

$\pi\pi$ scattering and the σ resonance

Lattice 2022

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WILLIAM & MARY
CHARTERED 1693

hadspec

Jefferson Lab
Thomas Jefferson National Accelerator Facility

$\pi\pi$

Pseudo-goldstone bosons

Spontaneous symmetry breaking

Populate final states

[Leskovec's talk]

Crucial for hadron physics

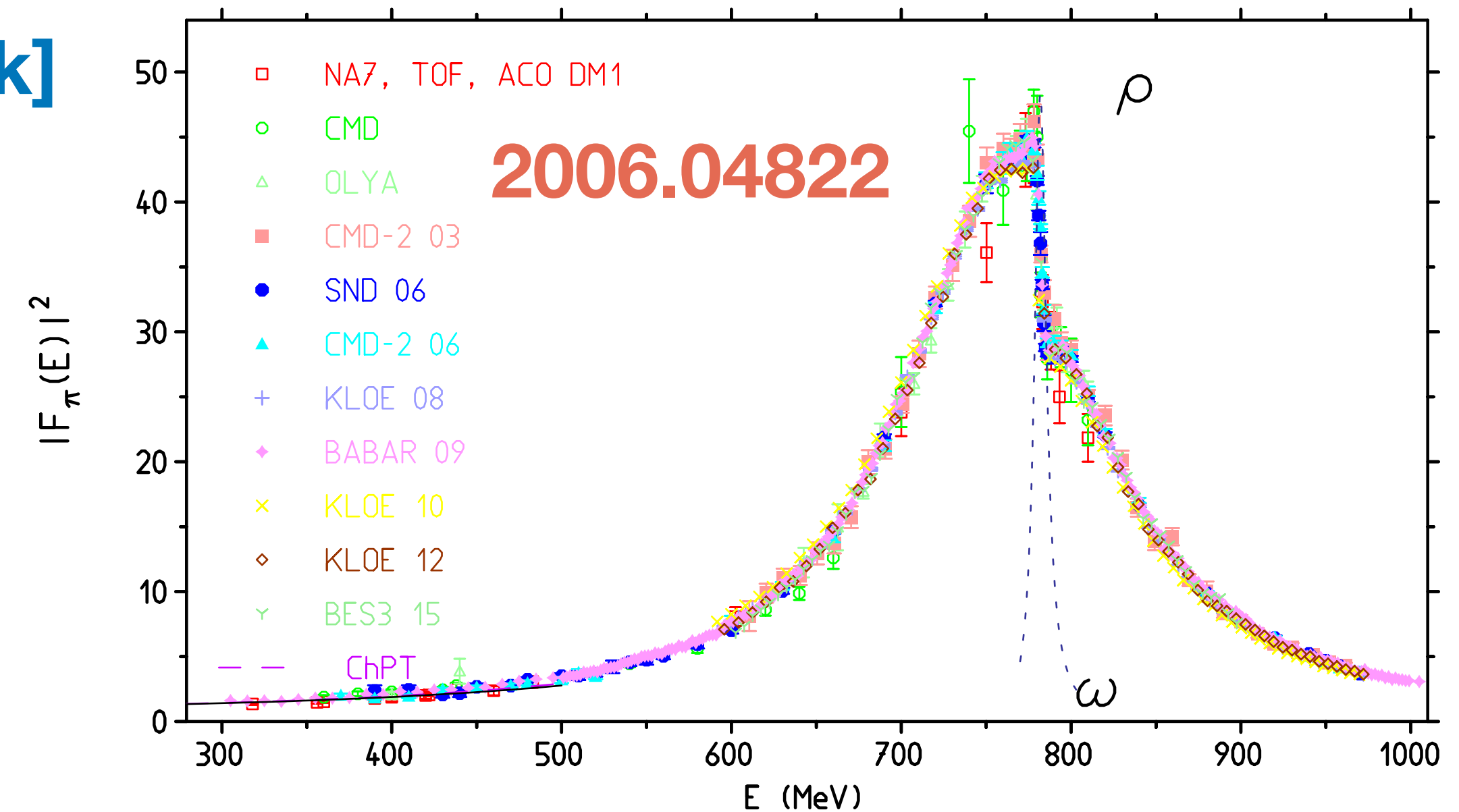
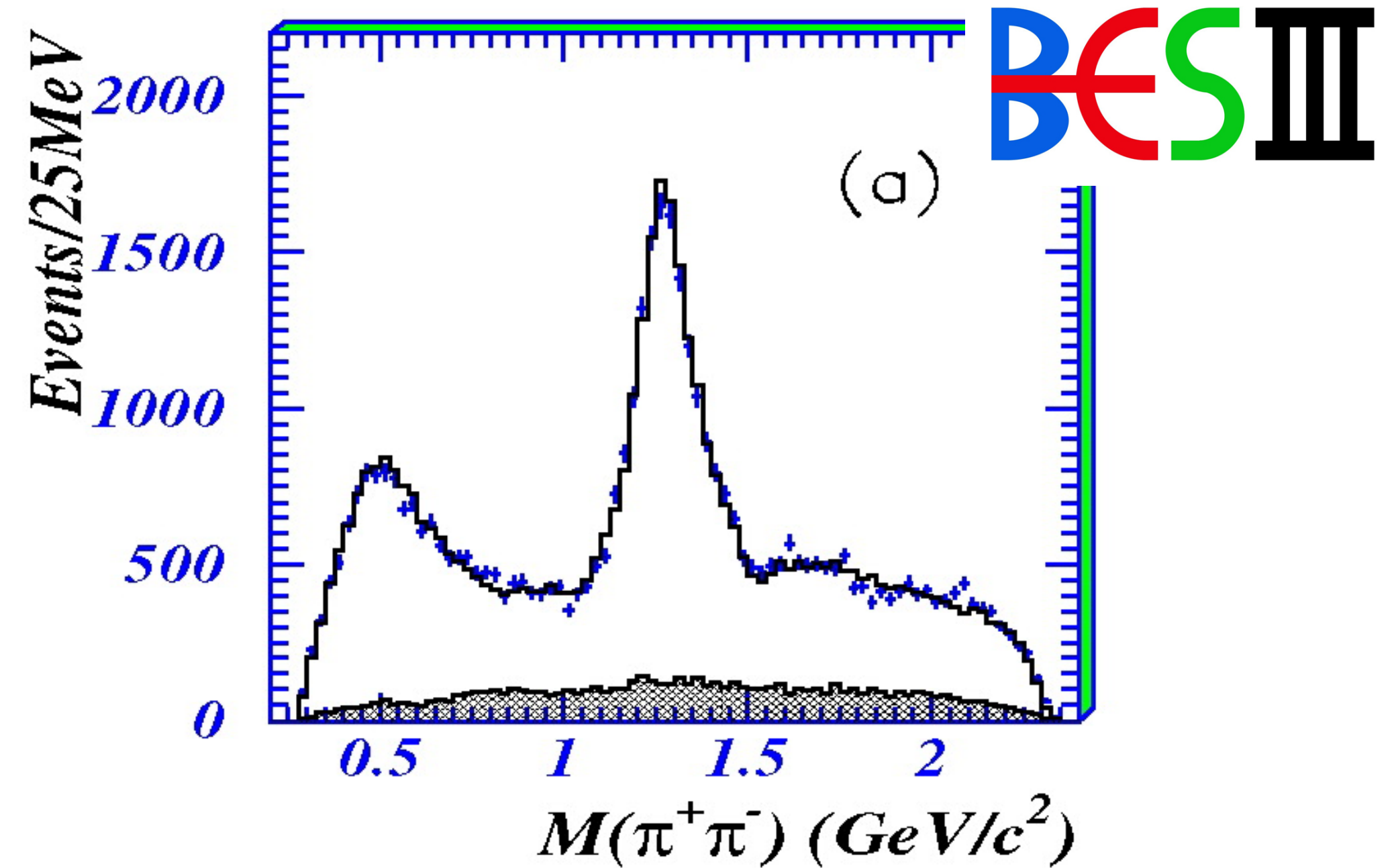
HLbL

HVP

[Hoferichter's talk]

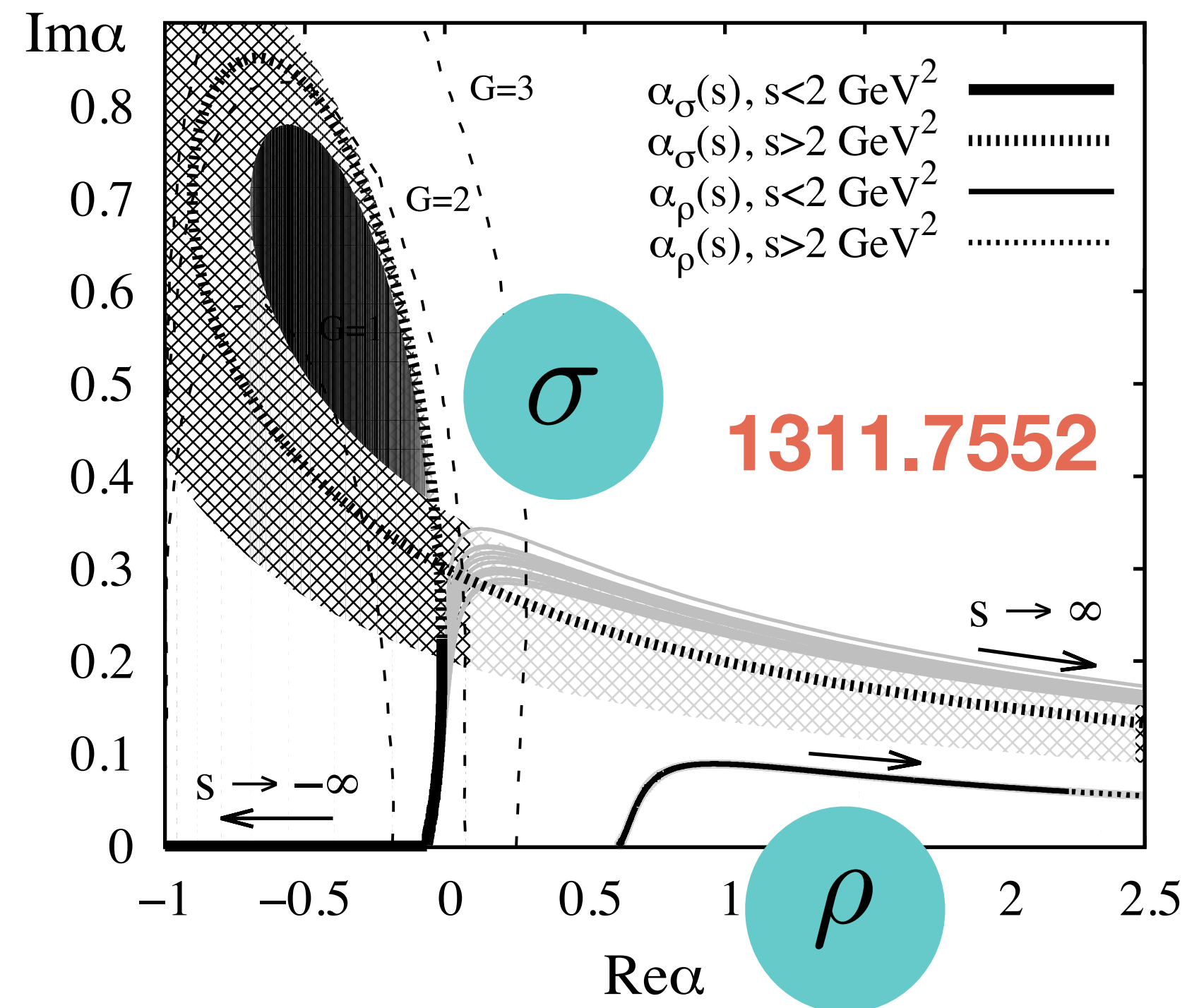
Resonances

Ordinary and non-ordinary

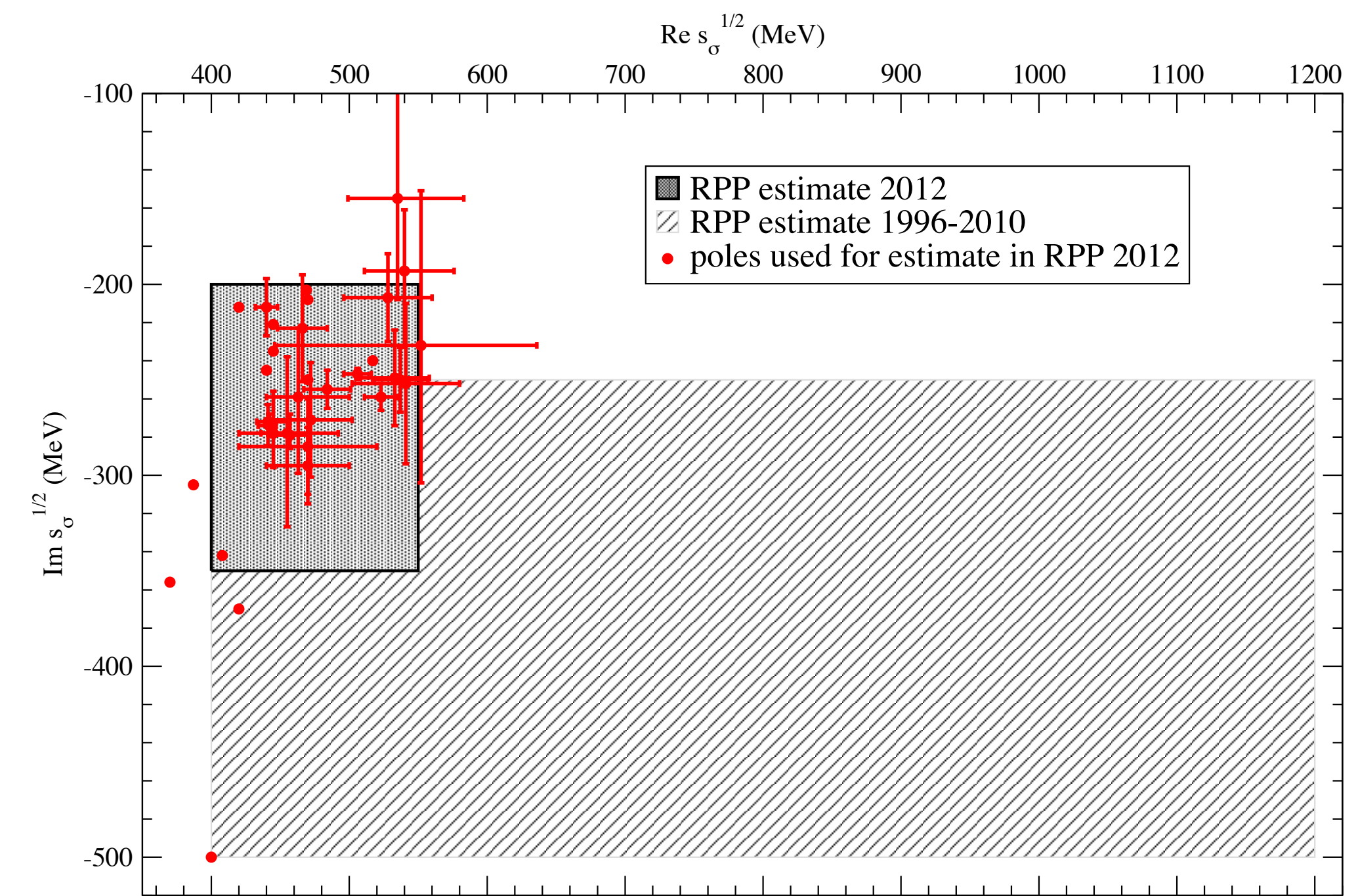


Light Scalars: the σ

Extremely broad object at phys. m_π



PDG



Non-ordinary resonance

Compositeness?

Debated for over half a century!!

The σ : Why lattice QCD?

Exp. “data” is not data

One beam (maybe)

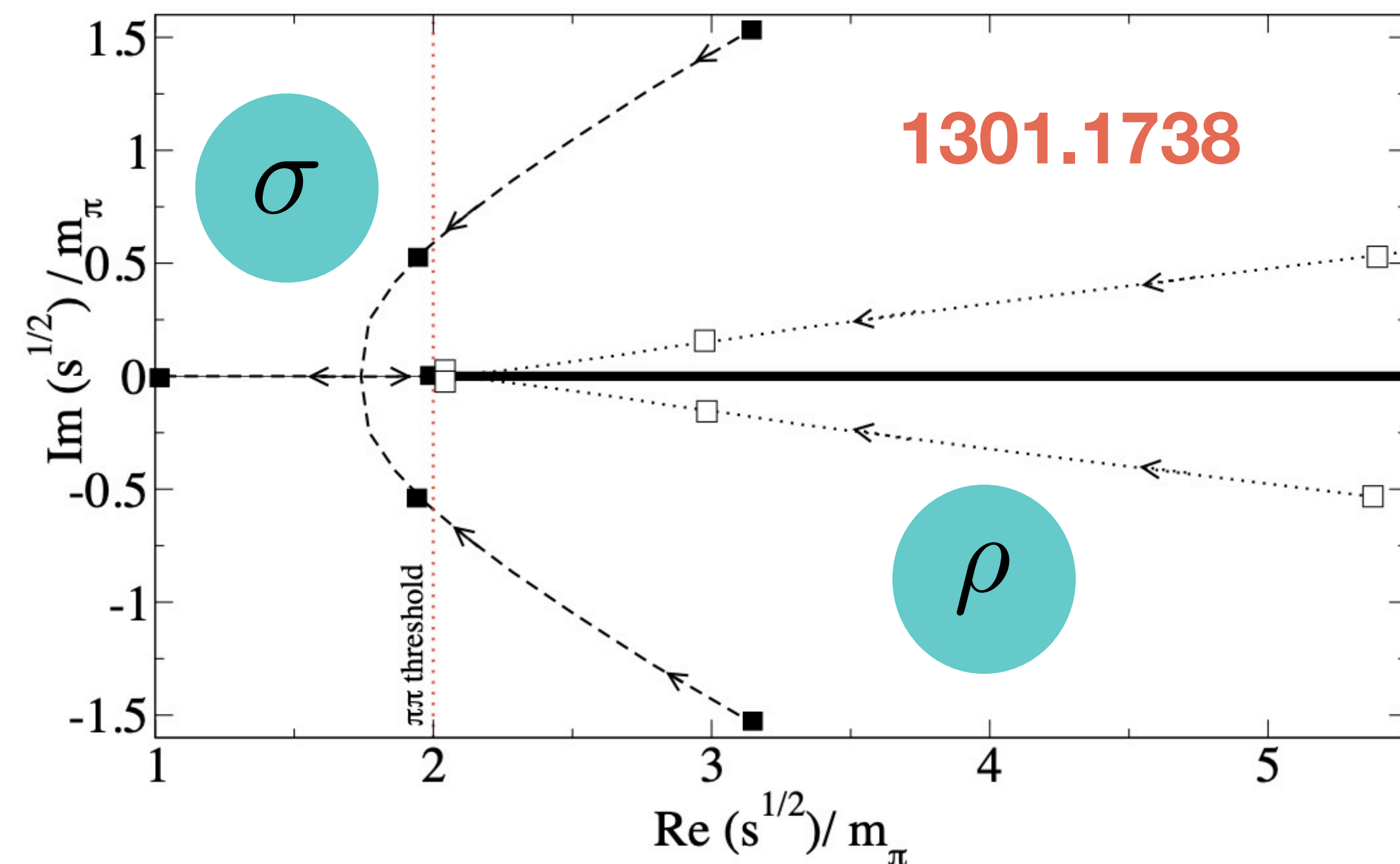
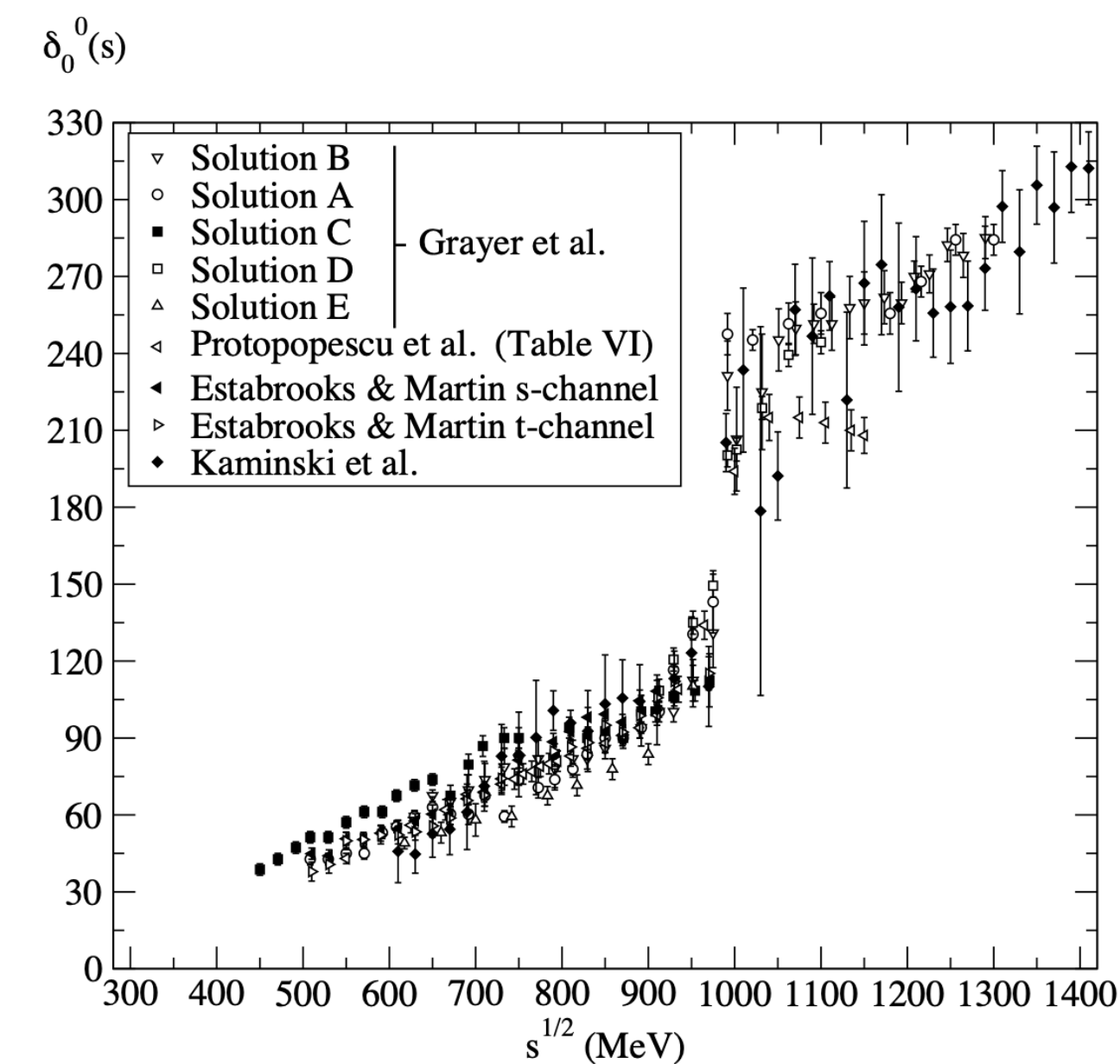
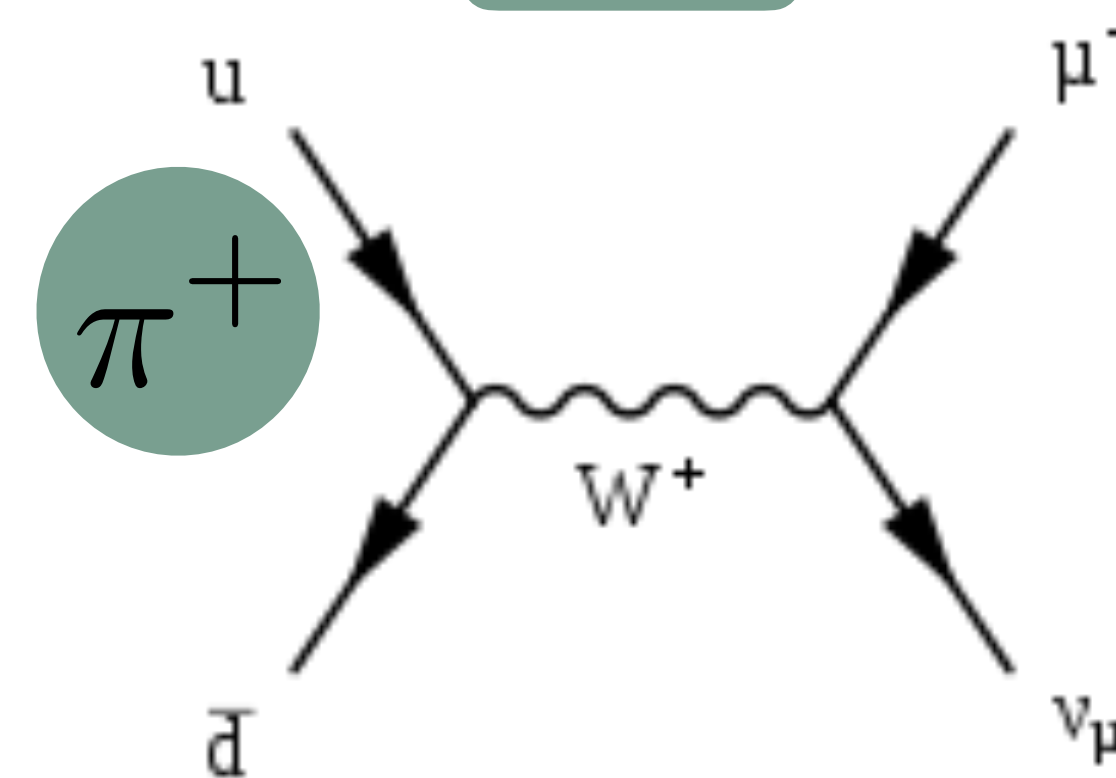
For meson-meson “data” is always modeled

Interesting “modeled” m_q dependence

Can be related to its compositeness

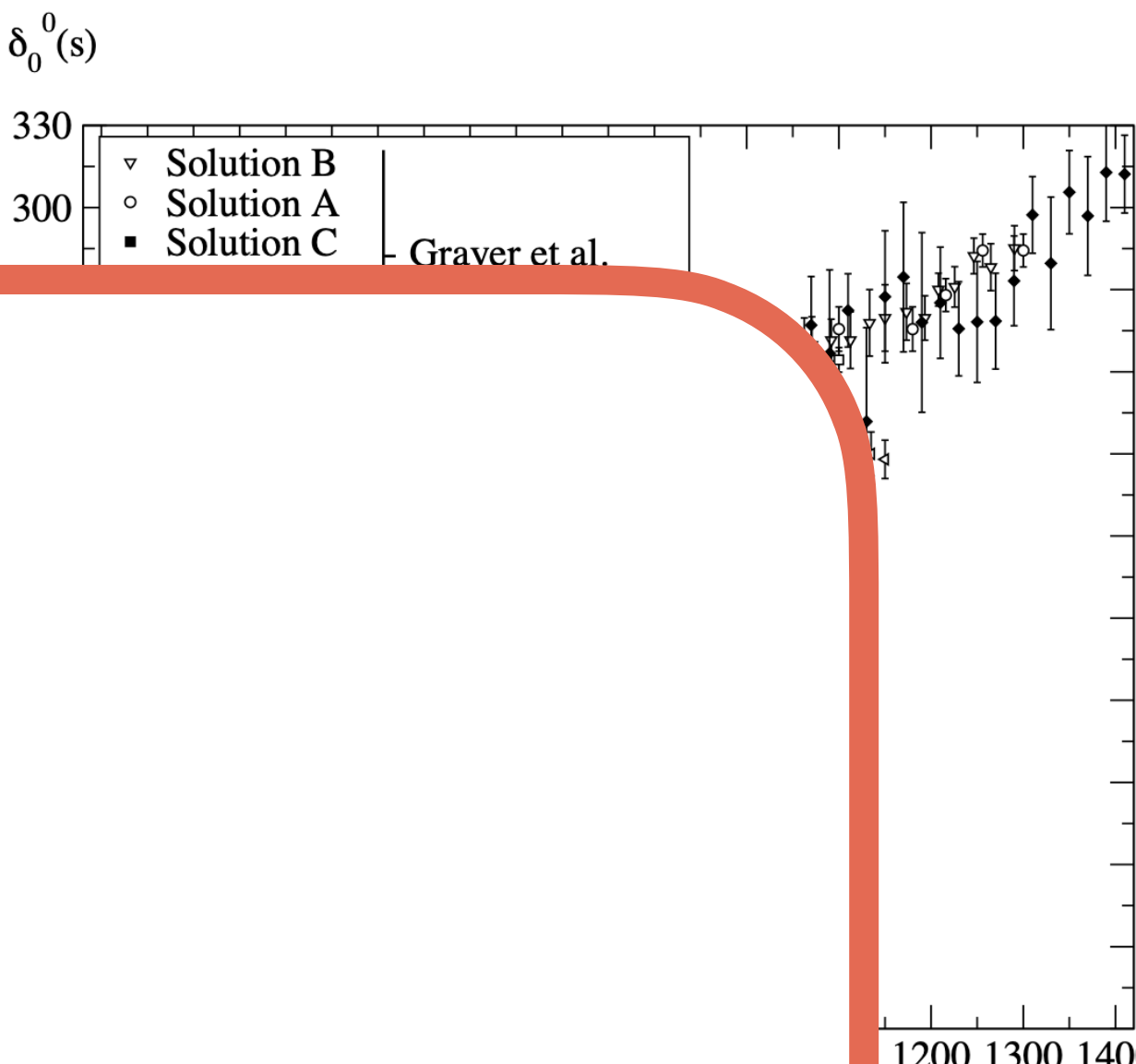
Studied through its final products

$$C\tau_{\pi^+} = 7.8 \text{ m}$$



The σ : Why lattice QCD?

$C\tau_{\pi^+} = 7.8 \text{ m}$



Exp.

