Semileptonic $b \to u$ and $b \to s$ decays of the B_c meson

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Abstract

This poster reviews our recent calculation of $B_c^+ \to D^0 \ell^+ \nu$ and $B_c^+ \to D_s^+ \ell^+ \ell^- (\bar{\nu}\nu)$ form factors [1]. We comment on prospects for experimental measurement of $B_c^+ \to D^{(*)0} \mu^+ \nu_\mu$ and implications for CKM matrix elements.

Motivation

- Longstanding discrepancies in inclusive vs exclusive determinations of CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.
- LHCb can measure decays of the B_c meson, e.g. the $b \to c$ decay $B_c^+ \to J/\psi \, \mu^+ \nu_\mu$.
- The production fraction of B_c mesons is not precisely known, but cancels in ratios of decay rates.
- A measurement of the $b \to u$ decay $B_c^+ \to D^0 \mu^+ \nu_\mu$ would provide a new determination of $|V_{ub}/V_{cb}|$.

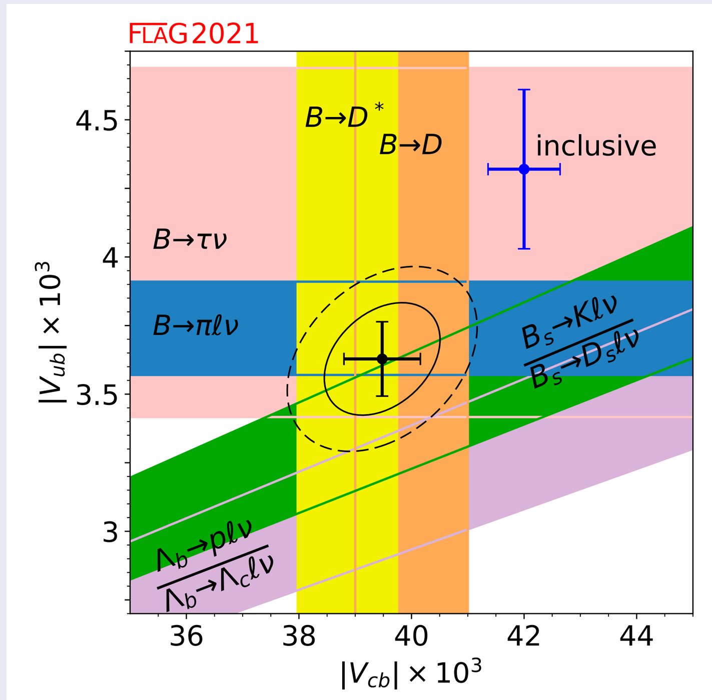

Figure: Constraints on $|V_{cb}|$ & $|V_{ub}|$

Form factors

The differential decay rate for $B_c \to D \ell \nu$ is given by

$$\frac{d\Gamma}{dq^2} = \eta_{\mathrm{EW}}^2 |V_{ub}|^2 \frac{G_F^2}{24\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\mathbf{q}| \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) |\mathbf{q}|^2 f_+^l(q^2)^2 + \frac{3m_\ell^2}{8q^2} \frac{(M_{B_c}^2 - M_D^2)^2}{M_{B_c}^2} f_0^l(q^2)^2 \right].$$

The form factors parametrize the hadronic matrix elements of the weak decay operator

$$\langle D_{l(s)}(p_2) | V^\mu | B_c(p_1) \rangle = f_0^{l(s)}(q^2) \left[\frac{M_{B_c}^2 - M_{D_{l(s)}}^2}{q^2} q^\mu \right] + f_+^{l(s)}(q^2) \left[p_2^\mu + p_1^\mu - \frac{M_{B_c}^2 - M_{D_{l(s)}}^2}{q^2} q^\mu \right]$$

For rare, FCNC decays such as $B_c \to D_s \ell^+ \ell^-$ we also need

$$\langle D_s(p_2) | T^{k0} | B_c(p_1) \rangle = \frac{2iM_{B_c} p_2^k}{M_{B_c} + M_{D_s}} f_T^s(m_b; q^2).$$

