The isentropic equation of state of (2+1)-flavor QCD: An update based on high precision Taylor expansion and Padé-resummed expansion at finite chemical potentials

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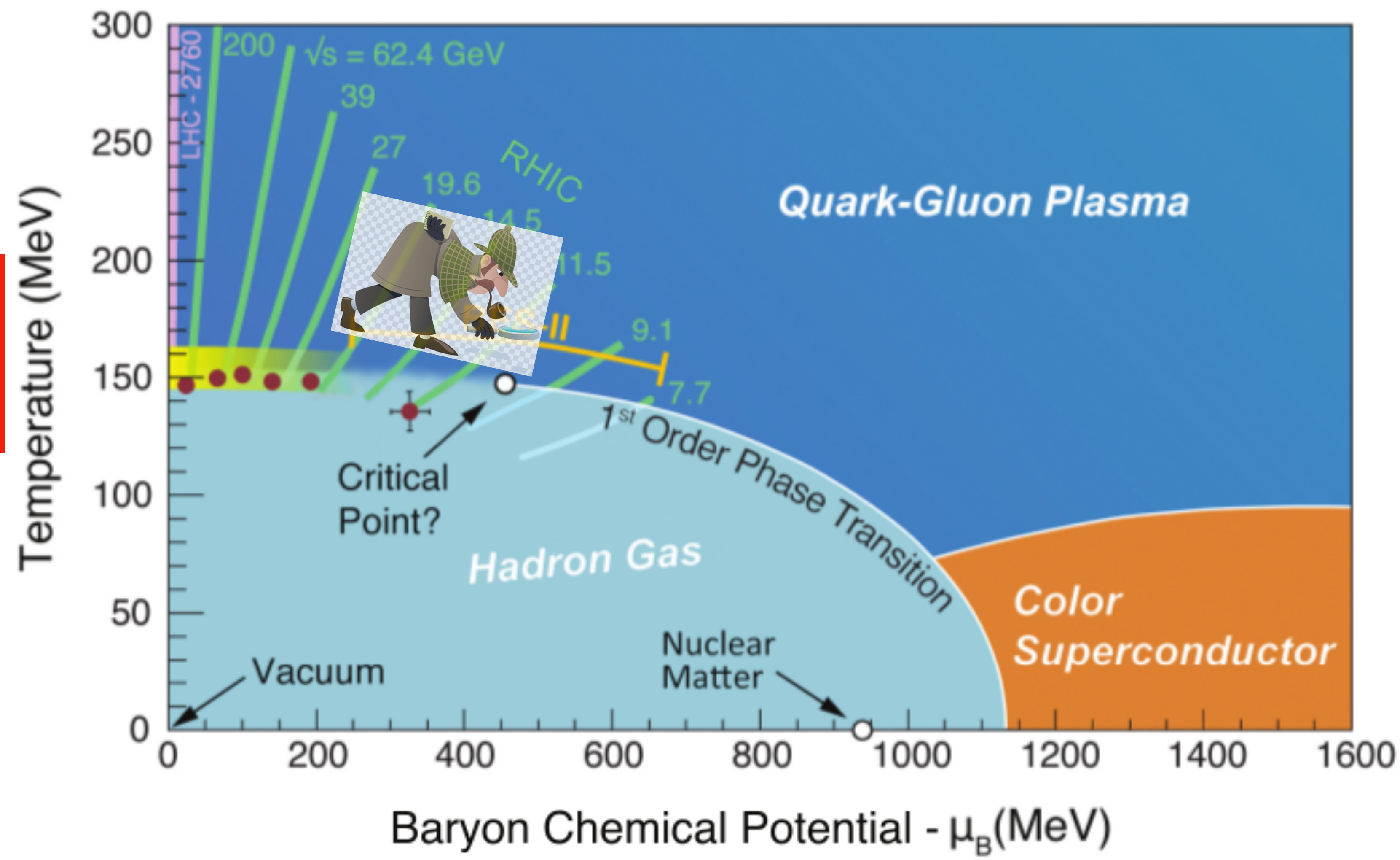
Acknowledgement: To all members of the HotQCD collaboration

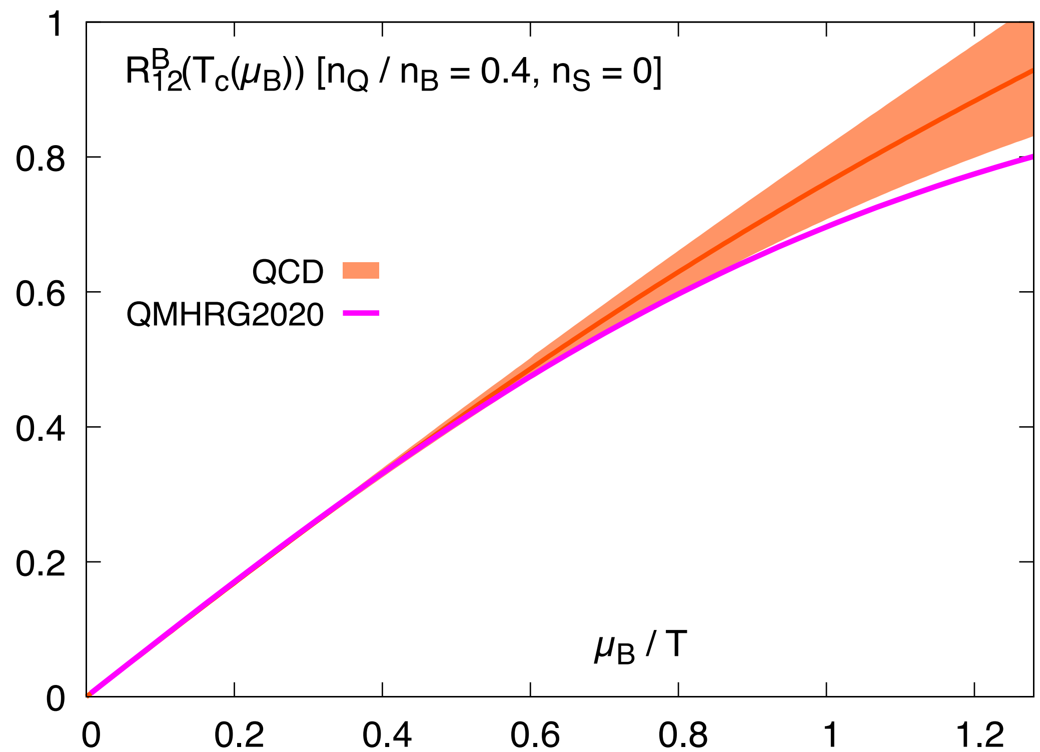
QCD phase diagram

Bigger Picture : Understand the thermodynamics at the QCD crossover, study the QCD phase diagram, indication of the location of the critical point.....