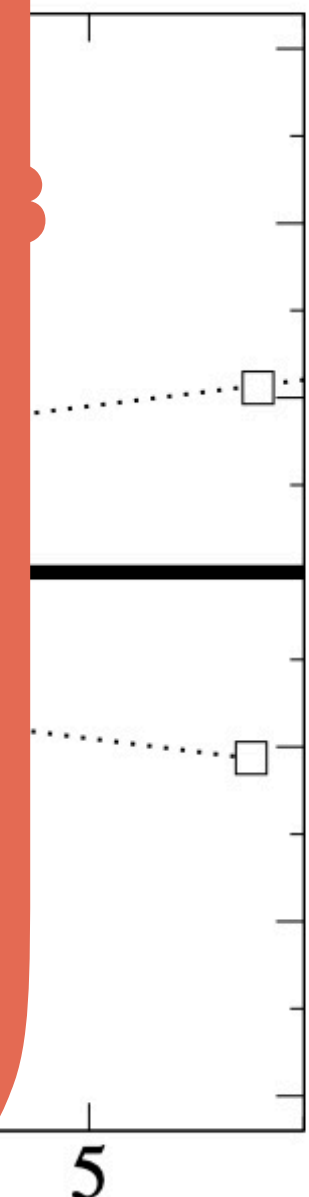
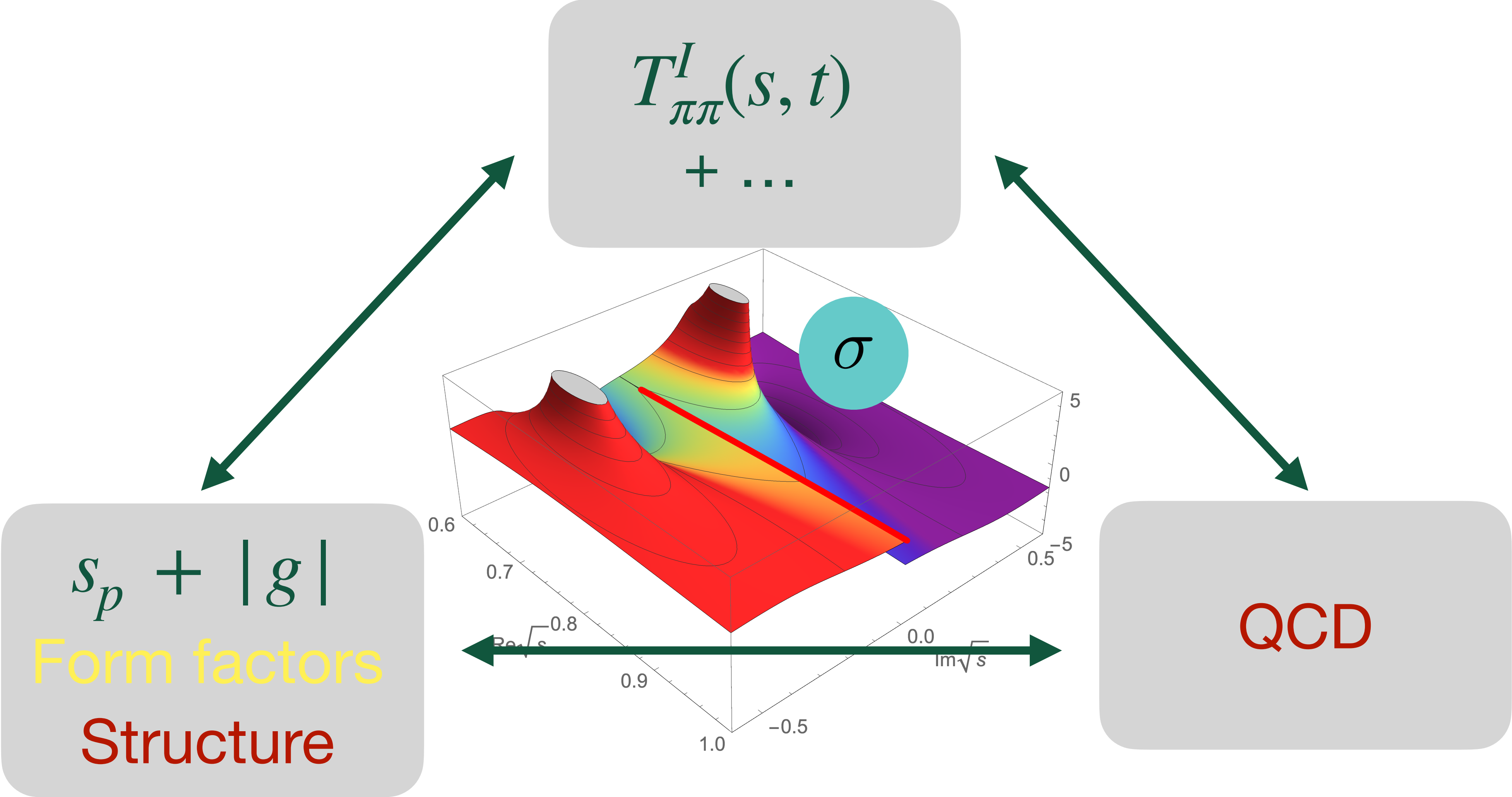
On

For

Inter

C

St



The σ as intermediate state

$$N_f = 2 + 1$$

✓ $m_\pi \sim 391 \text{ MeV} \rightarrow \text{BS}$

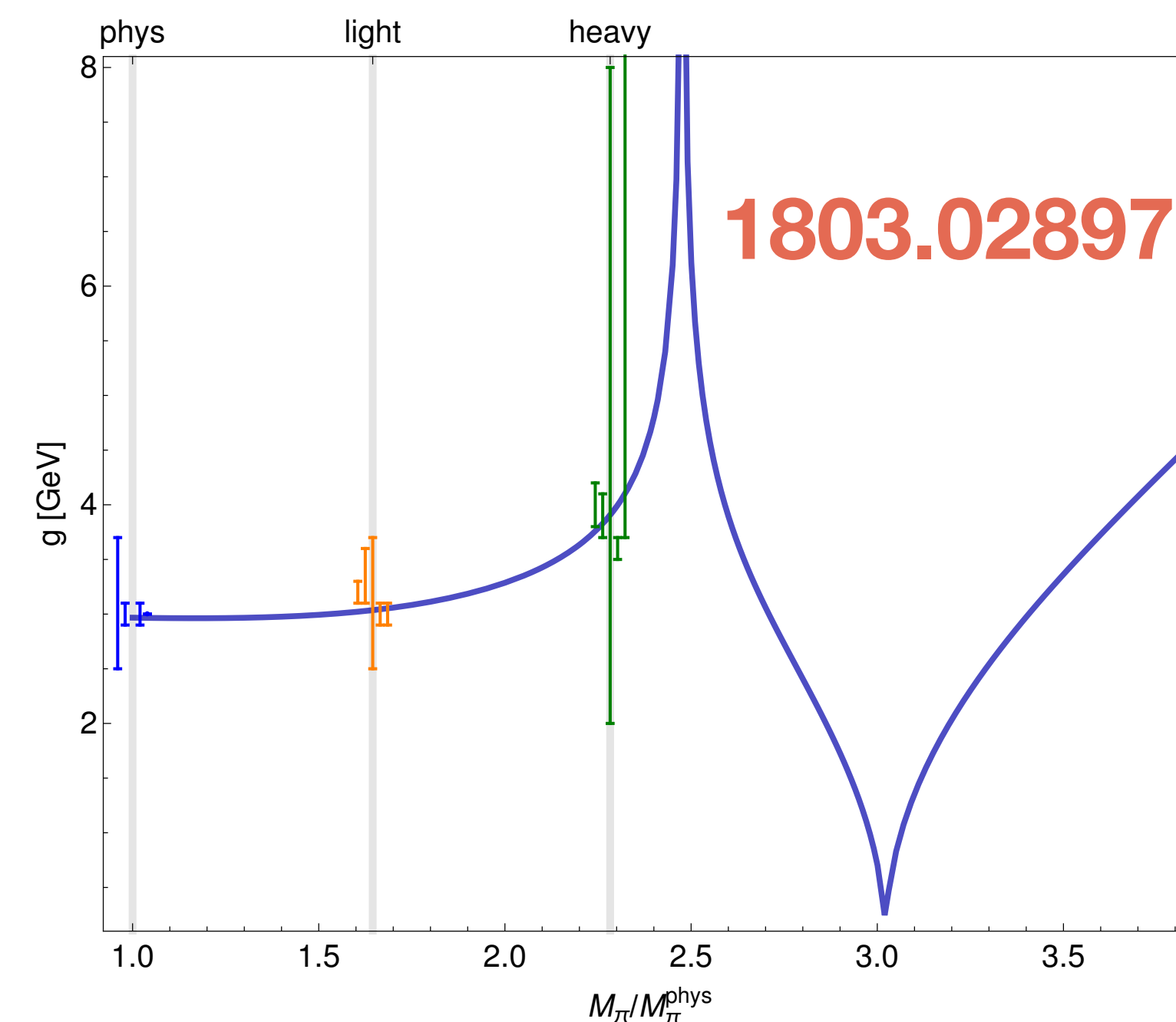
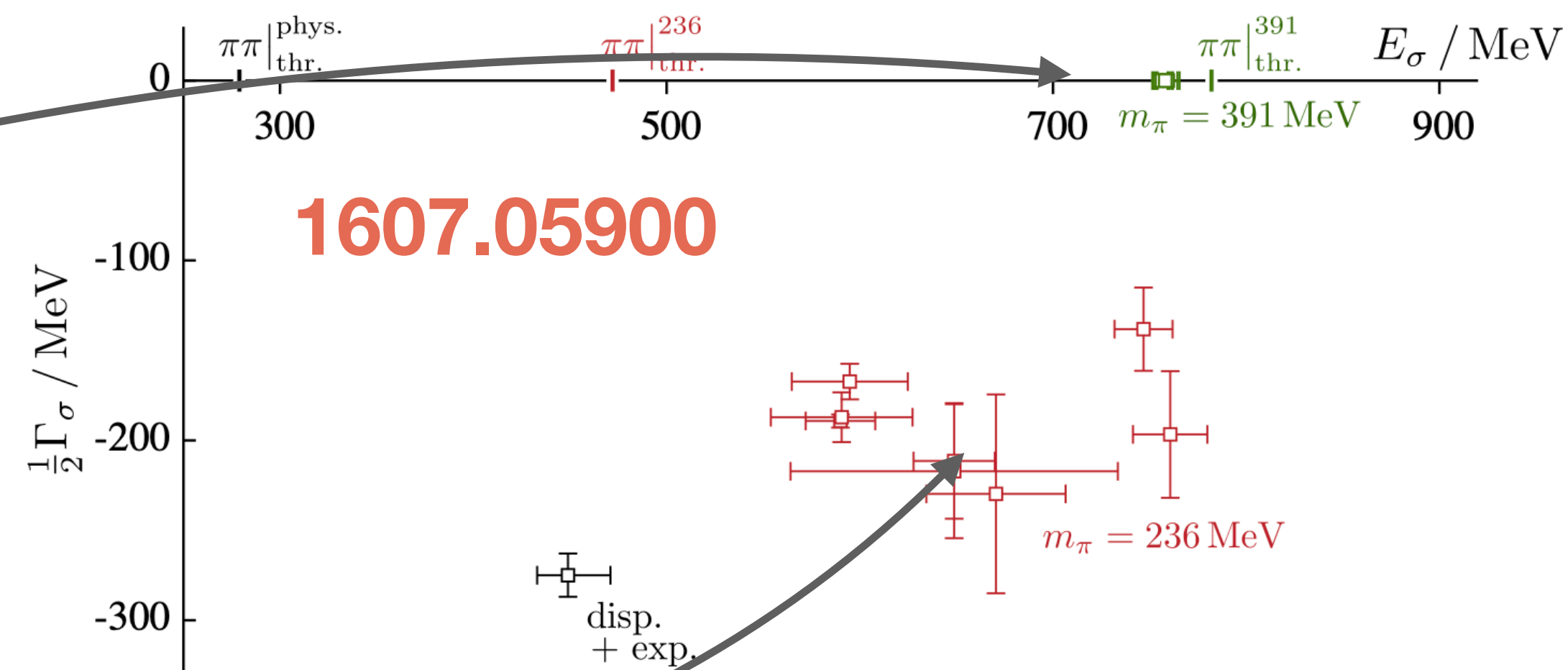
○ $m_\pi \sim ? \text{ MeV} \rightarrow \text{THIS TALK}$

✓ $m_\pi \sim 236 \text{ MeV} \rightarrow \text{Broad resonance}$

$$N_f = 2$$

$m_\pi \sim 315 \text{ MeV} \rightarrow \text{VB vs resonance}$

$m_\pi \sim 227 \text{ MeV} \rightarrow \text{Broad resonance}$

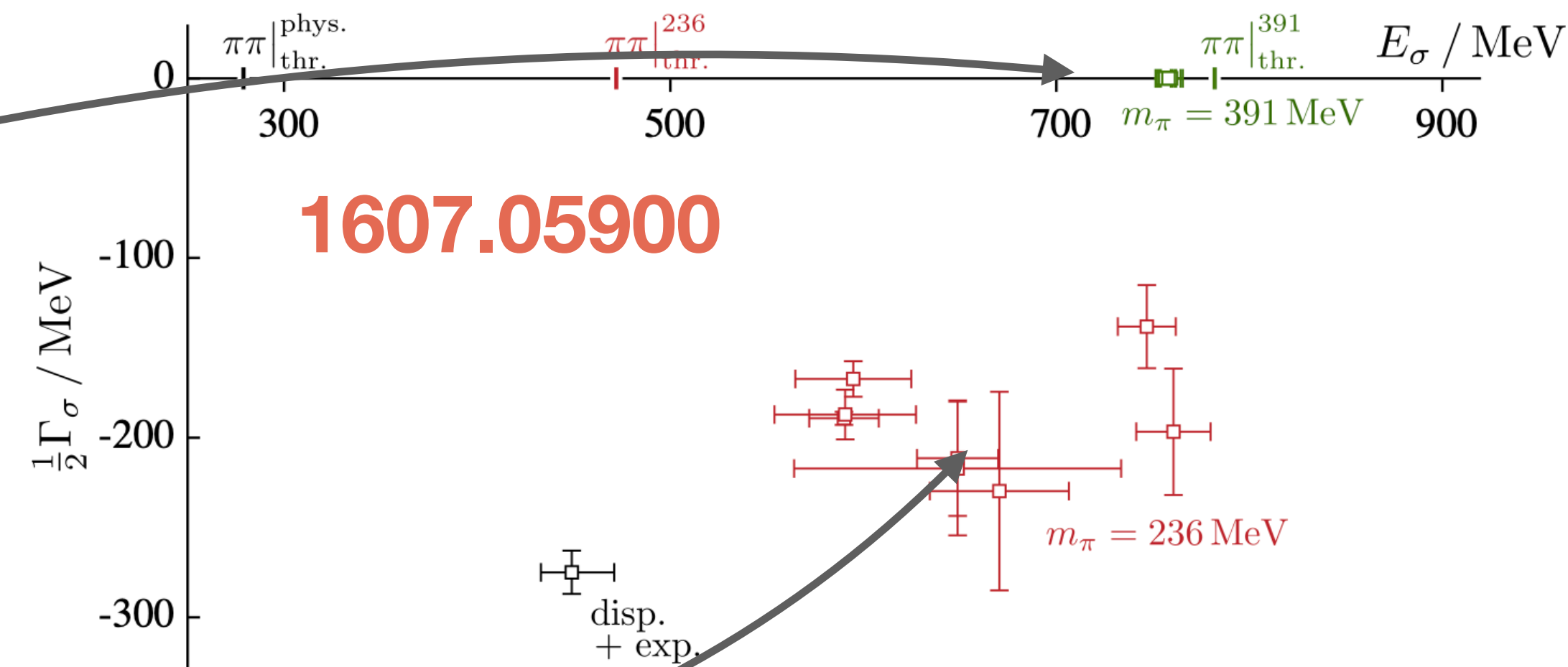


The σ as intermediate state

$$N_f = 2 + 1$$

✔ $m_\pi \sim 391 \text{ MeV} \rightarrow \text{BS}$

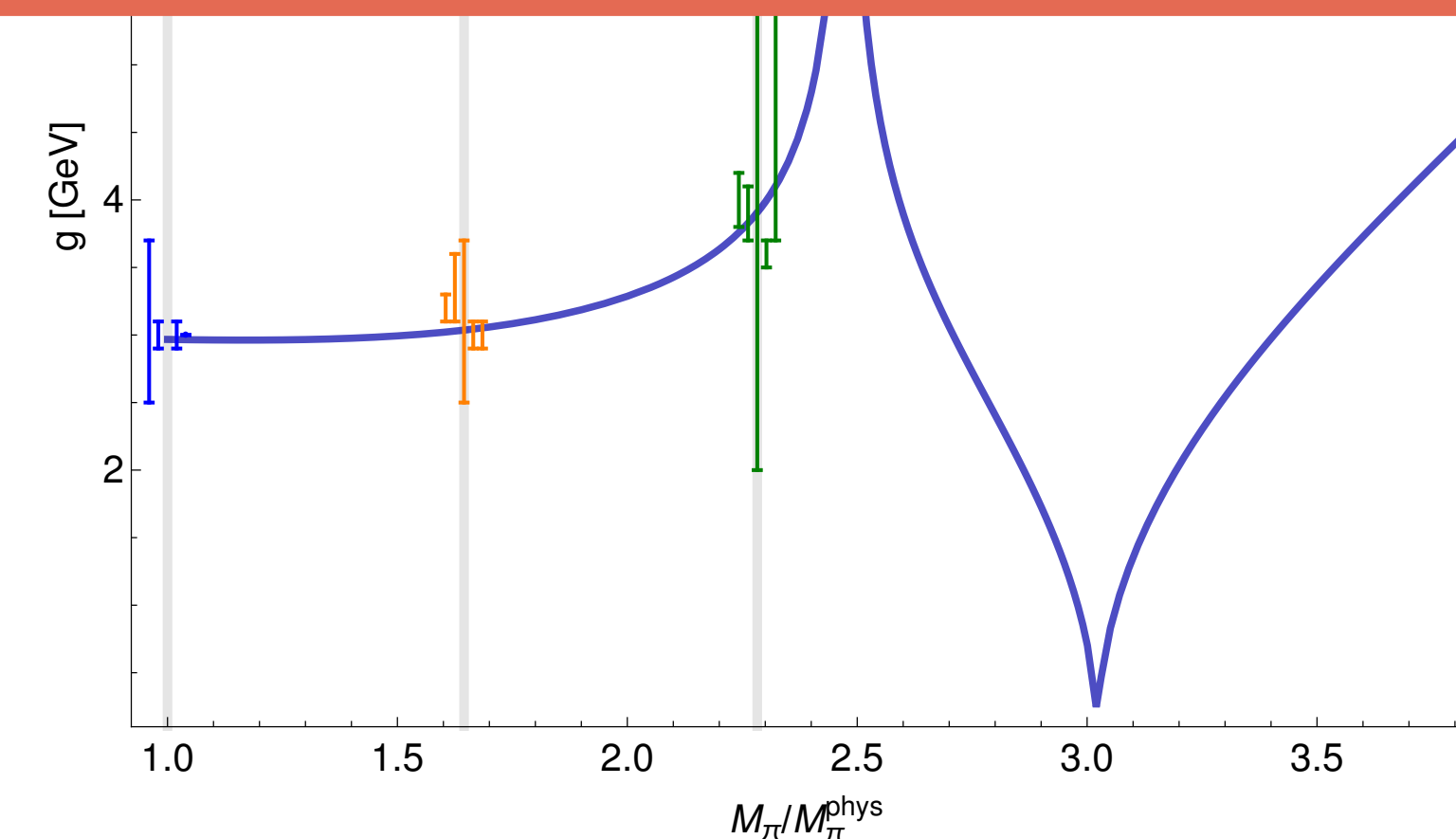
○ $m_\pi \sim ? \text{ MeV} \rightarrow \text{THIS TALK}$



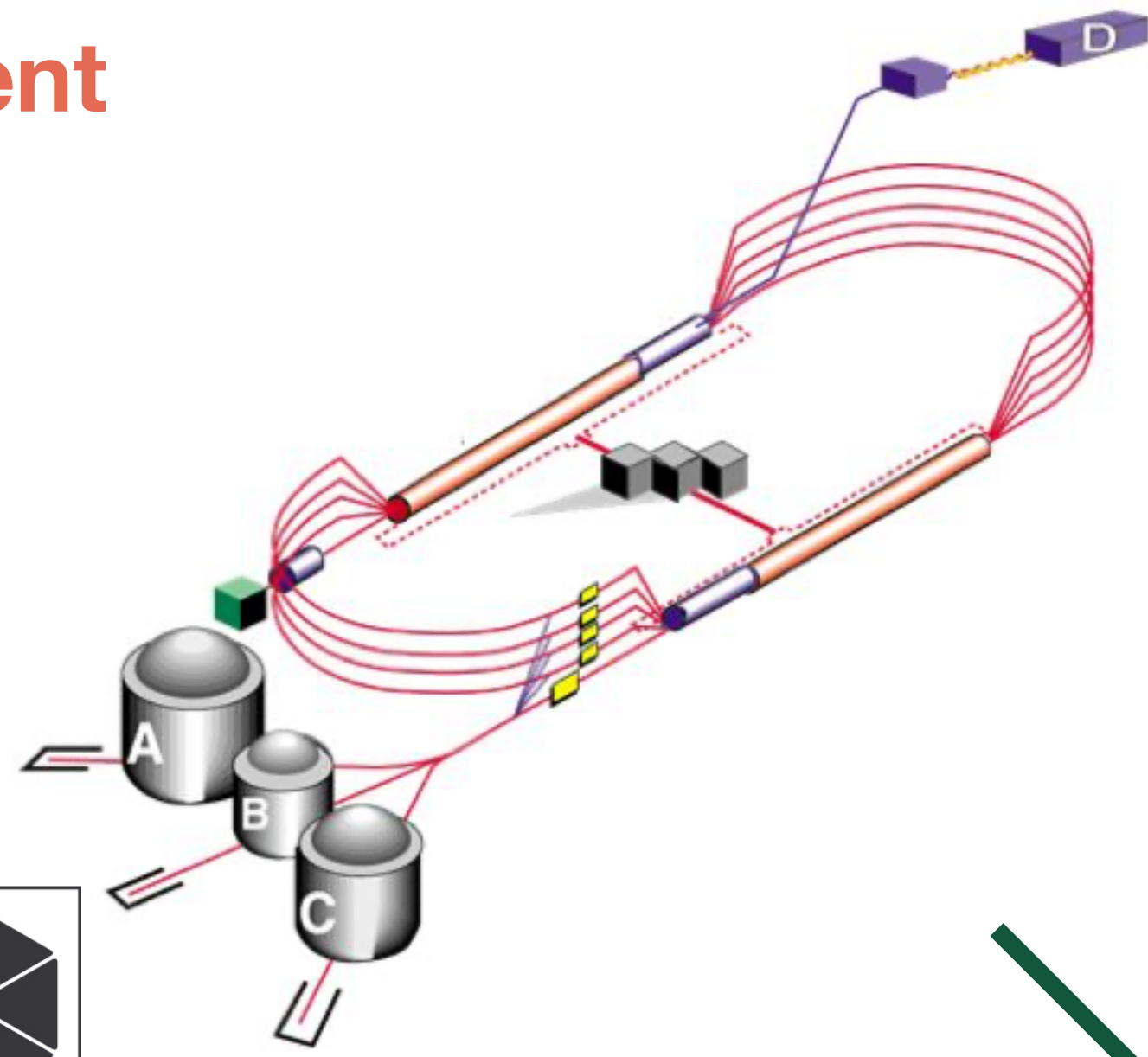
$-a_t m_\ell$	$(L/a_s)^3$	$\times T/a_t$	N_{cfigs}	N_{vecs}	N_{tsrc}	$a_t m_\pi$	$a_t m_K$	$a_t m_\eta$	$a_t m_\Omega$	ξ	m_π/MeV
0.0850	24^3	$\times 256$	482	162	8–16	0.05634(14)	0.09027(15)	0.09790(100)	0.2857(8)	3.463(7)	330
0.0856	$24^3, 32^3$	$\times 256$	400, 484	162, 256	8	0.04724(13)	0.08659(14)	0.09602(70)	0.2793(8)	3.461(5)	283

$m_\pi \sim 315 \text{ MeV} \rightarrow \text{VB vs resonance}$

$m_\pi \sim 227 \text{ MeV} \rightarrow \text{Broad resonance}$

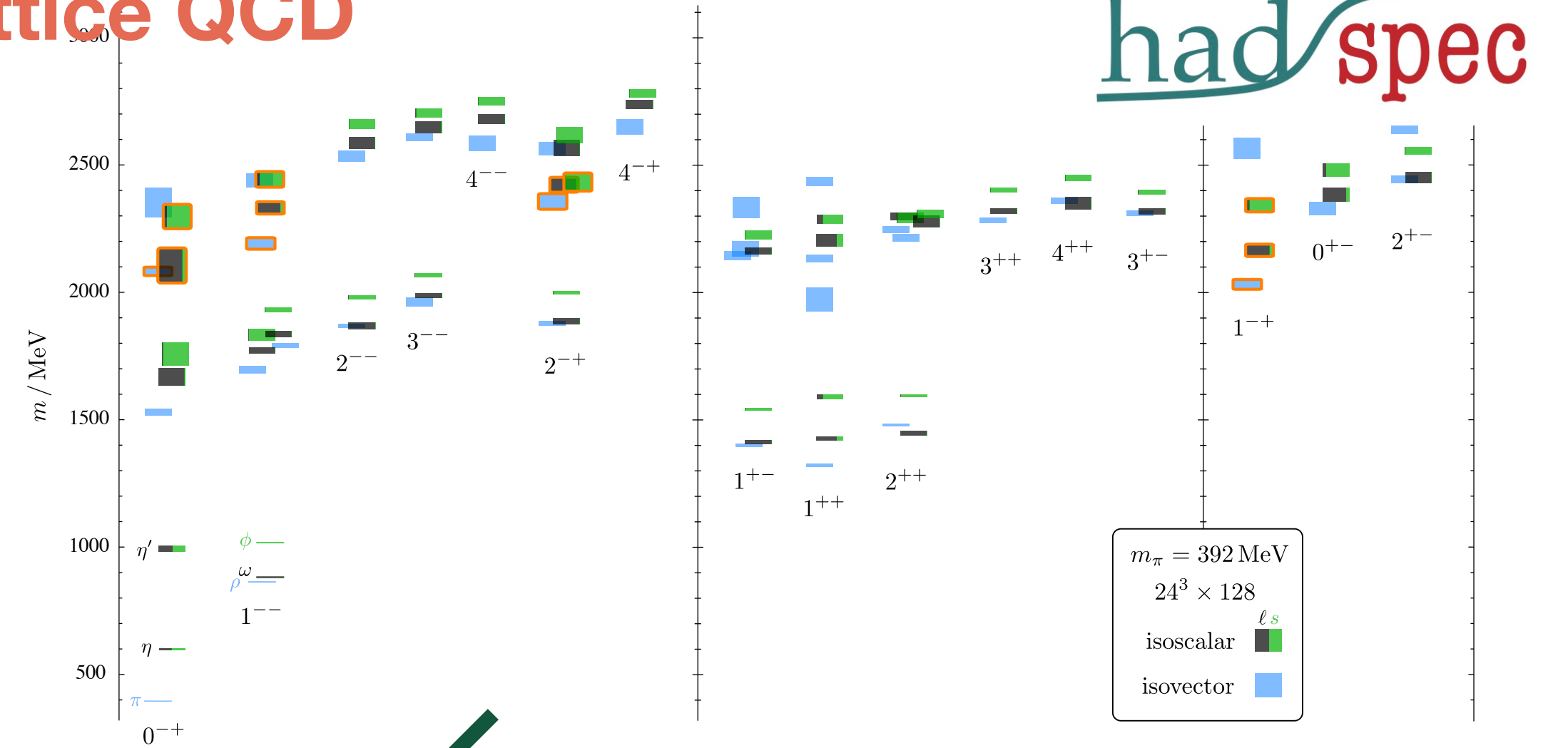


Experiment

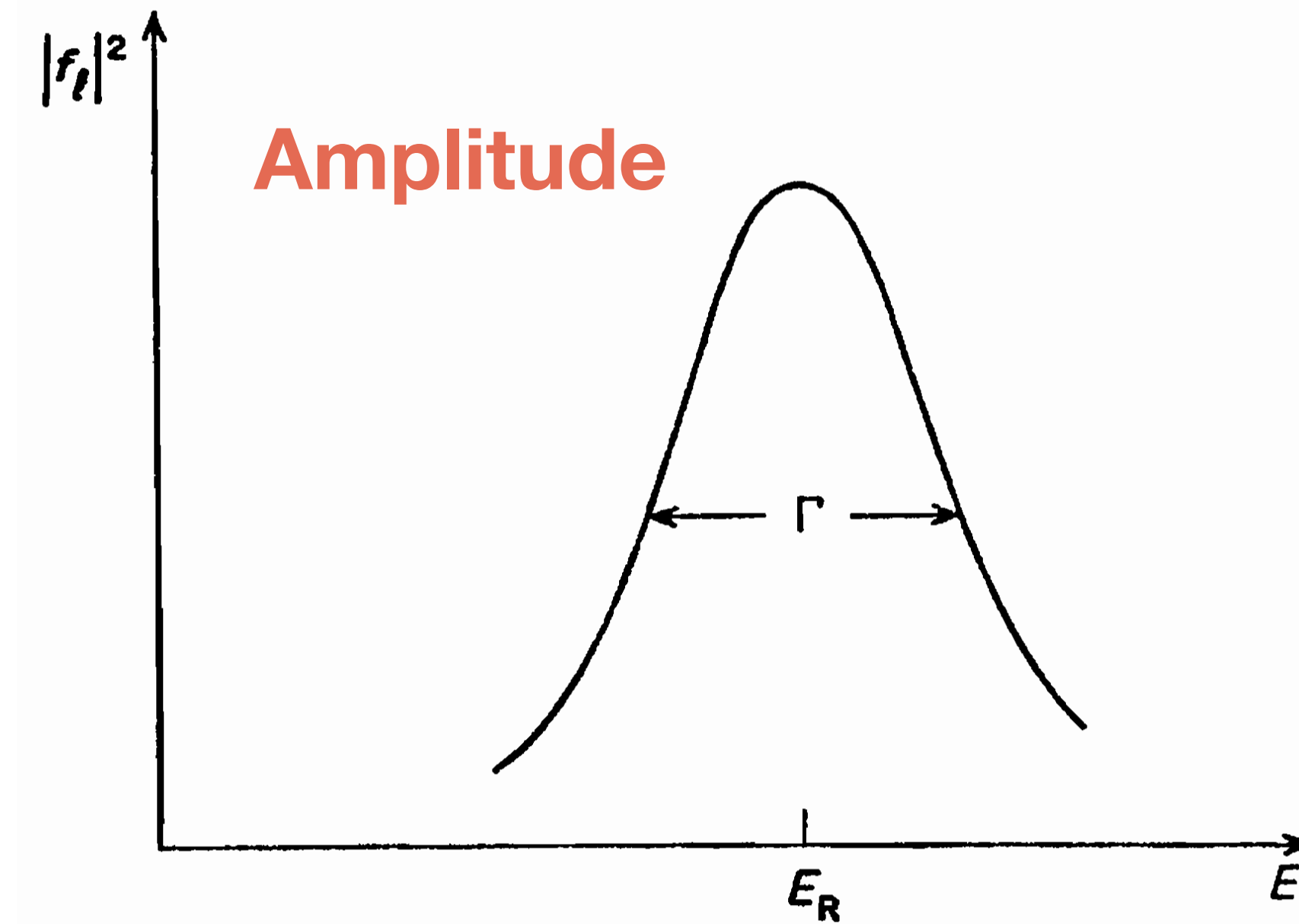


GLUEX

Lattice QCD



Resonances manifest
as amplitudes

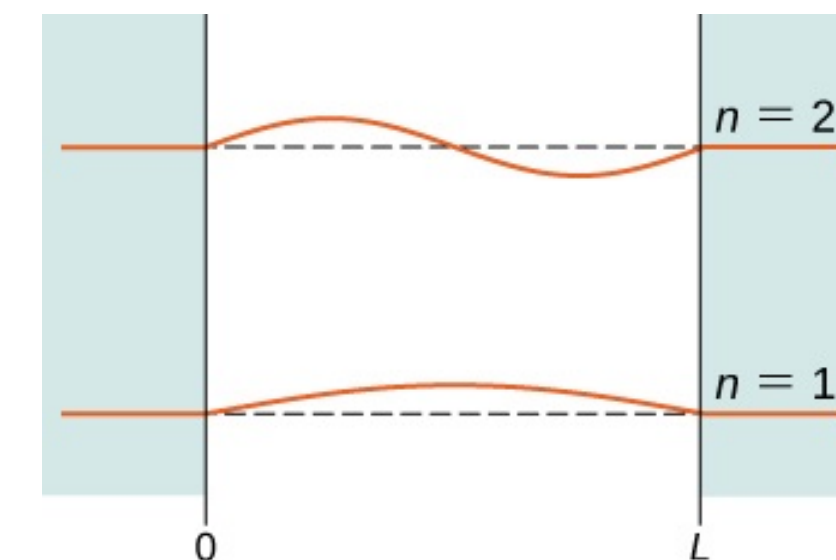


Lattice QCD

In

Out

Boundary

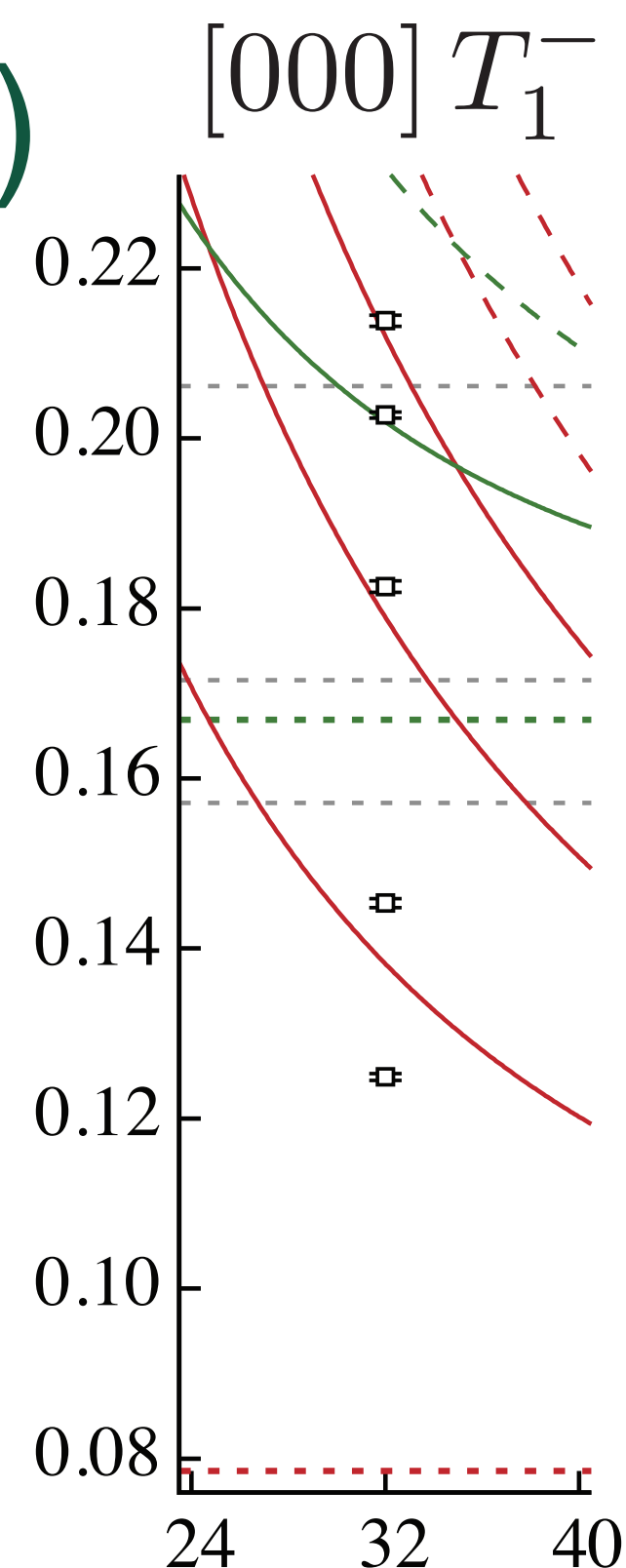


Lüscher

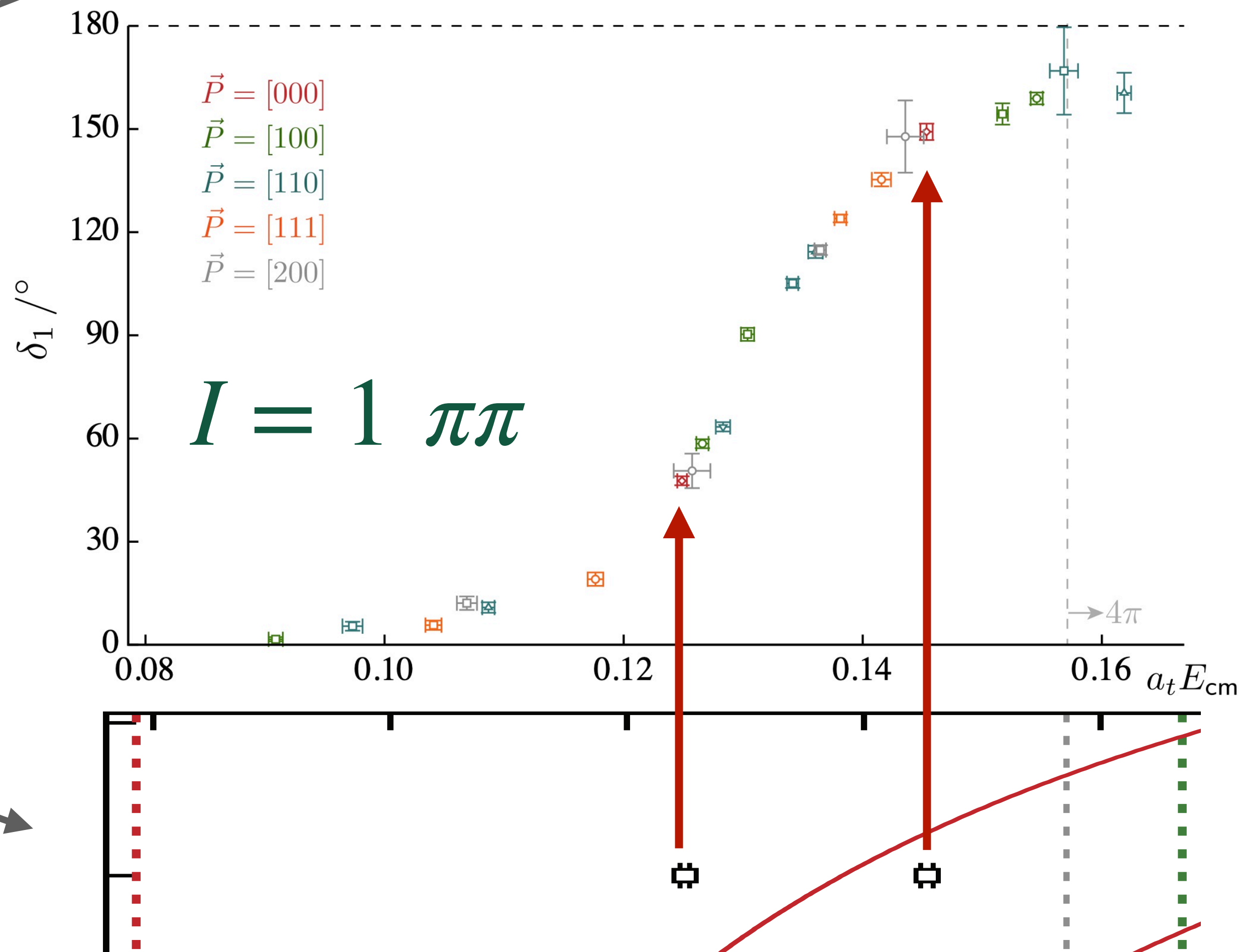
$$e^{ikL + 2i\delta(k)} = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$