Ensembles

Table: Parameters for the MILC ensembles [2] (and earlier). The lattice spacing a is determined from the Wilson flow parameter w_0 [3]. The physical value $w_0 = 0.1715(9)\ \mathrm{fm}$ was fixed from f_π in [4]. $M_\pi L$ and M_π values for each lattice are given in [5]. We give n_{cfg}, the number of configurations used for each set. On each we used four different positions for the source to increase statistics.

set	handle	w_0/a	$N_x^3 \times N_t$	$M_\pi L$	M_π MeV	n_{cfg}	am_l^{sea}	am_s^{sea}	am_c^{sea}	am_l^{val}	am_s^{val}	am_c^{val}	T
1	fine	1.9006(20)	$32^3 \times 96$	4.5	316	500	0.0074	0.037	0.440	0.0074	0.0376	0.450	14, 17, 20
2	fine-physical	1.9518(17)	$64^3 \times 96$	3.7	129	500	0.00120	0.0364	0.432	0.00120	0.036	0.433	14, 17, 20
3	superfine	2.896(6)	$48^3 \times 144$	4.5	329	250	0.0048	0.024	0.286	0.0048	0.0245	0.274	22, 25, 28
4	ultrafine	3.892(12)	$64^3 \times 192$	4.3	315	250	0.00316	0.0158	0.188	0.00316	0.0165	0.194	31, 36, 41