EoS of (2+1)-flavor QCD :
Study of the bulk
Thermodynamic observables

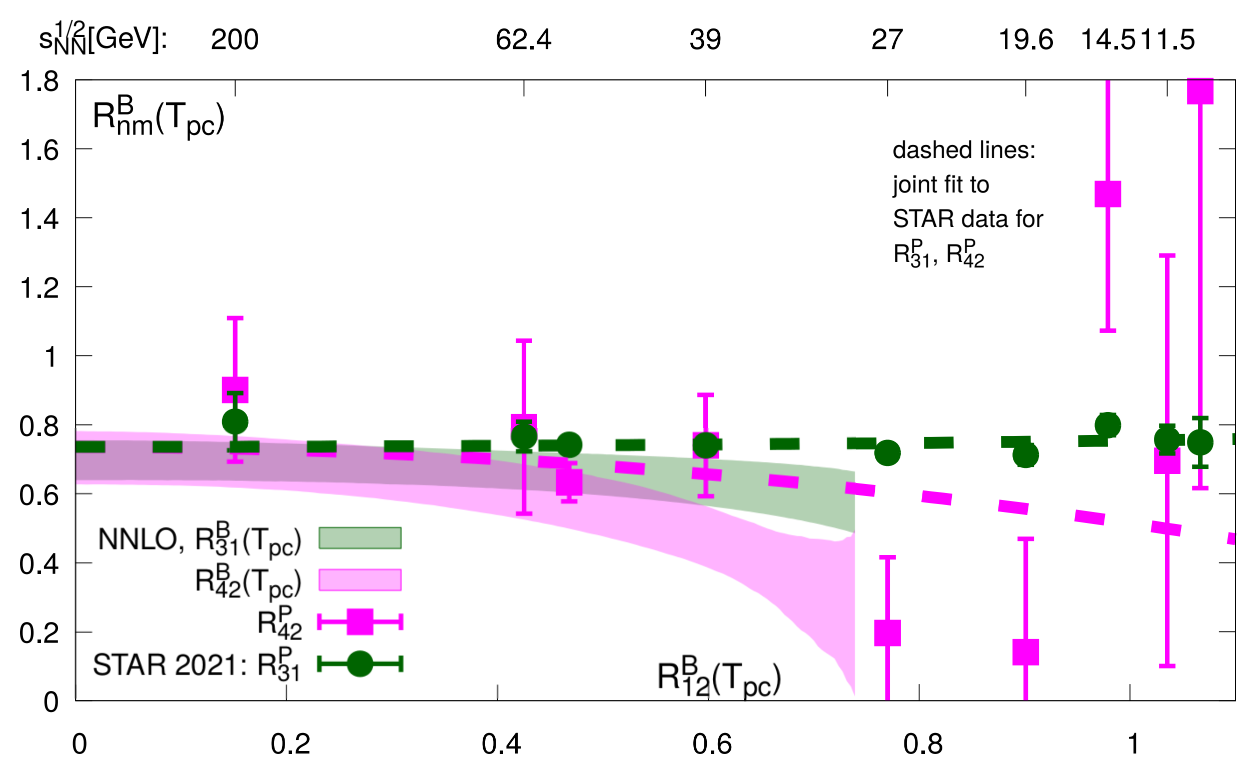


Success and limitation of Taylor expansions



- ❖ **Good control on lower order cumulants at Intermediate baryon chemical potential.**
- ❖ **Good control on higher order cumulants at smaller baryon chemical potential.**
- ❖ **Validity and reliability of Taylor expansion will be limited by its radius of convergence and on the number of terms.**
- ❖ **Calculating higher order terms using brute force method seems unreasonable.**

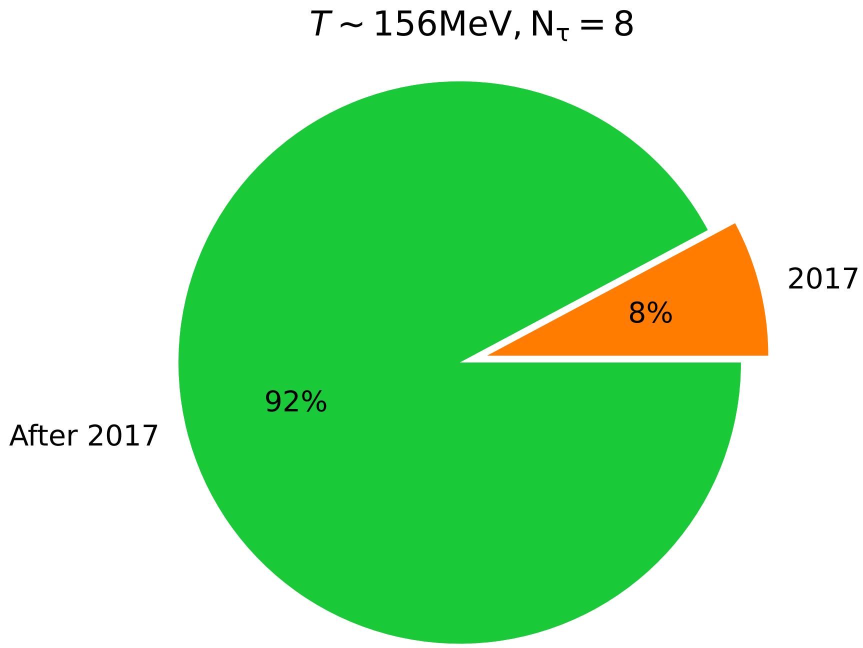
Era of resummation?

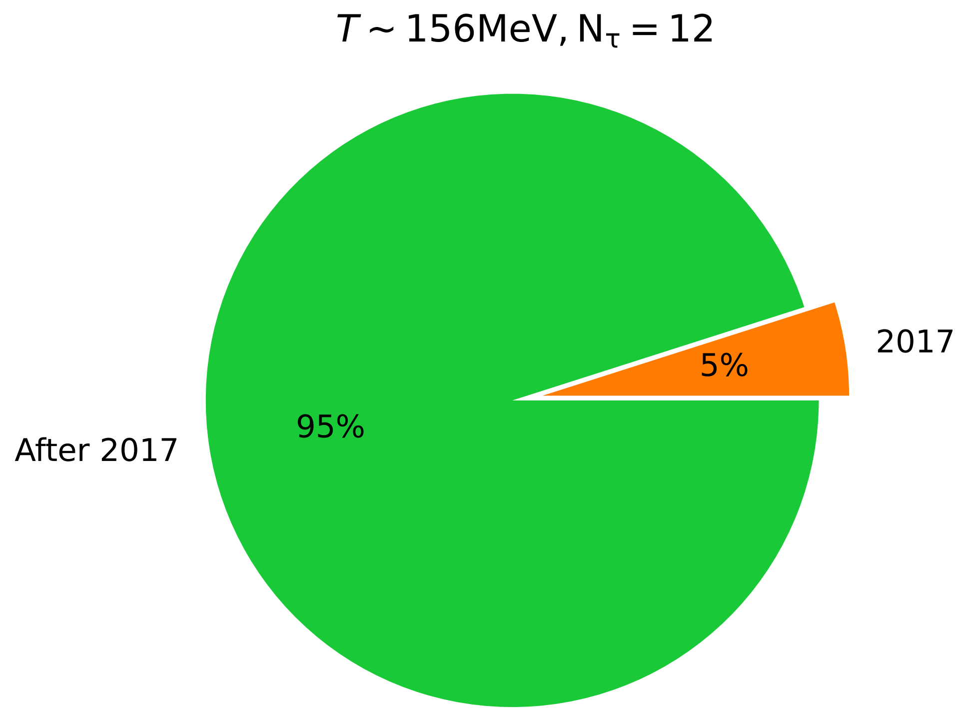


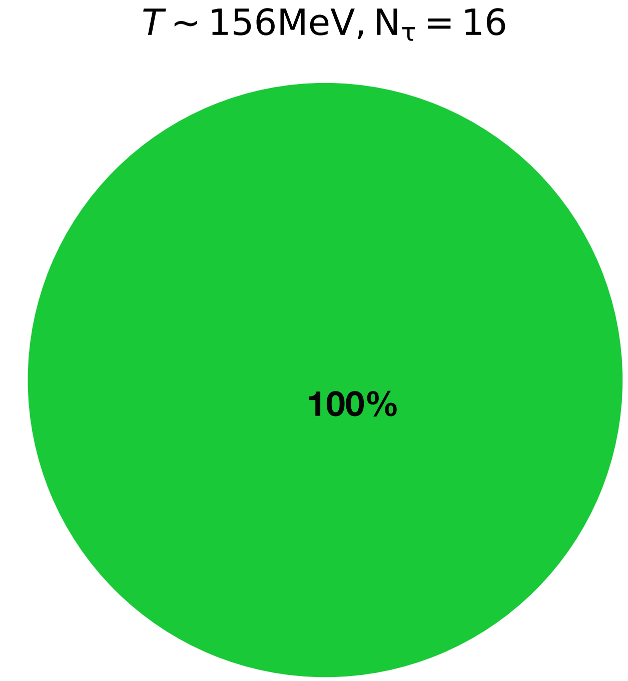
A. Bazavov et al, *Phys.Rev.D* 101 (2020) 7, 074502

