Setting the Scale Using Baryon Masses with Isospin-Breaking Corrections

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August 11, 2022

Motivation

- Increased interest in high-precision observables
  - Discrepency between theory and experiment on $a_\mu = \frac{(g-2)_\mu}{2}$
  - Theoretical uncertainty is dominated by QCD
    $\rightarrow$ Strongly influenced by the uncertainty of the lattice scale
  - CLS $N_f = 2 + 1$ ensembles
    $\rightarrow$ Isospin symmetric action leads to systematic uncertainty
    $\rightarrow$ Need to account for QED effects and difference in light quark masses

- Scale setting currently done using f_π and f_K[1]
  $\rightarrow$ Difficult to determine IB corrections[2] reliably
  $\rightarrow$ Use baryon masses instead

- Allows for calculations of mass splittings of isospin multiplets[3]

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[1] Bruno et al. 2017, *Phys. Rev. D* **95** no. 7, p. 074504.
[2] Carrasco et al. 2015, *Phys. Rev. D* **91** no. 7, p. 074506.
[3] Borsanyi 2015, *Science* **347**, pp. 1452–1455.

Operator Basis

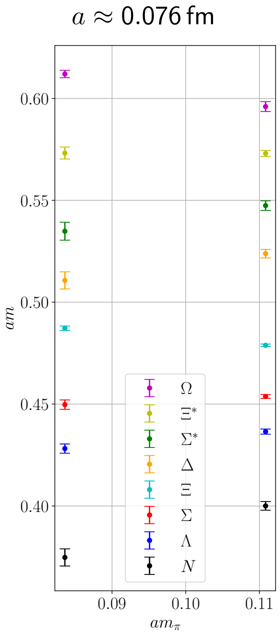
- Construction based on isospin-symmetric QCD following a procedure introduced by the *Lattice Hadron Physics Collaboration*[4]
- Classification by symmetries in flavor and spin (and parity eigenvalues)
- Example for symmetric spin and flavor indices (e.g. $\Omega^-_{ijk} = s_i s_j s_k$, $\Delta^+_{ijk} = \frac{1}{\sqrt{3}}(u_i u_j d_k + u_i d_j u_k + d_i u_j u_k)$, etc.)
  (Operators in Dirac-Pauli basis)

Embedding	S_z	gerade (even)	ungerade (odd)
1	$\frac{3}{2}$	Ω_{111}	$\sqrt{3}\Omega_{113}$
1	$\frac{1}{2}$	$\sqrt{3}\Omega_{112}$	$\Omega_{114} + 2\Omega_{123}$
1	$-\frac{1}{2}$	$\sqrt{3}\Omega_{122}$	$2\Omega_{124} + \Omega_{223}$
1	$-\frac{3}{2}$	Ω_{222}	$\sqrt{3}\Omega_{224}$
2	$\frac{3}{2}$	$\sqrt{3}\Omega_{133}$	Ω_{333}
2	$\frac{1}{2}$	$2\Omega_{134} + \Omega_{233}$	$\sqrt{3}\Omega_{334}$
2	$-\frac{1}{2}$	$\Omega_{144} + 2\Omega_{234}$	$\sqrt{3}\Omega_{344}$
2	$-\frac{3}{2}$	$\sqrt{3}\Omega_{244}$	Ω_{444}

H irrep for symmetric spin and flavor indices

[4] Basak et al. 2005, *Phys. Rev. D* **72**, p. 074501.

Baryon Spectrum at $m_\pi \in \{215\,\text{MeV}, 290\,\text{MeV}\}$



- Setup:
  - $\mathcal{O}(a)$-improved Wilson fermions
  - tree-level Lüscher–Weisz gauge action
  - Interpolators as described before
  - Wuppertal smeared point sources with smearing radius ~ 0.5 fm
  - APE smeared gauge links
- Correlation functions averaged over
  - Different S_z
  - Forward propagator and backwards parity partner
- Have access to correlator matrices allowing for GEVP
  - → Found that one operator is much less noisy than the others for each state
  - → GEVP mostly projects on less noisy correlator
  - → Data shown only from one correlation function

QCD+QED vs. QCD_{iso}

- Method based on a procedure introduced by the *RM123 collaboration*[5][6]
- Consider a QCD+QED action S with parameters:

$$\varepsilon = (\beta, e^2, m_u, m_d, m_s)$$

- Expand around isosymmetric action $S^{(0)}$ with parameters

$$\varepsilon^{(0)} = \left(\beta^{(0)}, 0, m_{ud}^{(0)}, m_{ud}^{(0)}, m_s^{(0)}\right)$$

- Dividing S into three parts, write

$$S[U, A, \psi, \bar{\psi}] = S_g[U] + S_\gamma[A] + S_q[U, A, \psi, \bar{\psi}]$$

- QED_L action S_γ[7] in Coulomb gauge
  $\rightarrow$ IR regularisation by setting $\sum_{\mathbf{x}} A^{\mathbf{x},t} = 0 \, \forall t$

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[5] Divitiis 2012, *JHEP* **04**, p. 124.
[6] Divitiis et al. 2013, *Phys. Rev. D* **87** no. 11, p. 114505.
[7] Hayakawa and Uno 2008, *Prog. Theor. Phys.* **120**, pp. 413–441.

