

B -meson Decay Constants Using RHQ

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- RBC/UKQCD collaboration

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Introduction

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Introduction

- Leptonic decays of B mesons are important in tests of the Standard Model and beyond

$$\Gamma(B^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 |V_{ub}|^2 f_B^2$$

- Experiment measures $\Gamma(B \rightarrow \ell \nu_\ell)$ ➔ higher precision needed
- A pseudoscalar decay constant is defined in the axial current matrix element:

$$\langle 0 | \mathcal{A}_\mu | B_q \rangle = i f_{B_q} p_\mu, \quad \mathcal{A}_\mu = \bar{b} \gamma_\mu \gamma_5 q$$

- For a precise determination of $|V_{ub}|^2$, we also need f_B
- f_B also used in other SM predictions:

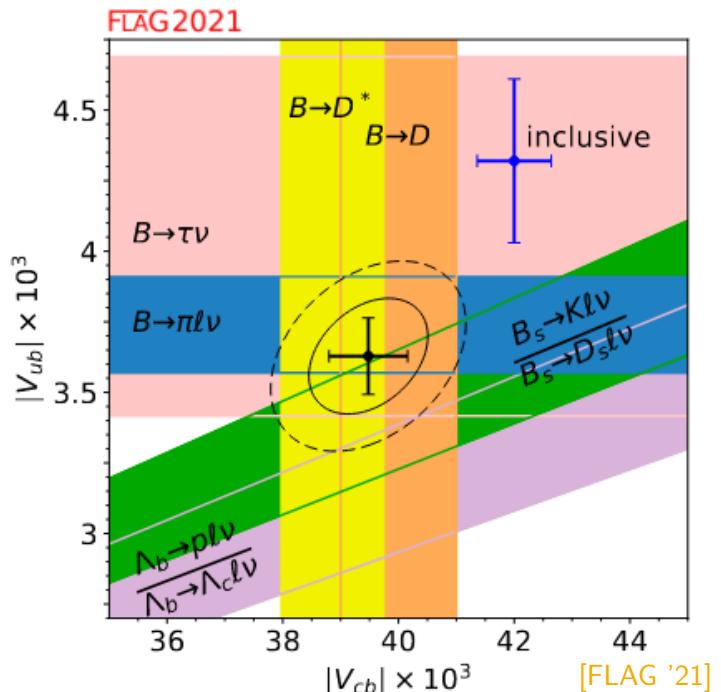
↳ B -mixing Bag parameters, e.g. $\langle Q_1 \rangle$, $q = d, s$

$$\langle Q_1 \rangle = \langle B_q | Q_1 | \bar{B}_q \rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 B_1(\mu_b)$$

➔ FCNC B -meson leptonic decays, $B_q \rightarrow \ell^+ \ell^-$

$$\Gamma(B_q \rightarrow \ell^+ \ell^-) = \frac{G_F^2 m_{B_q} m_\ell^2}{\pi} Y \left(\frac{\alpha}{4\pi \sin^2 \Theta_W} \right)^2 \left(1 - 4 \frac{m_\ell^2}{m_{B_q}^2} \right)^{1/2} |V_{tb}^* V_{tq}|^2 f_{B_q}^2$$

- Current tensions in $|V_{ub}|$, $|V_{cb}|$
- Inclusive semileptonic determination ($B \rightarrow X_{u,c}\ell\nu_\ell$)
 $|V_{ub}| = 4.25(30) \cdot 10^{-3}$, $|V_{cb}| = 42.2(8) \cdot 10^{-3}$
[PDG '21]
vs.
- Exclusive semileptonics, e.g. $B \rightarrow \pi\ell\nu_\ell$
 $|V_{ub}| = 3.63(14) \cdot 10^{-3}$, $|V_{cb}| = 39.48(68) \cdot 10^{-3}$
[FLAG '21]
- Leptonic modes (e.g. $B \rightarrow \tau\nu$) are not measured precisely enough to be competitive