EoS: 2017 vs 2022 $[N_\sigma = 4N_\tau]$

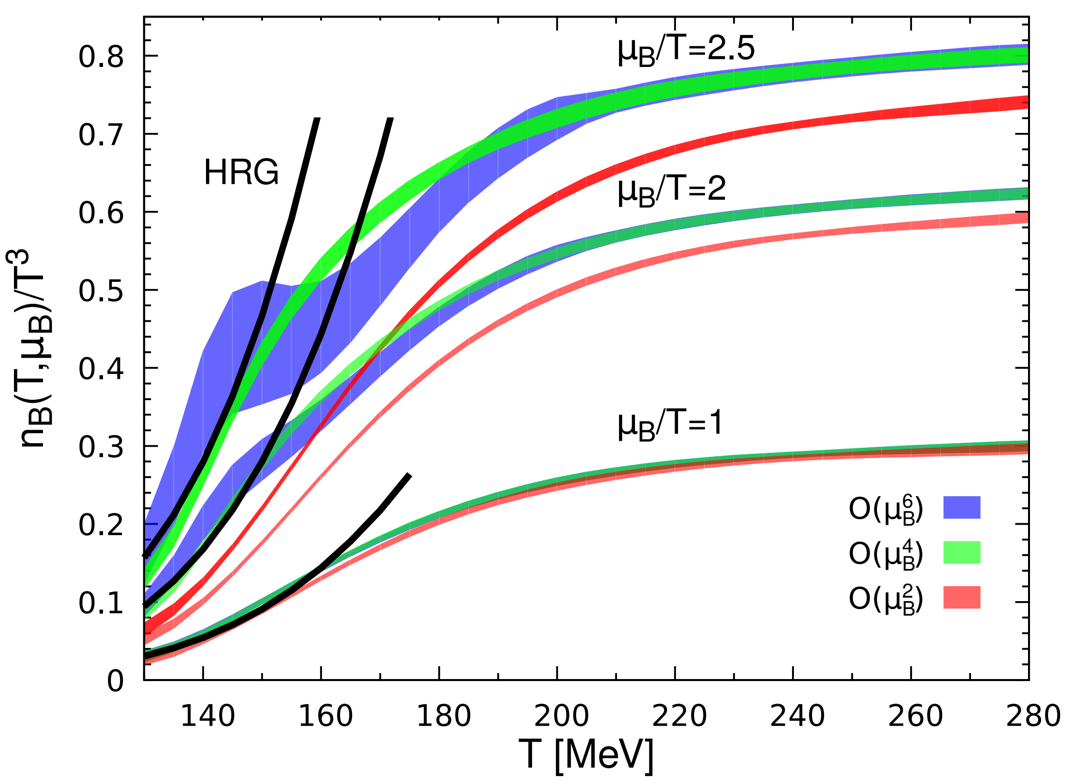
#Gauge configs. = 1602k **300k** **22k**

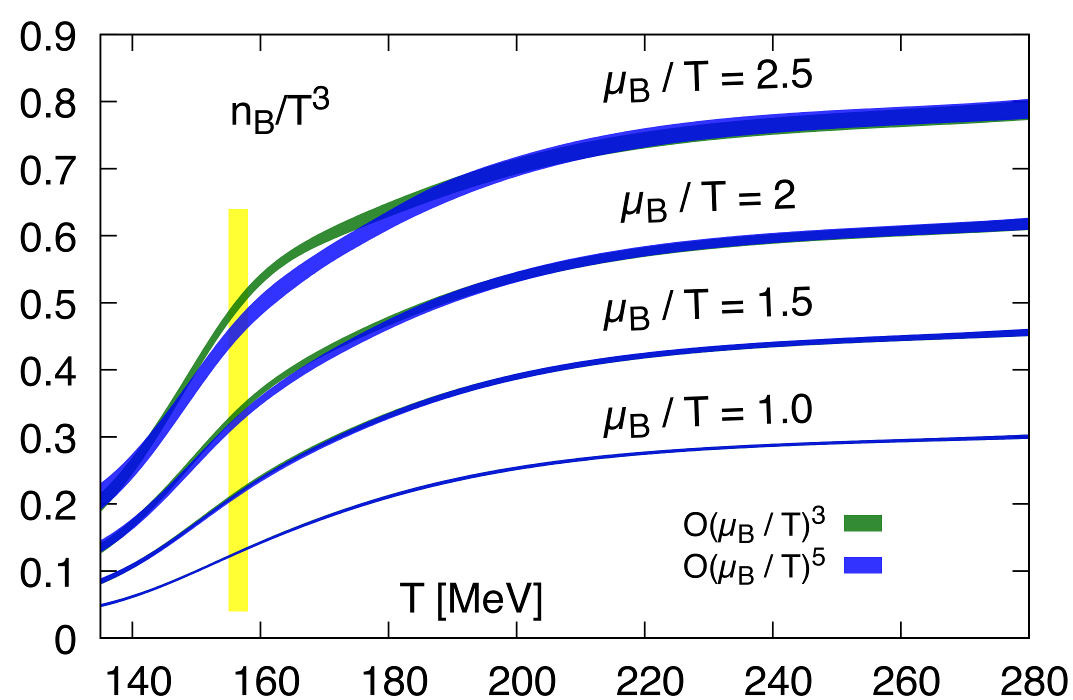






S. Mitra, 11 August, 10:00





Thanks to : Juwels, Marconi, Piz-daint, Summit and Bielefeld gpu clusters.

Thermodynamics using Lattice QCD

Expansion of the QCD pressure:

$$\frac{P(T,\overrightarrow{\mu})}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k \ , \ \hat{\mu} = \mu/T,$$

$$\chi_{ijk}^{BQS}(T,0) = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_X^{i,j,k}} \right|_{\mu_X=0} , \ X = B, Q, S$$

Constrained series expansion :

