The long-distance behaviour of the vector correlator from π-π scattering at the physical point

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    - Talk by Ferenc Pittler on πN contamination on nucleons.
    - Nucleon structure talks, dealing with excited state contaminations especially Nπ.
    - $B\pi$ excited states in B-meson observables.
    - ......

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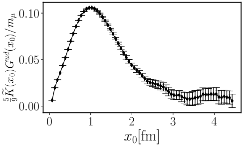

[1807.09370]

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[1807.09370]

- Precursor to calculating timelike pion form factor, one way to estimate finite-size effects in a_μ^{hvp} .

[1808.05007]

Lattice Setup

- CLS simulations $N_f = 2 + 1$ E250 Ensemble (periodic)
  $O(a)$ Improved Wilson fermions,
  $m_\pi = 129.60(97)$ MeV, (lower than physical)
  $L \approx 6.2$ fm, $96^3 \times 192$, $m_\pi \, L = 4.1$
  Enables $0 \leq |\vec{P}|^2 \leq 4$
  $a = 0.06426$ fm (fine)
  $N_{configs} = 353$

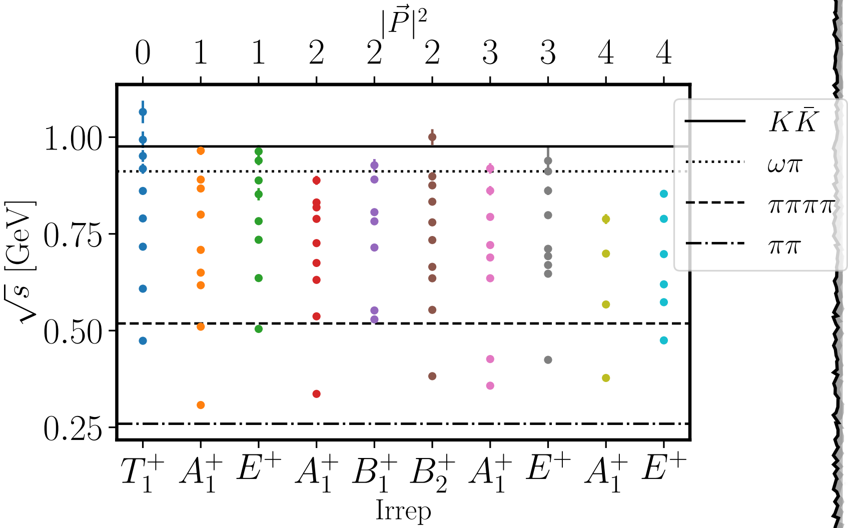
[1712.04884]

- Distillation Setup:
  $N_{eigenvector} = 1536$, $N_{noise} = 6$
  $N_{src} = 4$

[Hadron Spectroscopy]

- Construct local multihadron operators $I = 1$ channel $0 \leq |\vec{P}|^2 \leq 4$.
- Compute correlation matrices.
- Project onto irreps.
- Extract spectrum and principal eigenvectors using GEVP.

Reconstructing the long distance regime of the vector-vector correlator



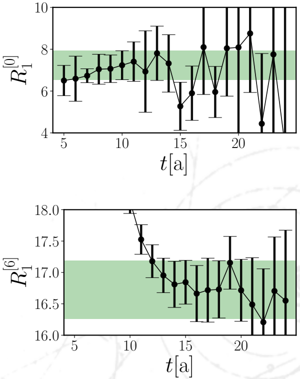
Reconstructing the long distance regime of the vector-vector correlator

$$R_1^{(n)}(t) = \left| \frac{\hat{D}_n(t)}{\sqrt{\hat{C}_n(t)\mathrm{e}^{-E_n t}}} \right|, \quad R_2^{(n)}(t) = \left| \frac{\hat{D}_n(t)}{A_n \mathrm{e}^{-E_n t}} \right|,$$

$$R_3^{(n)}(t) = \left| \frac{\hat{D}_n(t) A_n}{\hat{C}_n(t)} \right|$$

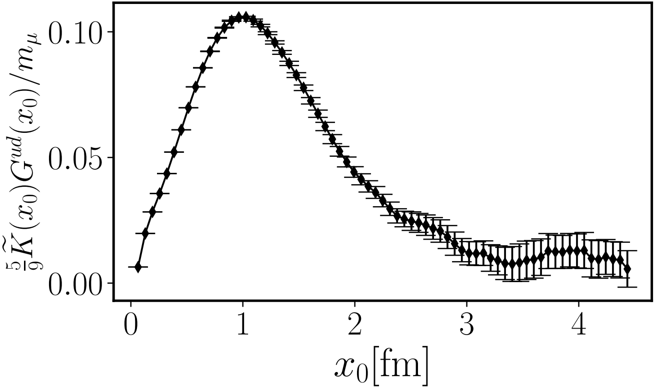
- $\hat{D}_n(t)$ Current overlap π-π
- $\hat{C}_n(t)$, A_n from ratio fits.
- Visual plateau selection
- Plateau average
- R_2, R_3 for comparison
- Source of systematic uncertainty.