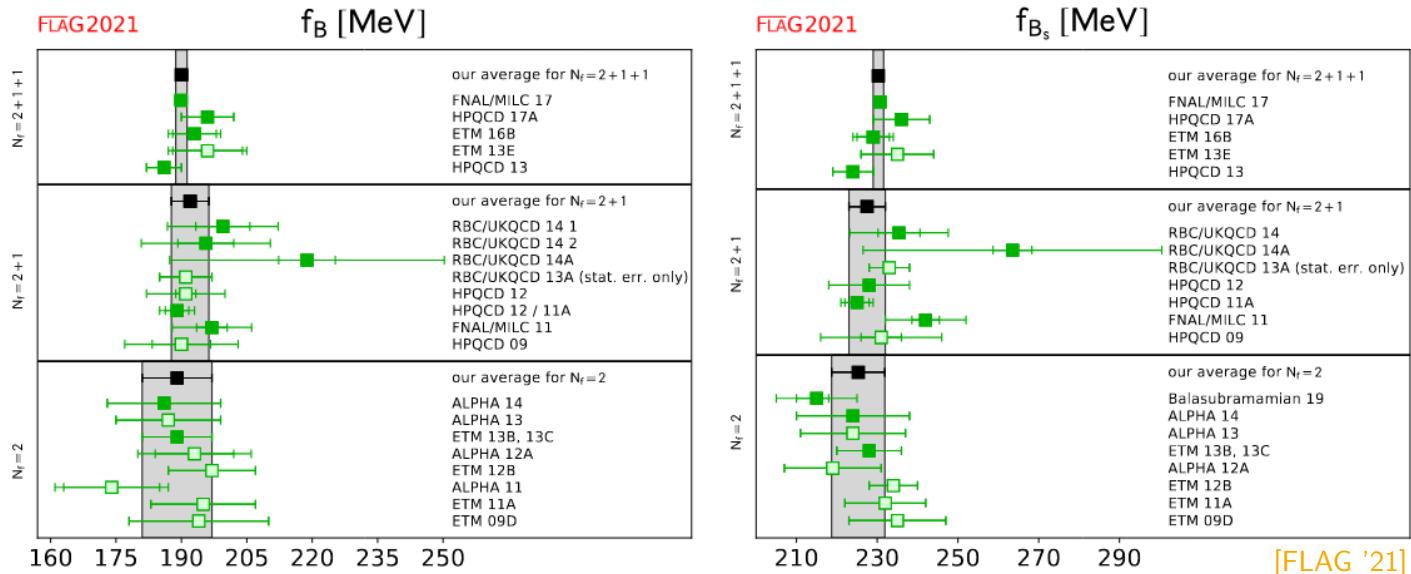


	[Gelhausen et al. '13]	[Lucha et al. '10]	[Pullin, Zwicky '21]
$f_B :$	207^{+17}_{-9} MeV	$193.4(16.6)$ MeV	192^{+20}_{-19} MeV
$f_{B_s} :$	242^{+17}_{-12} MeV	$232.5(21.0)$ MeV	225^{+21}_{-20} MeV
$f_{B^*} :$	210^{+10}_{-12} MeV		209^{+23}_{-22} MeV
$f_{B_s^*} :$	251^{+14}_{-16} MeV		245^{+24}_{-23} MeV
$f_{B^T} :$			200^{+21}_{-20} MeV
$f_{B_s^T} :$			236^{+22}_{-21} MeV

- Pseudoscalar decay constants determined to $\mathcal{O}(10\%)$ precision
- First calculations of tensor decay constants ➔ tbd on the lattice..?

Current Status – Pseudoscalar Decay Constants (Lattice)

5



→ $f_B = 190(1.3) \text{ MeV}$ [FLAG '21]

→ $f_{B_s} = 230.3(1.3) \text{ MeV}$ [FLAG '21]

→ $f_{B_c} = 434(15) \text{ MeV}$ [HPQCD '15]

→ $f_{B_s}/f_B = 1.209(5)$ [FLAG '21]

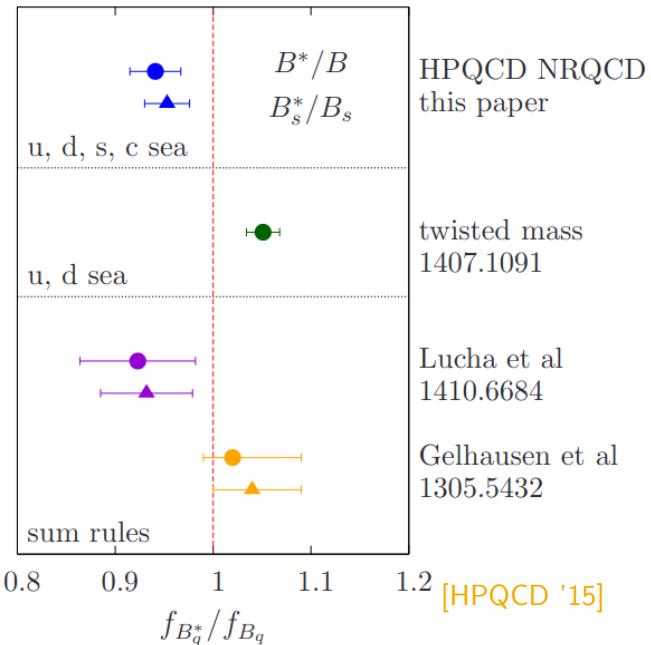
- Fewer Lattice collaborations have calculated f_{B^*}

➥ tension between HPQCD and ETMC

- Vector meson decay constant related to the matrix element:

$$\langle 0 | \mathcal{V}_i | B_j^*(0) \rangle = f_{B^*} M_{B^*} \delta_{ij}, \quad i, j = x, y, z$$