Parametrization of $\mu_Q,\ \mu_S\ [n_Q/n_B = r, n_S = 0]$ **:**

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + \ldots..$$

$$\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + \ldots..$$

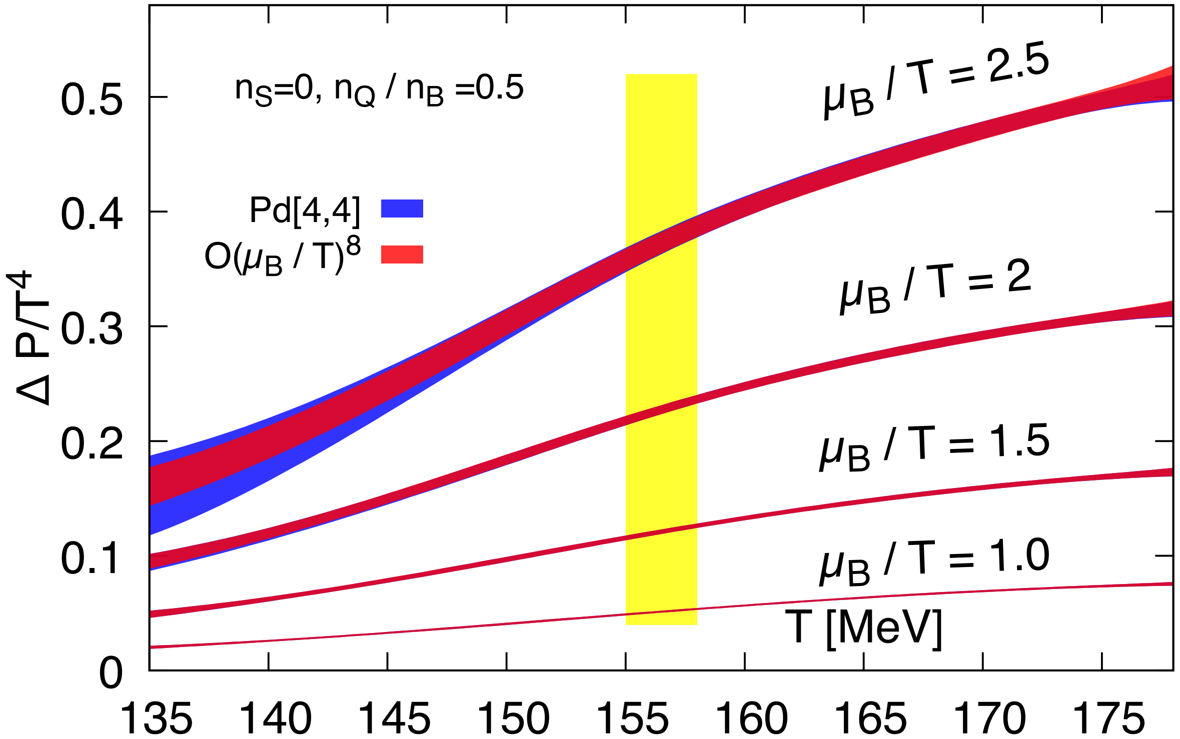
We will discuss, $n_S = 0,\ \ n_Q/n_B = 0.5,\ \ \Rightarrow \mu_B, \mu_S \neq 0,\ \mu_Q = 0,$

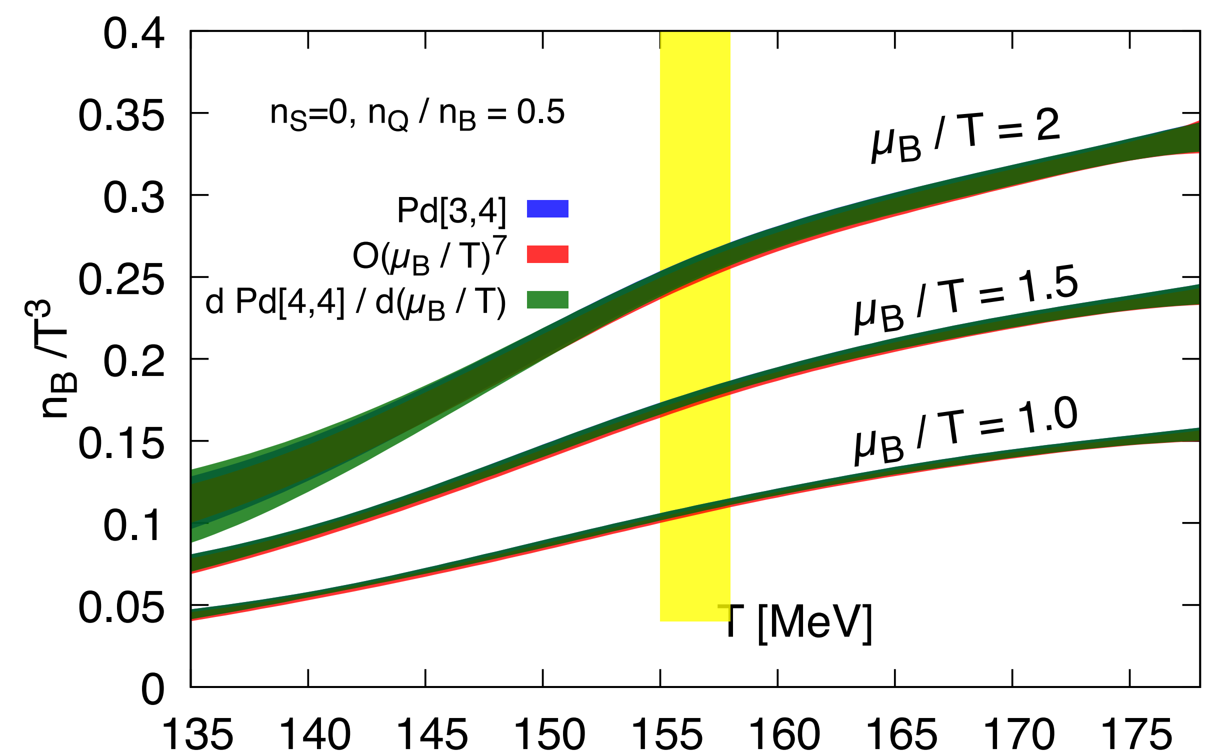
Condition in HIC, $n_S = 0,\ \ n_Q/n_B = 0.4,\ \ \Rightarrow \mu_B, \mu_S \neq 0,\ \mu_Q \ll \mu_B$

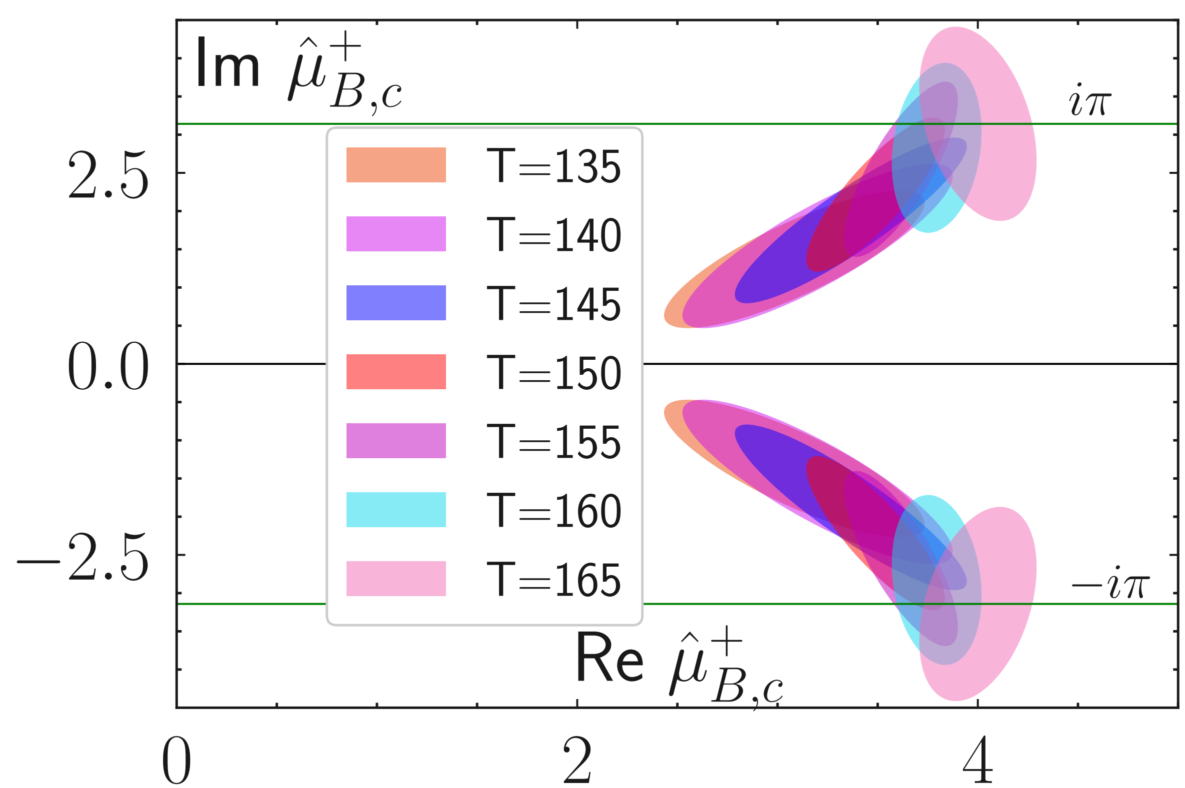
Padé resummed expansion of pressure and number density

