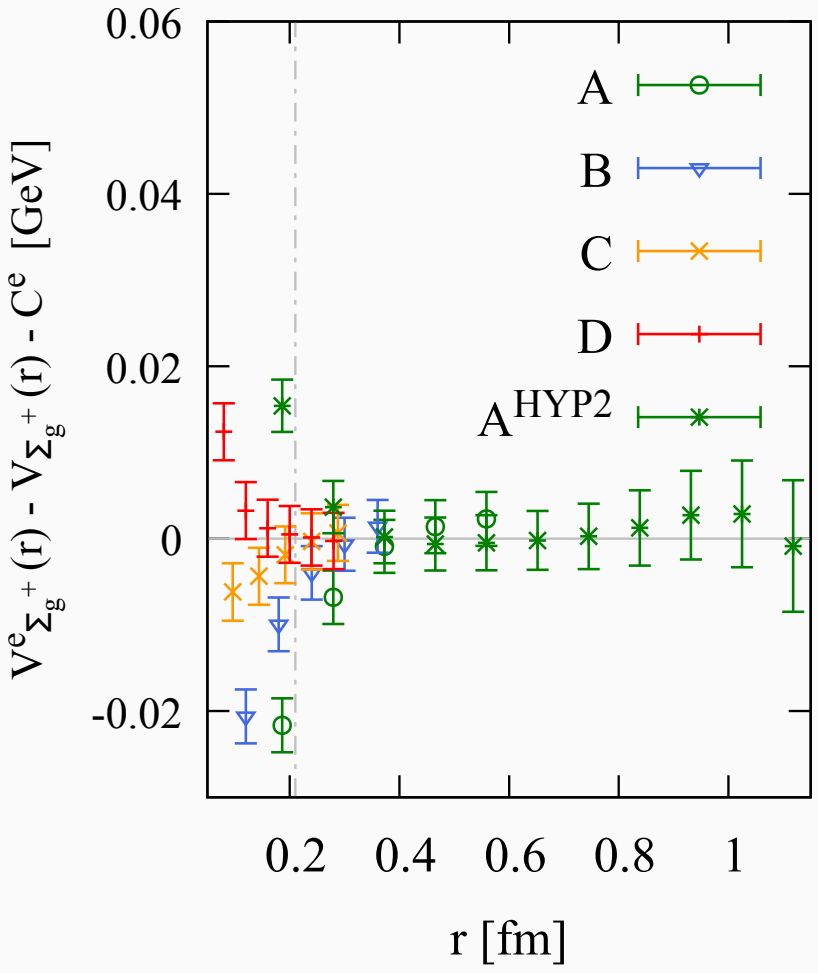
black data point = parametrization $V_{\Sigma_g^+,\Pi_u}(r = 0.24\,\text{fm})$