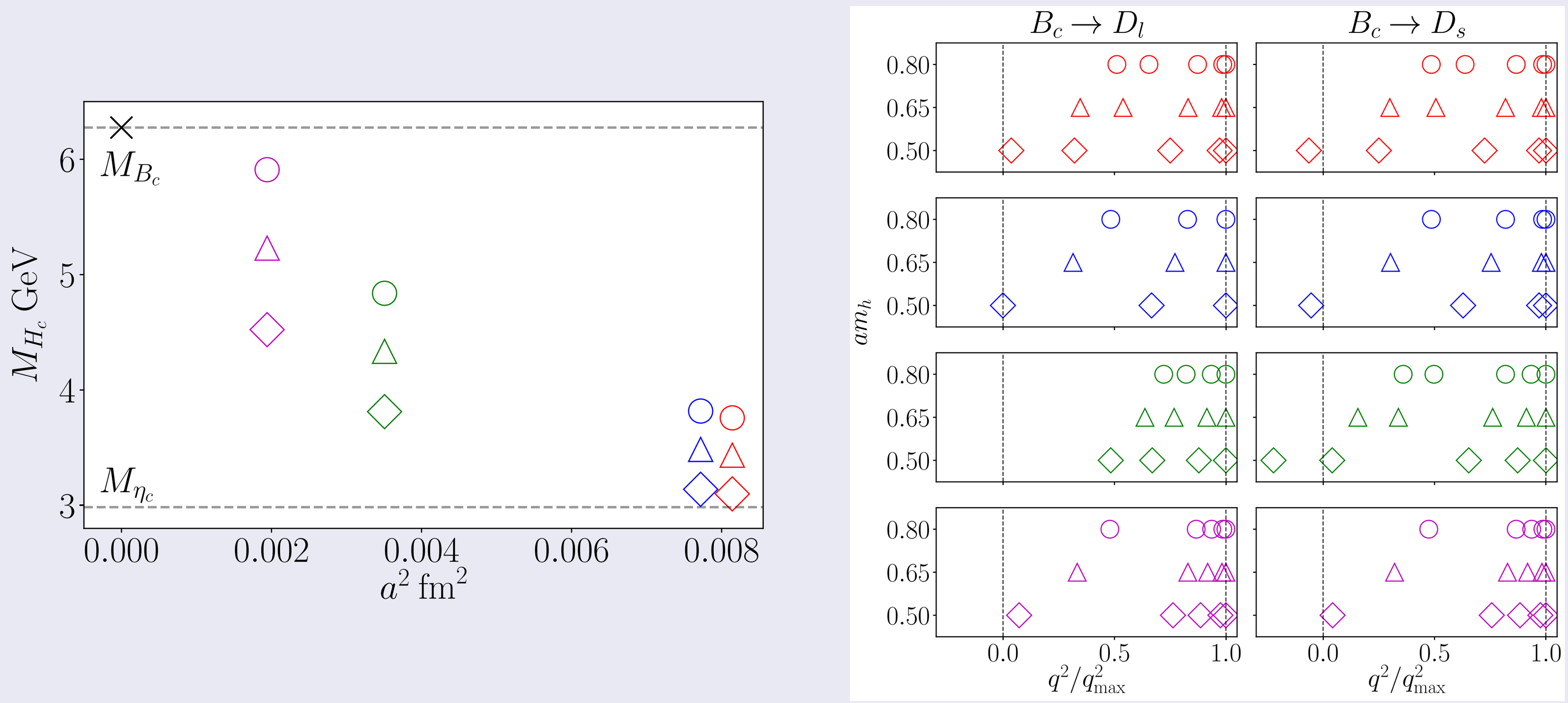

Figure: Range of heavy masses at each lattice spacing.

Correlation functions

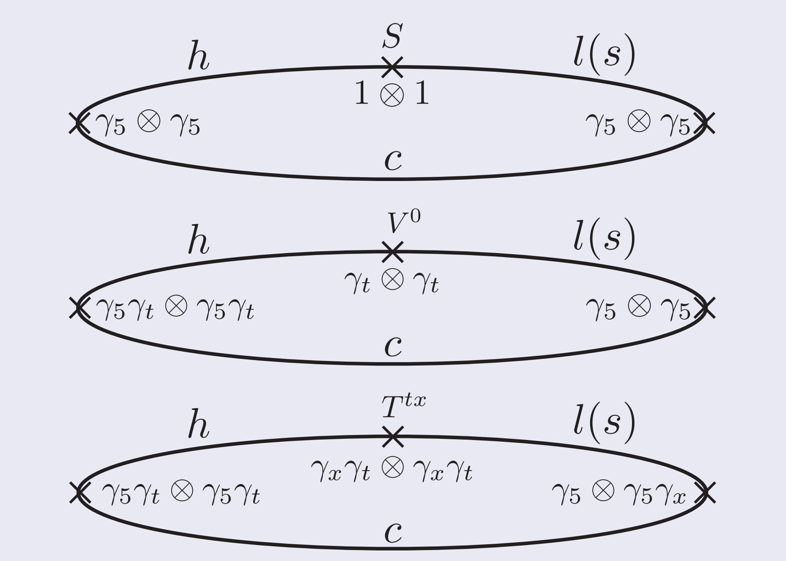

Figure: Illustration of the three-point correlation functions calculated.

Form factors from correlator amplitudes use the Ward identity

$$q_\mu \langle D_{l(s)} | V^\mu_{\mathrm{local}} | H_c \rangle Z_V = (m_h - m_{l(s)}) \langle D_{l(s)} | S_{\mathrm{local}} | H_c \rangle$$

So we have

$$f_0^{l(s)}(q^2) = \langle D_{l(s)} | S_{\mathrm{local}} | H_c \rangle \frac{m_h - m_{l(s)}}{M_{H_c}^2 - M_{D_{l(s)}}^2}$$

$$f_+^{l(s)}(q^2) = \frac{Z_V \langle D_{l(s)} | V^0_{\mathrm{local}} | H_c \rangle - q^0 f_0^{l(s)}(q^2) \frac{M_{H_c}^2 - M_{D_{l(s)}}^2}{q^2}}{p_2^0 + p_1^0 - q^0 \frac{M_{H_c}^2 - M_{D_{l(s)}}^2}{q^2}}$$

$$f_T^s(q^2) = \frac{Z_T \langle D_s | T^{1,0}_{\mathrm{local}} | H_c \rangle (M_{H_c} + M_{D_s})}{2iM_{H_c} p_2^1}$$