Estimated radius of convergence from 8th order pressure series, $T \geq 130$ MeV : $\mu_B/T \leq 2.5$

Poles of the $[4,4]$ Pade approximants : Complex in the temperature range $[135 : 165]$







For , $\mu_Q = 0$, $n_B = \dfrac{d}{d\hat{\mu}_B} \sum_{n=1}^{4} P_{2n} \hat{\mu}_B^{2n}$

Poles of the [4,4] -Padé approximant of pressure

arXiv:2202.09184

Padé resummed expansion of bulk Thermodynamic observables

$n_Q/n_B = 0.5,\ n_S = 0$

Estimated radius of convergence from 8th order Pressure series $T \geq 130$ MeV : $\mu_B/T \leq 2.5$

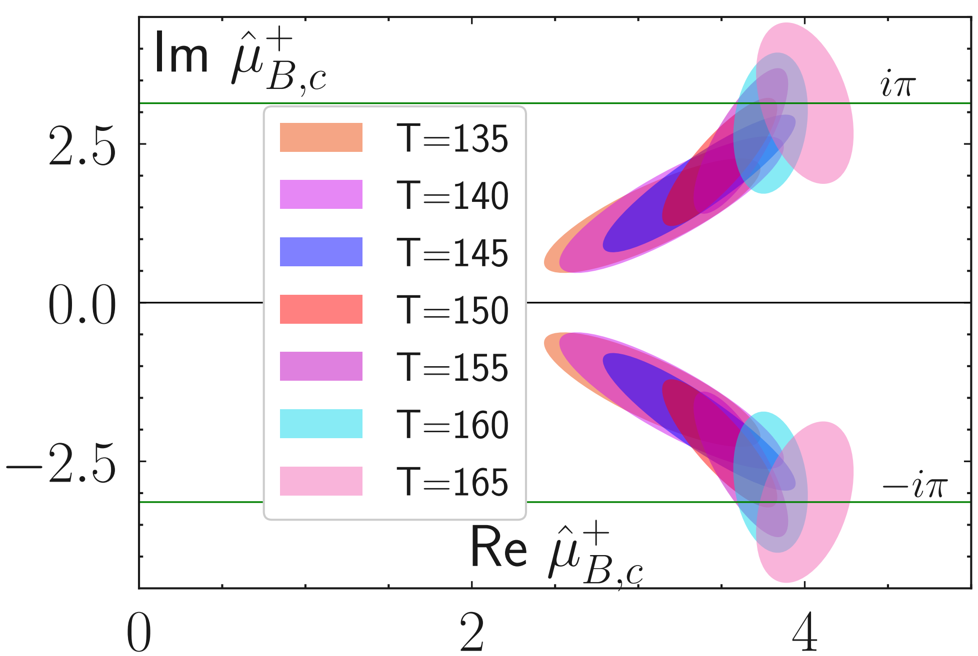
Poles of the $[4{,}4]$ Pade approximants : Complex in the temperature range $[135 : 165]$

Energy density , entropy density and specific heat:

$$\hat{\epsilon}(T,\hat{\mu}_B) = 3\hat{P}(T,\hat{\mu}_B) + T\frac{\partial \hat{P}(T,\hat{\mu}_B)}{\partial T}$$

After separating out the zeroth order,

$$\Delta\hat{\epsilon}(T,\hat{\mu}_B) = 3\Delta\hat{P}(T,\hat{\mu}_B) + T\frac{\partial \Delta\hat{P}(T,\hat{\mu}_B)}{\partial T}$$



Entropy density,

$$\hat{S} = \hat{P} + \hat{\epsilon} - \hat{\mu}_B\hat{n}_B - \cancelto{0}{\hat{\mu}_Q\hat{n}_Q} - \cancelto{0}{\hat{\mu}_S\hat{n}_S}$$

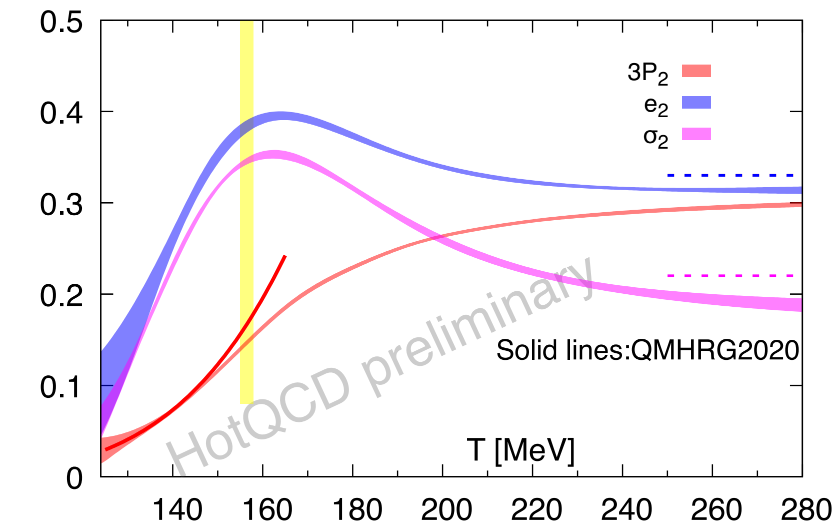
Similarly,

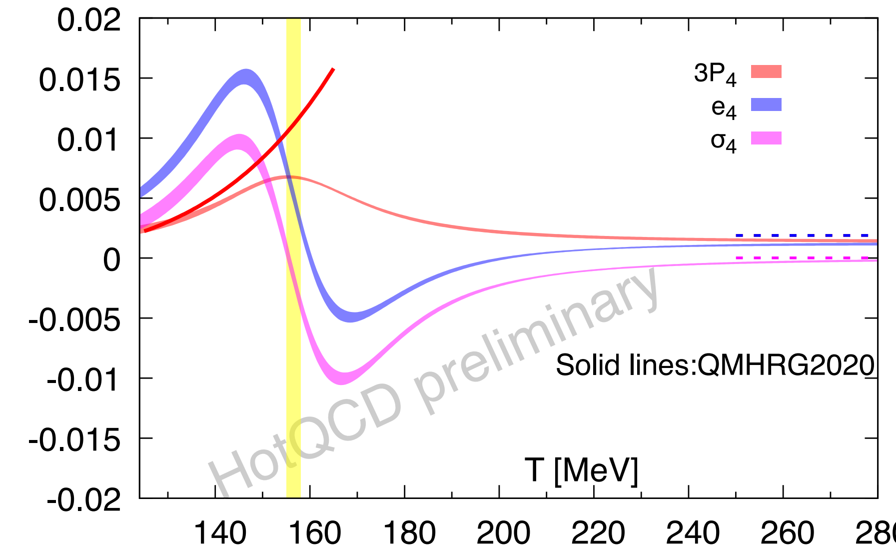
$$\Delta\hat{S} = \Delta\hat{P} + \Delta\hat{\epsilon} - \hat{\mu}_B\hat{n}_B$$