Pathway towards $t_\ell^I(s)$

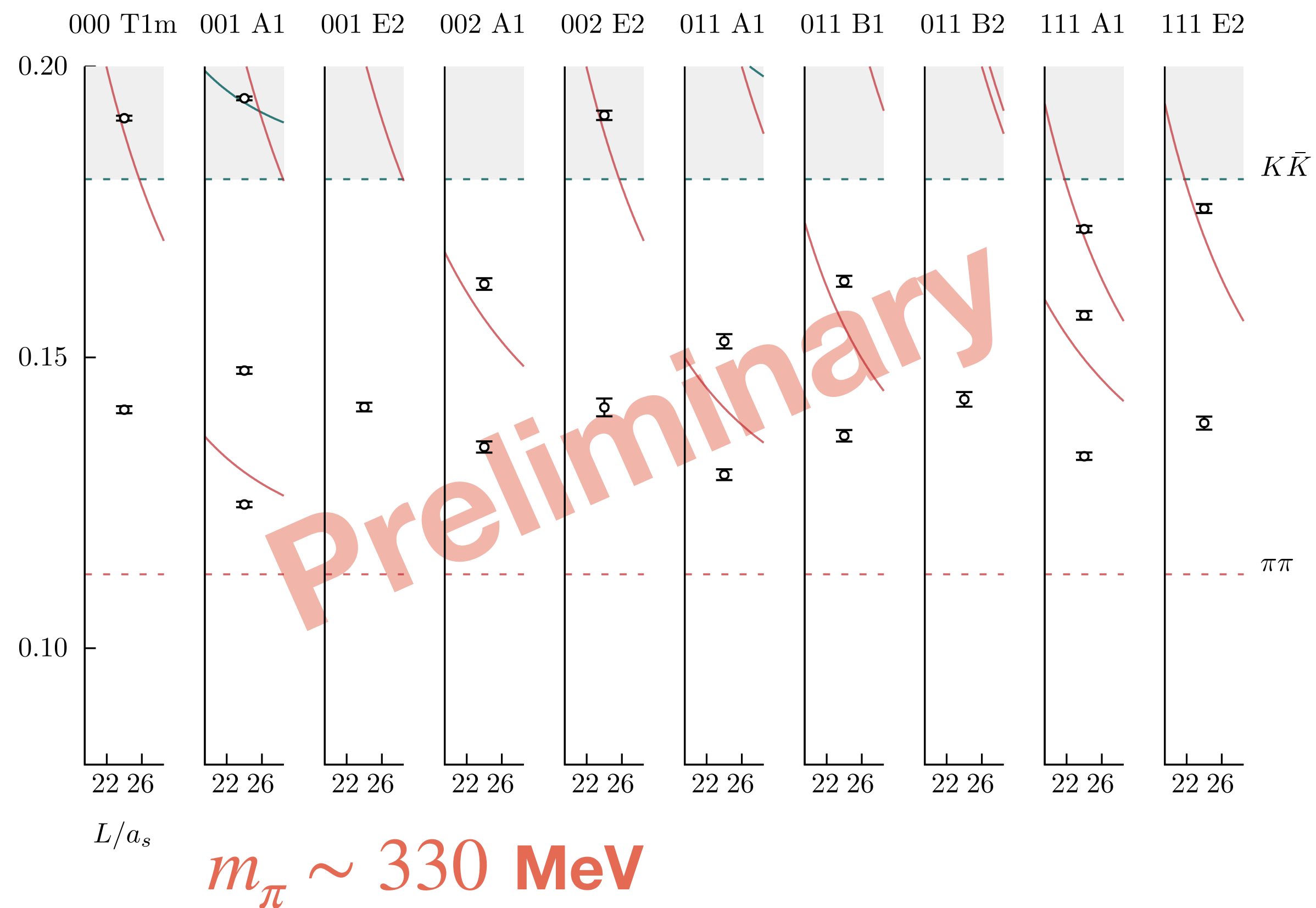
Spectrum (GEVP)