$\rightarrow$ consistent with continuum limit

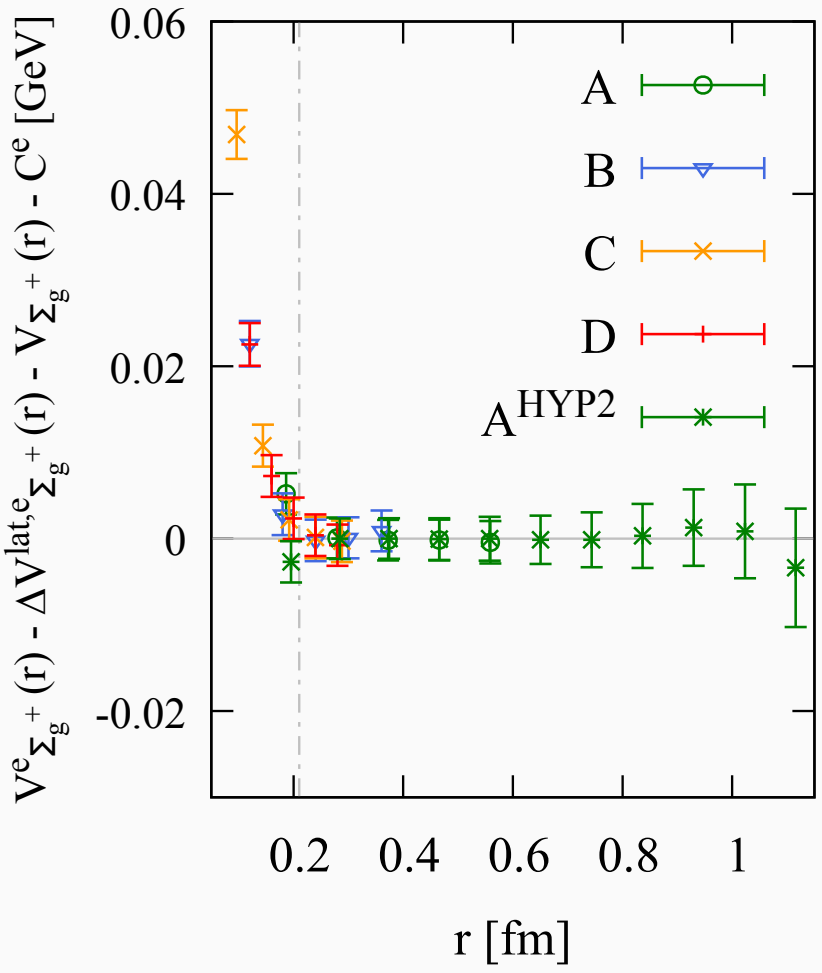


Tree-level improvement

Unimproved data $V^e_{\Sigma_g^+}(r)$



Improved data $\tilde{V}^e_{\Sigma_g^+}(r)$



Fit parameters

$$V^{\mathsf{fit},e}_{\Sigma_g^+}(r) = -\frac{\alpha}{r} + \sigma r + C^e + \alpha' \left(\frac{1}{r} - \frac{G^e(r/a)}{a} \right) \tag{6}$$

$$V^{\mathsf{fit},e}_{\Lambda_\eta^\epsilon}(r) = V_{\Lambda_\eta^\epsilon}(r) + C^e + \Delta V^{\mathsf{lat},e}_{\mathsf{hybrid}}(r) + A'^e_{2,\Lambda_\eta^\epsilon} a^2 \tag{7}$$

$$V_{\Pi_u}(r) = \frac{A_1}{r} + A_2 + A_3 r^2 \tag{8}$$

$$V_{\Sigma_u^-}(r) = \frac{A_1}{r} + A_2 + A_3 r^2 + \frac{B_1 r^2}{1 + B_2 r + B_3 r^2}, \tag{9}$$

α [GeV fm]	σ [GeV/fm]	α' [GeV fm]	χ^2_{red}
0.0571(4)	1.064(4)	0.0735(23)	0.7

	A_1 [GeV fm]	A_2 [GeV]	A_3 [GeV fm^2]	B_1 [GeV fm^2]	B_2 [fm^{-1}]	B_3 [fm^{-2}]	χ^2_{red}
Fit 1	0.0124(9)	1.135(8)	0.372(7)	1.56(15)	1.2(3)	2.1(2)	1.2
Fit 2	0.0147(18)	1.126(11)	0.381(7)	1.57(17)	1.0(4)	2.3(2)	0.8
Fit 3	0.0065(16)	1.190(14)	−0.092(91)	1.15(4)	-	-	0.5

		Fit 1		Fit 2		Fit 3		
ensemble	C^e [GeV]	A'^e_{2,Π_u}	A'^e_{2,Σ_u^-}	A'^e_{2,Π_u}	A'^e_{2,Σ_u^-}	A'^e_{2,Π_u}	A'^e_{2,Σ_u^-}	[GeV/fm^2]
A	1.398(2)	3.1(7)	6.7(8)	3.0(9)	6.5(9)	3.4(8)	5.7(9)	
B	2.059(2)							
C	2.472(2)							
D	2.862(2)							
A^{HYP2}	0.340(2)	1.0(7)	5.0(5)	0.9(9)	4.7(9)	1.6(7)	4.4(6)	