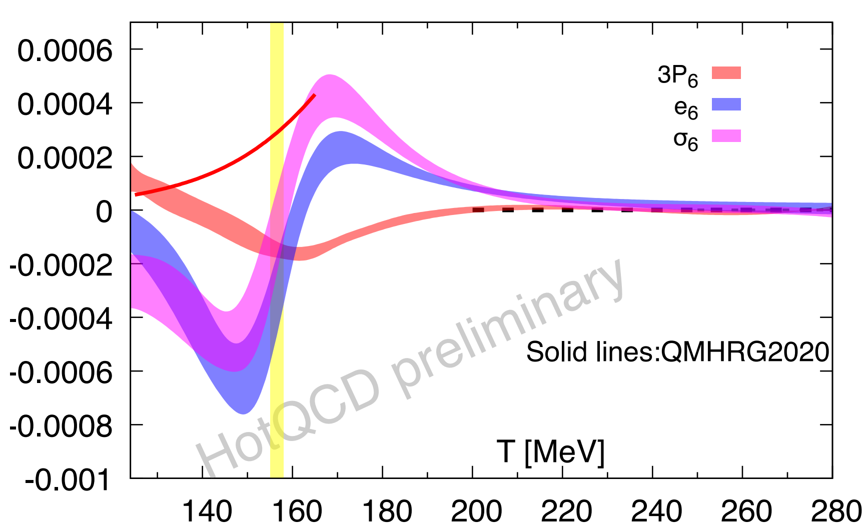
$$\Delta\hat{C}_V = 4\Delta\hat{\epsilon} + T\frac{\partial \Delta\hat{\epsilon}}{\partial T}$$

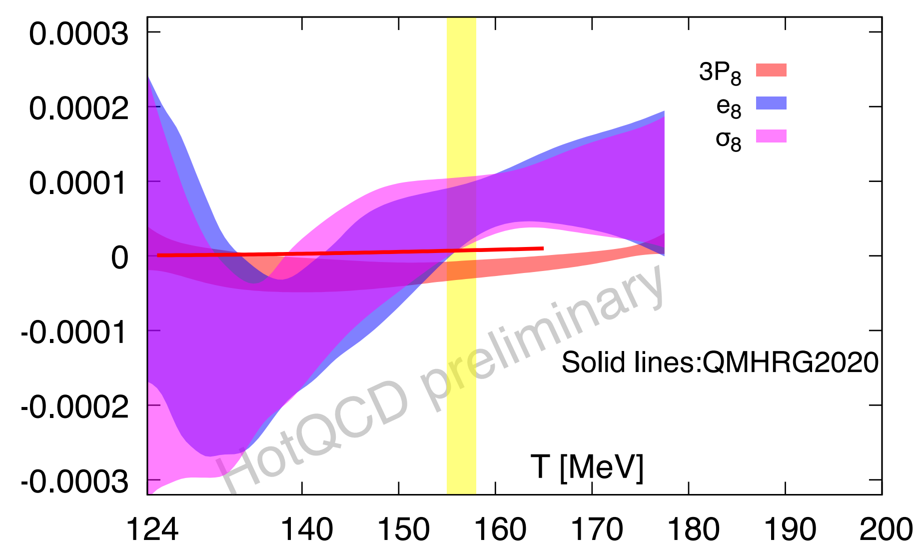
Expansion co-efficients of the Thermodynamic observables

$n_Q/n_B = 0.5, n_S = 0$









We evaluate the bulk Thermodynamic observables at non-zero temperature and baryon chemical potential using these expansion coefficients.

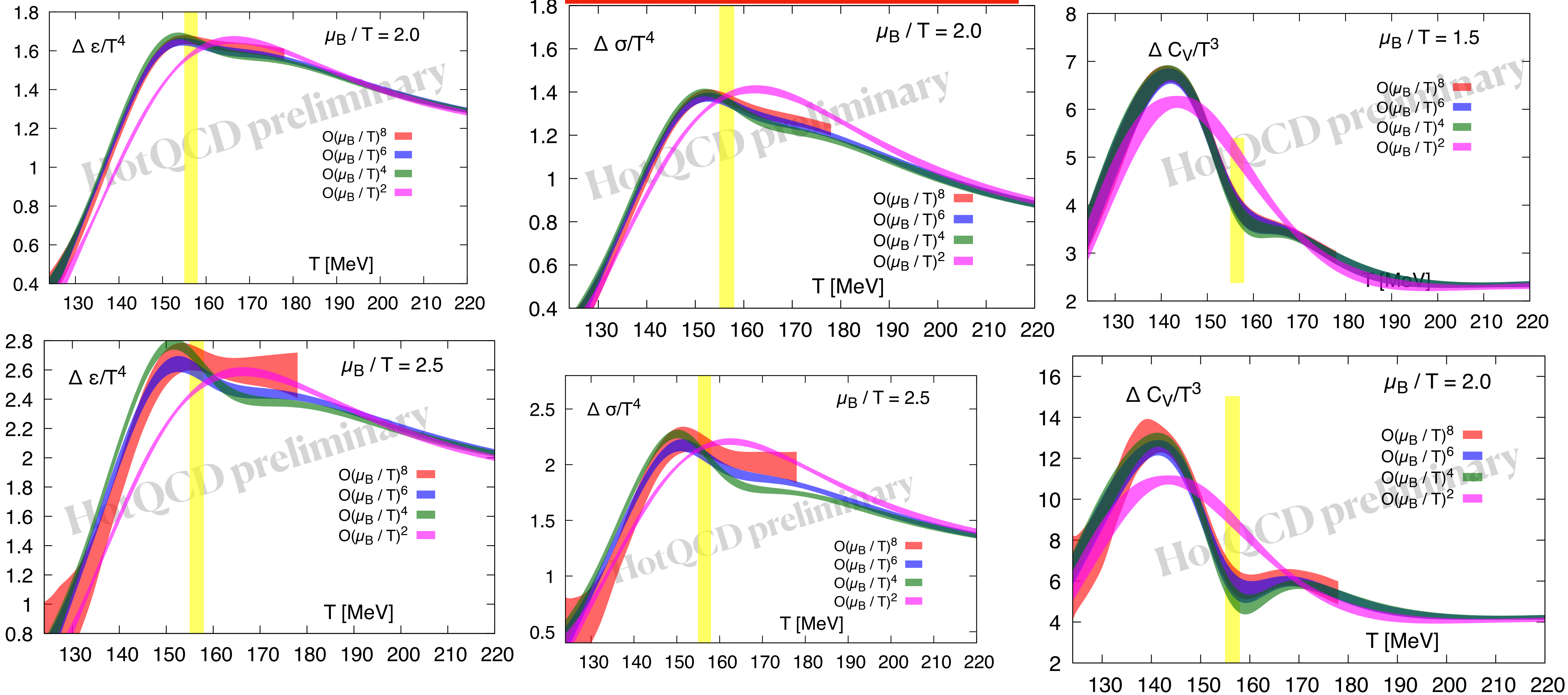
$$\Delta\hat{P} \equiv \frac{P(T,\mu_B) - P(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\hat{\mu}_B^{2k}$$

$$\Delta\hat{\epsilon} \equiv \frac{\epsilon(T,\mu_B) - \epsilon(T,0)}{T^4} = \sum_{k=1}^{\infty} \epsilon_{2k}(T)\hat{\mu}_B^{2k}$$

$$\Delta\hat{S} \equiv \frac{S(T,\mu_B) - S(T,0)}{T^3} = \sum_{k=1}^{\infty} \sigma_{2k}(T)\hat{\mu}_B^{2k}$$