- We aim to provide a new independent lattice calculation



Lattice Setup

B-meson Decay Constants
Using RHQ

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- We use RBC-UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles
- Light, strange, and charm valence quarks simulated as DWFs
- For light quarks (u,d,s), $am_q \ll 1 \Rightarrow$ similar for c quarks, $am_c \sim 0.1 - 0.5$
- For heavy b quarks, $am_b > 1 \Rightarrow$ large discretisation effects
- To simulate b quarks on current lattices, either:
 - ↳ Extrapolate from multiple charm-like masses
 - ↳ Use an effective action tuned to physical values
- Use Relativistic Heavy Quarks [Christ, Lin '06] [Christ, Li, Lin '06]
 - ↳ Variant of Fermilab action [El Khadra et al. '96]
 - ↳ Non-perturbatively tune parameters [Aoki et al. '12]
 - ↳ To reduce cut-off effects, calculate $O(a)$ -improvement terms to (axial-)vector currents
 - ↳ Mostly non-perturbative renormalisation [El Khadra et al. '01]

$$Z_\phi = \rho_A^{bl} \sqrt{Z_V^{ll} Z_V^{bb}}$$

- Extract f_B from non-perturbative matrix elements calculated in Lattice QCD

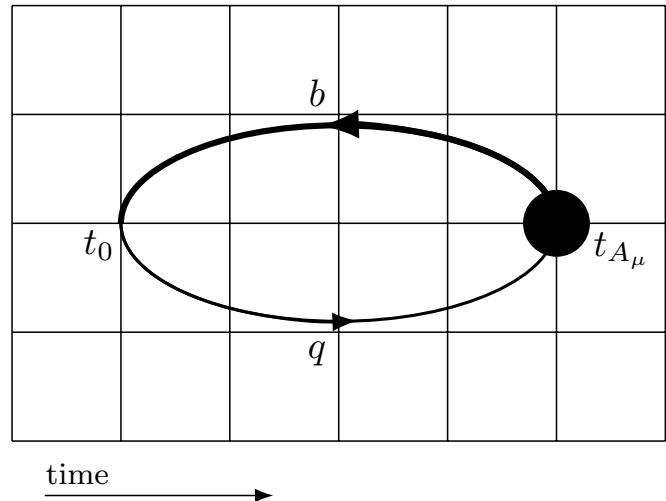
$$\langle 0 | \mathcal{A}_\mu | B_q(0) \rangle = i f_{B_q} M_{B_q}, \quad \mathcal{A}_\mu = \bar{b} \gamma_\mu \gamma_5 q$$

- For operators $\mathcal{O}_J = \bar{b} \Gamma_J q$, we calculate

$$C_{IJ}(t, t_0) = \sum_{\vec{y}} \langle \mathcal{O}_I^\dagger(\vec{y}, t) \mathcal{O}_J(\vec{0}, t_0) \rangle = \sum_n \frac{\langle B_{q,n} | \Gamma_I^\dagger | 0 \rangle \langle 0 | \Gamma_J | B_{q,n} \rangle}{2E_{B_{q,n}}} (e^{-E_{B_{q,n}}(t-t_0)} + e^{-E_{B_{q,n}}(T-t)})$$

- Fit data to $C_{IJ}(t, t_0)$

- for $t \gg t_0$, ground state dominates
- can also include excited states in fit



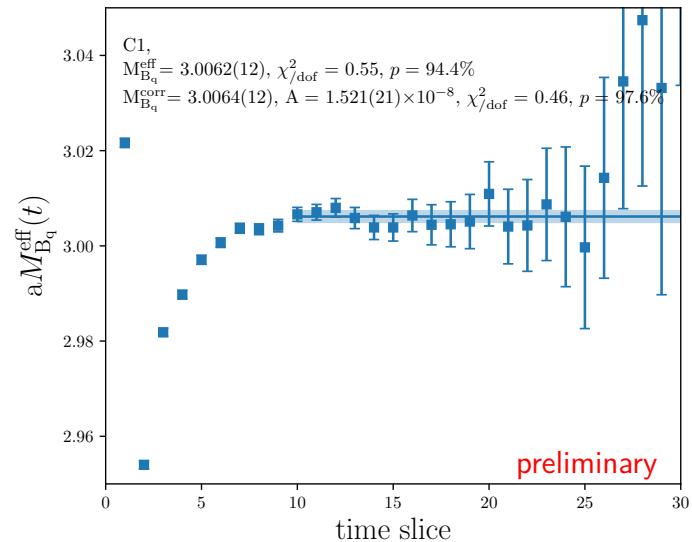
Work In Progress:

- We start with existing data to set up analysis
- Now creating new data with better statistics and at a finer lattice spacing
- Plateaus, signals etc to be settled once we have a full dataset
- y-ticks removed from plots

- Consider the *effective mass*:

$$M_{B_q}^{\text{eff}} = \lim_{t_0 \ll t \ll T} \cosh^{-1} \left(\frac{C_{IJ}(t, t_0) + C_{IJ}(t+2, t_0)}{2C_{IJ}(t+1, t_0)} \right)$$

- Earlier time: smaller statistical error, excited state contamination
- Later time: more noise, excited states decayed
- We **do not** yet include excited states in fits