Our E_n become the x axis



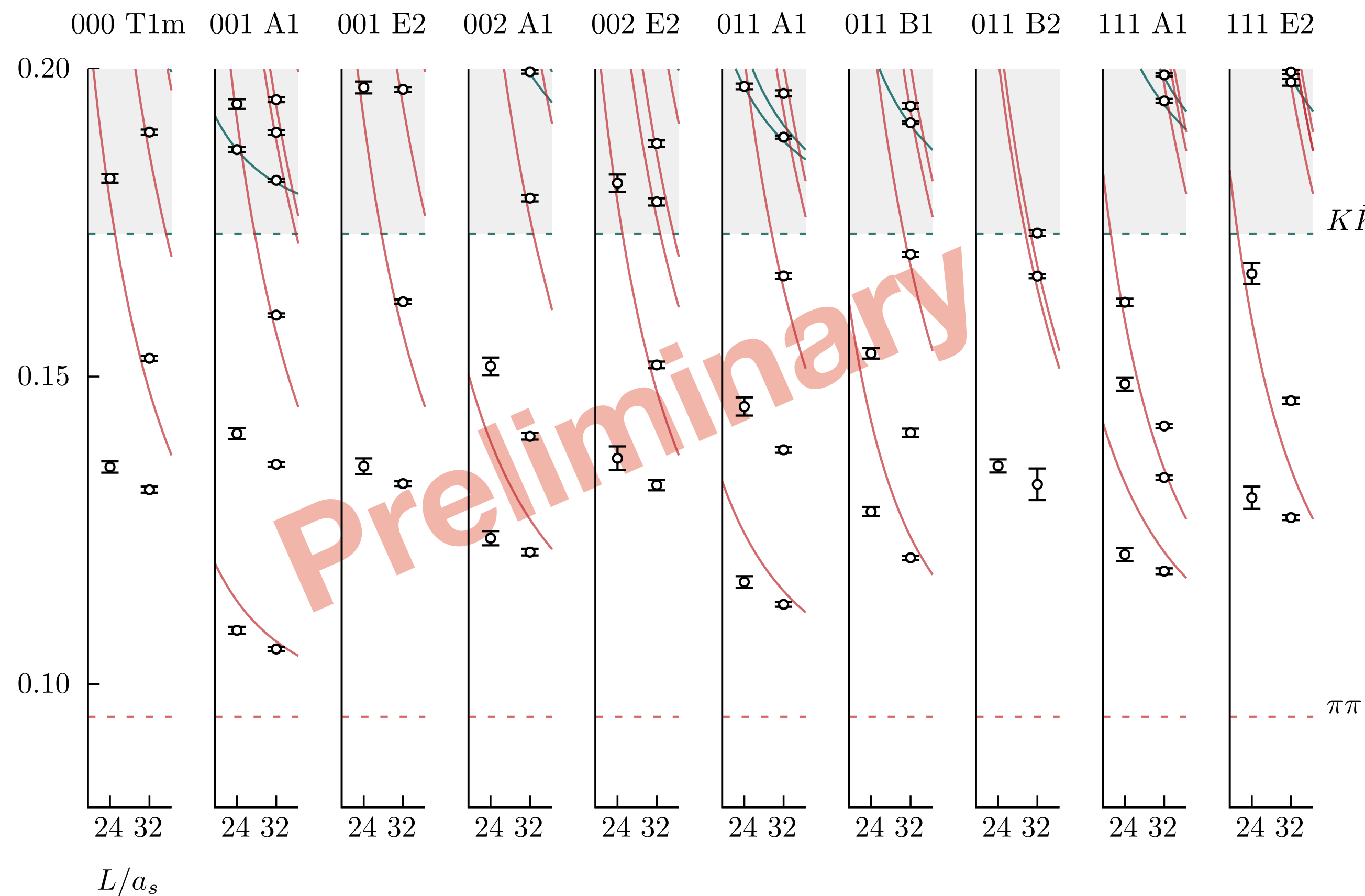
$I = 1 \pi\pi$



Allows us to study the
 ρ resonance m_q dependence

Similar spectrum to
previous masses

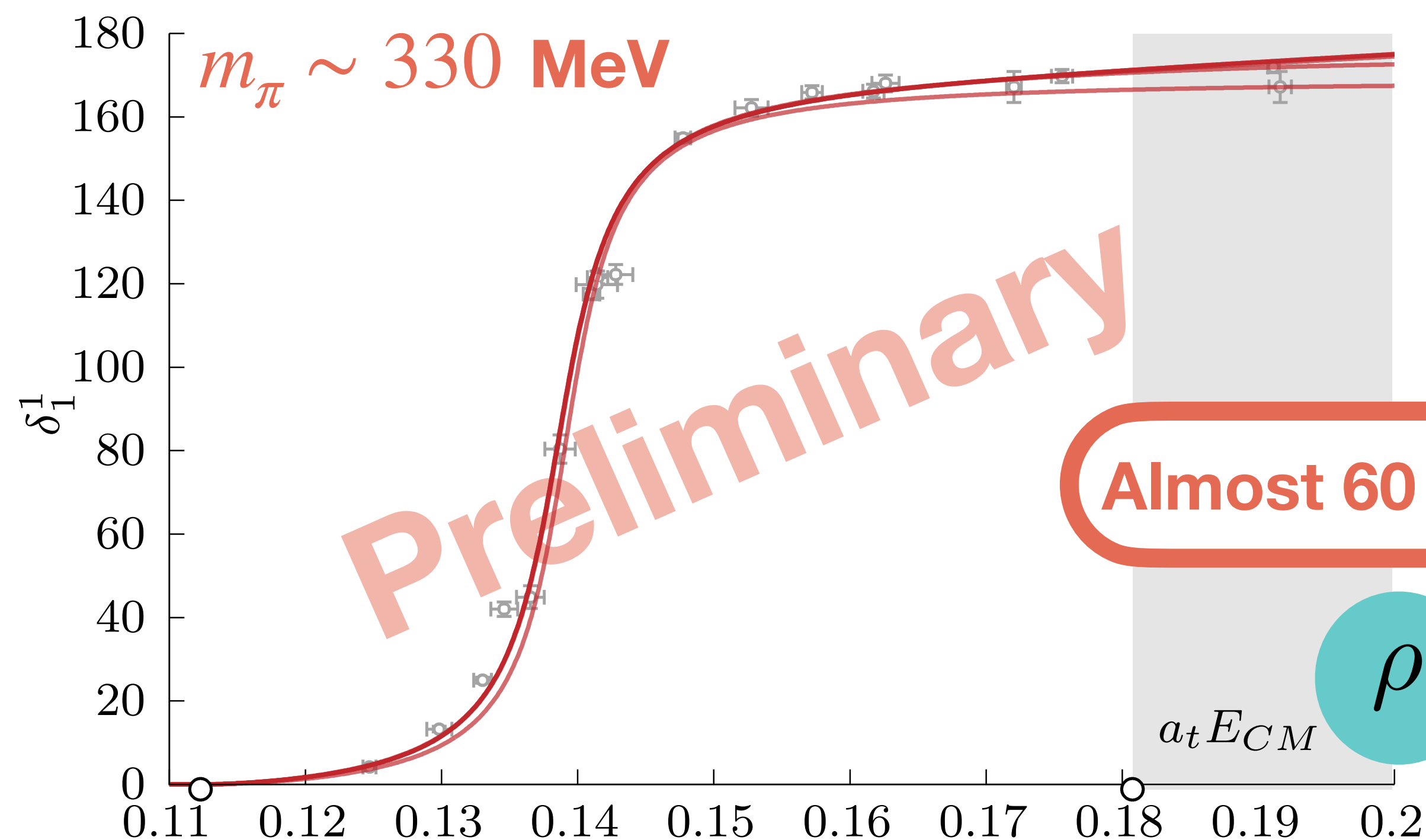
$m_\pi \sim 283 \text{ MeV}$



$I = 1 \pi\pi$

had spec

$I = 1 \pi\pi$



Percent error for $\delta(s)$

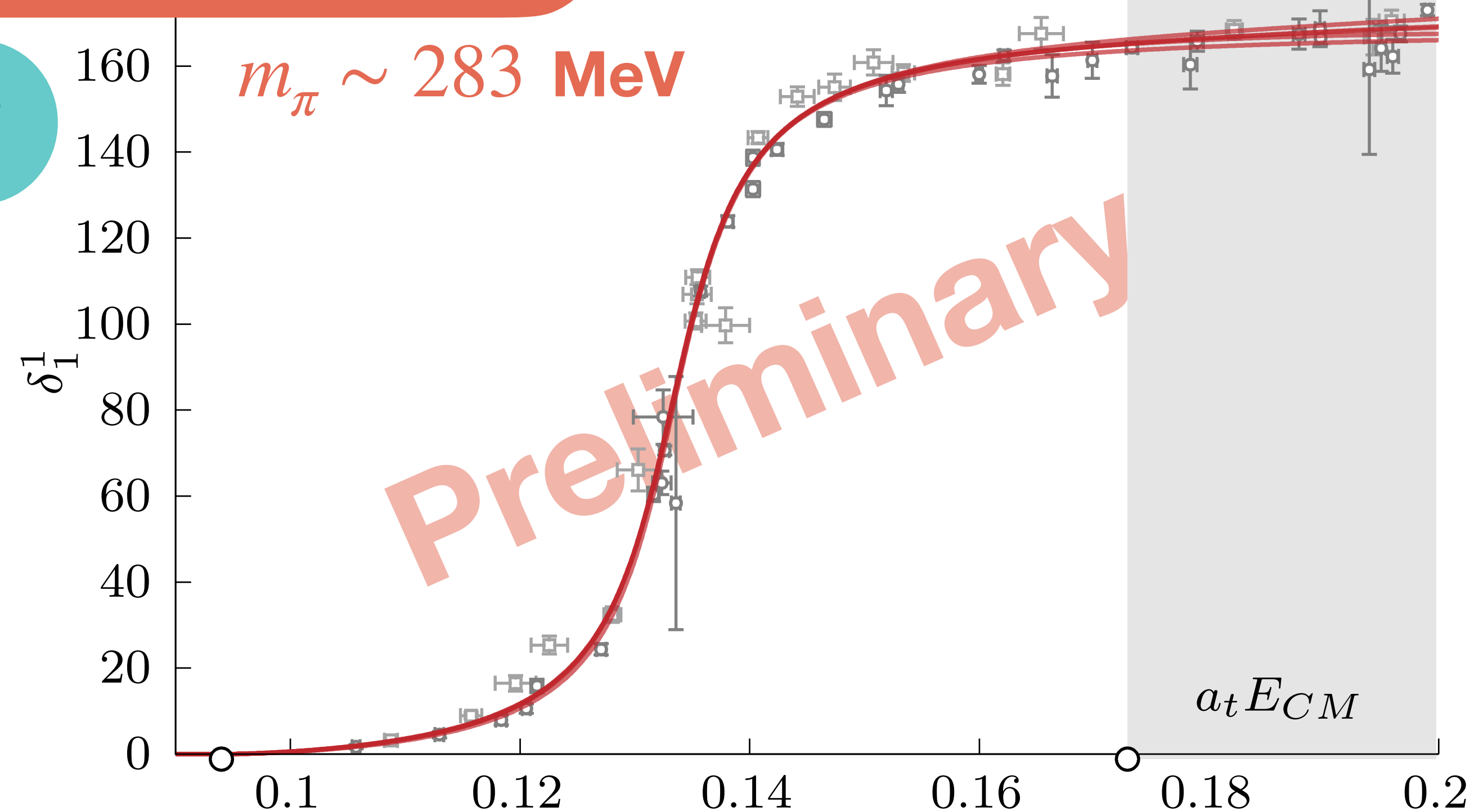
Almost 60 elastic data points

ρ

$a_t E_{CM}$

Over 10 parameterizations

Very consistent amplitude fits



$a_t E_{CM}$

$$I = 1 \pi\pi$$

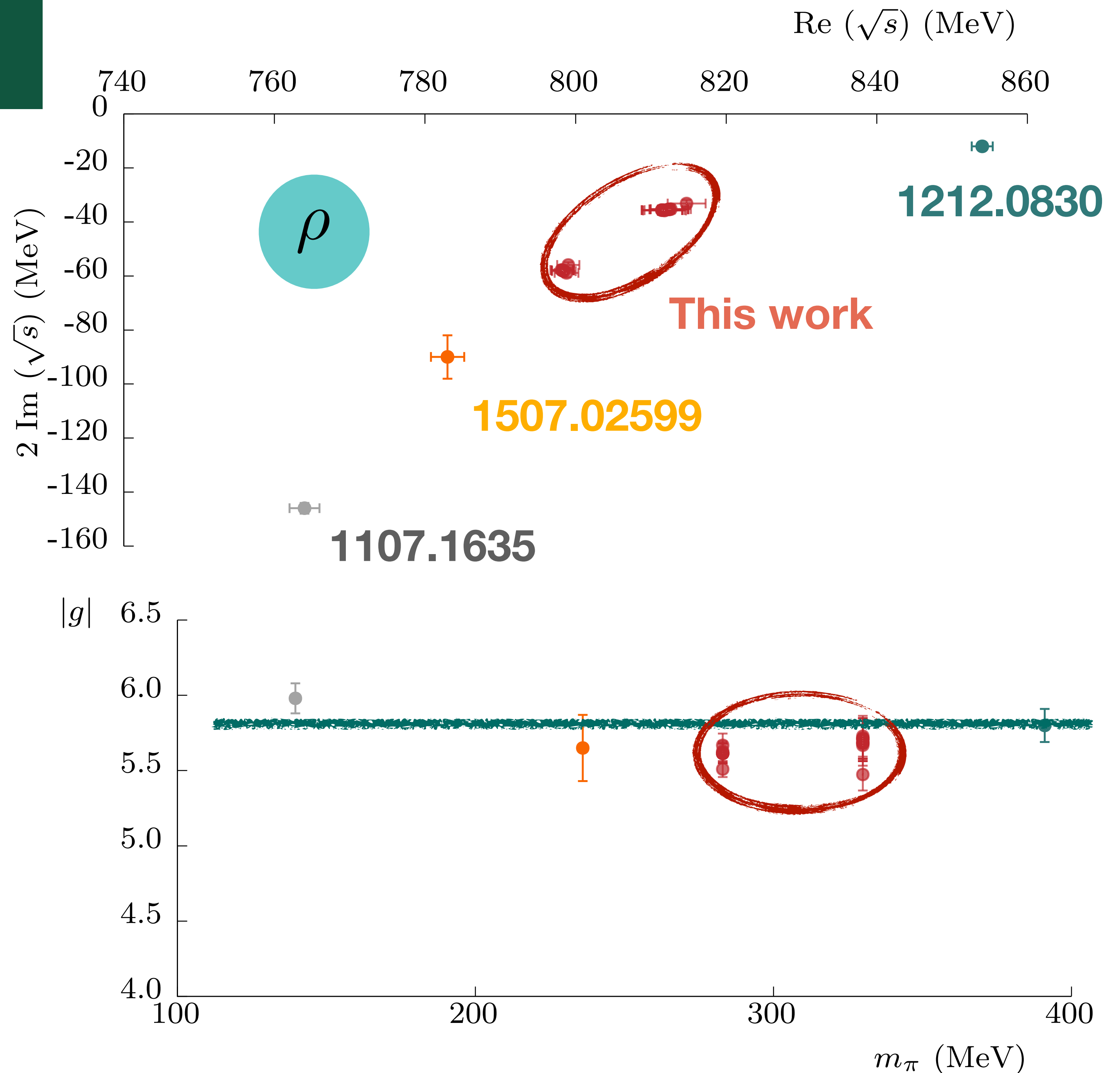
had spec

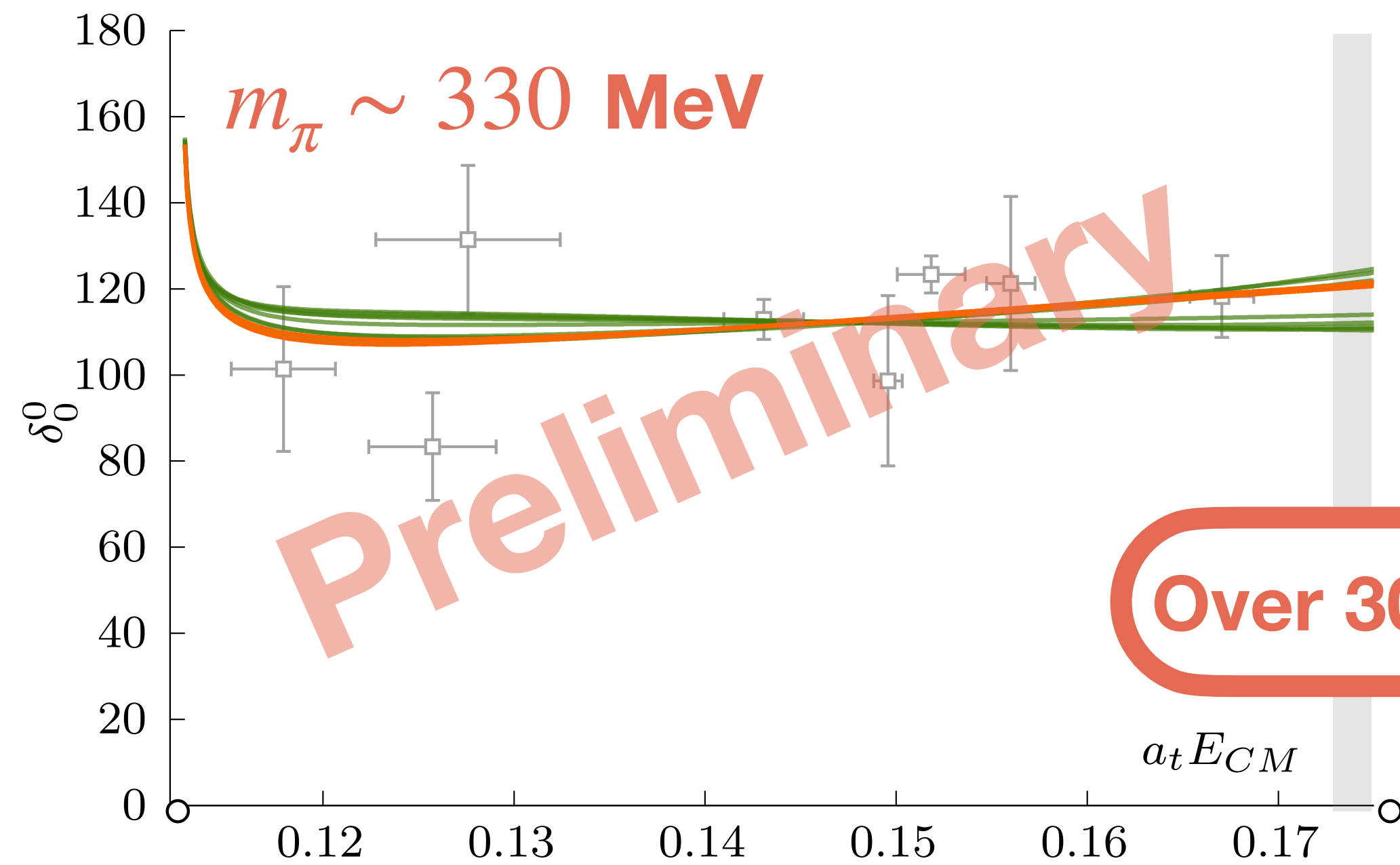
Ordinary m_q dependence

$$M \sim a + bm_q$$

$$\Gamma \sim q^2$$

g constant





Extremely different behavior

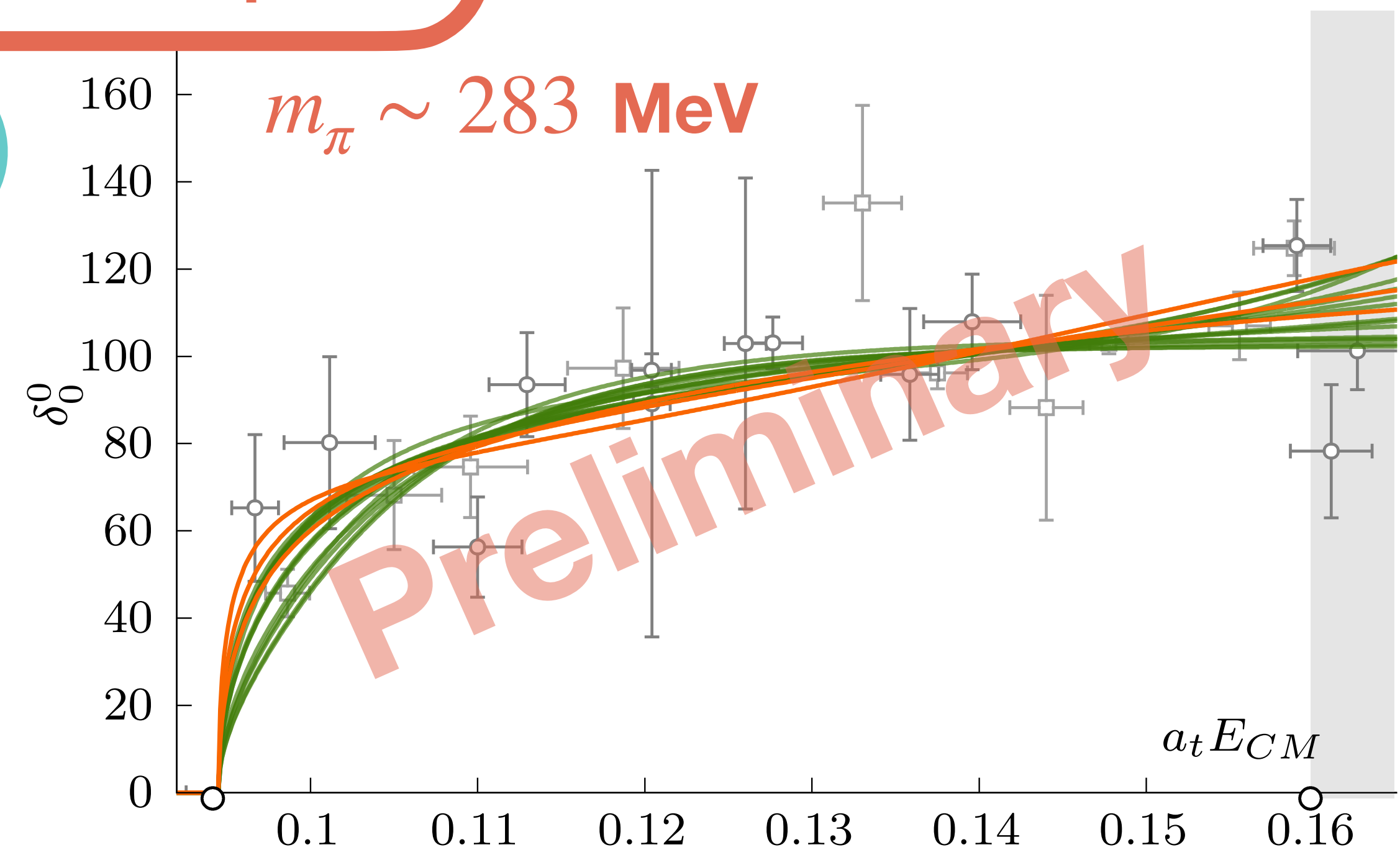
Large negative derivative at threshold

Over 30 elastic data points

σ

Large positive derivative at threshold

Crosses $\delta(s) = \pi/2$ far from threshold



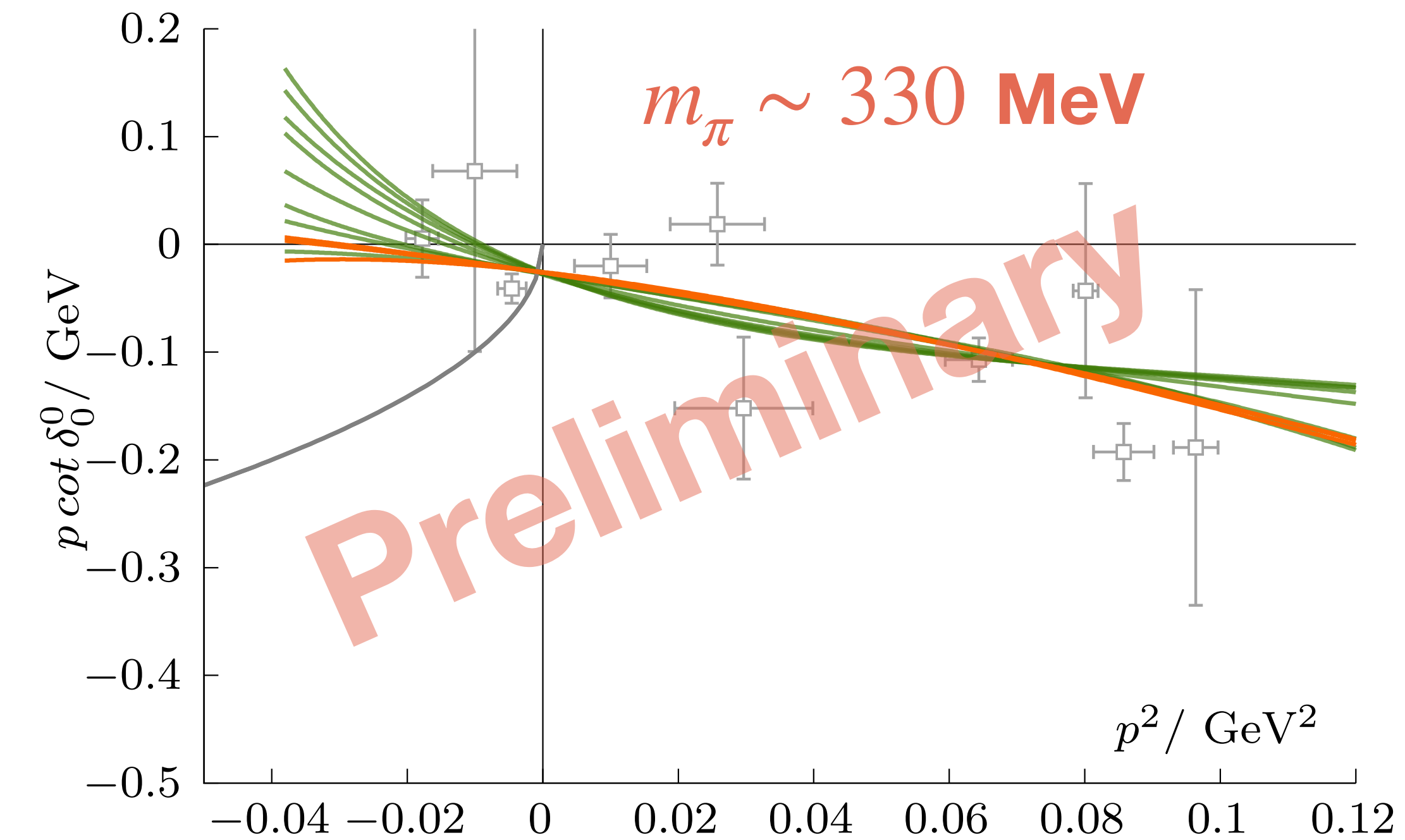
$I = 0 \ \pi\pi$

$$p \cot \delta(s) \sim \frac{1}{a}$$

Large, negative a

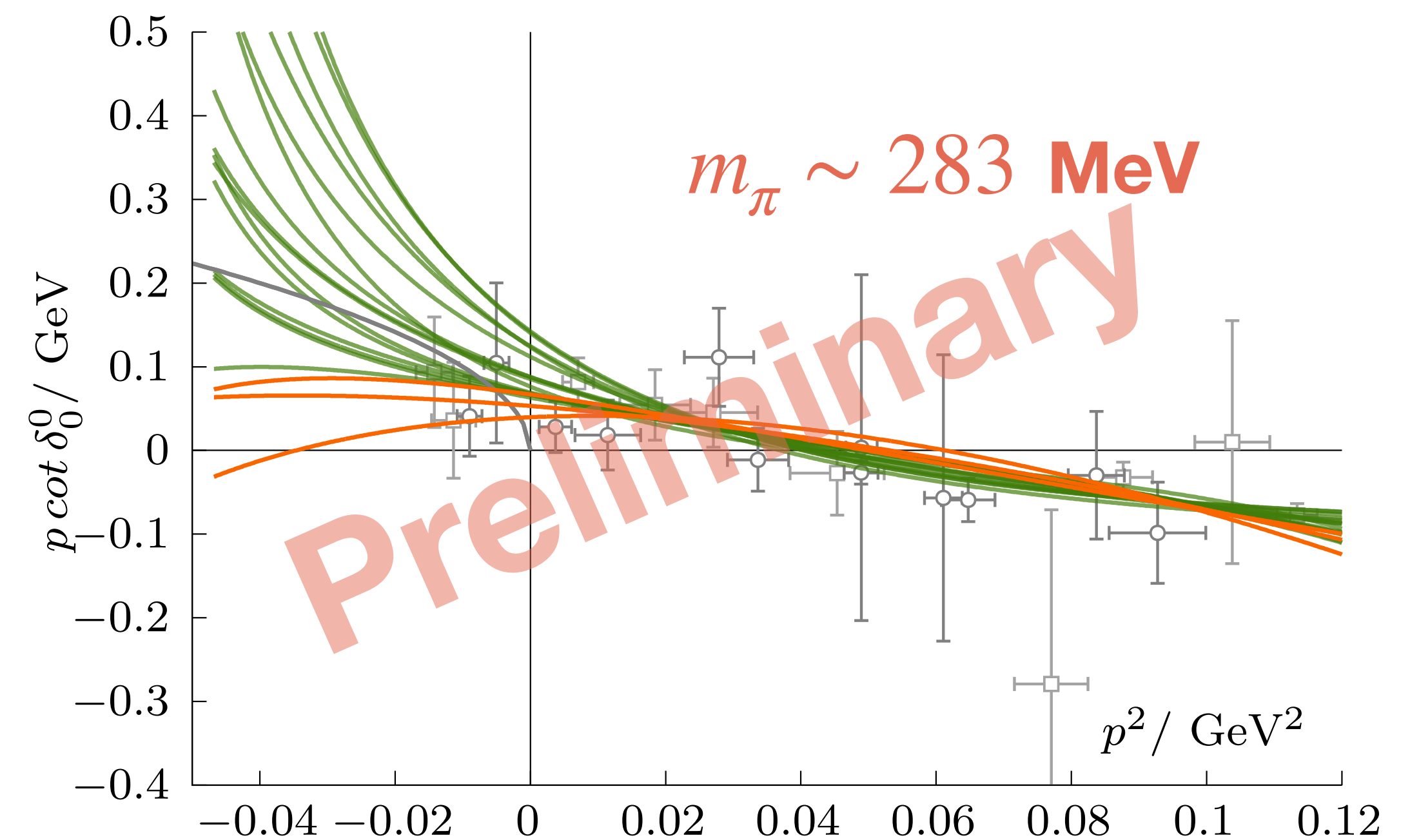
Extremely precise at threshold

σ

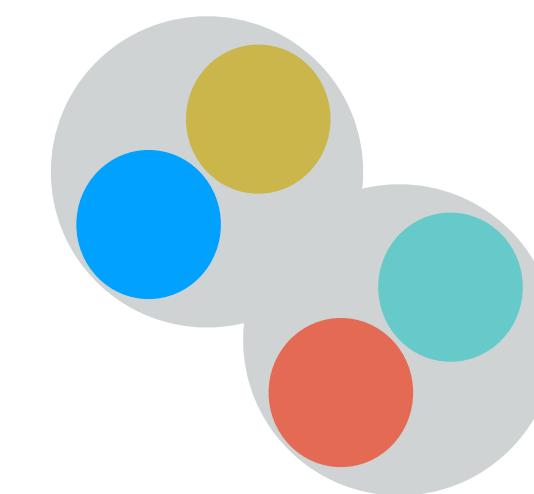
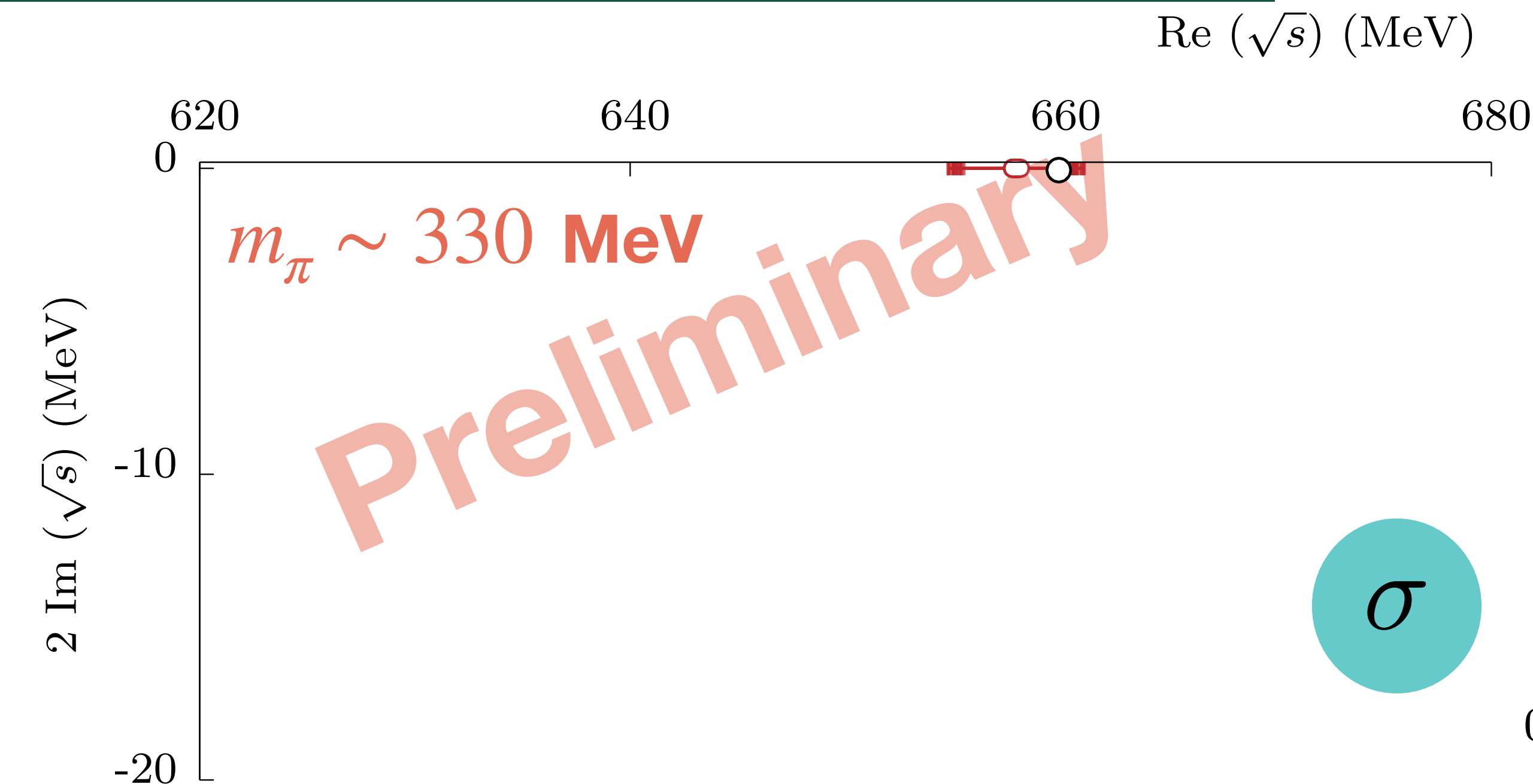


Large, positive a

Large spread at threshold



Light Scalars: the σ



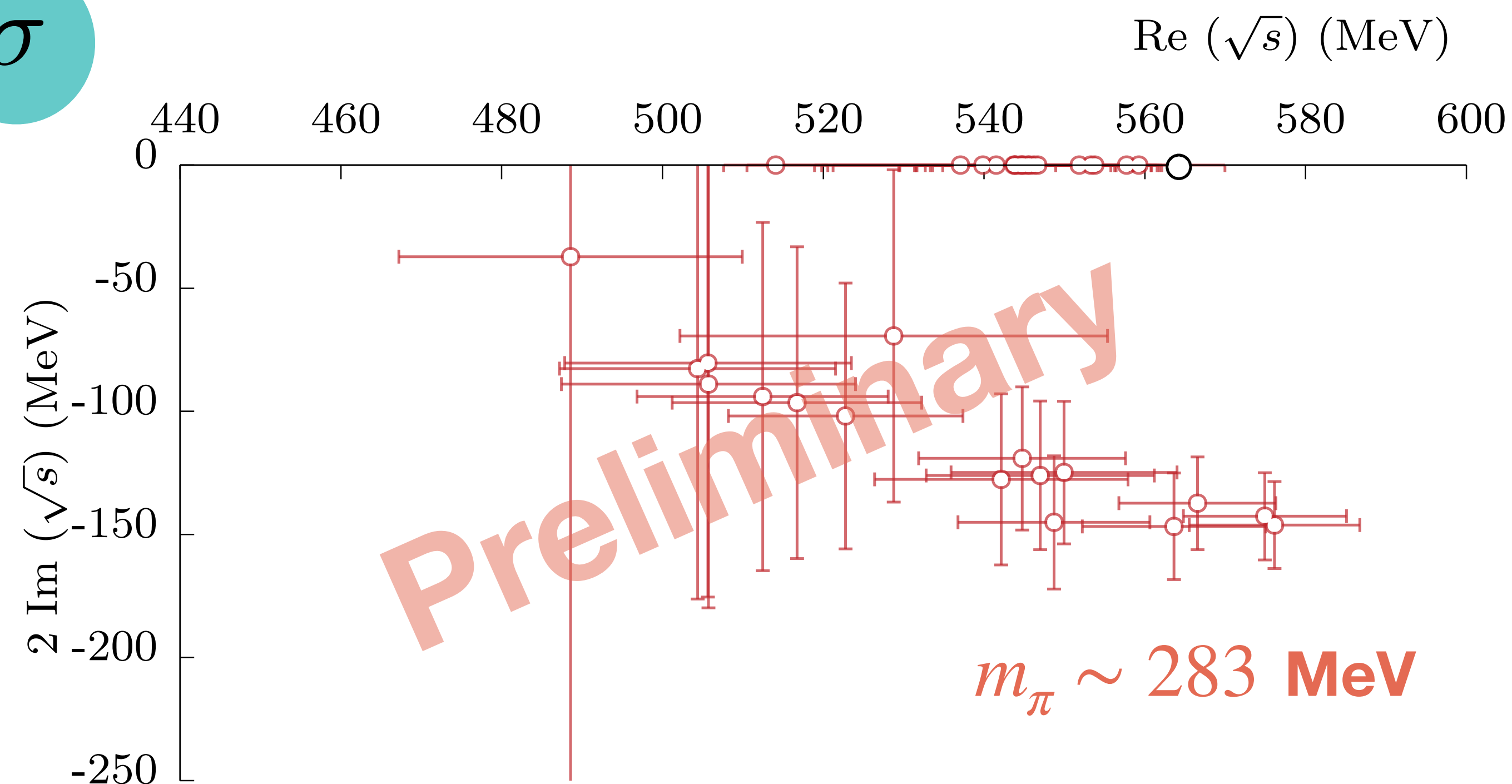
2nd Riemann sheet

VB vs resonance

Lower $a \rightarrow$ broader σ

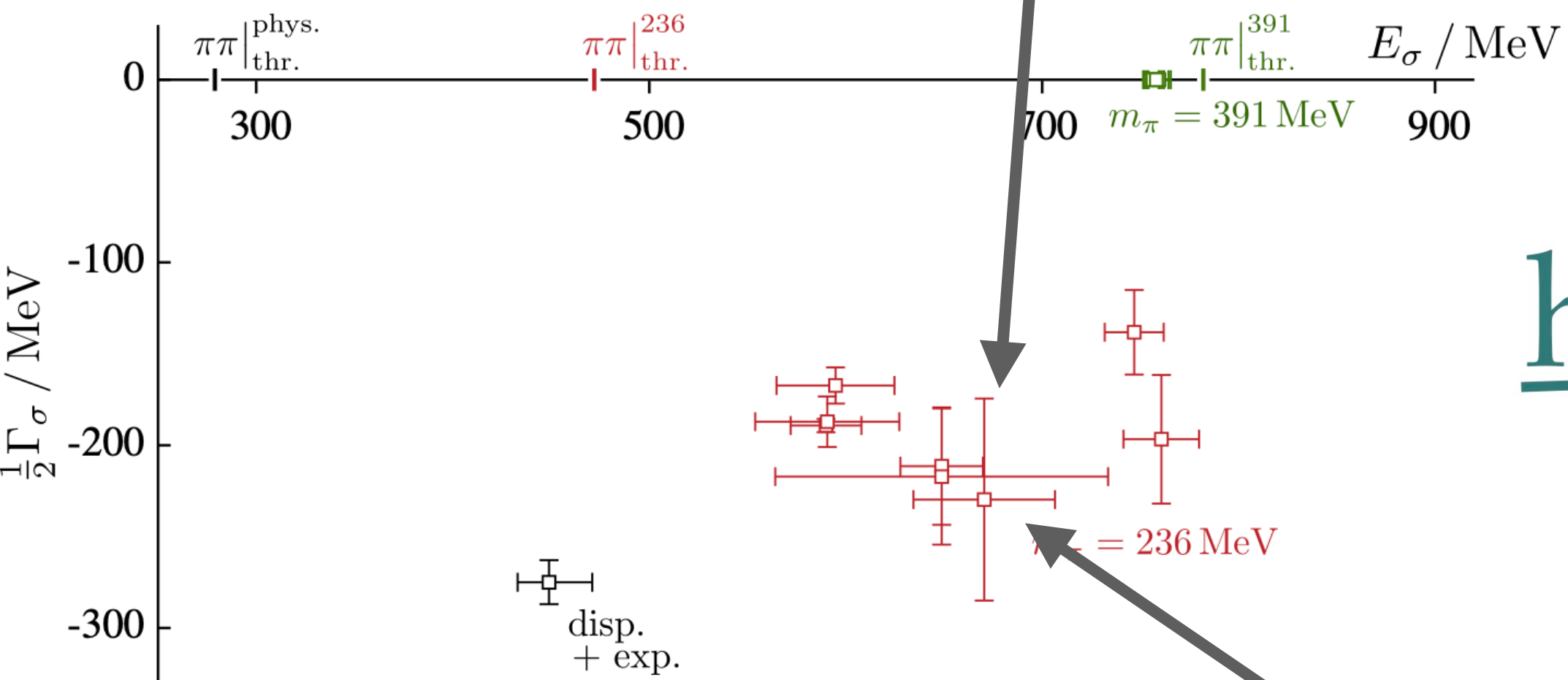
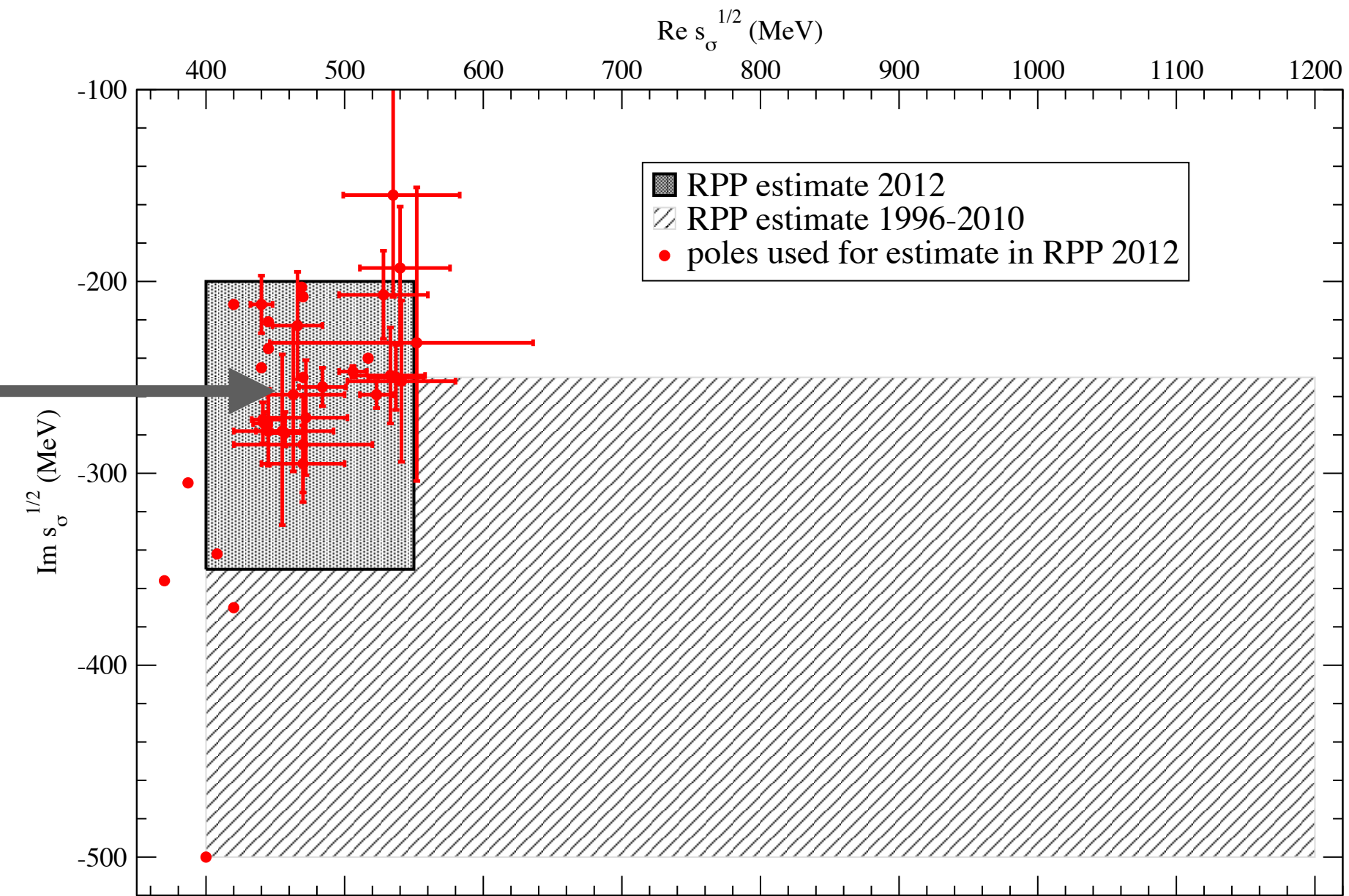
Extremely “noisy”

σ



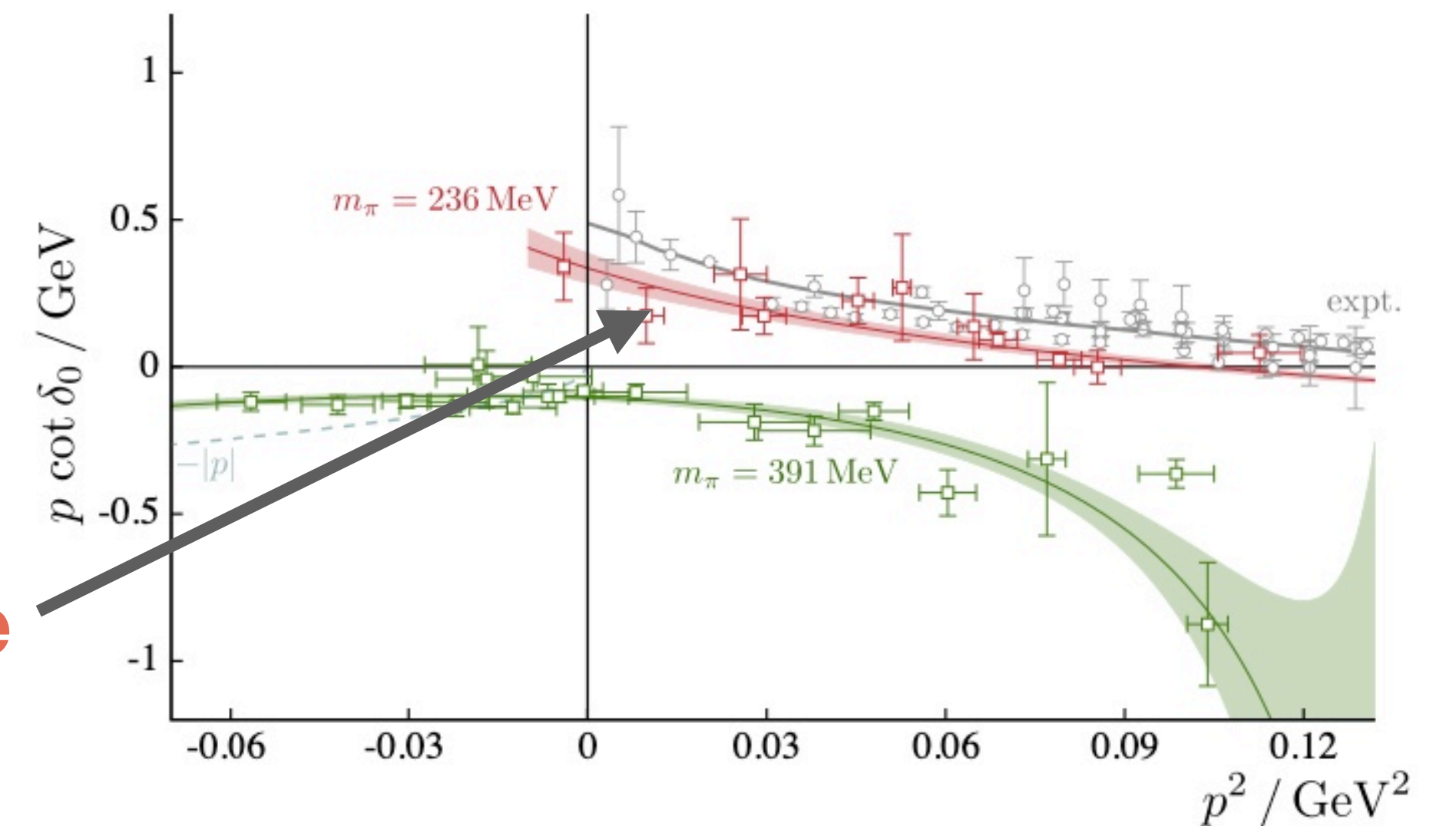
Light Scalars: the σ

It affects both experiment
and lattice QCD



had spec
1607.05900

But data is precise



New, precise data for $m_\pi = 330,283$ MeV

Robust systematic analysis

Small statistic uncertainties

ρ trajectory at various m_q points

Ordinary behavior

Sub-percent errors

σ found as a BS, VB and resonance

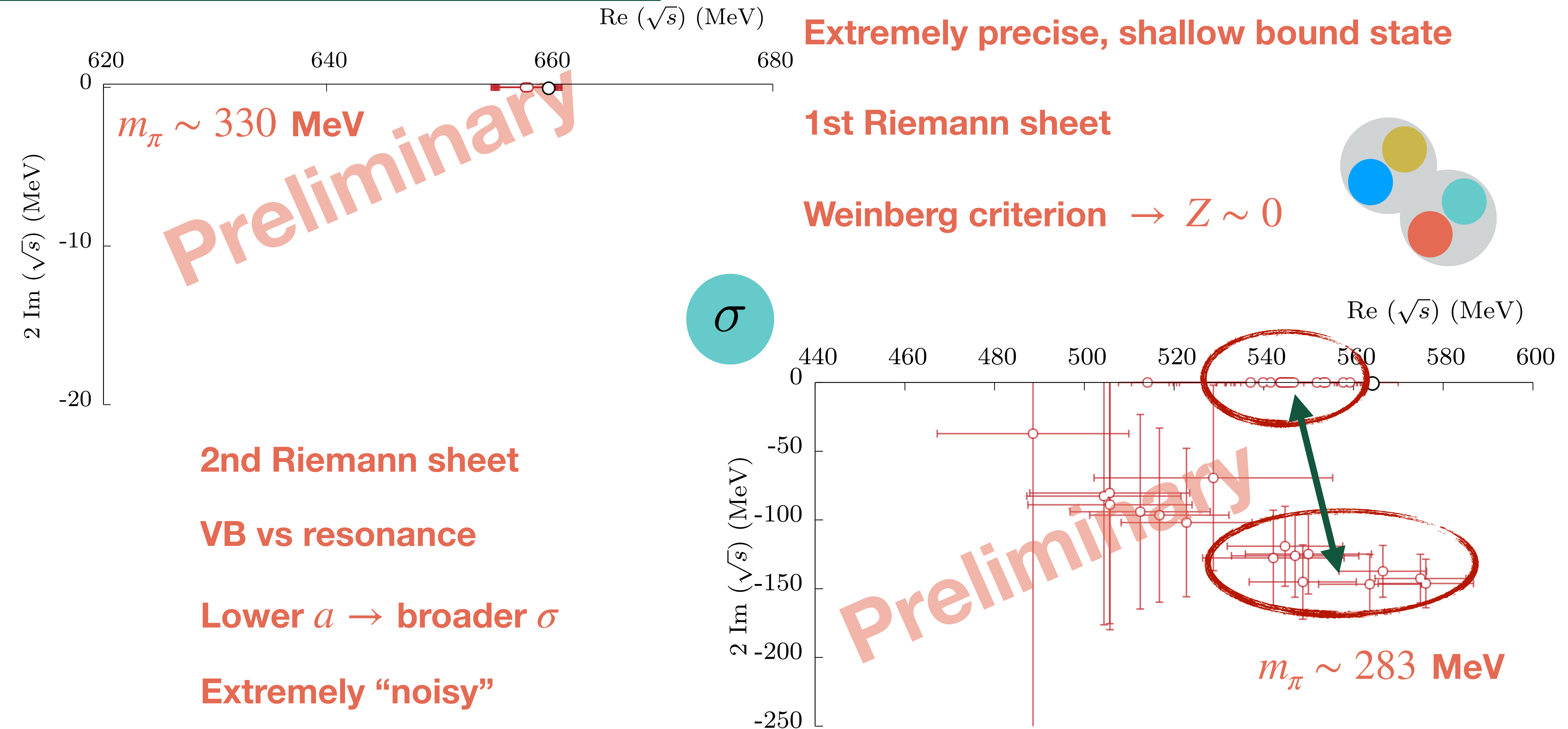
Systematics $\rightarrow$ param. model dependence

Lighter $m_\pi \rightarrow$ problem worsens

A “model-free” σ resonance??

Extra slides

Light Scalars: the σ



$$I = 0 \pi\pi$$

Many ops. for a good GEVP

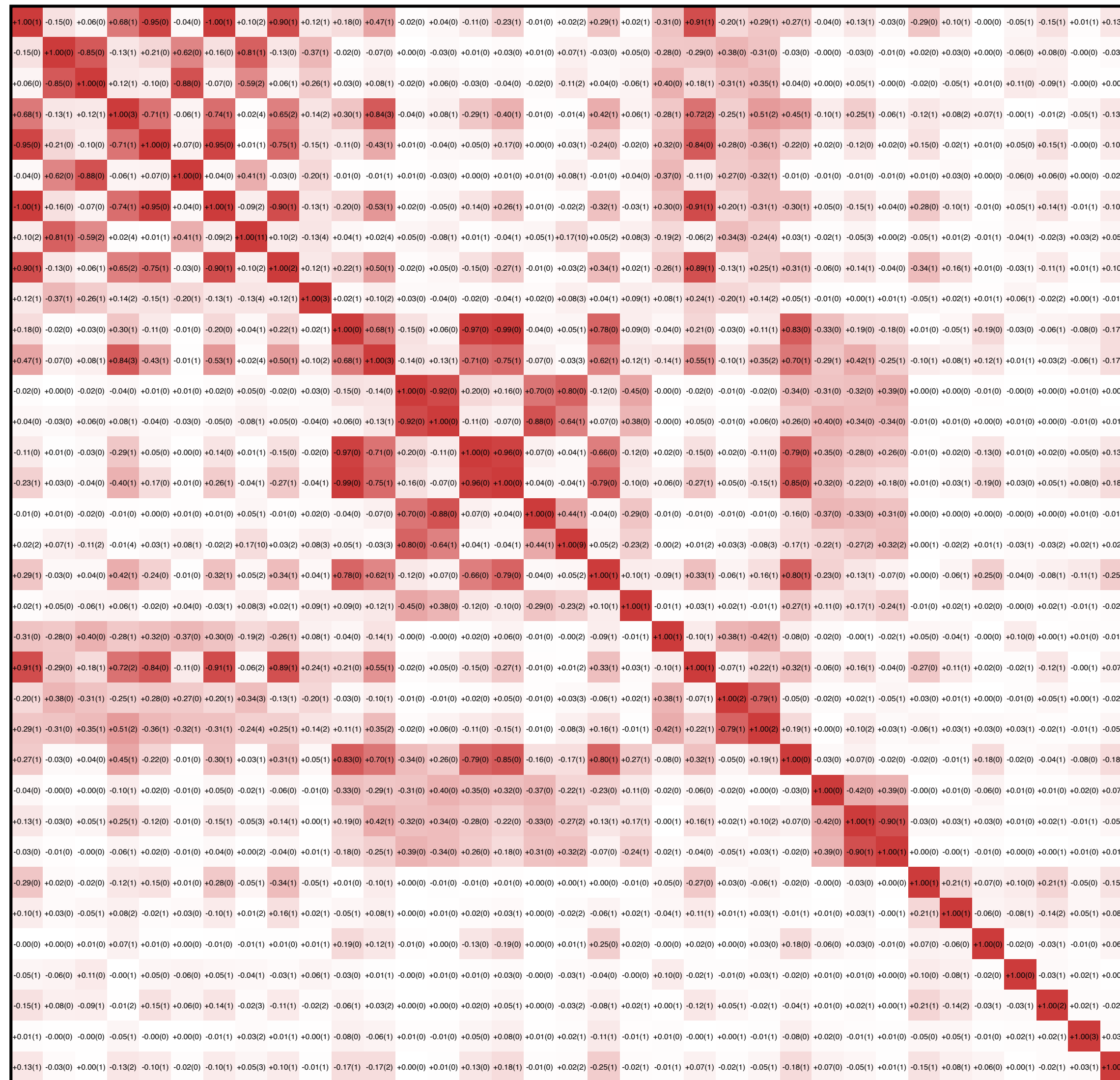
Distillation $\rightarrow$ 0905.2160

Time Avg. correlations

Some highly correlated

More than a few relevant ops.

op0
op1
op2
op3
op4
op5
op6
op7
op8
op9
op10
op11
op12
op13
op14
op15
op16
op17
op18
op19
op20
op21
op22
op23
op24
op25
op26
op27
op28
op29
op30
op31
op32
op33
op34