Taylor expansion of bulk Thermodynamic observables

$n_Q/n_B = 0.5,\ n_S = 0$

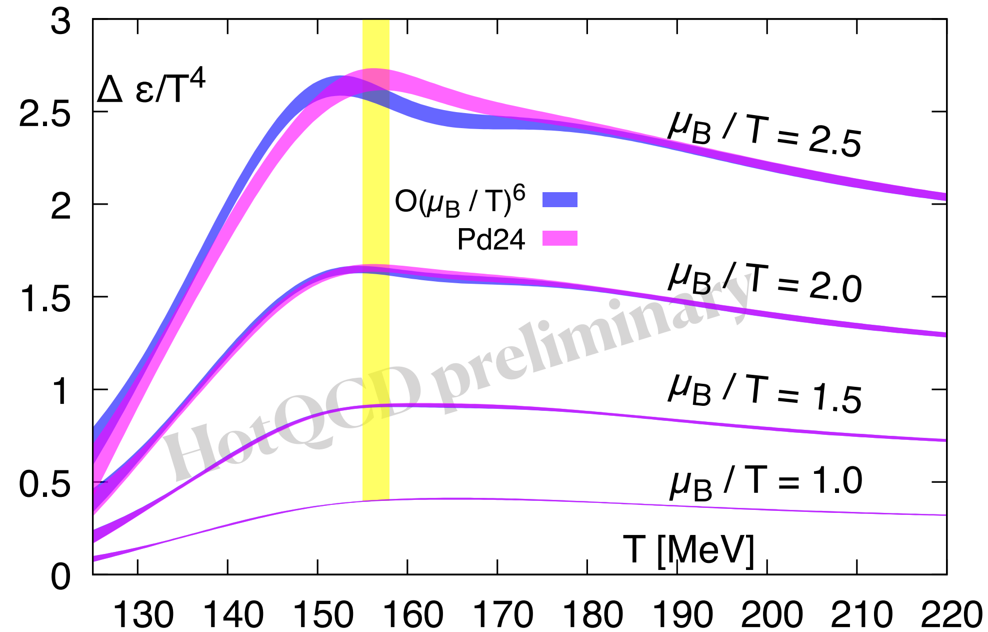


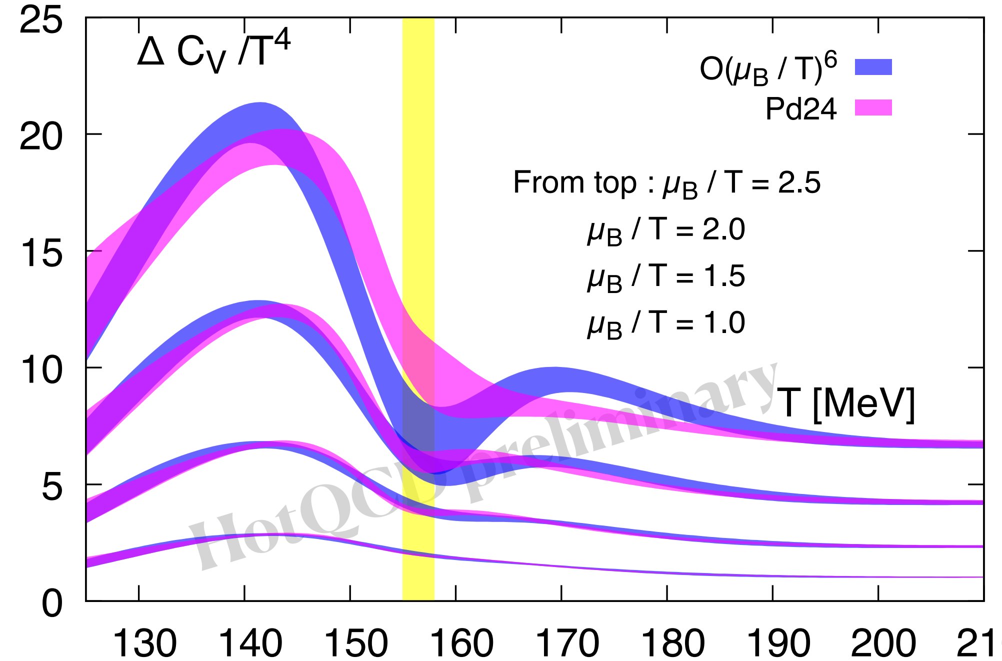
HotQCD collaboration: arXiv:2202.09184

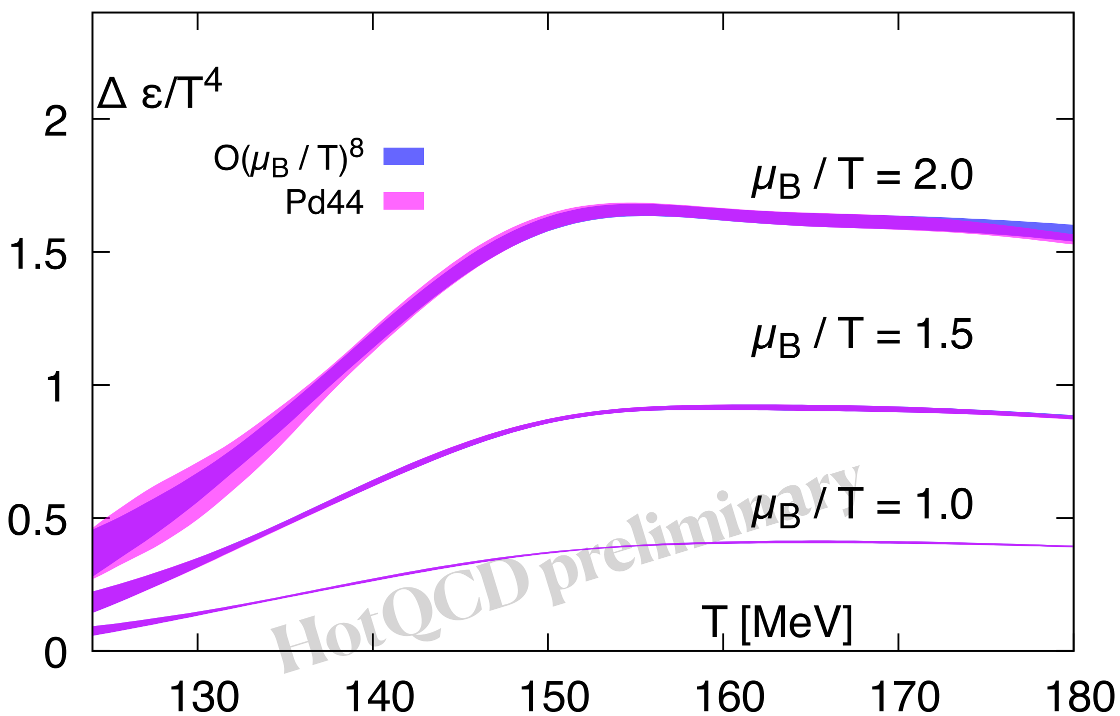
Reliability of the expansion: For , $T \geq 130$ MeV , number density is considered upto $\mu_B/T < 2$ while the pressure series is reliable upto $\mu_B/T < 2.5$.

Reliability of the expansion: For , $T \geq 130$ MeV , energy density and entropy is considered to be reliable for $\mu_B/T \sim [2.5-2]$ while the specific heat is considered to be reliable for $\mu_B/T \sim [1.5-2]$.