Expectation values

$$
\begin{aligned}
\langle \mathcal{O} \rangle^{\varepsilon} &= \left\langle \langle \mathcal{O} \rangle_{q\gamma} \right\rangle^{\varepsilon}_{\text{eff}} \\
\langle \mathcal{O} \rangle_{q\gamma}[U] &= \frac{1}{Z_{q\gamma}[U]} \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\, \mathcal{O}[U, A, \psi, \bar{\psi}] e^{-S_\gamma[A]-S_q[U,A,\psi,\bar{\psi}]} \\
\langle \mathcal{O}[U] \rangle^{\varepsilon}_{\text{eff}} &= \frac{1}{Z} \int \mathcal{D}U\, \mathcal{O}[U] Z_{q\gamma}[U] e^{-S_g[U]} \\
&= \frac{1}{Z} \int \mathcal{D}U\, \mathcal{O}[U] e^{-S_{\text{eff}}[U]}
\end{aligned}
$$

Can relate $\langle \mathcal{O} \rangle^{\varepsilon}$ to $\langle \mathcal{O} \rangle^{\varepsilon^{(0)}}$ via reweighting:

$$
\langle \mathcal{O} \rangle^{\varepsilon} = \frac{\left\langle R \langle \mathcal{O} \rangle_{q\gamma} \right\rangle^{\varepsilon^{(0)}}_{\text{eff}}}{\langle R \rangle^{\varepsilon^{(0)}}_{\text{eff}}}, \quad R = \frac{\exp(-S_{\text{eff}})}{\exp\left(-S^{(0)}_{\text{eff}}\right)} = \frac{\exp(-S_g) Z_{q\gamma}}{\exp\left(-S^{(0)}_g\right) Z^{(0)}_q}
$$

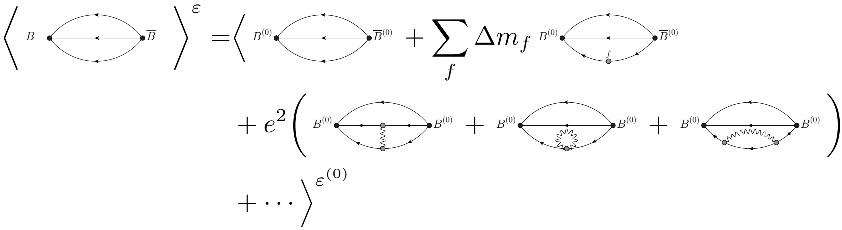
Perturbative Expansion – Expectation Values

Expand expectation values

$$\langle \mathcal{O} \rangle^{\varepsilon} = \langle \mathcal{O} \rangle^{\varepsilon^{(0)}} + \sum_{\varepsilon_i \in \varepsilon} \underbrace{\left(\varepsilon_i - \varepsilon_i^{(0)} \right)}_{=:\Delta\varepsilon_i} \frac{\partial \langle \mathcal{O} \rangle^{\varepsilon}}{\partial \varepsilon_i} \bigg|_{\varepsilon = \varepsilon^{(0)}} + O\left(\Delta\varepsilon^2\right)$$

$$\left.\begin{array}{l} \varepsilon = \left(\beta, e^2, m_u, m_d, m_s\right) \\ \varepsilon^{(0)} = \left(\beta, 0, m_{ud}^{(0)}, m_{ud}^{(0)}, m_s^{(0)}\right) \end{array}\right\} \Delta\varepsilon = \left(0, e^2, \Delta m_u, \Delta m_d, \Delta m_s\right)$$

Baryon correlation function:



Perturbative Expansion – Spectroscopy

- Correlation functions asymptotically behave like

$$C(t) = ce^{-mt}$$

- Expansion in isospin breaking parameters ($\Delta\varepsilon_i := \varepsilon_i - \varepsilon_i^{(0)}$)