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

1

0.8

0.6

0.4

0.2

0

$$I = 0 \pi\pi$$

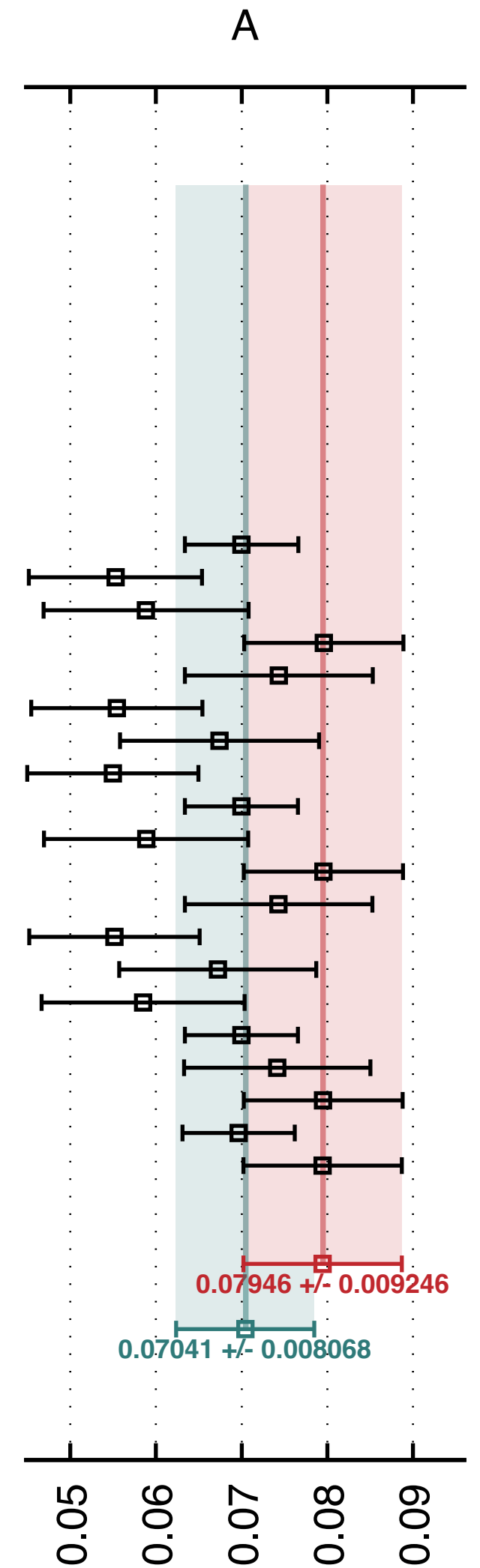
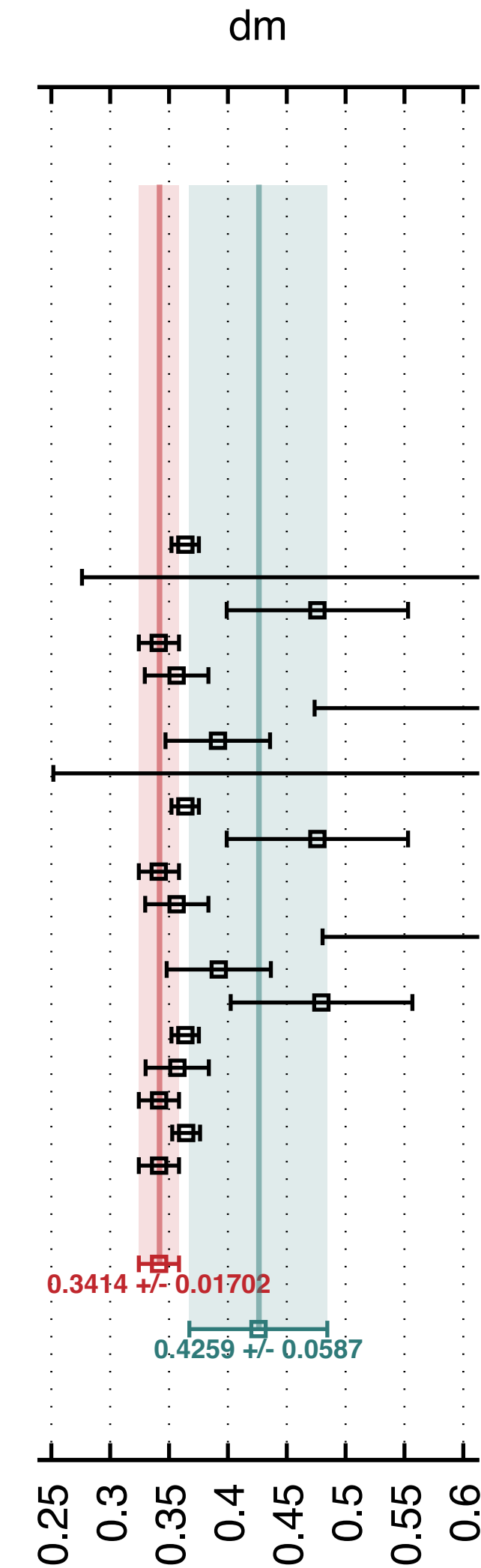
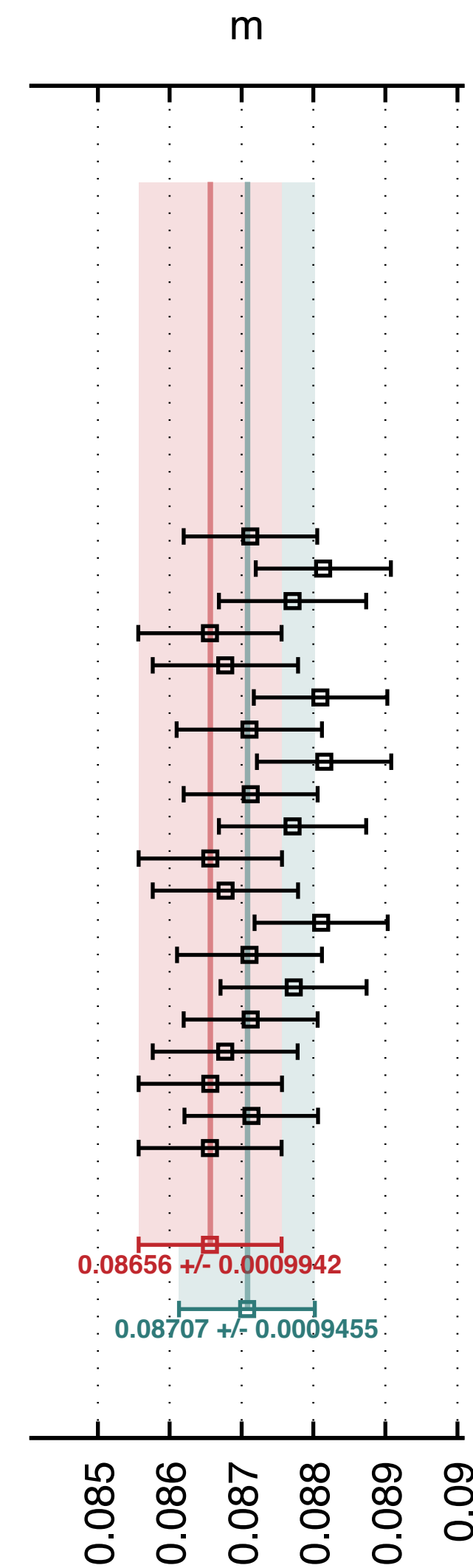
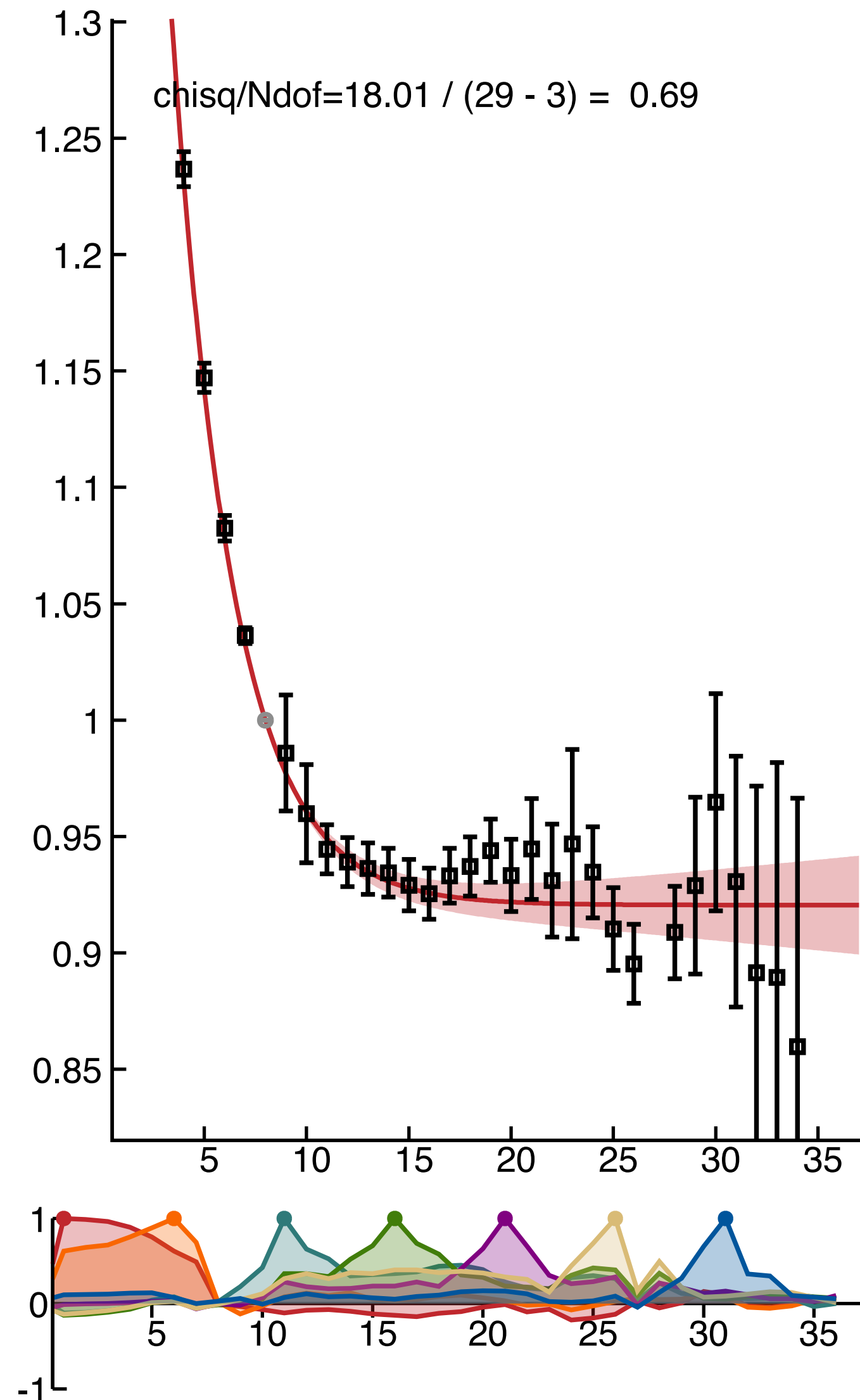
Many fits for a
good E_n

Many fits for diff.

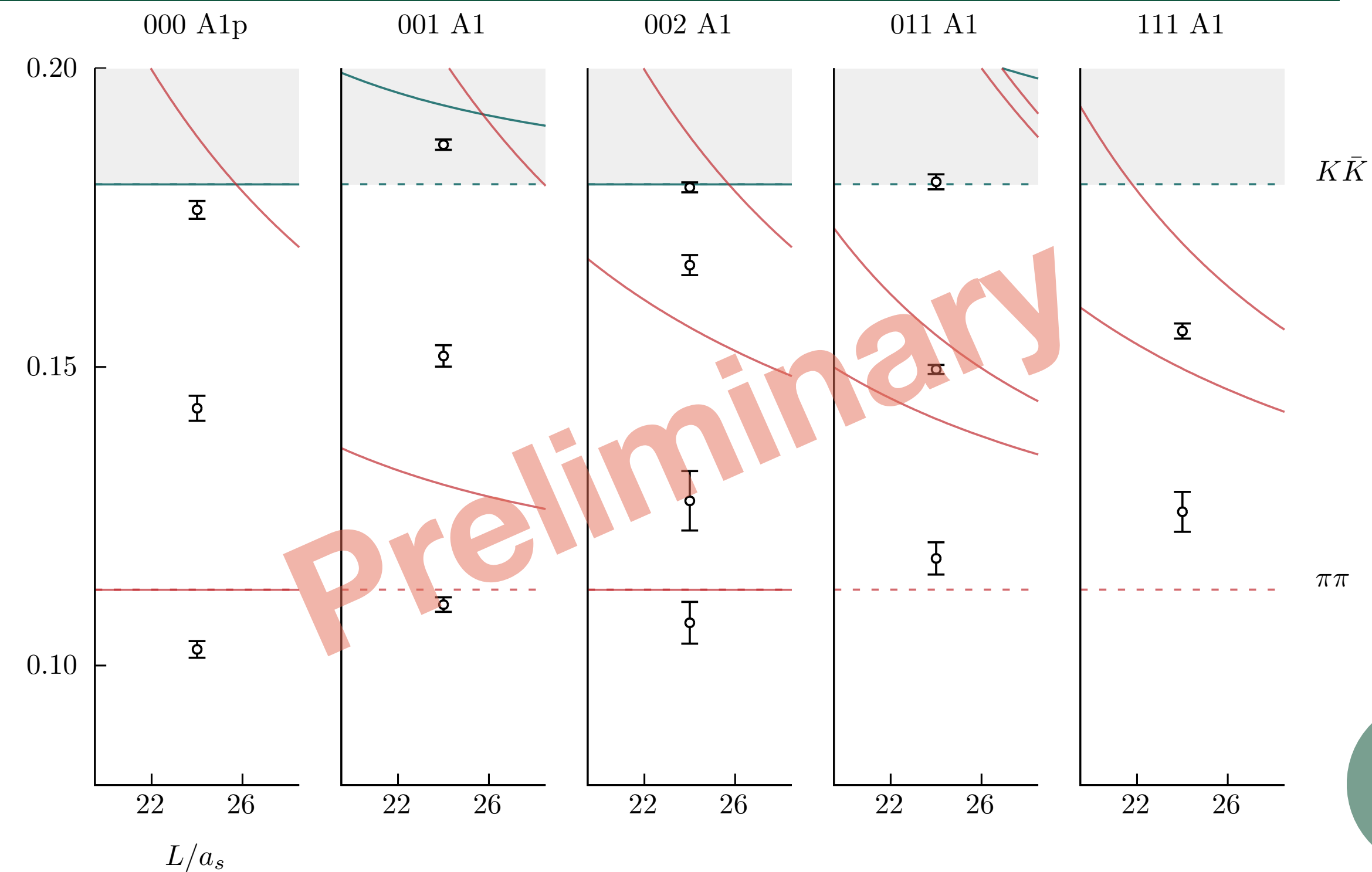
- t_0
- t_{min}, t_{max}
- N_{exp}

Model averaging
technique

2008.01069



$$I = 0 \ \pi\pi$$

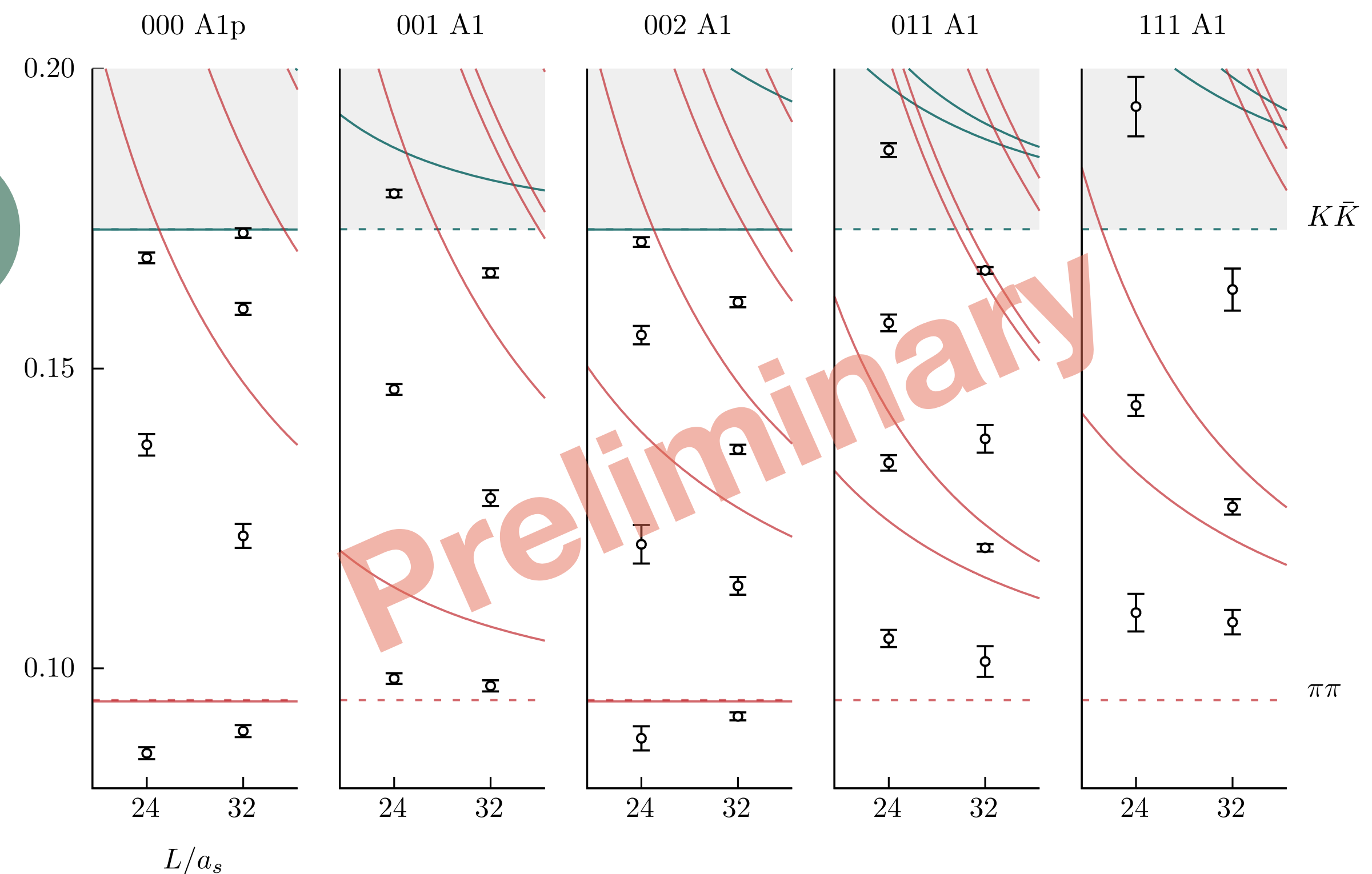
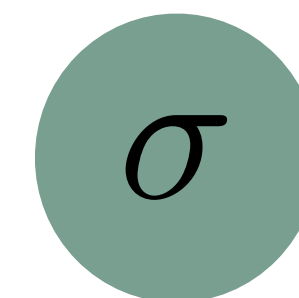


$m_\pi \sim 330 \text{ MeV}$

Around 60 “elastic” levels

Similar spectrum to previous masses

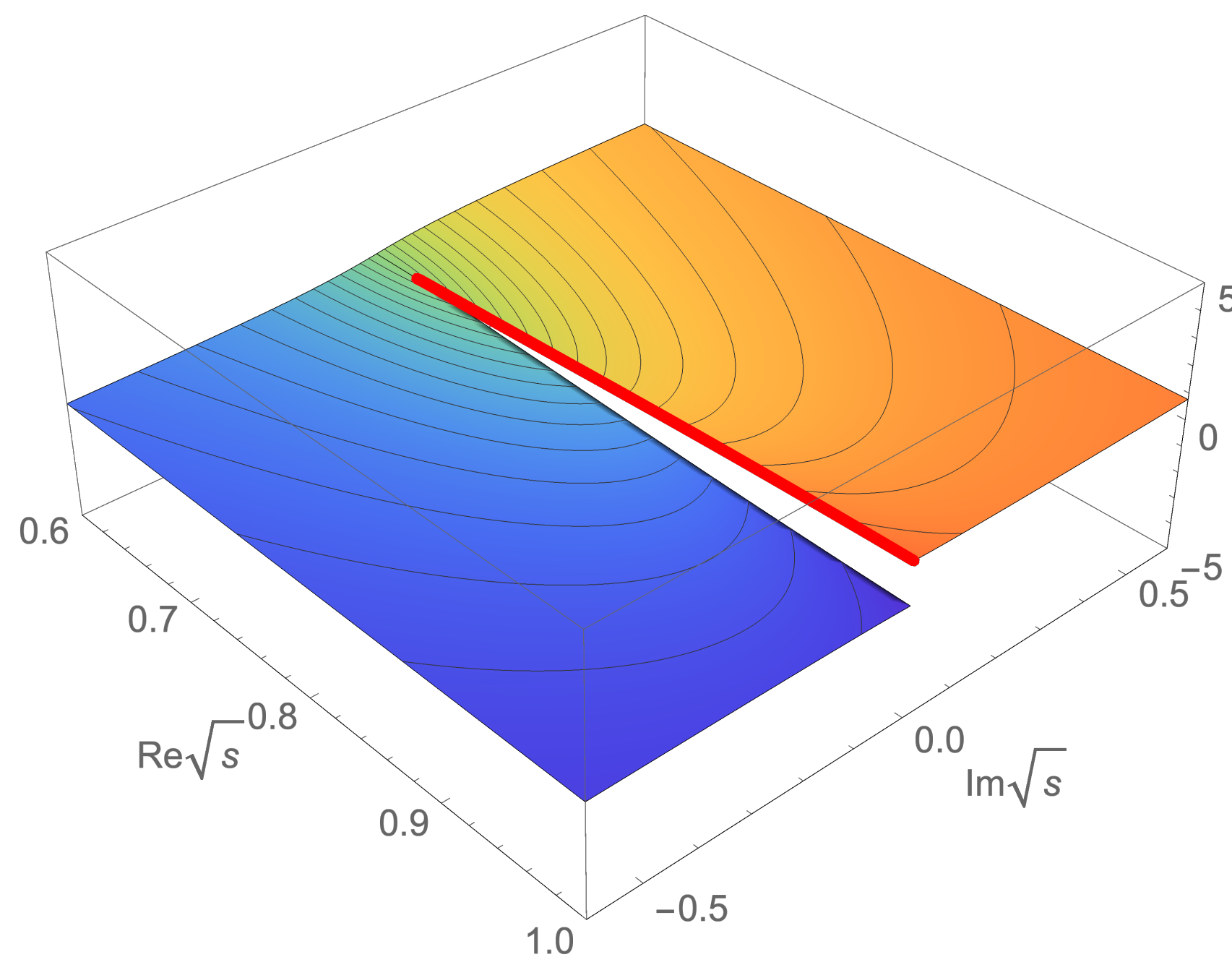
$m_\pi \sim 283 \text{ MeV}$



Scattering

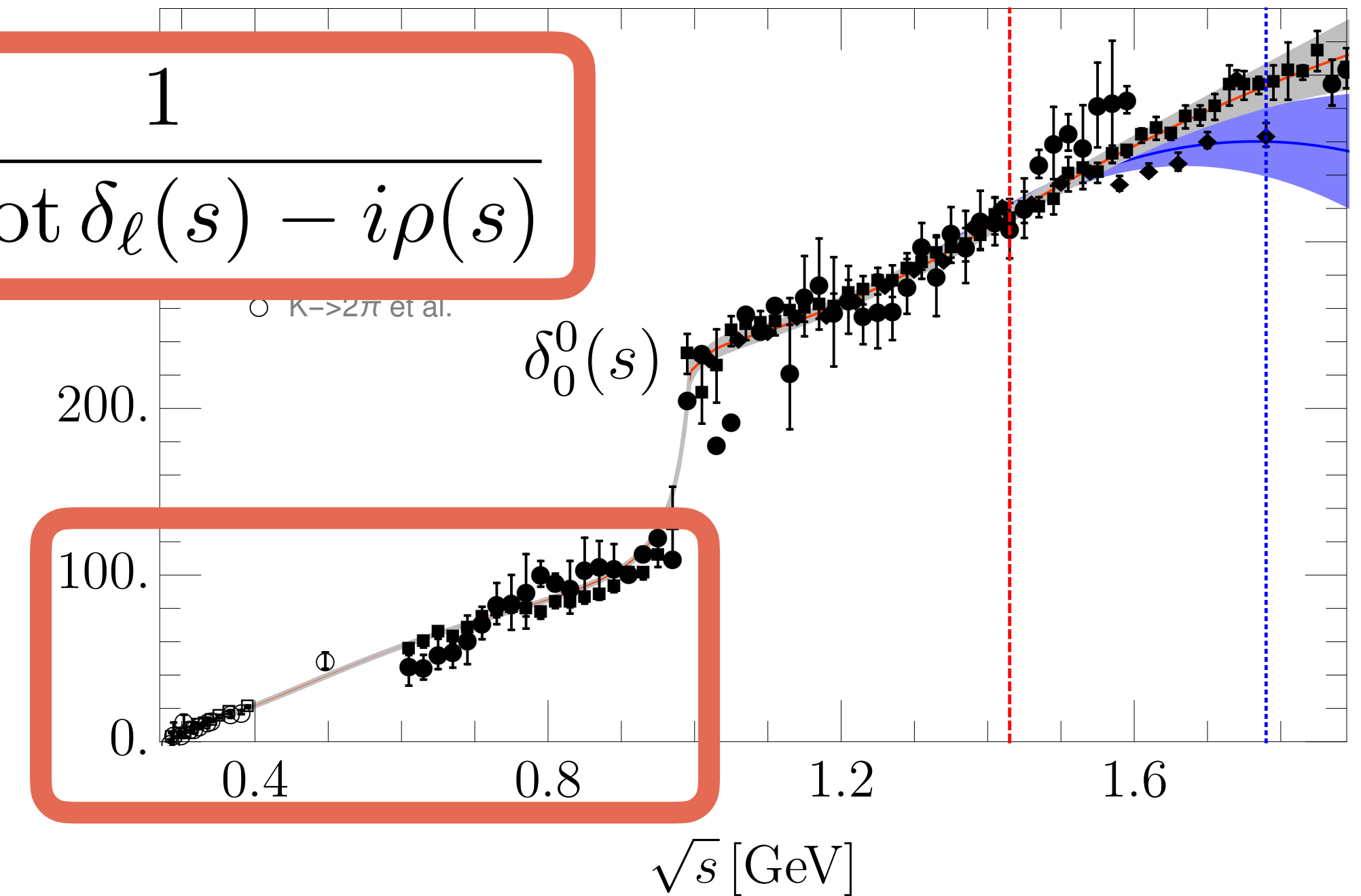
My fits

I sheet clear of poles



Analiticity

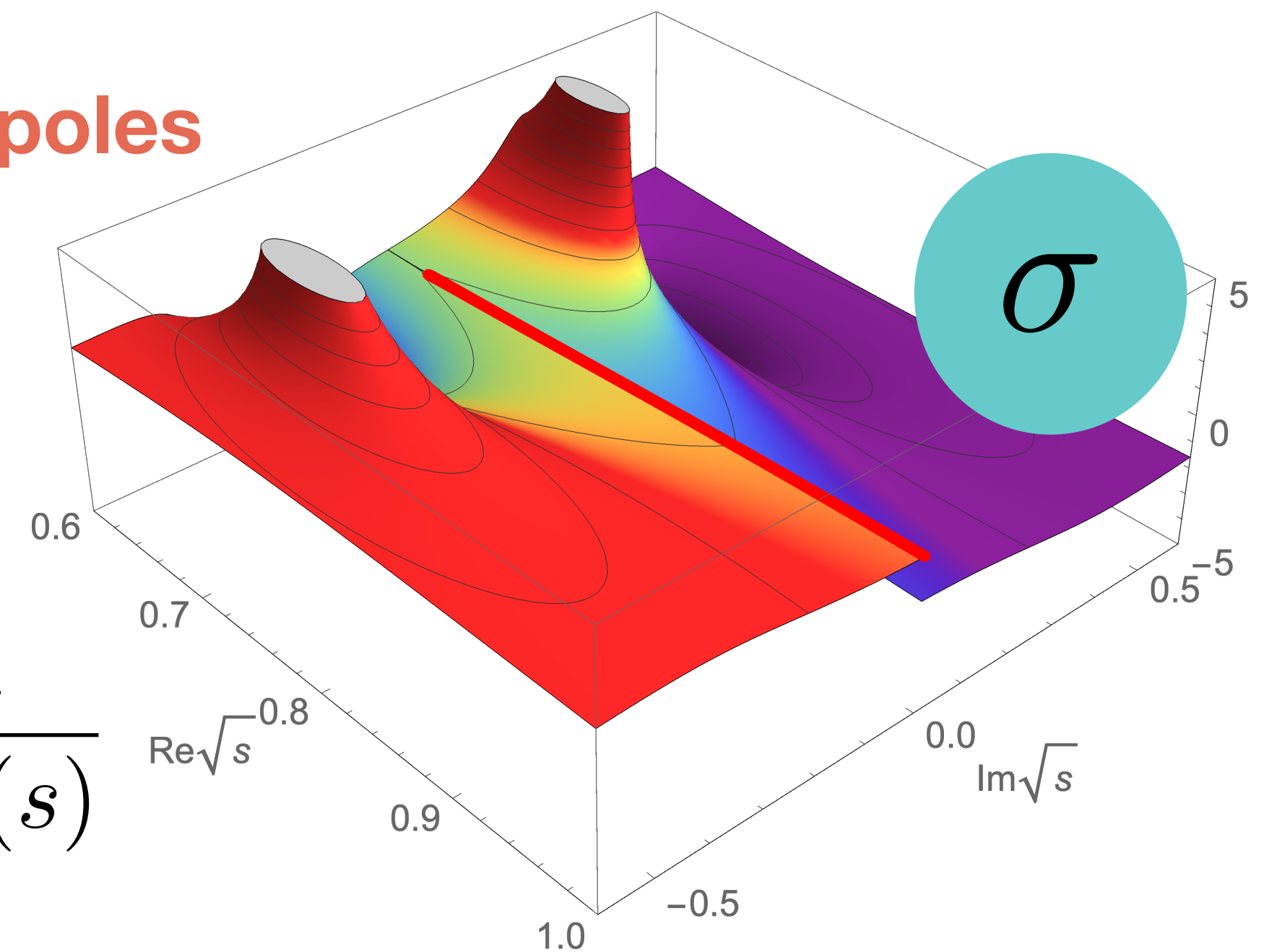
$$t_\ell(s) = \frac{1}{\rho(s) \cot \delta_\ell(s) - i\rho(s)}$$



II sheet poles

Unitarity

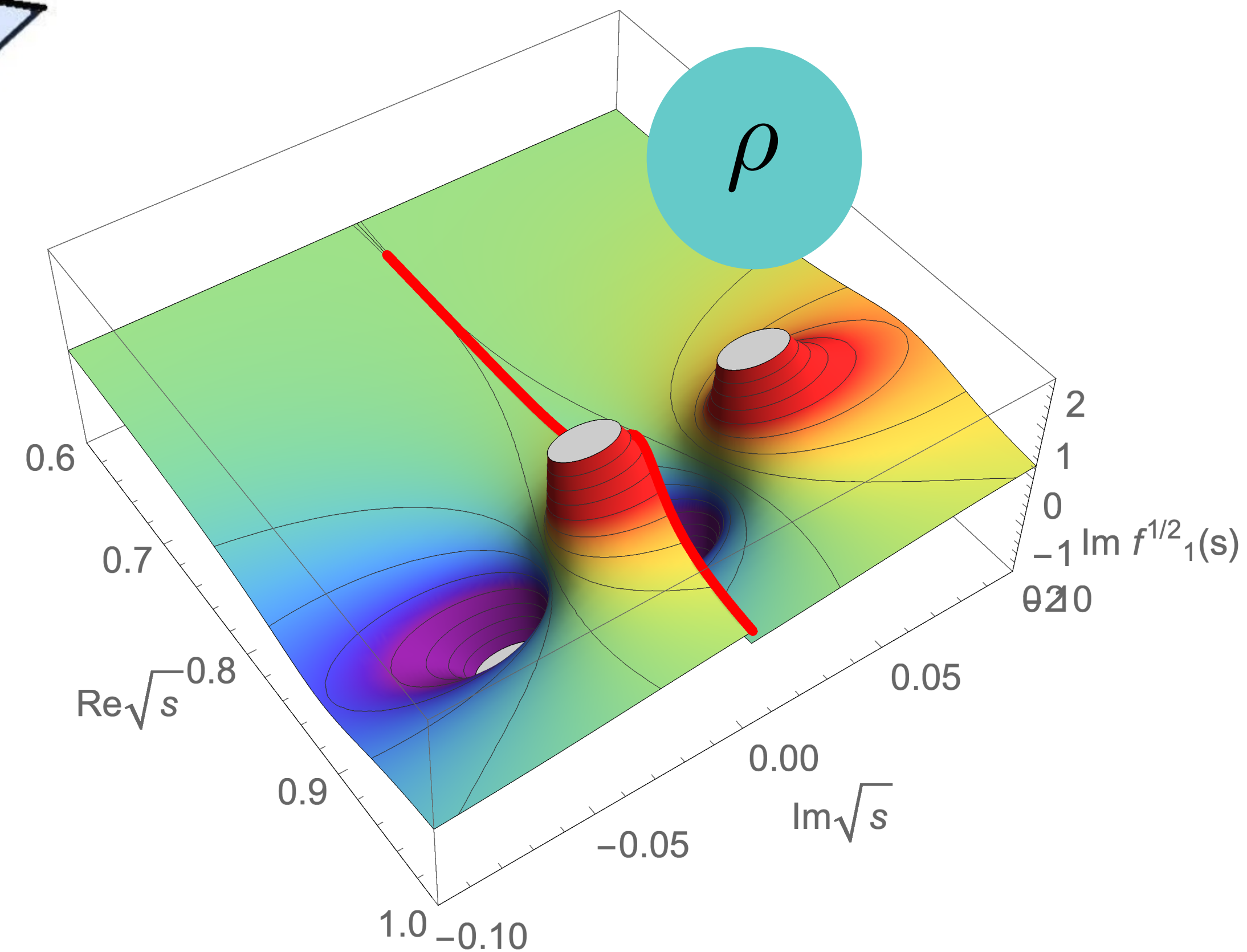
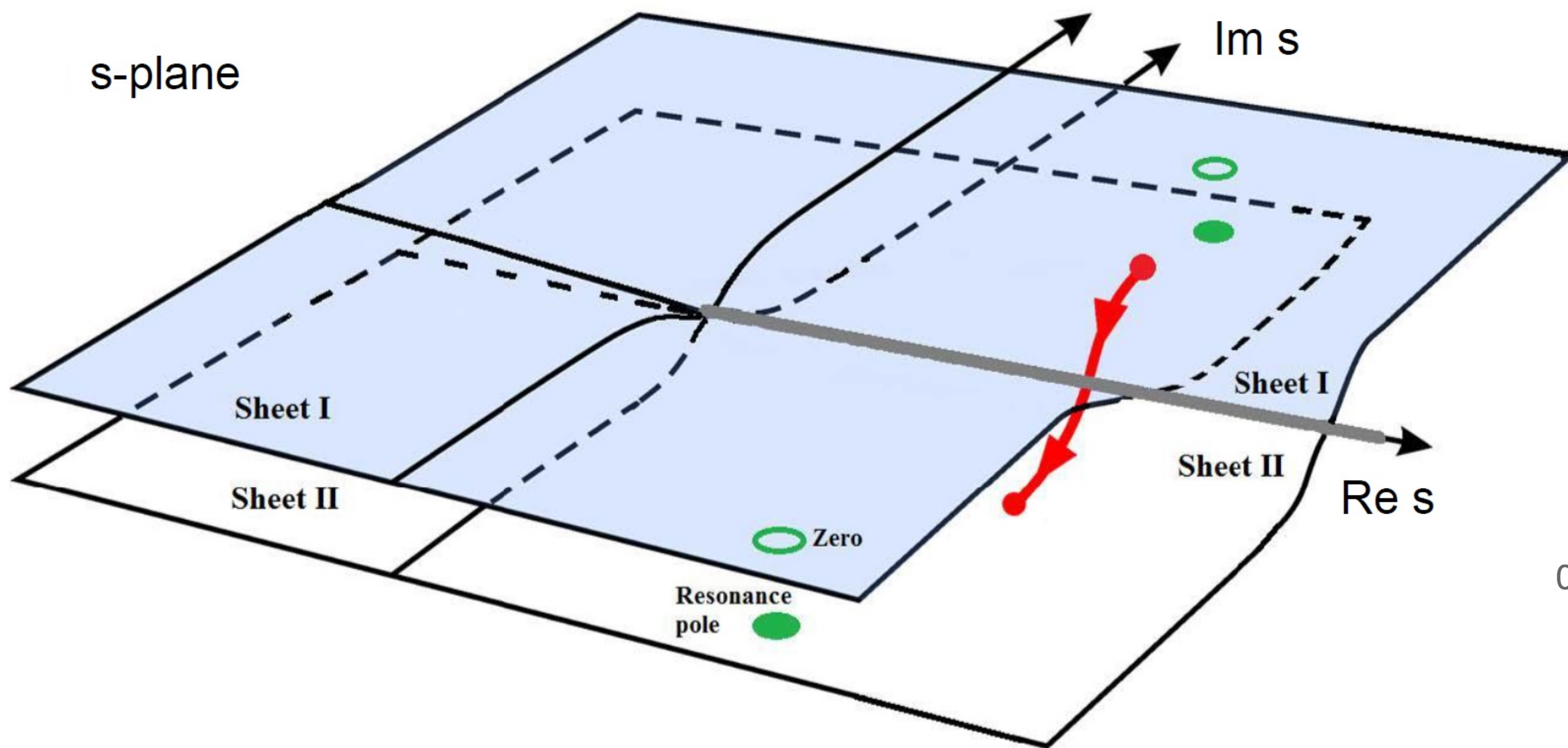
$$S_\ell^{II}(s) = \frac{1}{S_\ell^I(s)}$$