Logos

Fit form

We fit the form factors, with a pole term removed, to the following form

$$P(q^2) f(q^2) = \mathcal{L} \sum_{n=0}^{N_n} \sum_{r=0}^{N_r} \sum_{j=0}^{N_j} \sum_{k=0}^{N_k} A^{(nrjk)} \hat{z}^{(n,N_n)} \left(\frac{\Lambda}{M_{H_{l(s)}}} \right)^r \Omega^{(n)} \left(\frac{am_h}{\pi} \right)^{2j} \left(\frac{am_c}{\pi} \right)^{2k} \mathcal{N}_{\mathrm{mis}}^{(n)}.$$

where $\mathcal{L}$ contains the chiral logarithms

$$\mathcal{L} = 1 + \left(\zeta^{(0)} + \zeta^{(1)} \frac{\Lambda}{M_{H_l}} + \zeta^{(2)} \frac{\Lambda^2}{M_{H_l}^2} \right) x_\pi \log x_\pi$$

The $\Omega^{(n)}$ factors are given by

$$\Omega^{(n)} = 1 + \rho^{(n)} \log \left(\frac{M_{H_{l(s)}}}{M_{D_{l(s)}}} \right).$$

$\Omega^{(n)}$ allows for heavy quark mass dependence that appears as a prefactor to the expansion in inverse powers of the heavy mass. From HQET this prefactor could include fractional powers of the heavy quark mass and/or logarithmic terms which vary in different regions of q^2 [6]. We allow for this with a logarithmic term with a variable coefficient that depends on the form factor and the power of z in the z-expansion. We take priors for the $\rho^{(n)}$ of 0(1). The mistuning terms are given by

$$\mathcal{N}_{\mathrm{mis}}^{(n)} = 1 + \frac{\delta m_c^{\mathrm{sea}}}{m_c^{\mathrm{tuned}}} \kappa_1^{(n)} + \frac{\delta m_c^{\mathrm{val}}}{m_c^{\mathrm{tuned}}} \kappa_2^{(n)} + \frac{\delta m_l}{10 m_s^{\mathrm{tuned}}} \kappa_3^{(n)} + \frac{\delta m_s^{\mathrm{sea}}}{10 m_s^{\mathrm{tuned}}} \kappa_4^{(n)} + \frac{\delta m_s^{\mathrm{val}}}{10 m_s^{\mathrm{tuned}}} \kappa_5^{(n)}.$$

Form factor results

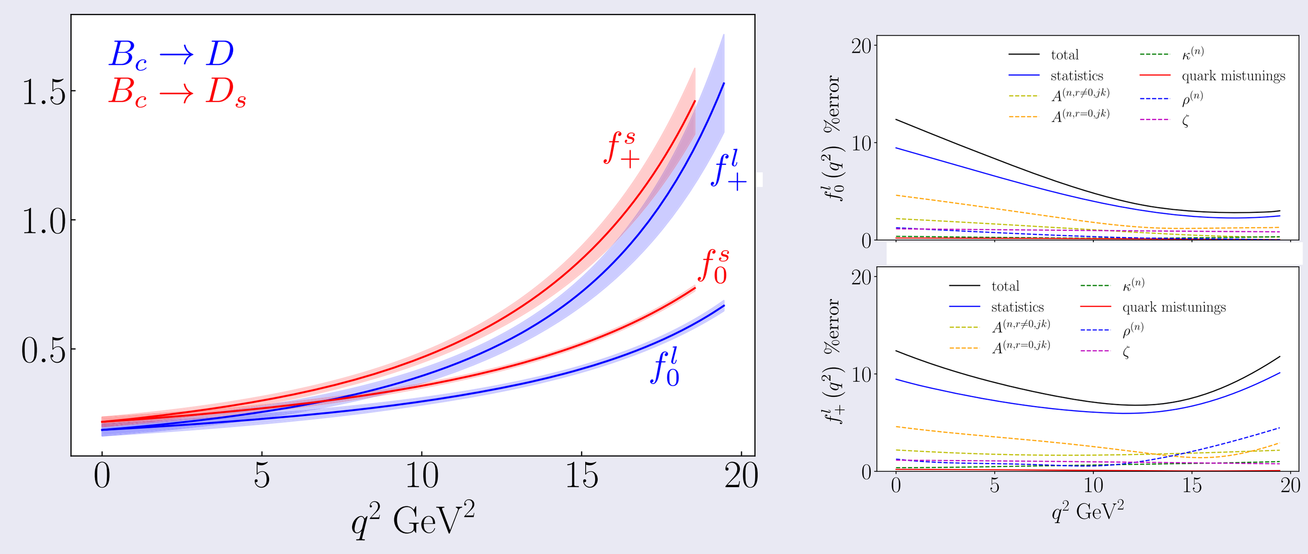

Figure: Results for the form factors in the continuum, physical mass limit.