- To find the decay constant,

$$\langle 0 | \mathcal{A}_0 | B_q(0) \rangle = i f_{B_q} M_{B_q},$$

we look for the decay amplitude

$$\Phi_{B_q} = f_{B_q} \sqrt{M_{B_q}}$$

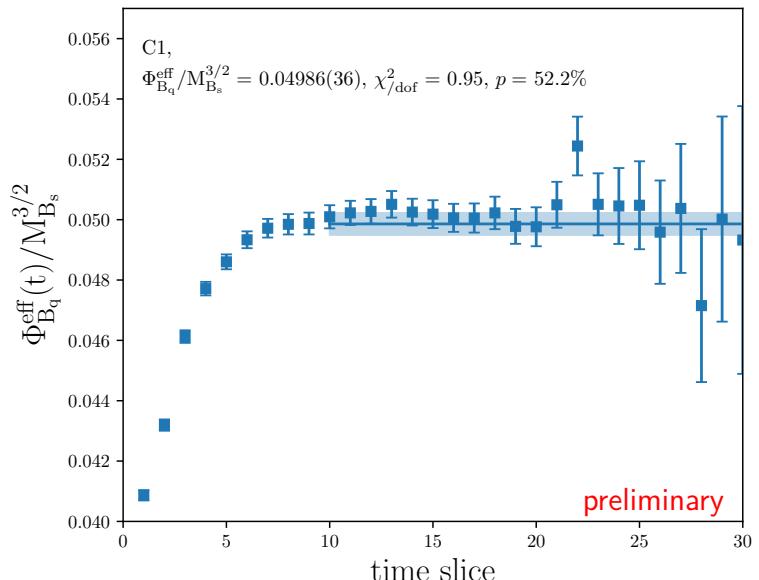
- Form a ratio of correlators:

$$\Phi_{B_q} = \sqrt{2} Z_\Phi \lim_{t_0 \ll t \ll T} \frac{|C_{A_0 P}(t, t_0) + c_A C_{A_0^{(1)} P}(t, t_0)|}{\sqrt{C_{PP}(t, t_0) e^{-M_{B_q}(t-t_0)}}}$$

$$\mathcal{O}_P = \bar{b} \gamma_5 q, \quad \mathcal{O}_{A_0} = \bar{b} \gamma_0 \gamma_5 q$$

$$\mathcal{O}_{A_0^{(1)}} = \bar{b} \gamma_0 \gamma_5 \sum_i \gamma_i (2 \overleftrightarrow{D}_i) q$$

- $\mathcal{O}_{A_0^{(1)}}$ → $O(a)$ improvement term

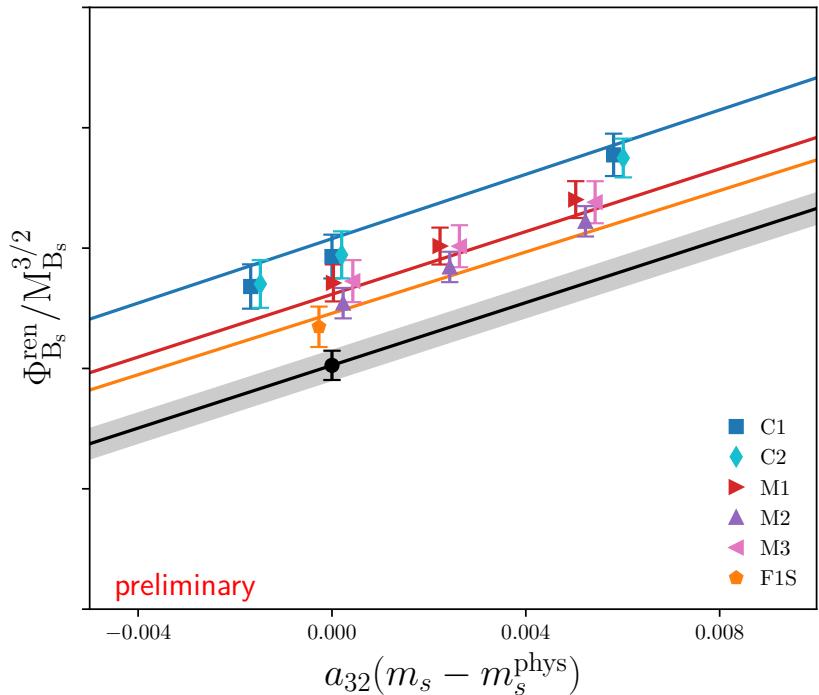


- Combine all lattice data in a global fit
- Simultaneously extrapolate to physical strange mass and continuum limit
- Fit ansatz:

$$\Phi = \varphi + \alpha a^2 + \beta \Delta m_s$$

$$\Delta m_s = (m_s - m_s^{\text{phys}}) / m_s^{\text{phys}}$$

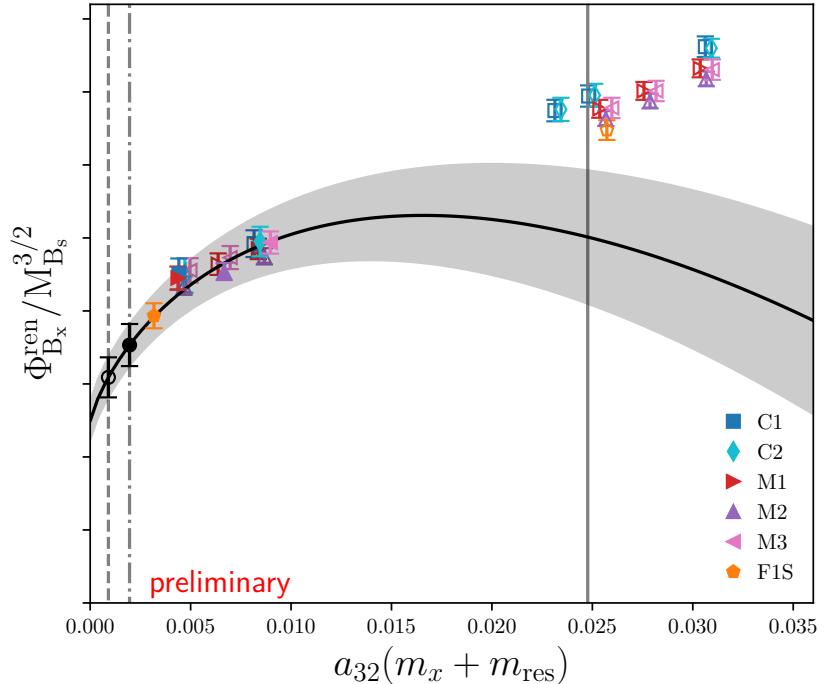
- 3 lattice spacings, up to 3 different light sea quark masses



*statistical errors only

- Use SU(2) Heavy Meson Chiral Perturbation Theory
 - ➔ For sufficiently light valence quarks
 - ➔ Fit to *unitary* light masses (solid points)
- Extrapolate to physical pion mass and continuum simultaneously
- HM χ PT ➔ $f_B \rightarrow f_{B_0}, f_{B^+}$ (IB, no QED)

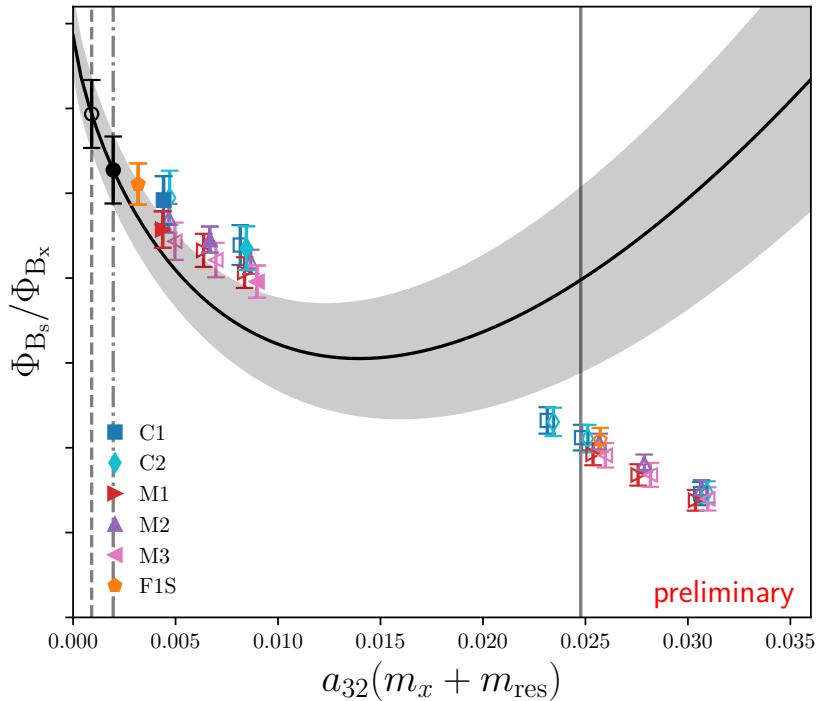
$$\Phi_{B_x} = \Phi_0 \left\{ 1 - \frac{1 + 3g_b^2}{(4\pi f_\pi)^2} \frac{3}{4} \cdot M_{xx}^2 \ln(M_{xx}^2/\Lambda_\chi^2) + c_{\text{sea}} \cdot \frac{4Bm_x}{(4\pi f_\pi)^2} + c_a \cdot \frac{a^2}{(4\pi f_\pi)^2} \right\}$$



*statistical errors only

- Now consider ratio f_{B_s}/f_B
- First get $\Phi_{B_s}(a, m_l)$ from global fit
- Again use $\text{HM}\chi\text{PT}$ for chiral-continuum extrapolation

$$\frac{\Phi_{B_s}}{\Phi_{B_x}} = R_{\Phi^{(2)}} \left\{ 1 + \frac{1 + 3g_b^2}{(4\pi f_\pi)^2} \frac{3}{4} \cdot M_{xx}^2 \ln(M_{xx}^2/\Lambda_\chi^2) \right\} \\ + d_{\text{sea}}^{(2)} \cdot \frac{4Bm_x}{(4\pi f_\pi)^2} + d_a^{(2)} \cdot \frac{a^2}{(4\pi f_\pi)^2}$$