Unitarity

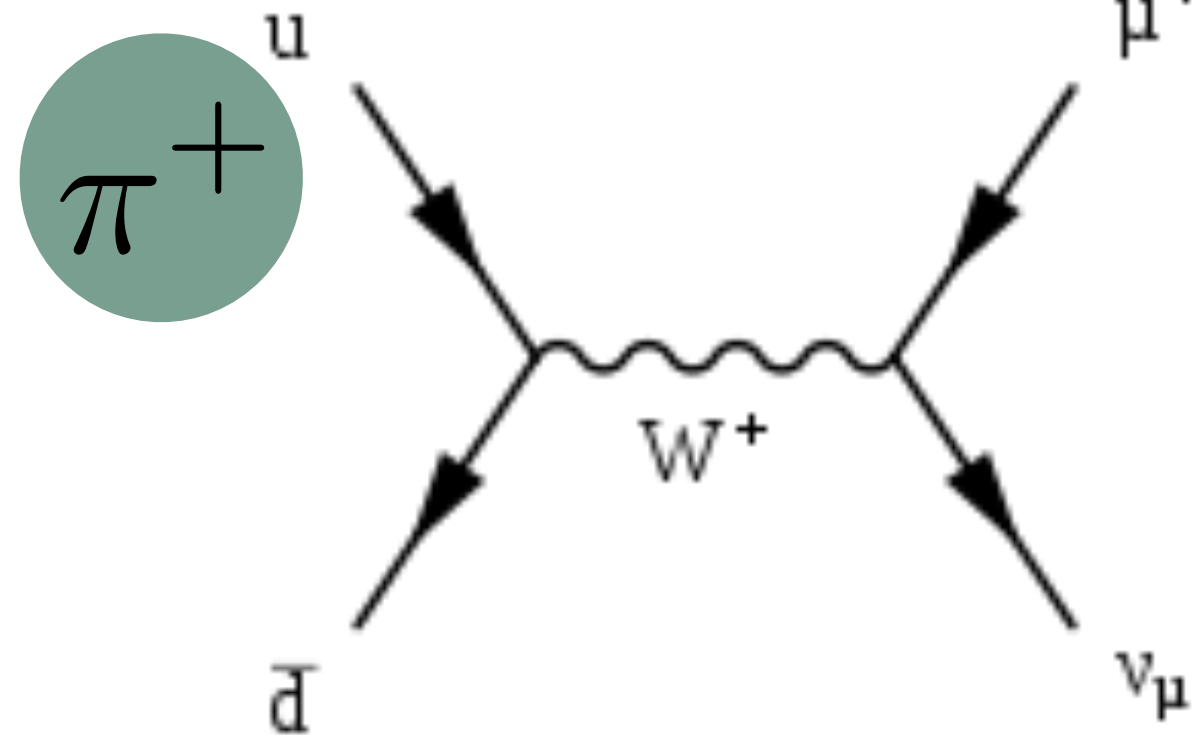
Exercise

$$S_{\ell}^{II}(s) = \frac{1}{S_{\ell}^I(s)}$$



Exp. Problem

$$c\tau_{\pi^+} = 7.8 \text{ m}$$

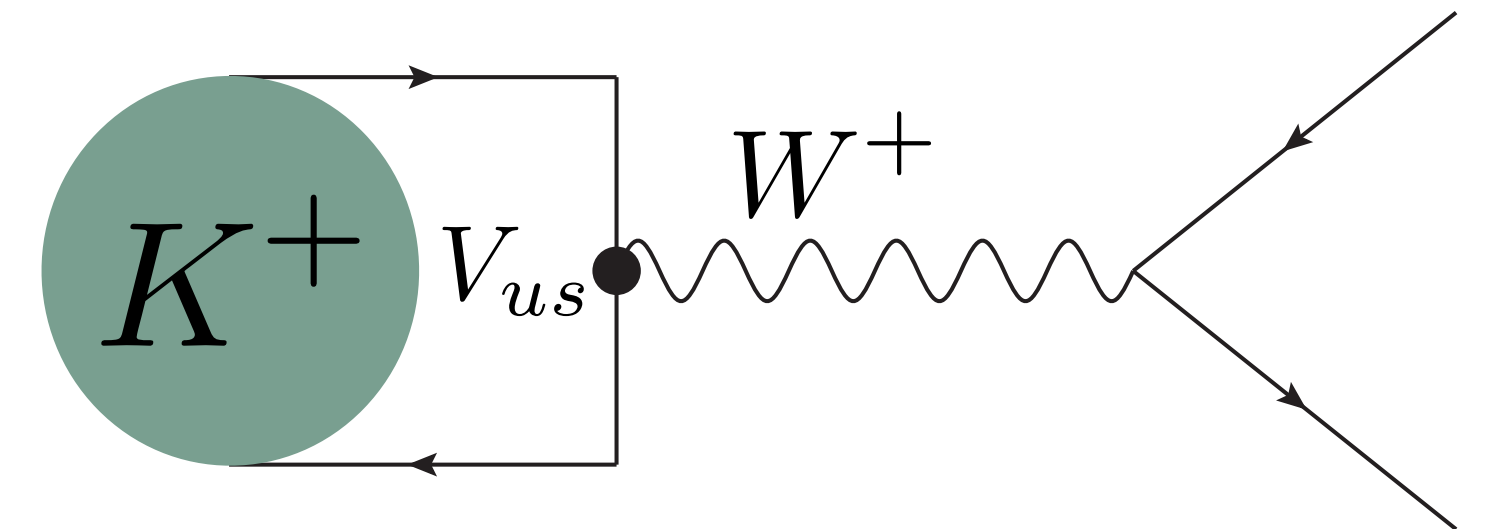


Mesons decay

One beam (maybe)

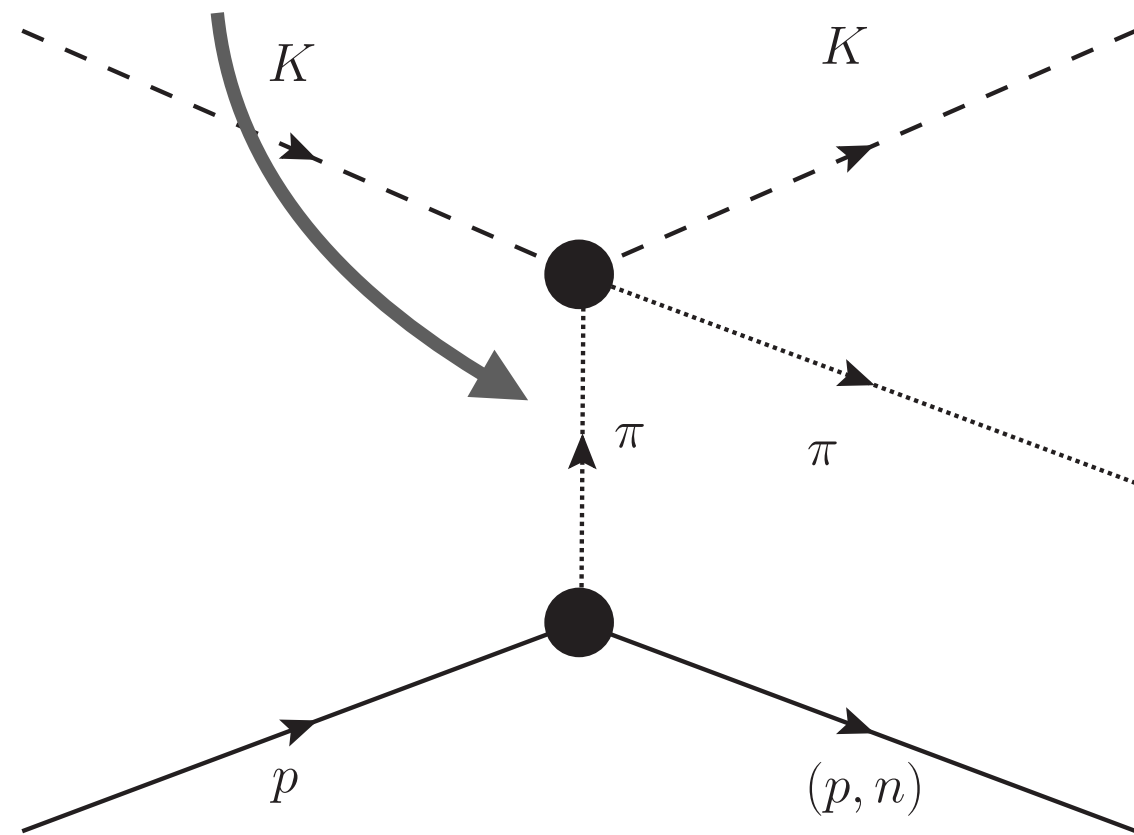
Not two beams

$$c\tau_K = 3.7 \text{ m}$$



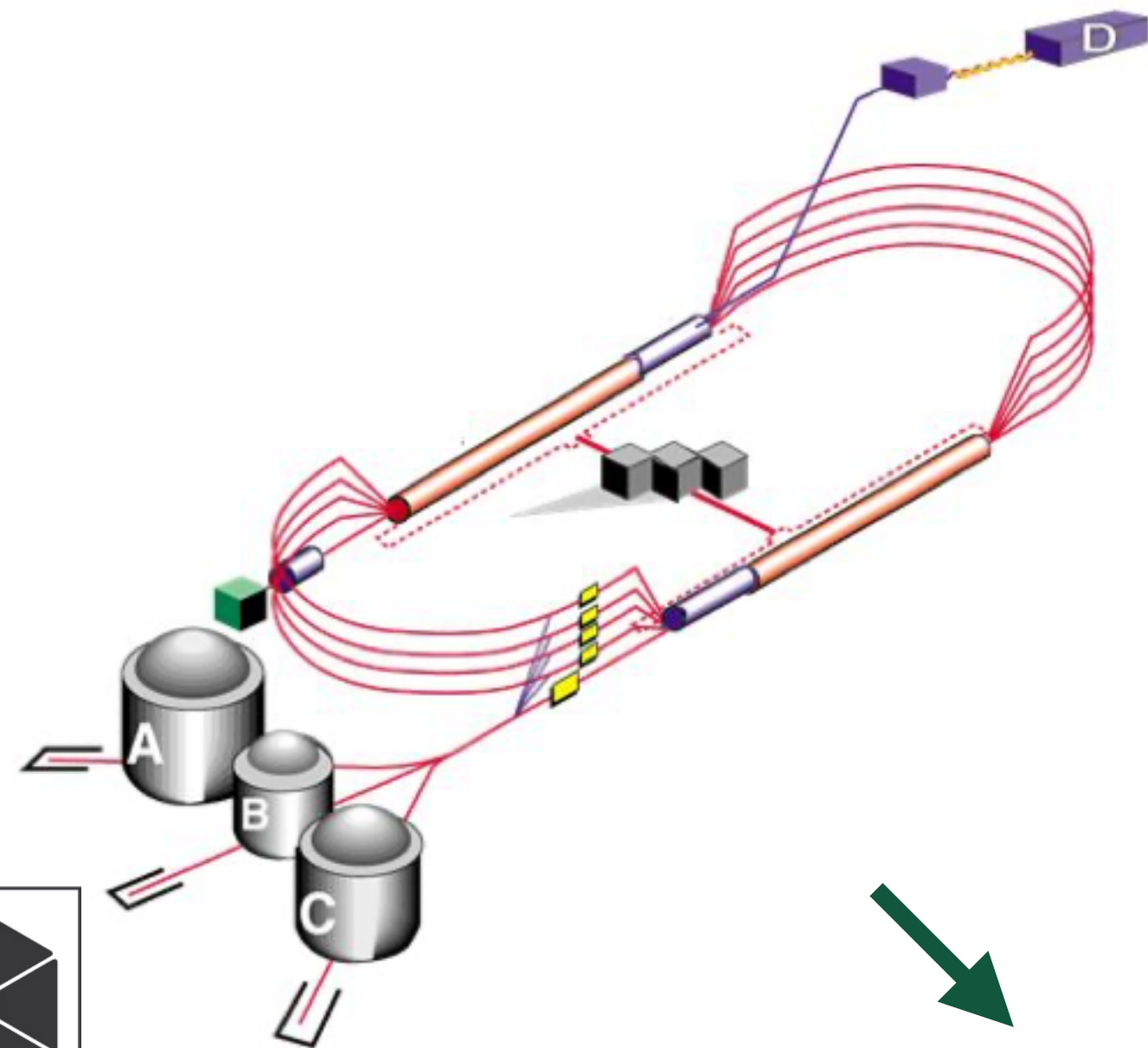
“Data” is not data

Virtual pion

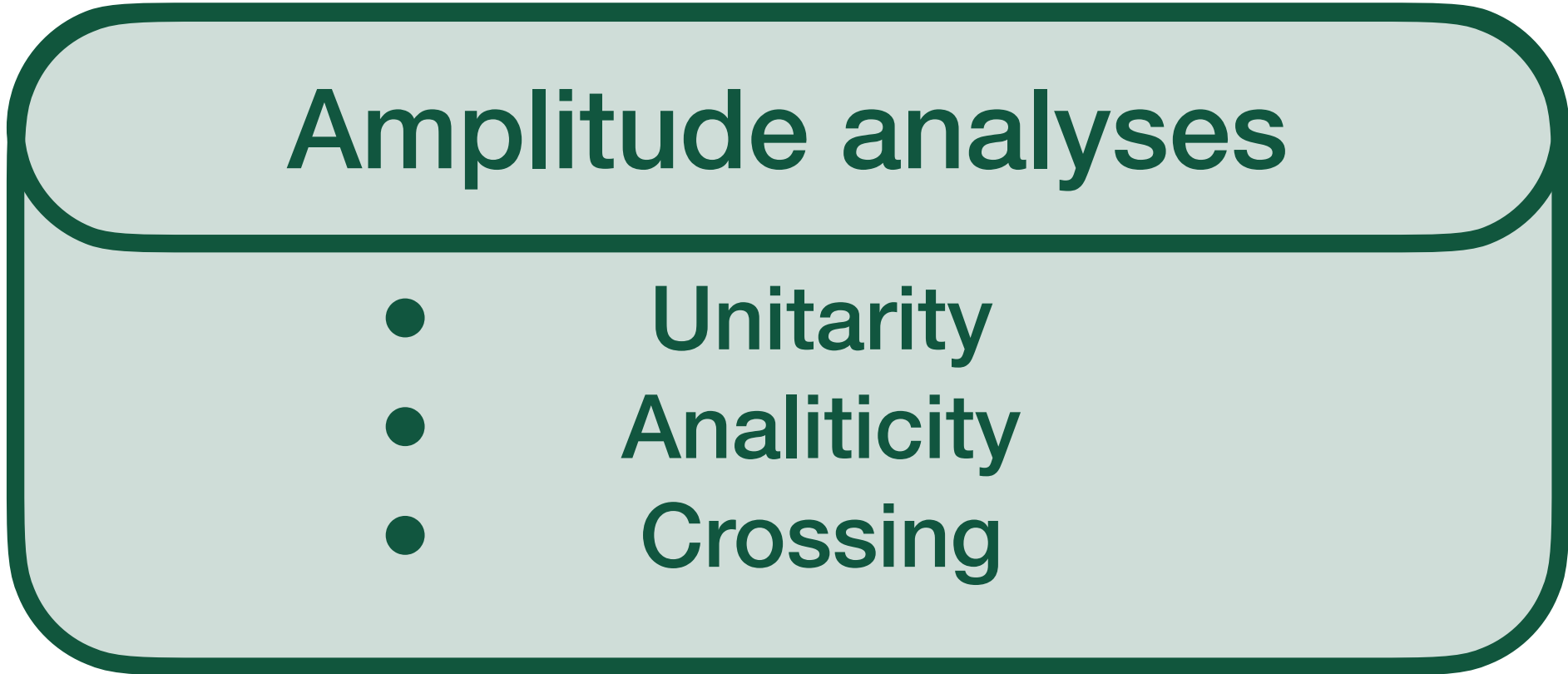
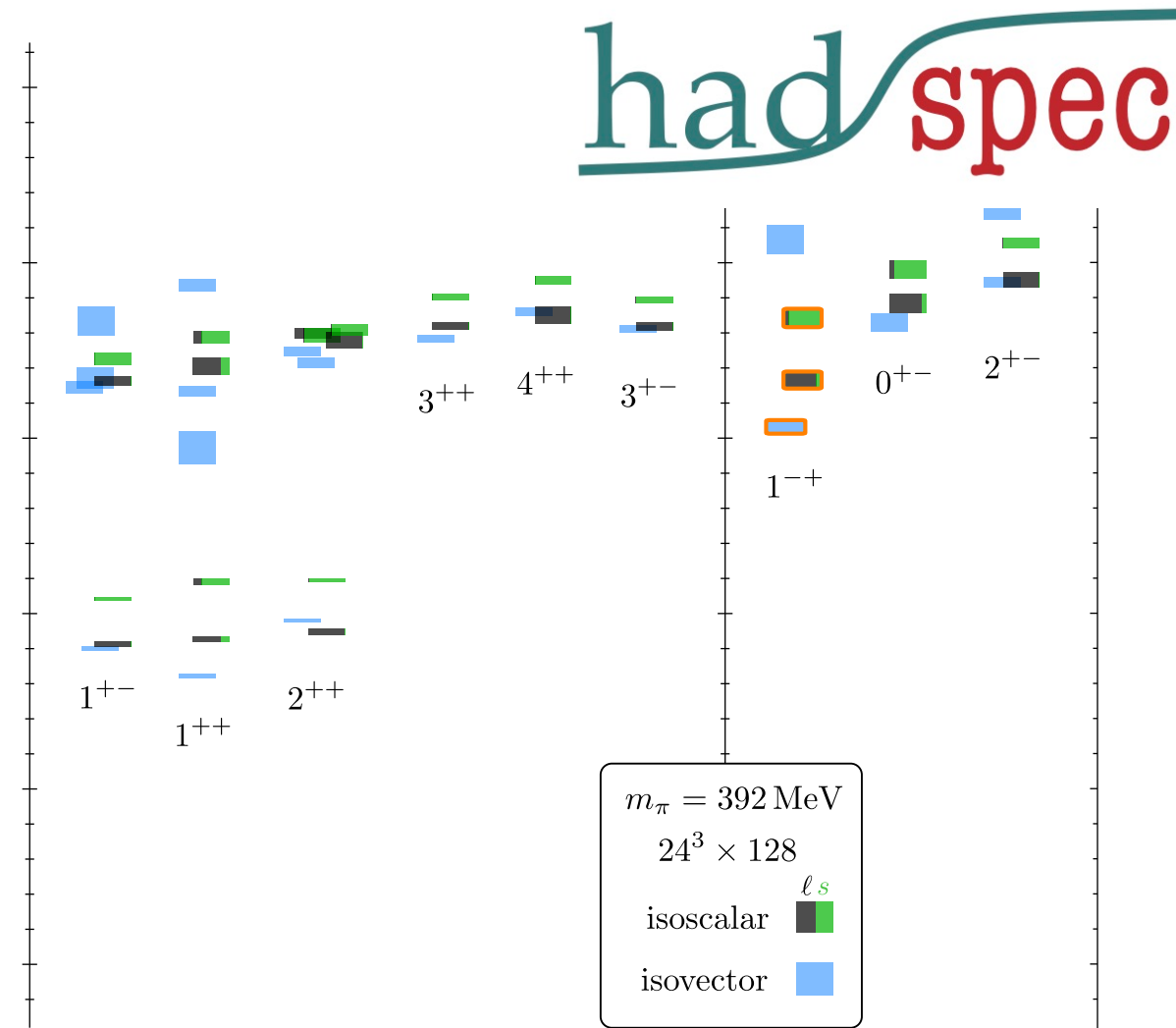
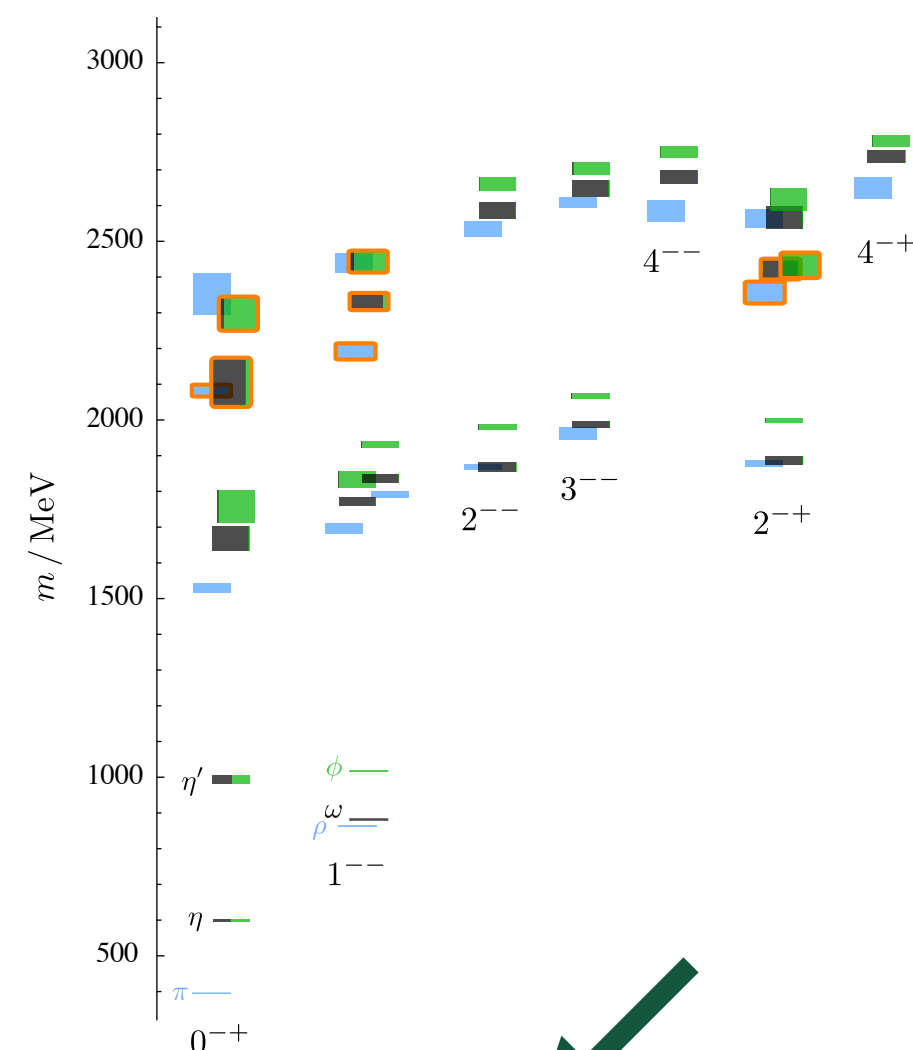


For meson-meson data must be always modeled

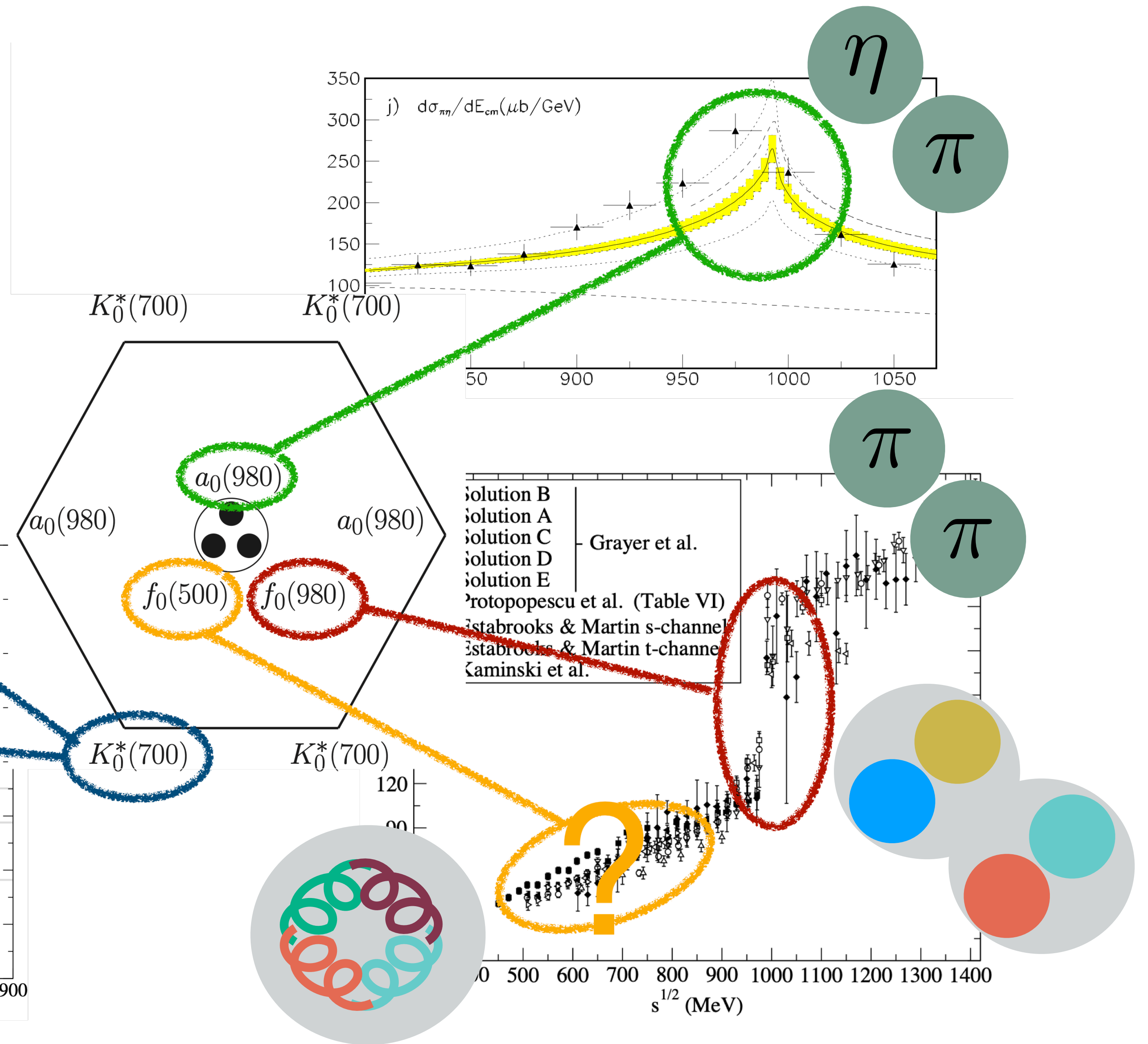
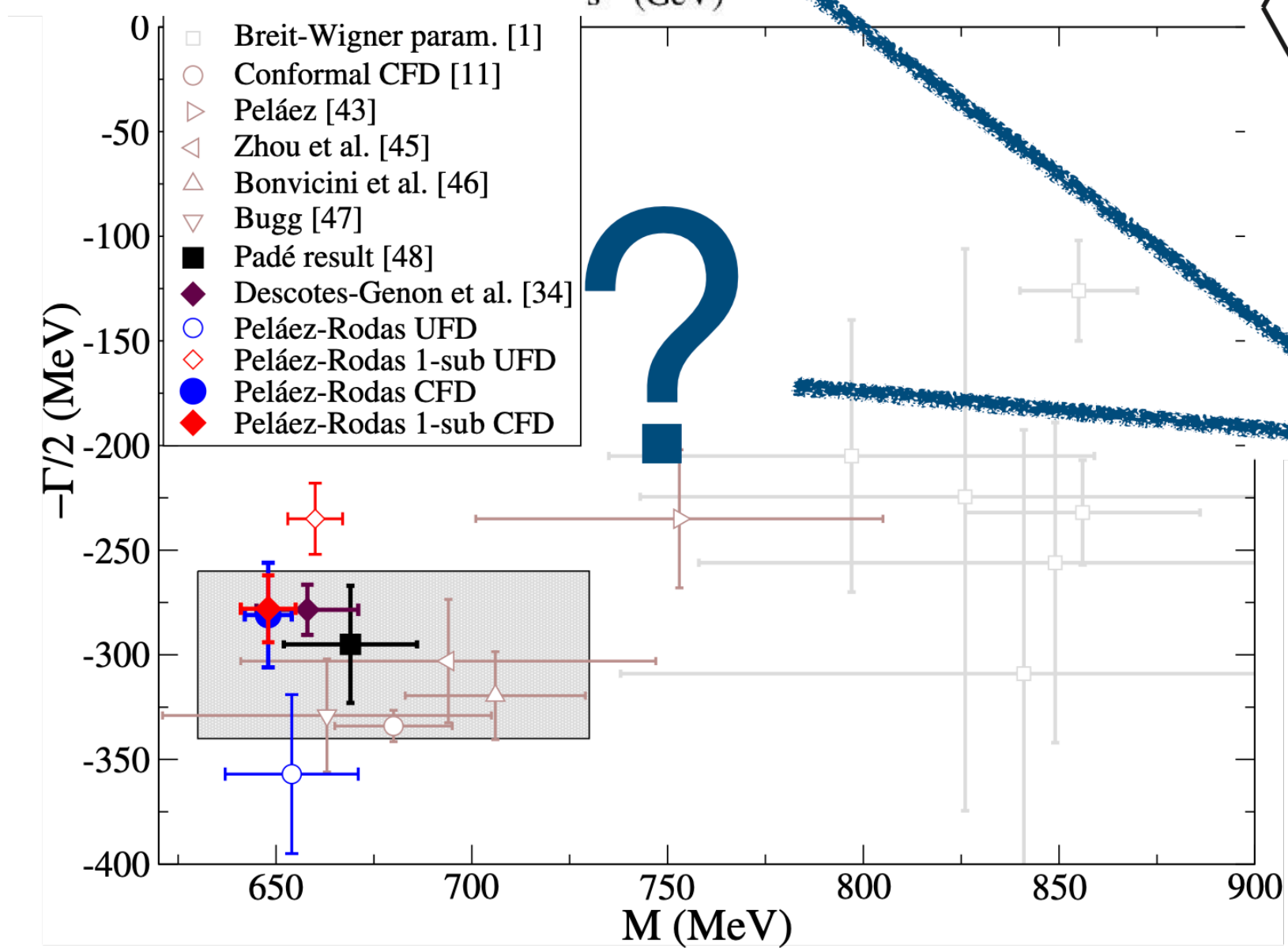
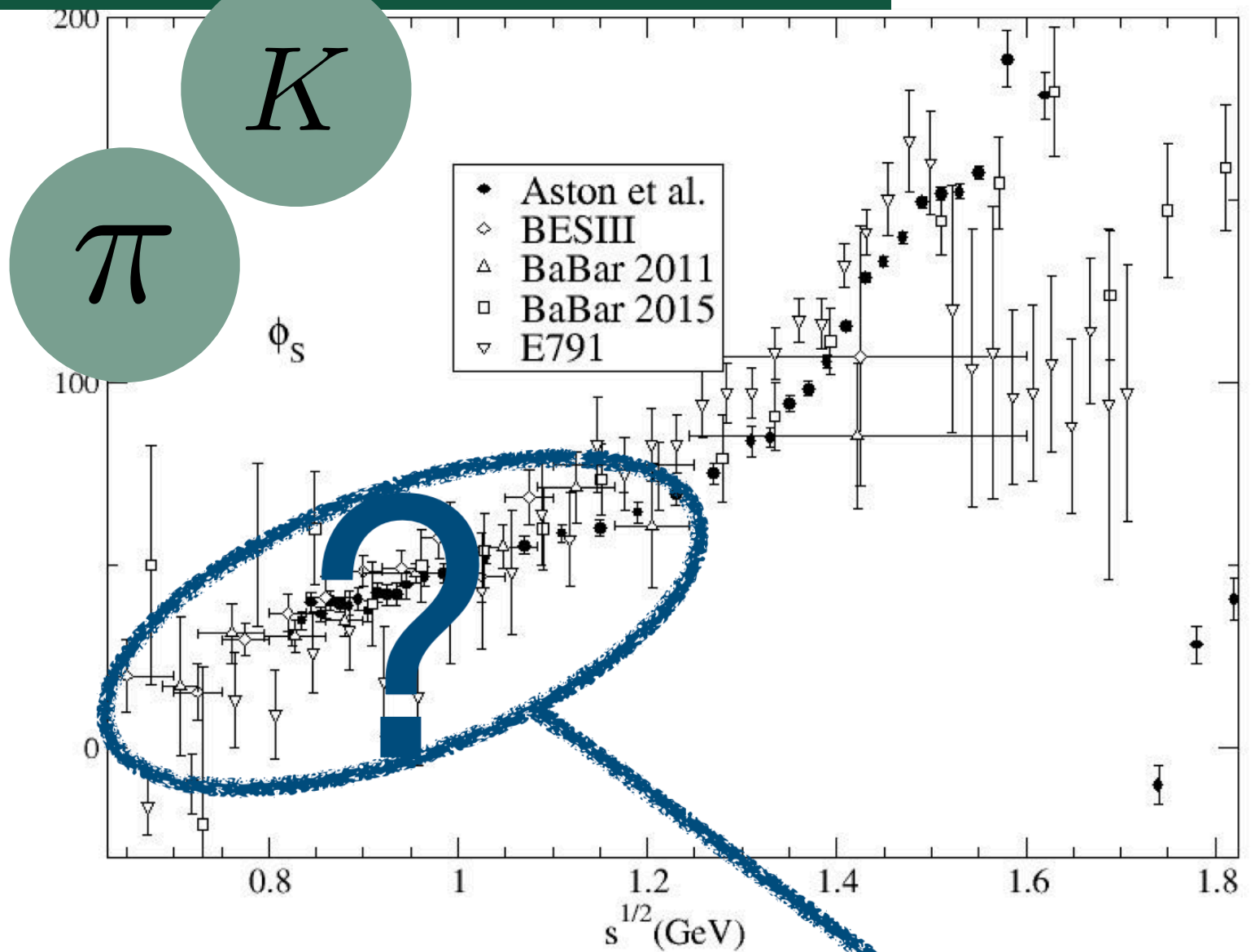
Some systematic uncertainties unknown