$B_c^+ \to D^0 \ell^+ \nu$ decay rates

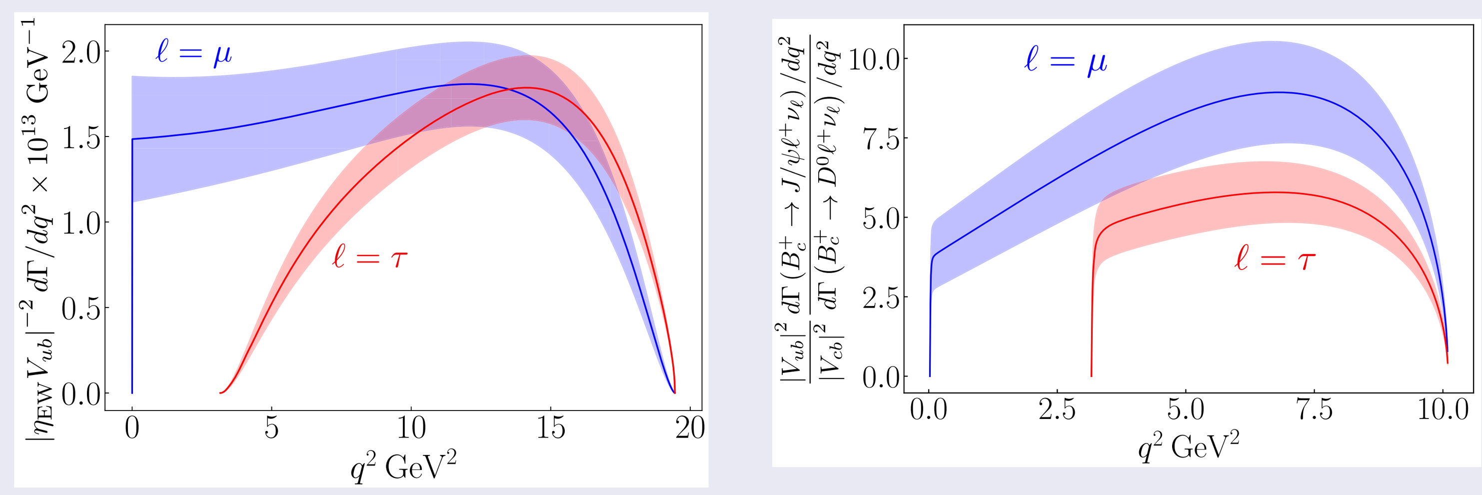

Figure: Differential decay rates of $B_c^+ \to D^0 \mu^+ \nu_\mu$ and $B_c^+ \to D^0 \tau^+ \nu_\tau$ (left) and ratio of differential decay rates normalized by $B_c^+ \to J/\psi \, \ell^+ \nu_\ell$.

Experimental prospects for $B_c^+ \to D^{(*)0} \ell^+ \nu$

LHCb is in the progress of analyzing $B_c^+ \to D^{(*)0} \mu^+ \nu$ decays [7]. These $b \to u$ decays are CKM-suppressed compared to $b \to c$ decays of the B_c^+, so the first measurements are likely to come from the semi-exclusive combination of the pseudoscalar D^0 and vector D^{*0} final states. In order to cancel experimental uncertainties associated with B_c-production, the branching fraction is normalized to that for the decay $B_c^+ \to J/\pi \, \mu^+ \nu_\mu$.

$$\frac{\mathcal{B}(B_c^+ \to D^{(*)0} \mu^+ \nu_\mu)}{\mathcal{B}(B_c^+ \to J/\psi \, \mu^+ \nu_\mu)} \propto \frac{|V_{ub}|^2}{|V_{cb}|^2}$$

In order to use such a measurement, form factors for $B_c \to D^* \ell \nu$ are needed, in addition to the $B_c \to D$ form factors presented here and $B_c \to J/\psi$ form factors published in Ref. [8].