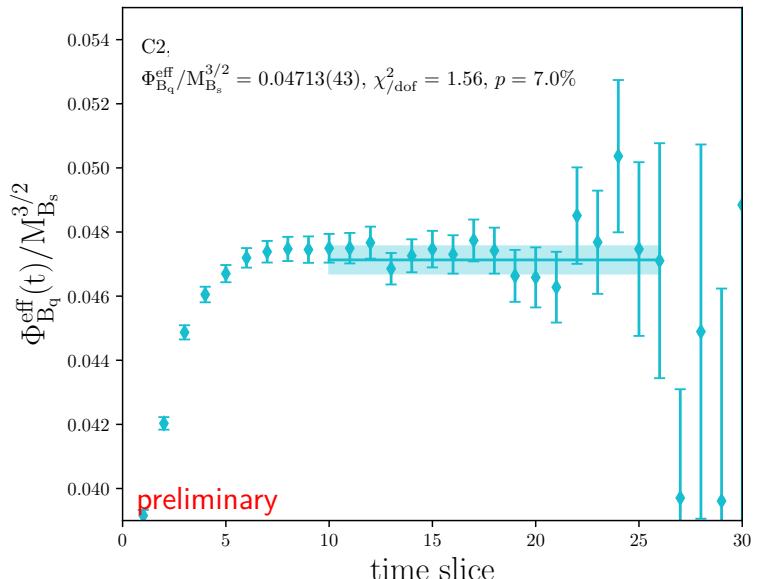
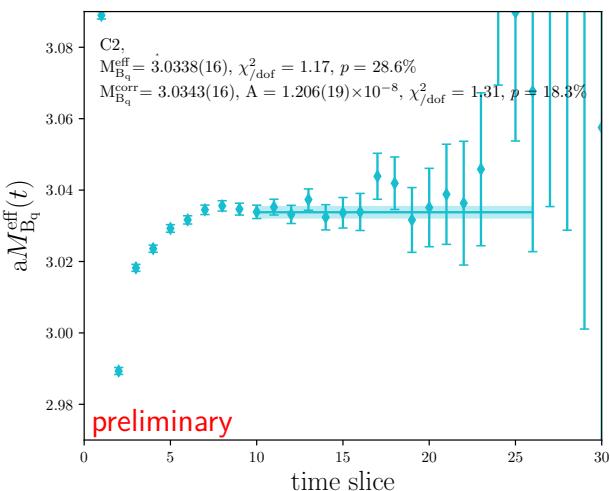
*statistical errors only

- Chiral-continuum fits
 - ➥ exclude/include different data, e.g. partially quenched, introduce M_π cutoff
 - ➥ try different fit Ansätze
- RHQ parameter uncertainty and implicit lattice scale dependence
- Account for sea s-quark mistuning
- Discretisation effects: light quarks, RHQ, gluons
- Renormalisation constants
- Finite volume effects, isospin breaking and QED

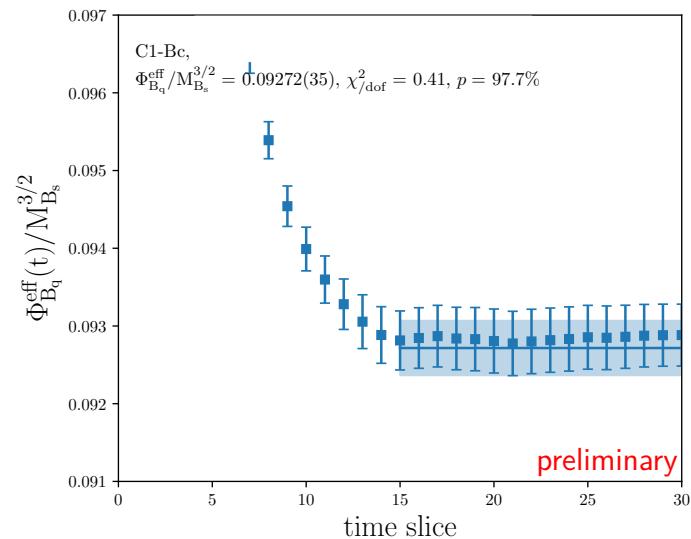
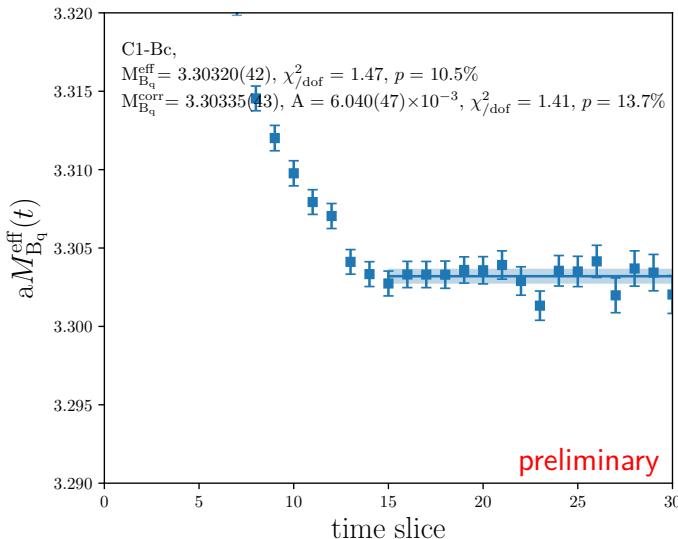
$$\Phi_{B^*} = \sqrt{2} Z_\Phi \lim_{t \gg t_0} \frac{|C_{V_j V_j} + c_s^3 C_{V_j^{(3)} V_j} + c_s^4 C_{V_j^{(4)} V_j}|}{\sqrt{C_{V_j V_j} e^{-M_{B^*}(t-t_0)}}}$$