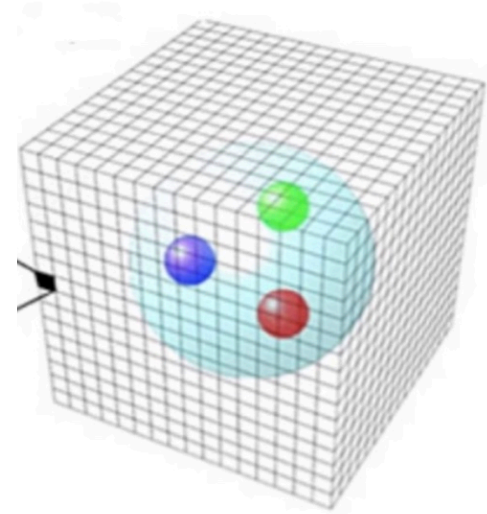
GLUEX



Light Scalars

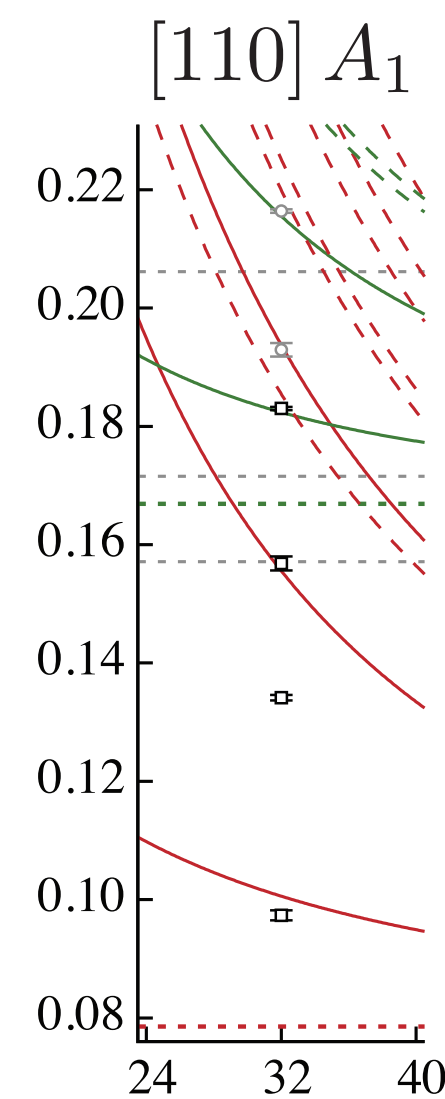
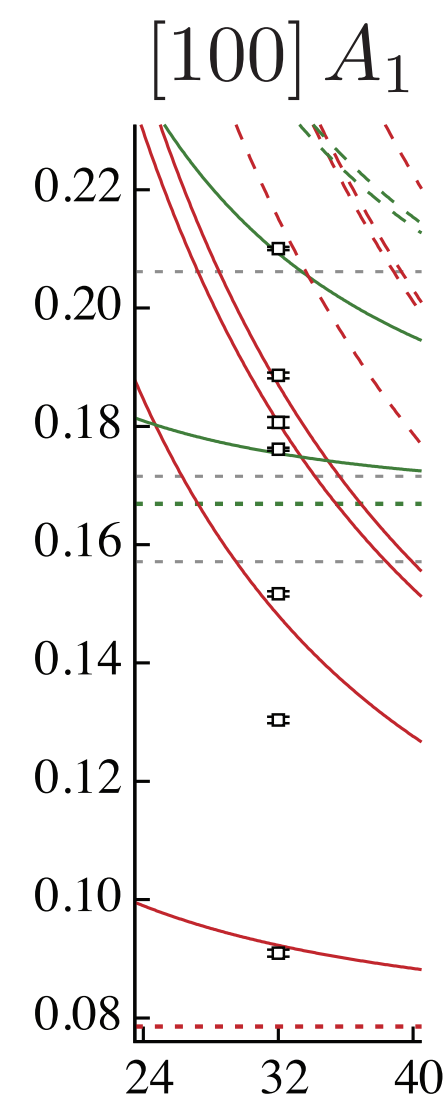
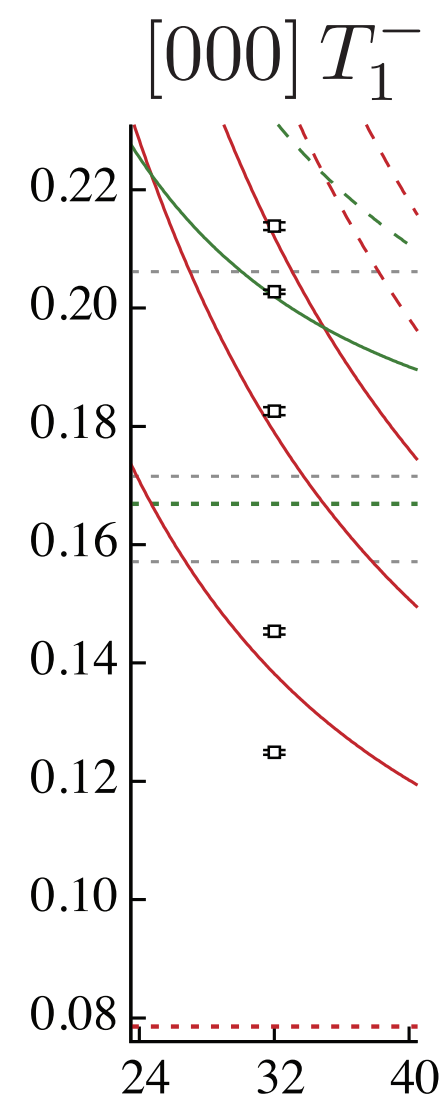


$\pi\pi$ scattering on the lattice



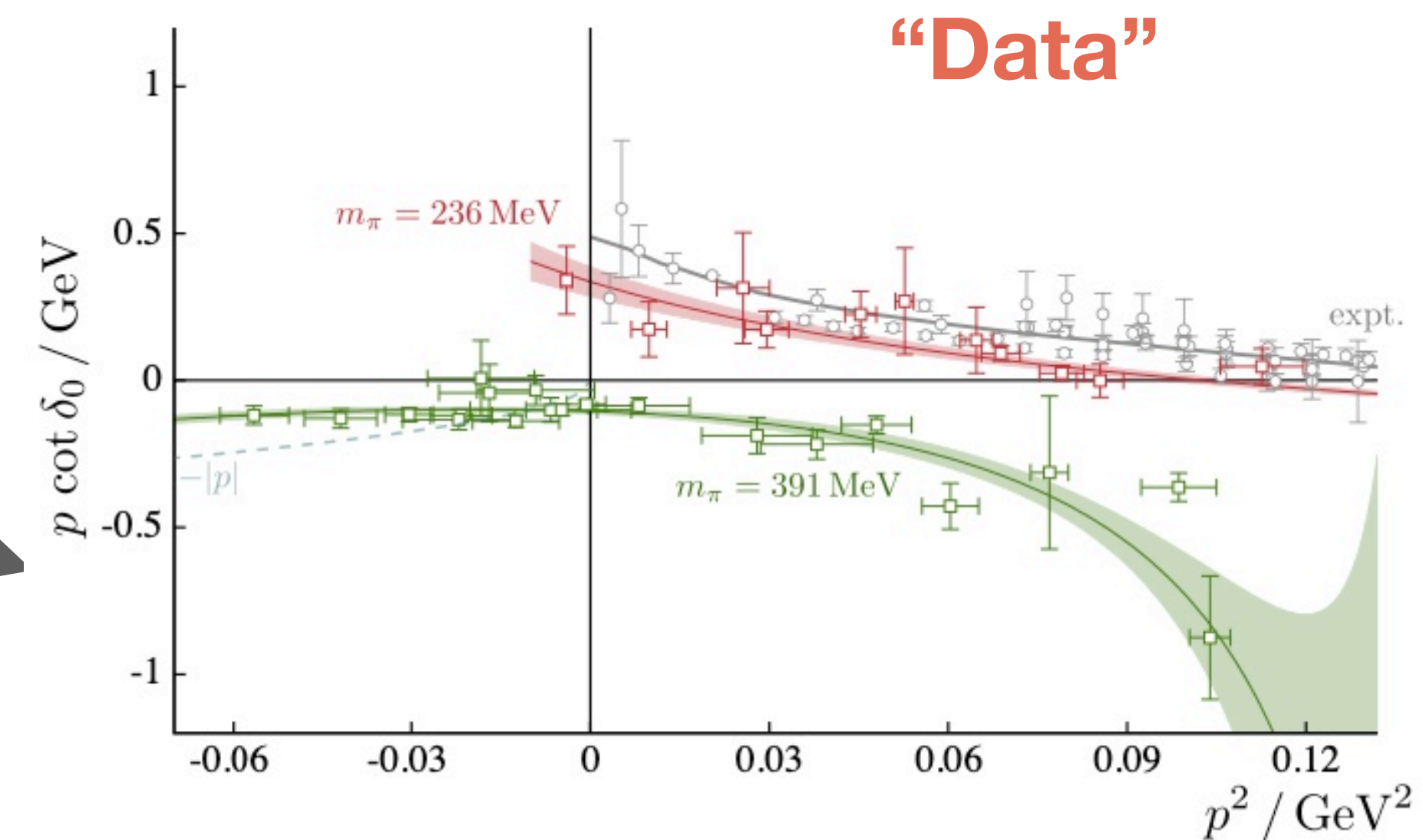
$$t \rightarrow -it$$

$$-iS = -i \int d^3x dt \mathcal{L} \rightarrow - \int d^3x dt \mathcal{L}_E = -S_E$$



Quantized levels in a box

Lüscher

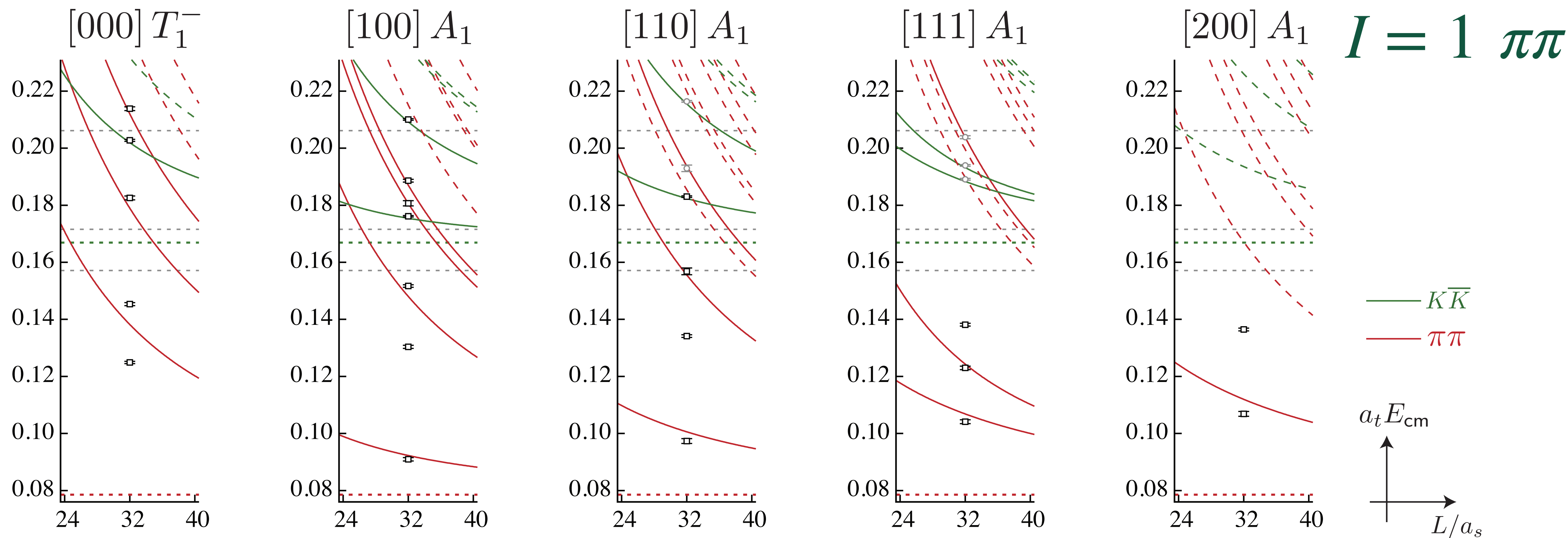


Lattice QCD

Hamiltonian evolution

$$e^{-iHt} \xrightarrow{t \rightarrow -it} \sum_n e^{-E_n t} \langle 0 | O_f(0) | n \rangle \langle n | O_i^\dagger(0) | 0 \rangle$$

Desired energies from GEVP



Lattice QCD

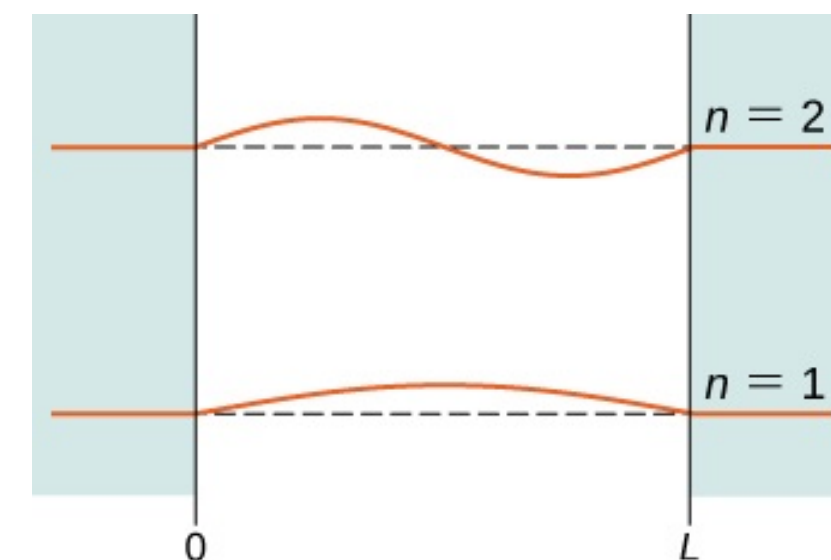
In a 1D box

$$R \ll L$$

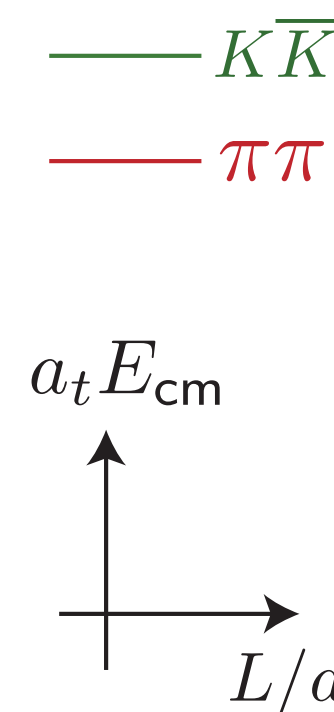
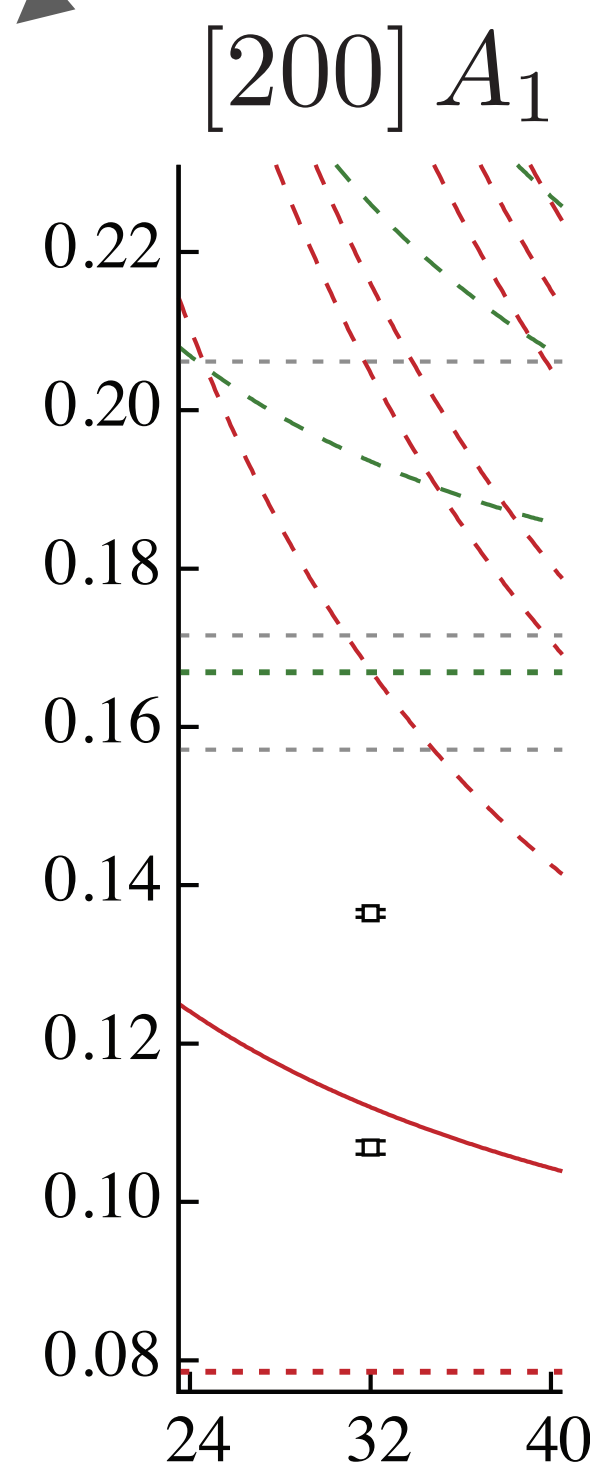
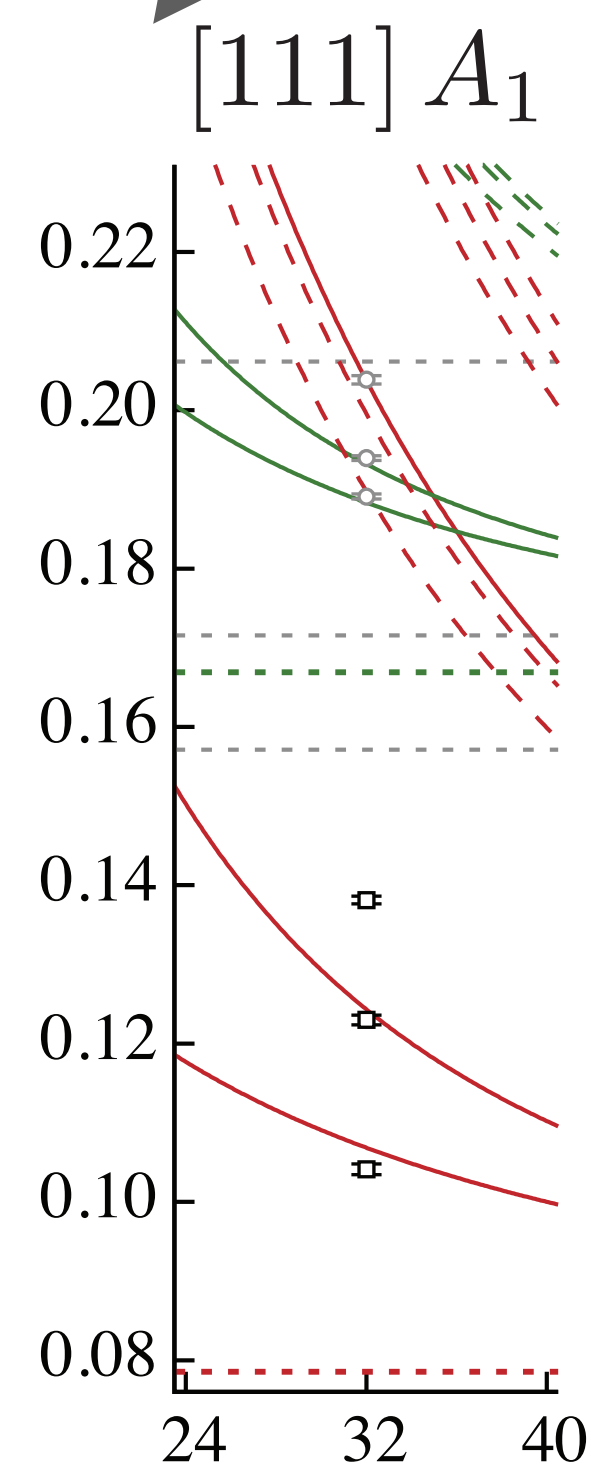
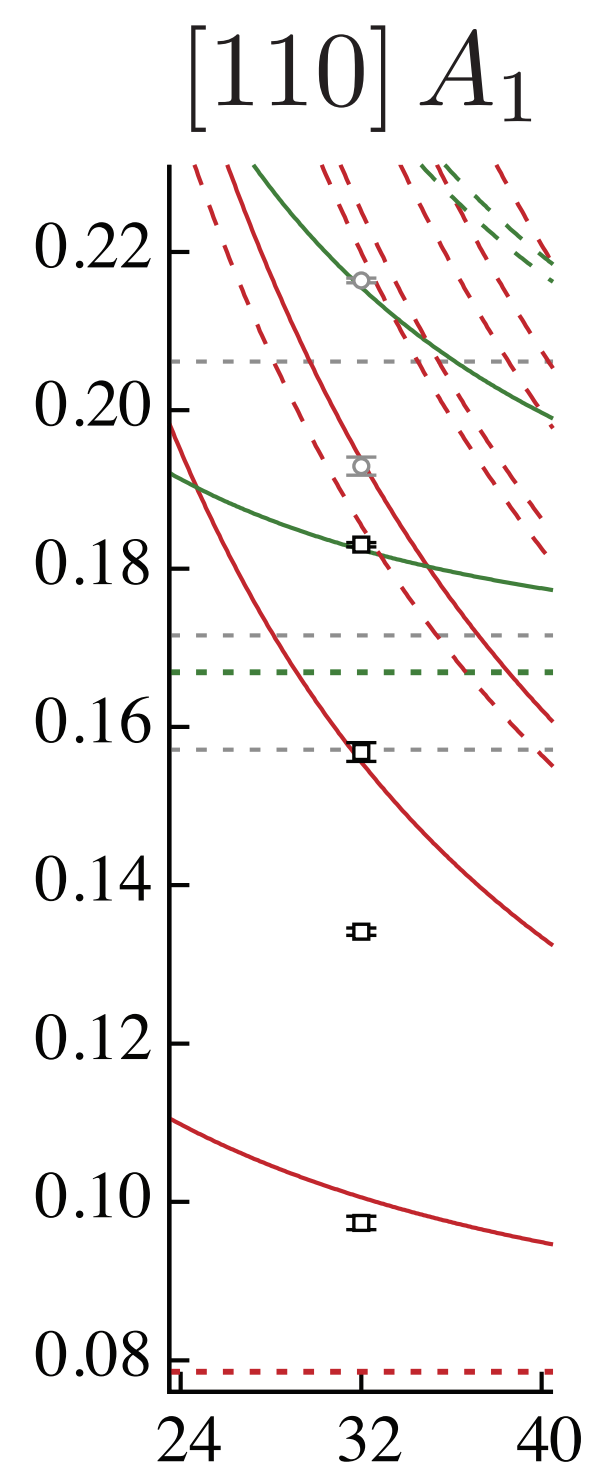
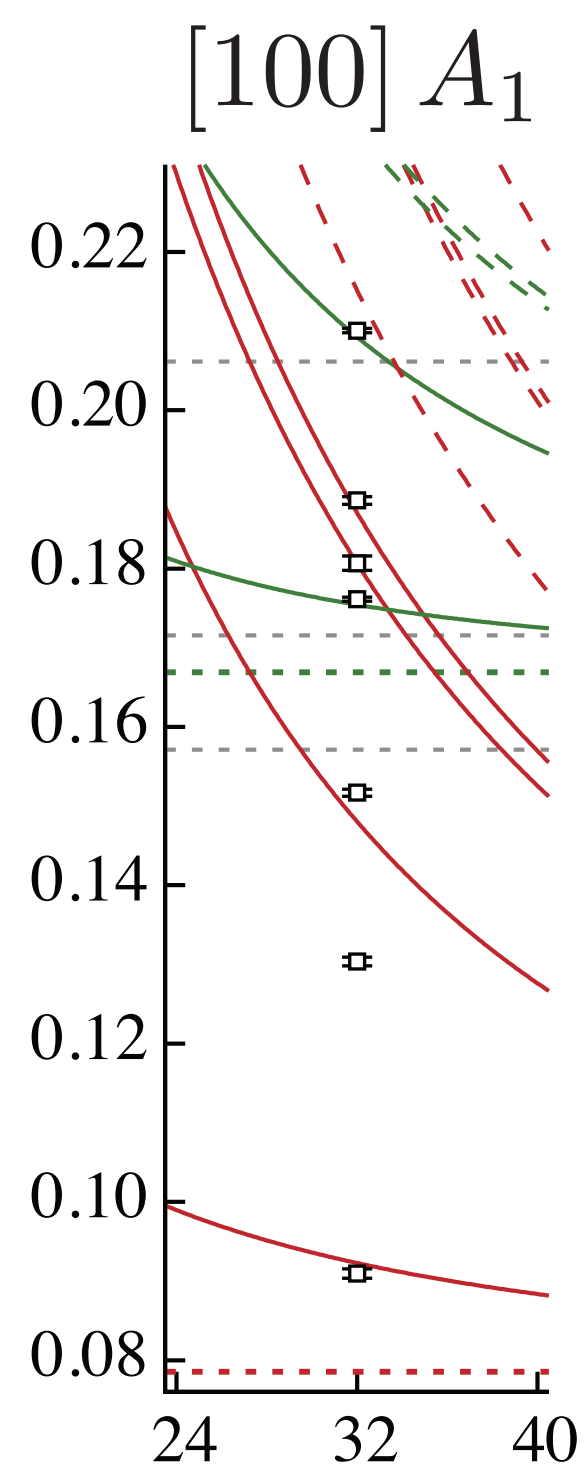
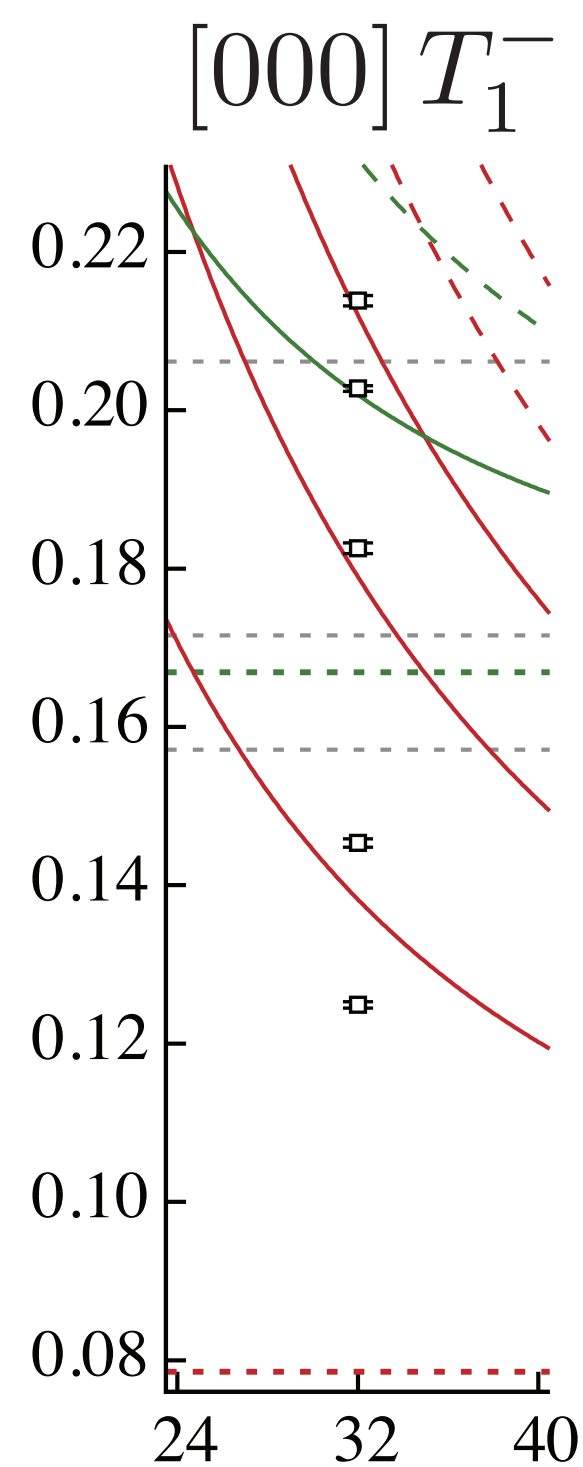
In