[1808.05007]



Reconstructing the long distance regime of the vector-vector correlator

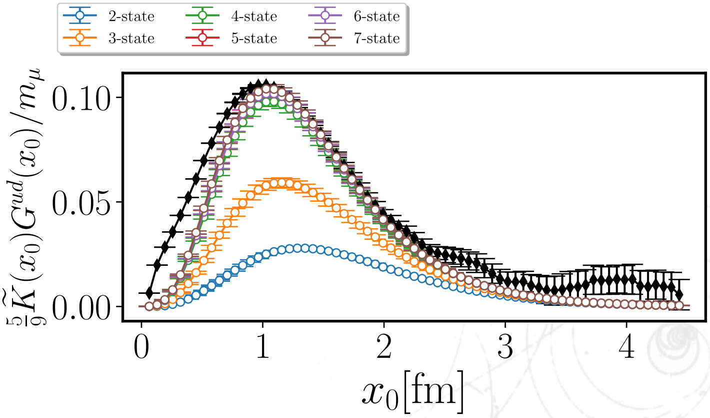
$$\left(a_\mu^{\rm hvp}\right)^f = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0 \frac{G\left(x_0\right)\widetilde{K}\left(x_0\right)}{m_\mu}$$



Reconstructed long distance regime

$$G^{ud}_{n_{\max}}(x_0) = \frac{10}{9} \sum_{n=0}^{n_{\max}} \left| R_1^{(n)} \right|^2 e^{-E_n x_0}$$

[1705.01775]



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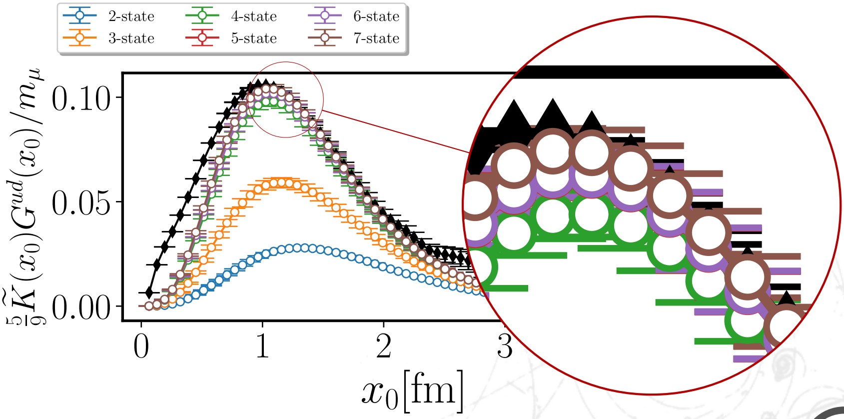
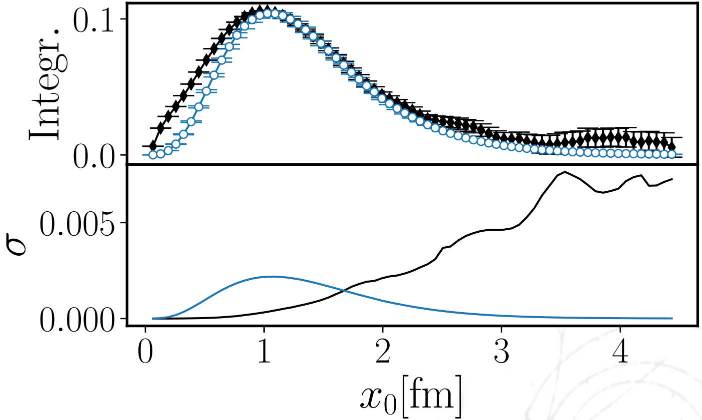


Reconstructed long distance regime : error comparison



Exponential noise reduction in the tail.

Another way of reconstructing the VV correlator

$$G^{\rho\rho}\left(x_0\right)_{\text{ext}} = \int_0^\infty d\omega\omega^2\rho\left(\omega^2\right)e^{-\omega x_0}$$

where

$$\rho\left(\omega^2\right) = \frac{1}{48\pi^2}\left(1-\frac{4m_\pi^2}{\omega^2}\right)^{\frac{3}{2}}\left|F_\pi(\omega)\right|^2,$$

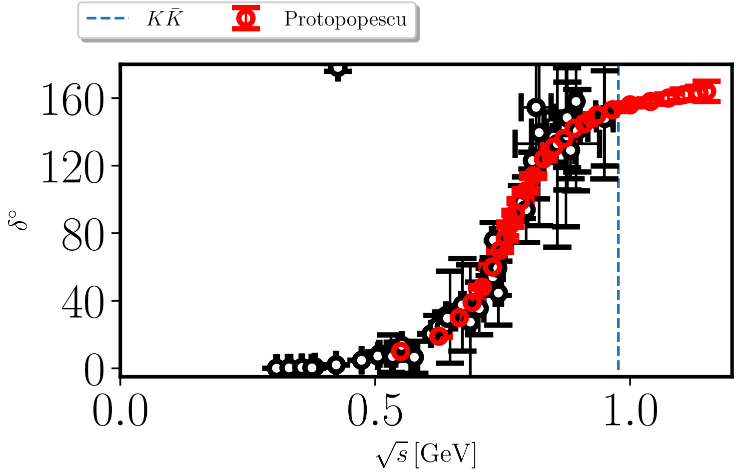
[1107.4388]

- Extract resonance parameters using moving frames.

Lüscher (1991)

- Construct Time-Like Pion form factor $F_\pi(\omega)$

Another way of reconstructing the VV correlator



In the process of extracting the resonance parameters with a t-matrix fit.

Outlook

- Take into contributions from higher partial waves.
- Different K-matrix parametrization fits for coupling and width.
- Extraction of F_π for inputs to estimating FV effects in a_μ^{hvp} .
- Compare with VV correlator using Low Mode Averaging. (g-2 Workshop, Edinburgh: Simon Kuberski)
- Another ongoing analysis on $J303$, CLS ensemble, to systematically reduce the uncertainties in $(g-2)$ calculations.