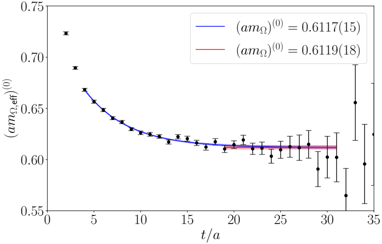
$$\begin{aligned} \rightarrow C(t) =& c^{(0)} e^{-m^{(0)}t} \\ &+ \sum_i \Delta\varepsilon_i \left(c_i^{(1)} - c^{(0)} m_i^{(1)} t \right) e^{-m^{(0)}t} \\ &+ \mathcal{O}(\Delta\varepsilon^2) \end{aligned}$$

$\rightarrow$ Can define effective mass at first order via

$$(am_{\text{eff}})^{(1)}_{\Delta\varepsilon_i} := -a \frac{\mathrm{d}}{\mathrm{d}t} \frac{C_i^{(1)}(t)}{C^{(0)}(t)}$$

- $\Delta\varepsilon_i$ can be set by matching different average multiplet masses and mass splittings

Correlators for Ω at $m_\pi = 215$ MeV, $a = 0.076$ fm

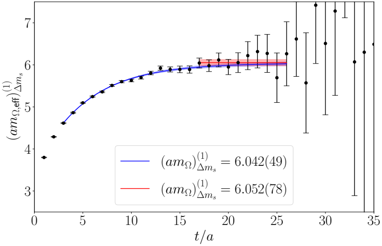


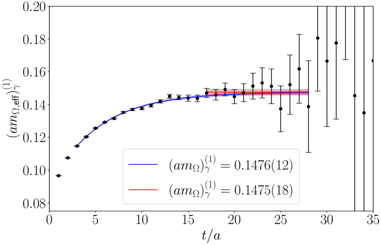
- At 0th order:

$$(am_{\text{eff}})^{(0)}(t) := -a\frac{\mathrm{d}}{\mathrm{d}t}\log C^{(0)}(t) \to \log\left(\frac{C^{(0)}(t)}{C^{(0)}(t+a)}\right)$$

- At 1st order:

$$(am_{\text{eff}})^{(1)}_{\Delta\varepsilon_i} := -a\frac{\mathrm{d}}{\mathrm{d}t}\frac{C_i^{(1)}(t)}{C^{(0)}(t)} \to \frac{C_i^{(1)}(t)}{C^{(0)}(t)} - \frac{C_i^{(1)}(t+a)}{C^{(0)}(t+a)}$$





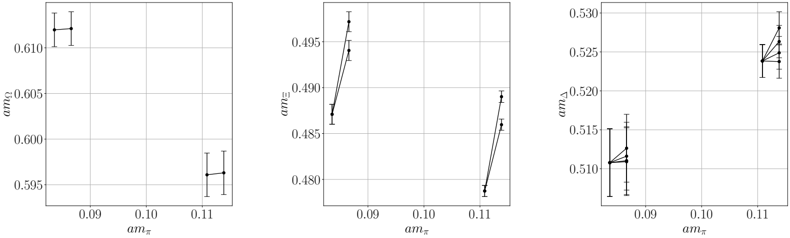
2-state-fit-ansatz[8]: $(am)_{\Omega,\text{eff}} = am_\Omega + ce^{-\Delta M t}$

[8] Del Debbio et al. 2007, *JHEP* **02**, p. 056.

Renormalization Scheme

Expansion coefficients provided by Andreas Risch using the renormalization scheme[9]

$$
\begin{aligned}
\left(m_{\pi^0}^2\right)^{\text{QCD+QED}} &= \left(m_{\pi^0}^2\right)^{\text{phys}} \\
\left(m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2\right)^{\text{QCD+QED}} &= \left(m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2\right)^{\text{phys}} \\
\left(m_{K^+}^2 - m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2\right)^{\text{QCD+QED}} &= \left(m_{K^+}^2 - m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2\right)^{\text{phys}} \\
\left(\alpha_{\text{em}}\right)^{\text{QCD+QED}} &= \left(\alpha_{\text{em}}\right)^{\text{phys}}
\end{aligned}
$$



[9]Risch and Wittig 2022, *PoS* **LATTICE2021**, p. 106.

Summary

- We perform a calculation of a complete set of octet and decuplet baryon operators
- We use a perturbative approach to include isospin breaking effects on isosymmetric ensembles
- Two Ensembles processed so far with sub-percent precision for some baryon masses (e.g. 0.25 % for Ω)
  - $\rightarrow$ Even high precision in isospin breaking corrections (e.g. 0.8 % for Ω)
  - $\rightarrow$ We intend to expand parameter space to further ensembles (N200 ($m_\pi = 285$ MeV), N203 ($m_\pi = 345$ MeV), D200 ($m_\pi = 200$ MeV), J303 ($m_\pi = 260$ MeV))
- We search for a suitable observable for setting the Scale
- We intend to investigate Σ-Λ-mixing