Out

Boundary



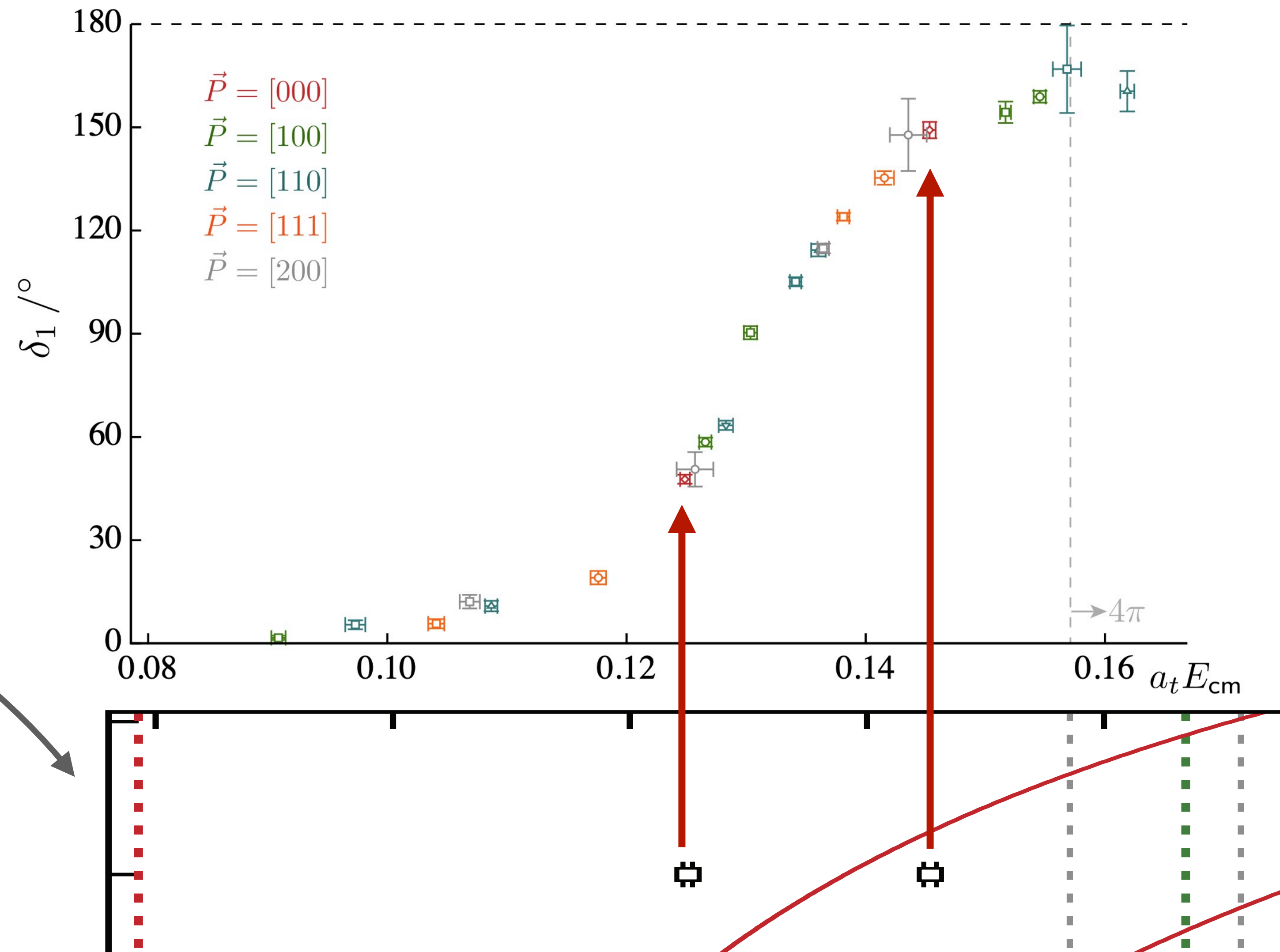
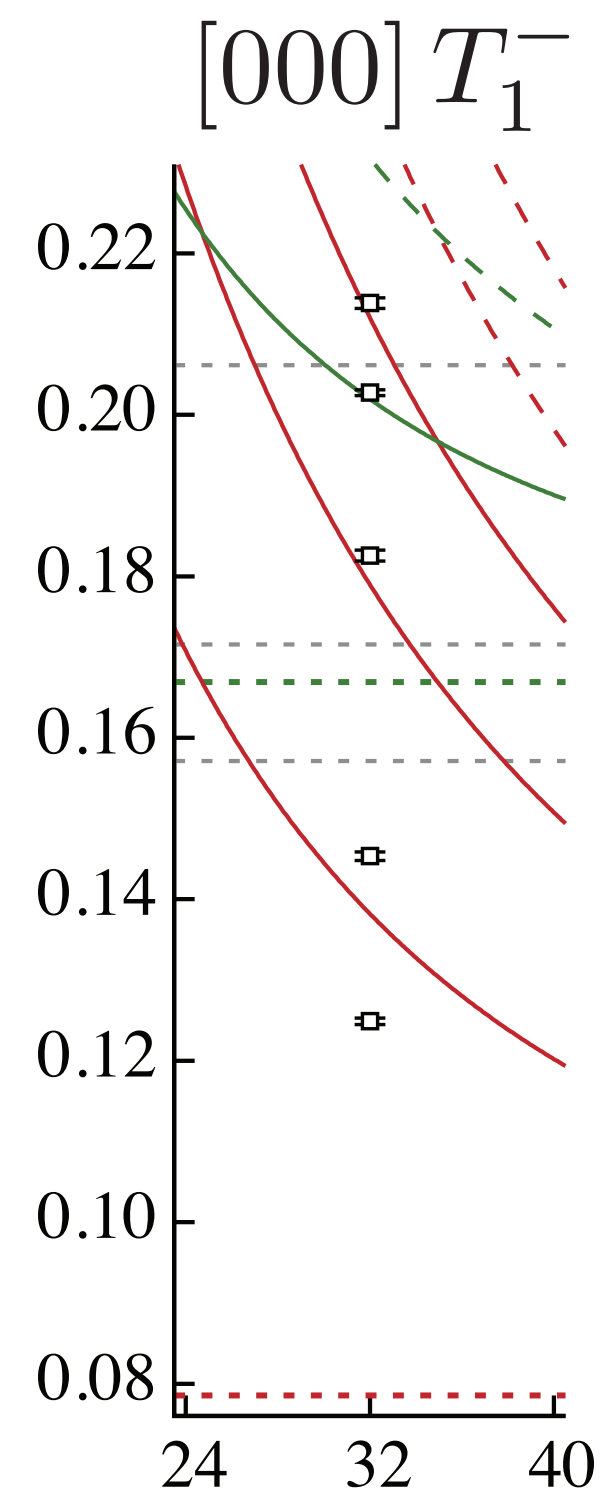
$$e^{ikL + 2i\delta(k)} = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$



$$e^{ikL+2i\delta(k)} = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$

We recover $\delta(s)$

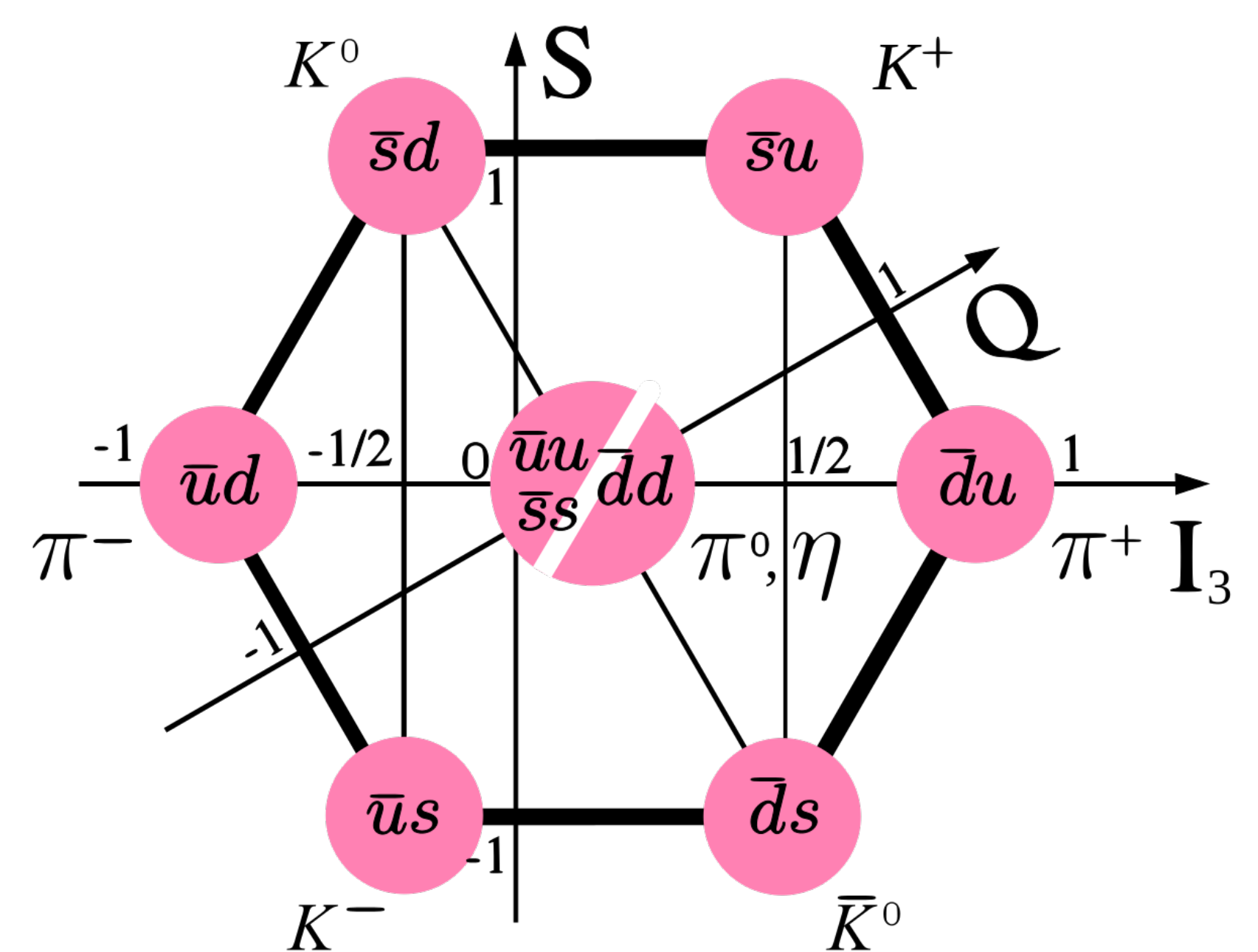
Our E_n become the x axis



Multiplets

Classify particles using symmetries

$$m_u \sim m_d \ll m_s \ll m_p$$

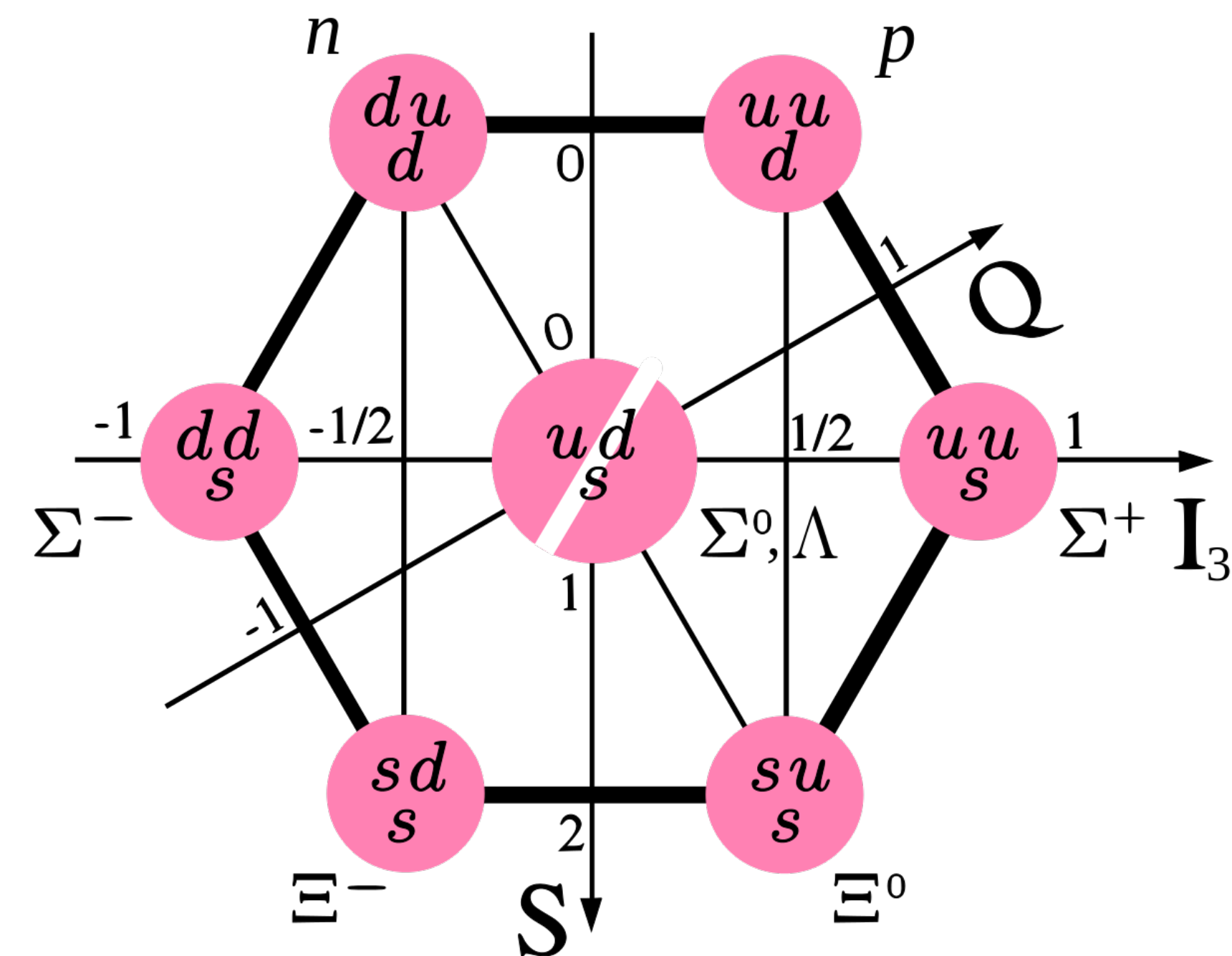


They are strongly related

Masses

Compsiteness

Sizes



Pseudo-scalars // Pseudo-goldstone

Chiral symmetry

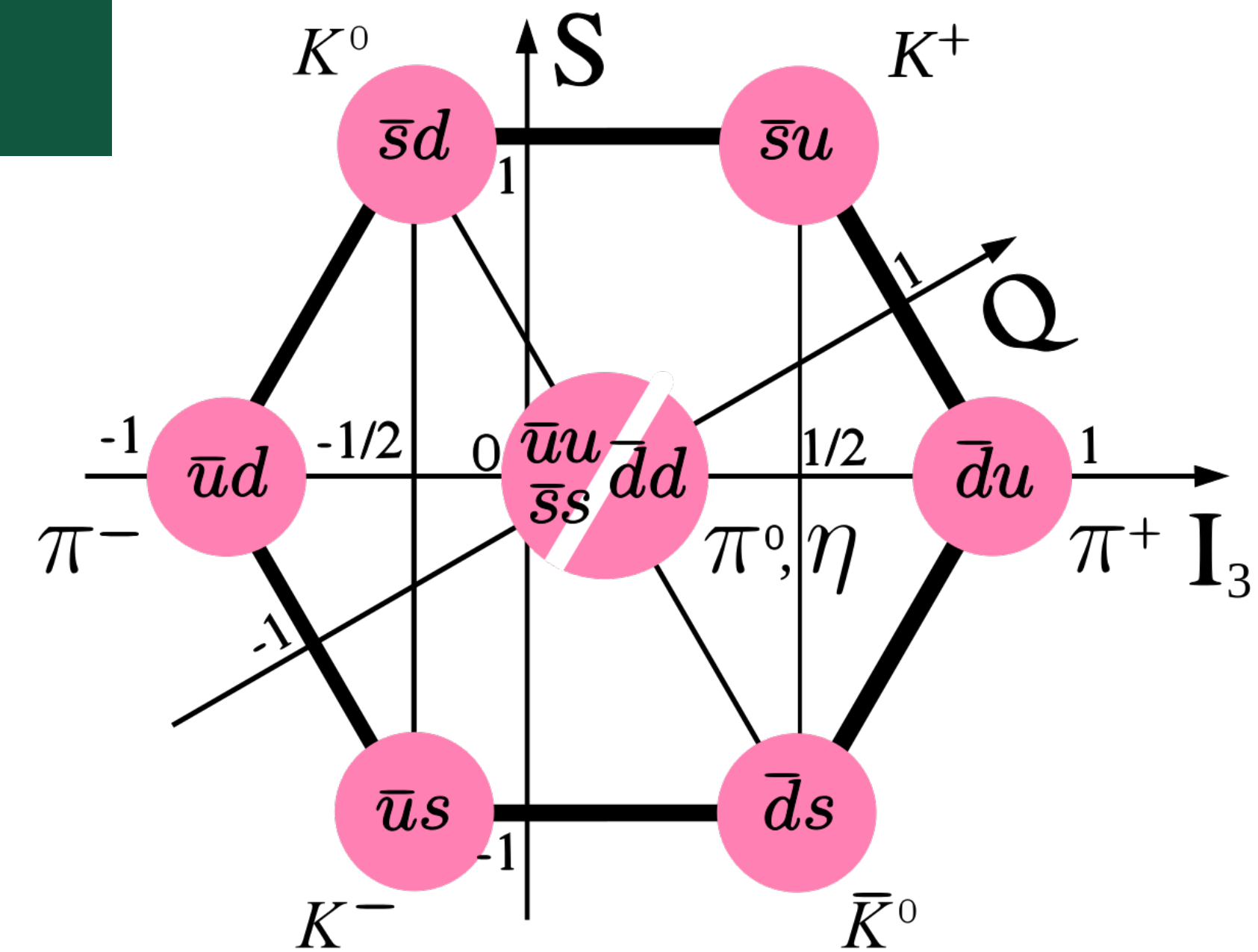
$$m_u \simeq m_d \simeq 0$$

$$q_{L,R} = \left(\frac{1 \mp \gamma_5}{2} \right) q$$

Invariant

$\mathcal{L}$

$$q_{L,R} \longrightarrow \exp \left(-i\theta_a^{L,R} \frac{T_a}{2} \right) q_{L,R}$$



Noether's theorem $\Rightarrow$ Conserved currents

$$V_a^\mu = \bar{q} \gamma^\mu T_a q$$

$$A_a^\mu = \bar{q} \gamma^\mu \gamma_5 T_a q$$

Pseudo-scalars // Pseudo-goldstone

Chiral symmetry

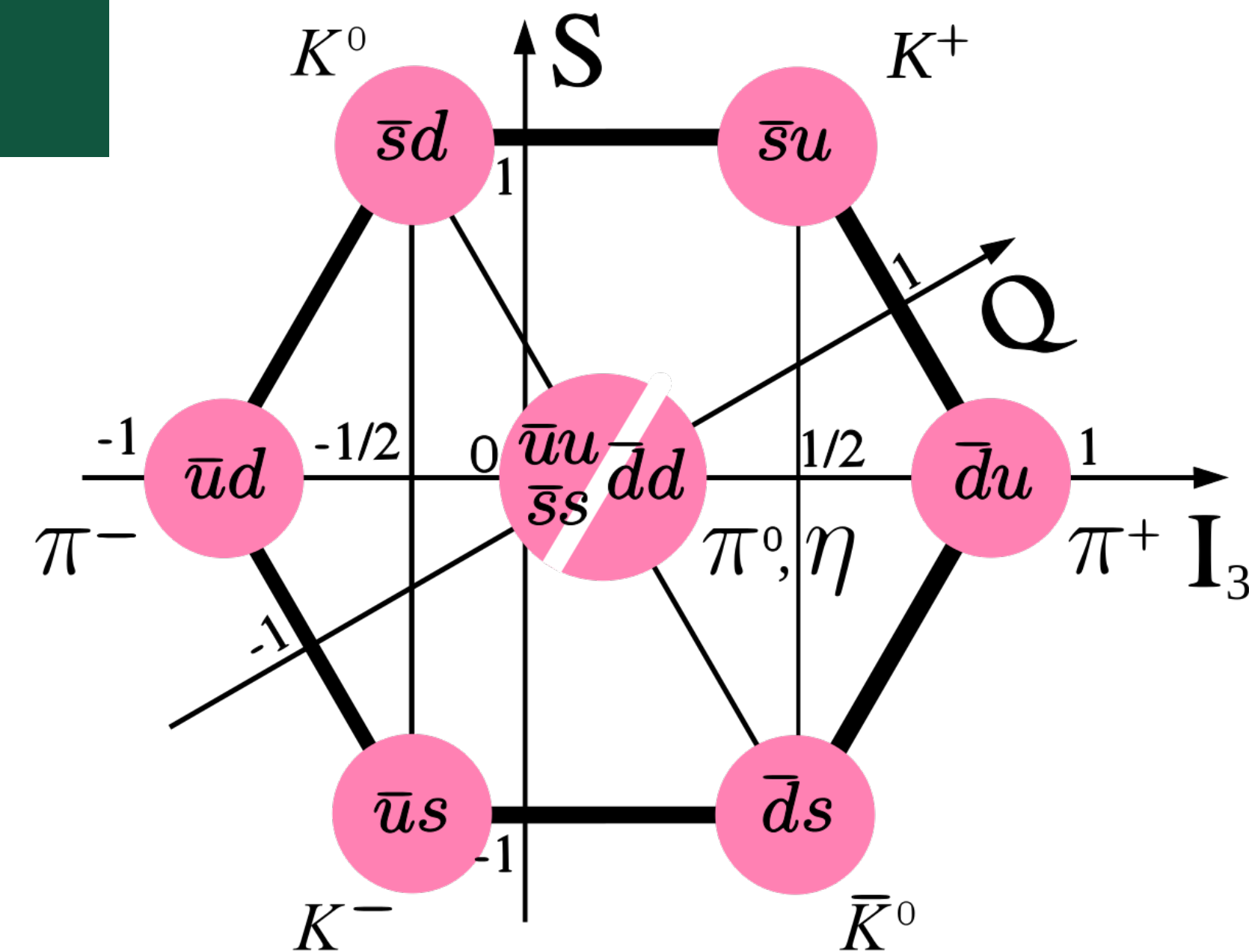
m_u

Spontaneously broken

Invariant

$$\left(\frac{1 \mp \gamma_5}{2} \right) q$$

$$q_{L,R}$$

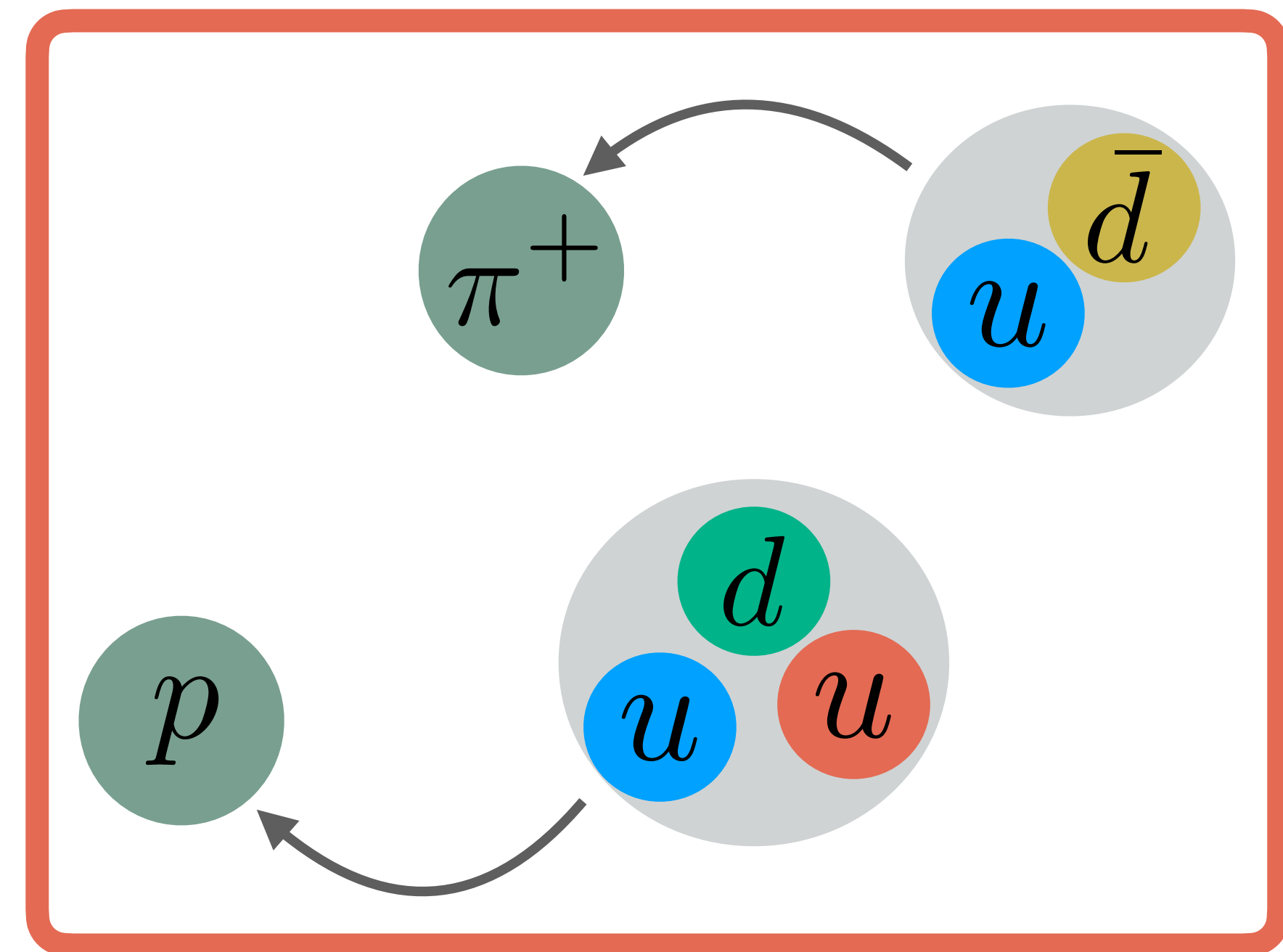


Noether's theorem $\Rightarrow$ Conserved currents

$$V_a^\mu = \bar{q} \gamma^\mu T_a q$$

$$A_a^\mu = \bar{q} \gamma^\mu \gamma_5 T_a q$$

$$m_\pi \ll m_p$$



Vectors

Ordinary mass hierarchy

Only light quarks

$$m_\rho \simeq 770 \text{ MeV}$$

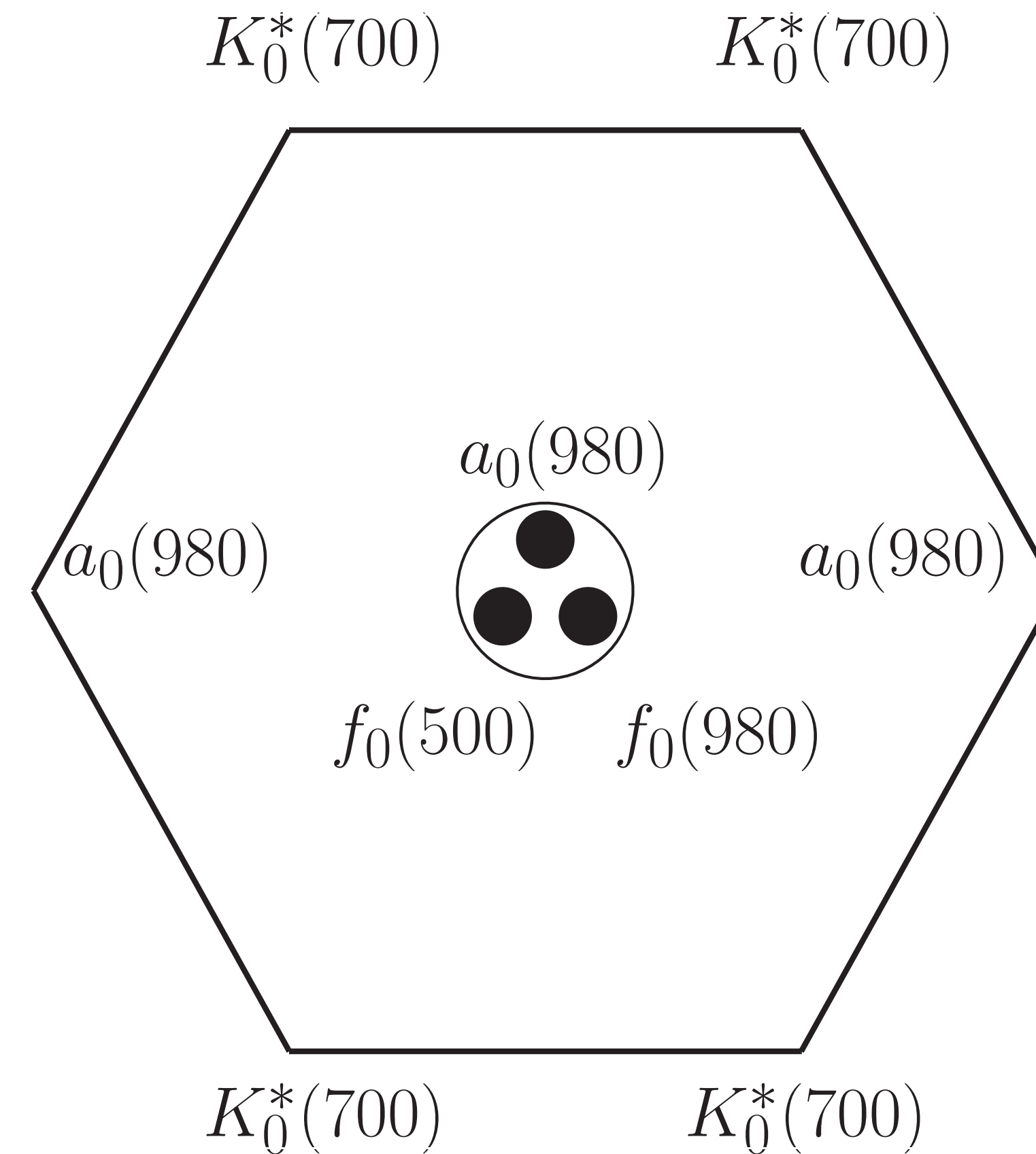
$$m_\omega \simeq 782 \text{ MeV}$$

Light+strange

$$m_K^* \simeq 892 \text{ MeV}$$

Only strange

$$m_\phi \simeq 1020 \text{ MeV}$$



Multiplets work...

Scalars

Non-ordinary mass hierarchy

Only light quarks

$$m_{\sigma} \sim 450 \text{ MeV}$$

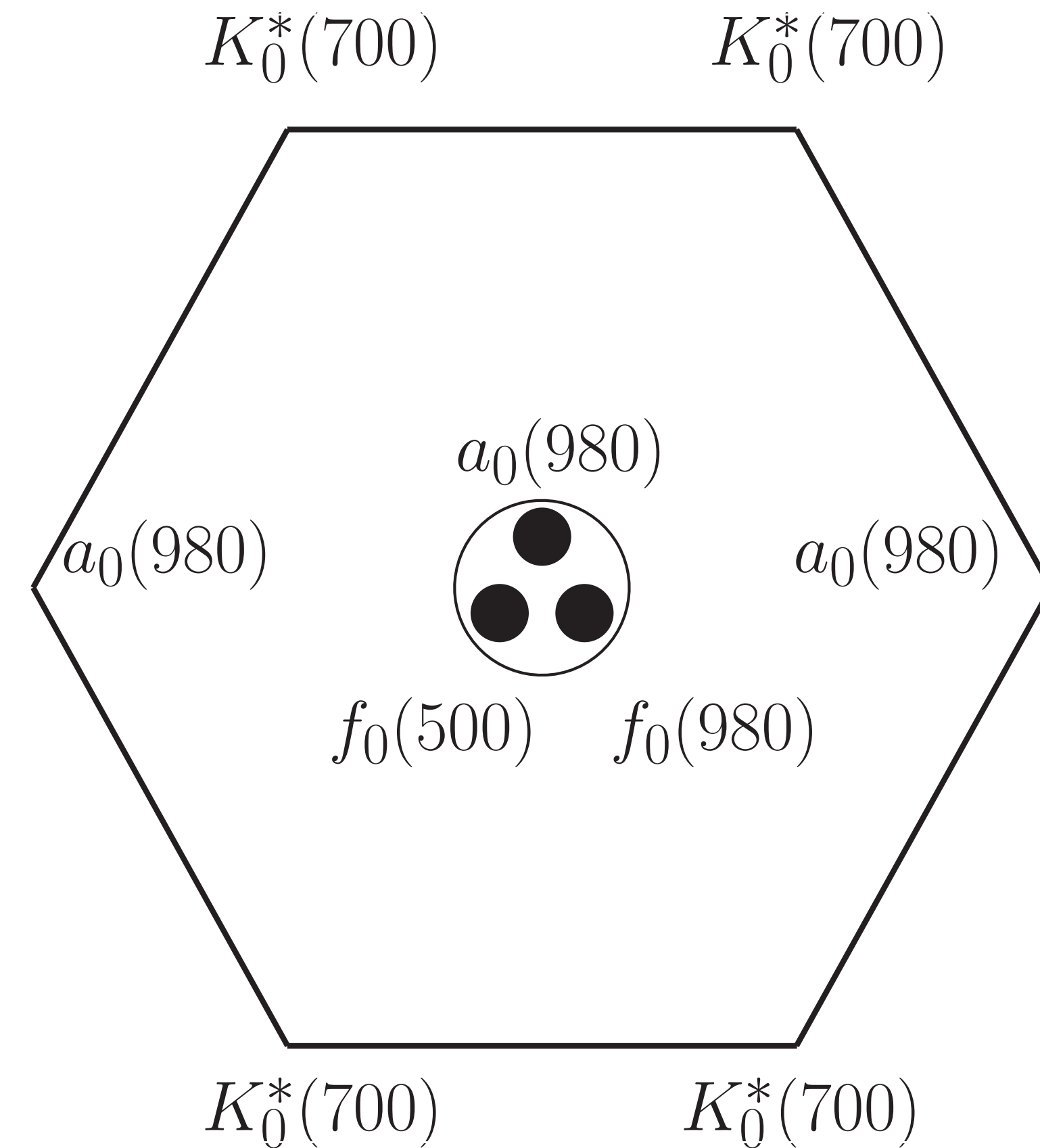
$$m_{f_0} \sim 980 \text{ MeV}$$

Light+strange

$$m_{\kappa} \sim 650 \text{ MeV}$$

Only strange

$$m_{a_0} \sim 980 \text{ MeV}$$



Multiplets work...

Sometimes