$B_c^+ \to D_s^+ \ell^+ \ell^- (\bar{\nu}\nu)$ decay rates

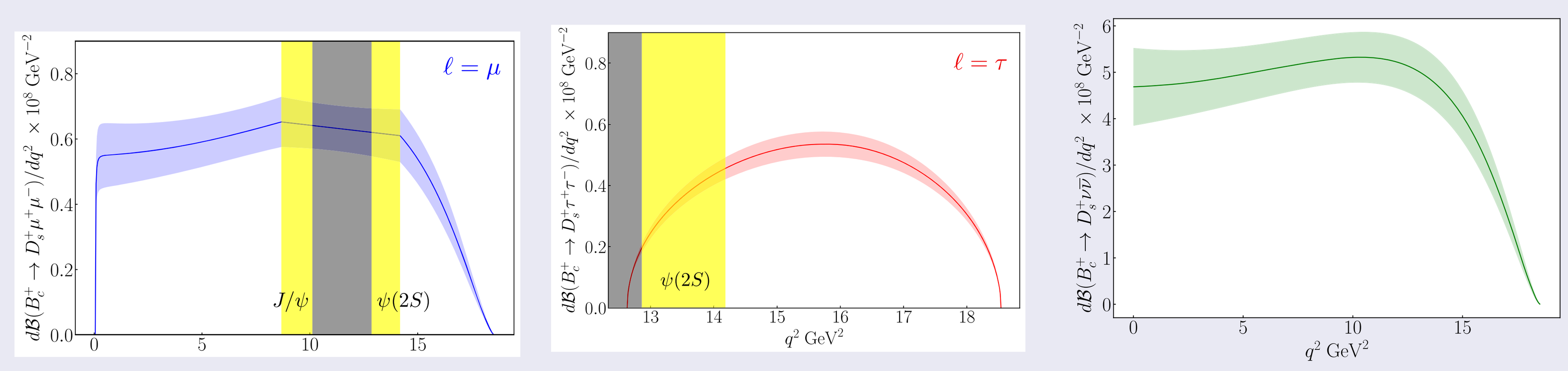

Figure: Decay rates respectively for $B_c \to D_s \mu^+ \mu^- / D_s \tau^+ \tau^- / D_s \bar{\nu} \nu$

Acknowledgements

We are grateful to the MILC collaboration for gauge configurations and code MILC-7.7.11 [9]. Computations were performed using the Cambridge Service for Data Driven Discovery (CSD3), part of which is operated by the University of Cambridge Research Computing on behalf of the STFC DiRAC HPC Facility. This work was partially supported by STFC consolidated grants.

References

[1] L. J. Cooper, C. T. H. Davies, and M. Wingate (HPQCD), Phys. Rev. D **105**, 014503 (2022), [arXiv:2108.11242].
[2] A. Bazavov *et al.* (MILC), Phys. Rev.Ḋ **93**, 094510 (2016), [arXiv:1503.02769].
[3] S. Borsanyi *et al.*, J. High Energy Phys. **09**, 010 (2012), [arXiv:1203.4469].
[4] R. J. Dowdall, C. T. H. Davies, G. P. Lepage, and C. McNeile, Phys. Rev. D **88**, 074504 (2013), [arXiv:1303.1670].
[5] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. D **98**, 074512 (2018), [arXiv:1712.09262].
[6] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **60**, 014001(1999), [arXiv:hep-ph/9812358].
[7] B. Delaney, *Determination of the CKM ratio $|V_{ub}|/|V_{cb}|$ using semileptonic B_c^+ decays at LHCb*, PhD thesis, University of Cambridge, privately communicated.
[8] J. Harrison, C. T. H. Davies, and A. Lytle, Phys. Rev. D **102**, 094518 (2020), [arXiv:2007.06957].
[9] MILC Code Repository, https://github.com/milc-qcd.