$$\mathcal{O}_{V_j} = \bar{b} \gamma_j q, \quad \mathcal{O}_{V_j^{(3)}} = \bar{b} \gamma_j \sum_i \gamma_i (2 \not{D}_i) q,$$

$$\mathcal{O}_{V_j^{(4)}} = \bar{b} \gamma_j \sum_i \gamma_i (2 \not{\overline{D}}_i) q$$



- Higher statistical precision for $B_c \rightarrow$ no light valence quarks
- More sensitivity to systematic effects



- Only one lattice calculation for f_{B_c} [HPQCD '15]

- Extrapolate/interpolate to the physical charm mass

- Calculate $\eta_c(1S)$ meson mass (connected)

$$M_{\eta_c} = 2983.9(4) \text{ MeV} \quad [\text{PDG '21}]$$

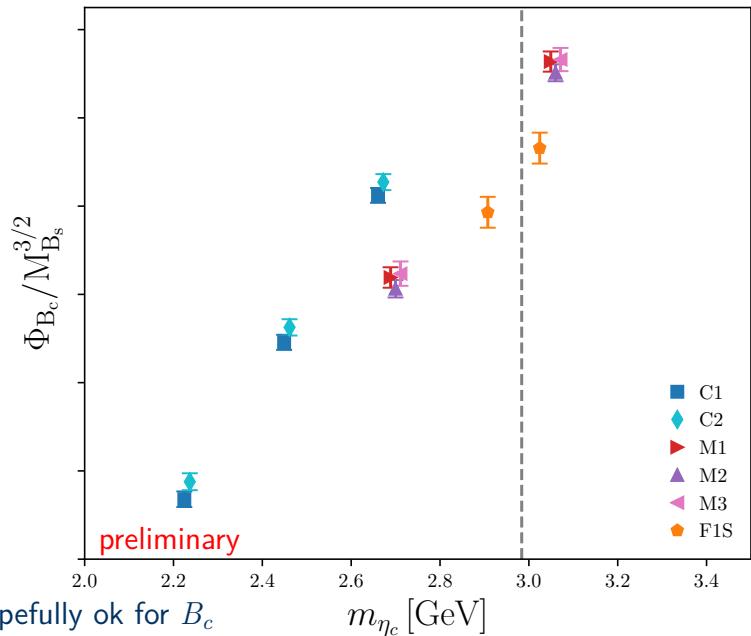
- Large cut-off effects visible on coarsest ensembles

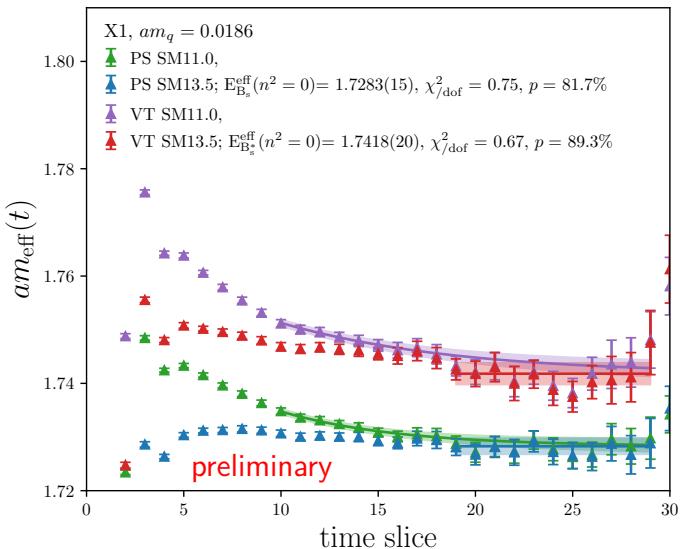
- Introduce a finer ensemble:

- $\rightarrow 32^3 \times 64, M_\pi = 371 \text{ MeV}, a^{-1} = 3.148 \text{ GeV}$

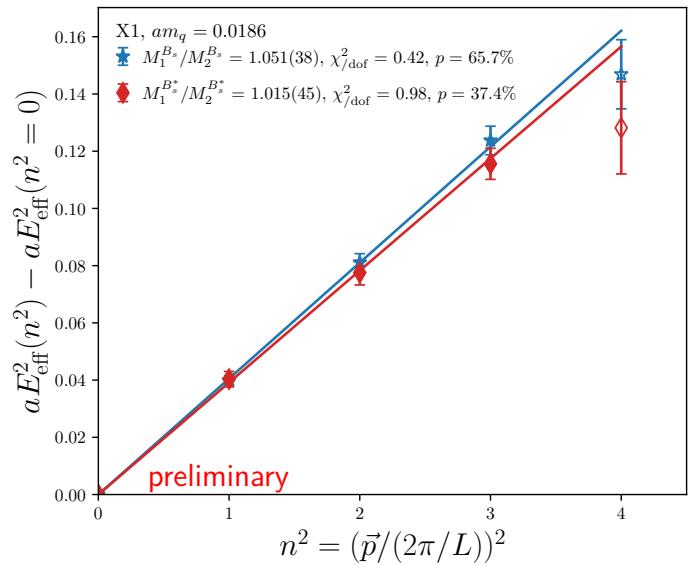
- \rightarrow heavy pions and small physical volume, but hopefully ok for B_c

- \rightarrow RHQ parameter tuning in progress...

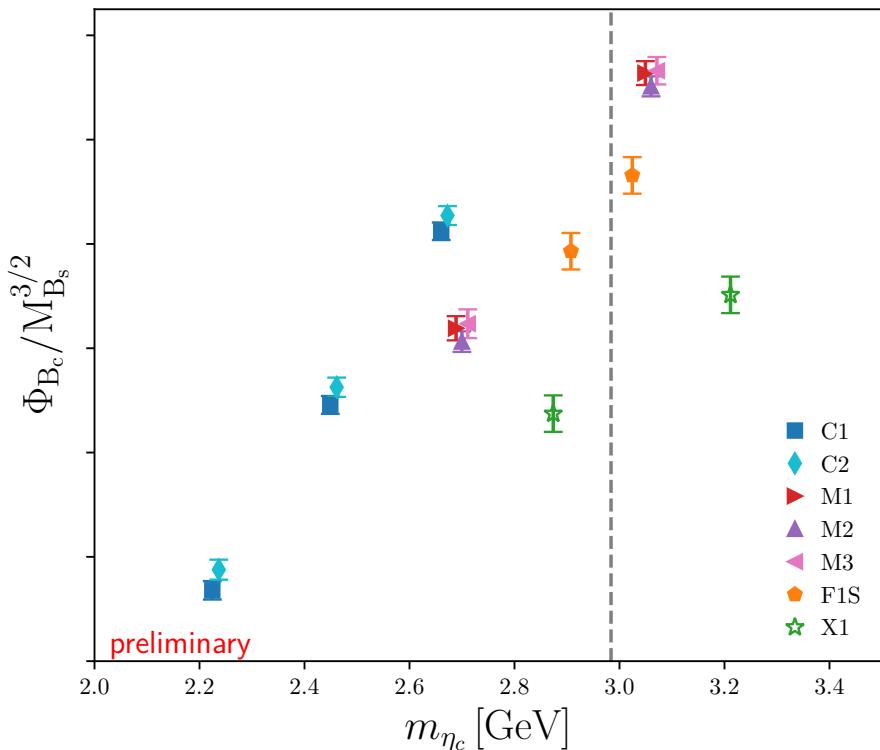




► Aim: tune RHQ parameters to $B_s^{(*)}$ masses



- Fit slope of dispersion relation (for $n^2 \leq 3$)
- Include in analysis for f_B, f_{B_s} etc once tuned



- We present preliminary work to extract B meson decay constants using Lattice QCD
- Some decay constants are well-known in literature ➔ further calculations to compare with and combine
- B_c meson decay constant has only one value in literature ➔ we aim to provide a second
- B^* vector meson decay constant has two calculations in literature from Lattice QCD, but there is tension between them ➔ we look to add another independent calculation
- We look now to further our analysis to extract continuum values for all decay constants

Computing resources:
➔ USQCD
➔ DiRAC, UK
➔ OMNI, Siegen

Thank you for your attention!

Gauge ensembles

	L	T	L_s	a^{-1}/GeV	am_l	am_s^{sea}	am_s^{phys}	M_π/MeV	# cfgs	# sources
C1	24	64	16	1.7848(50)	0.005	0.040	0.03224(18)	340	1636	1
C2	24	64	16	1.7848(50)	0.010	0.040	0.03224(18)	433	1419	1
M1	32	64	16	2.3833(86)	0.004	0.030	0.02477(18)	302	628	2
M2	32	64	16	2.3833(86)	0.006	0.030	0.02477(18)	362	889	2
M3	32	64	16	2.3833(86)	0.008	0.030	0.02477(18)	411	544	2
F1S	48	96	12	2.785(11)	0.002144	0.02144	0.02167(20)	267	98	24
X1	32	64	12	3.148(17)	0.0047	0.0186	0.01852(30)	371	40	16

- [Allton et al. '08]
- [Aoki et al. '10]
- [Blum et al. '14]
- [Boyle et al. '17]