Padé resummed expansion of bulk Thermodynamic observables



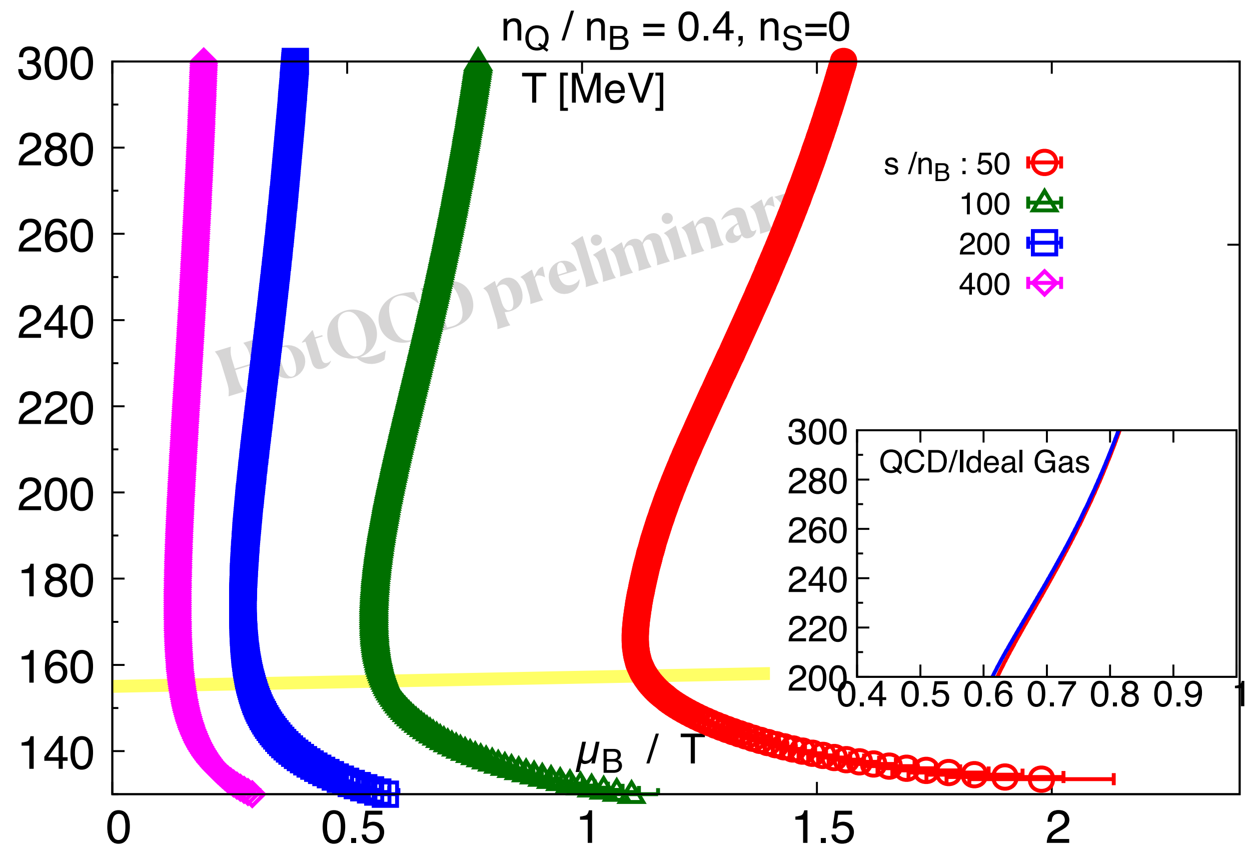


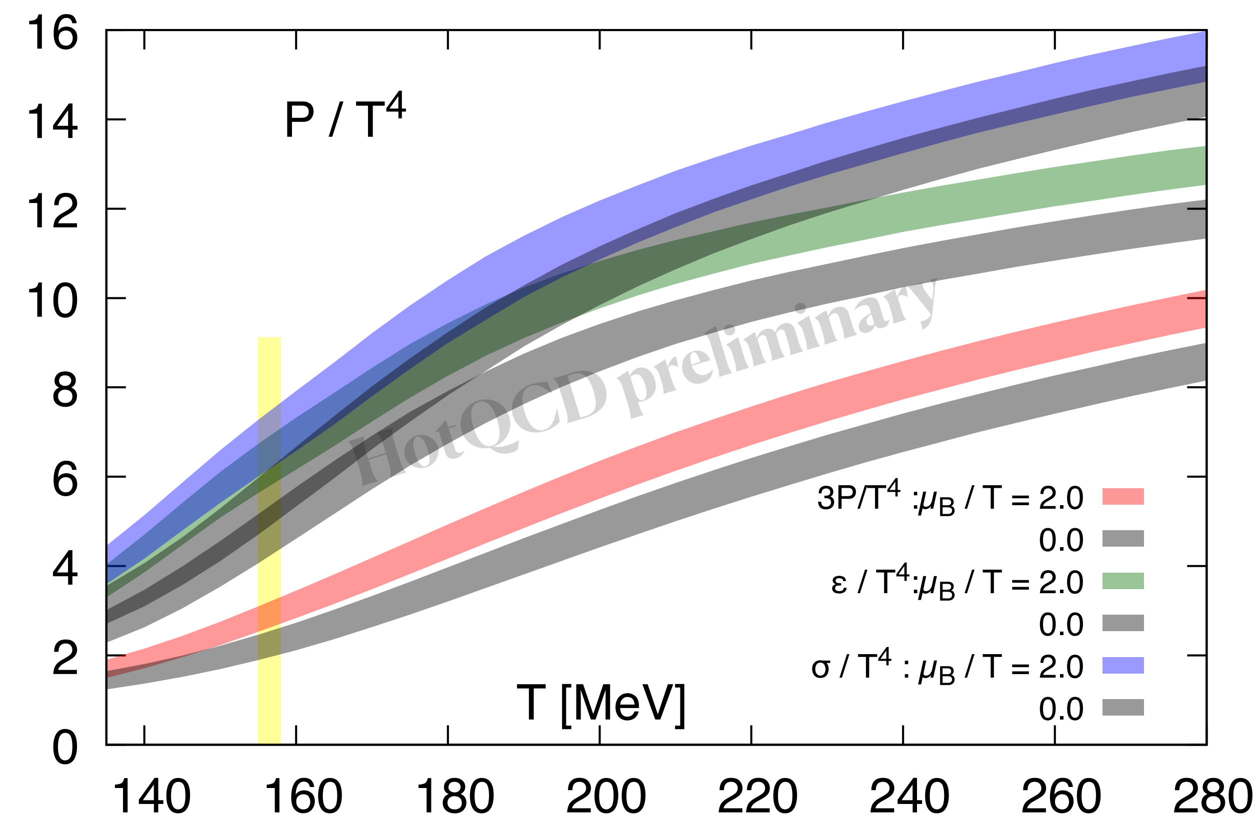


- **Sixth order Taylor series and $[2,4]-$Padé approximant agree with each other up to $\mu_B/T \leq 2$.**
- **Good agreement between eighth order Taylor series and $[4,4]-$Padé approximant for the energy density $\mu_B/T \leq 2$.**
- **Peak of the specific heat ($\hat{\mu}_B \neq 0$) can be understood from $O(4)$ scaling functions.**

Isentropic EoS of (2+1)-flavor QCD : relevant for HIC

Condition in HIC, $n_S = 0, \;\; n_Q/n_B = 0.4, \;\; \Rightarrow \mu_B, \mu_S \neq 0, \; \mu_Q \ll \mu_B$





$$\hat{s}/\hat{n}_B = (\hat{P} + \hat{\epsilon})/\hat{n}_B - \hat{\mu}_B - \hat{\mu}_Q \hat{n}_Q/\hat{n}_B$$

$$\hat{\mu}_Q = q_1 \hat{\mu}_B + q_3 \hat{\mu}_B^3 + \ldots$$

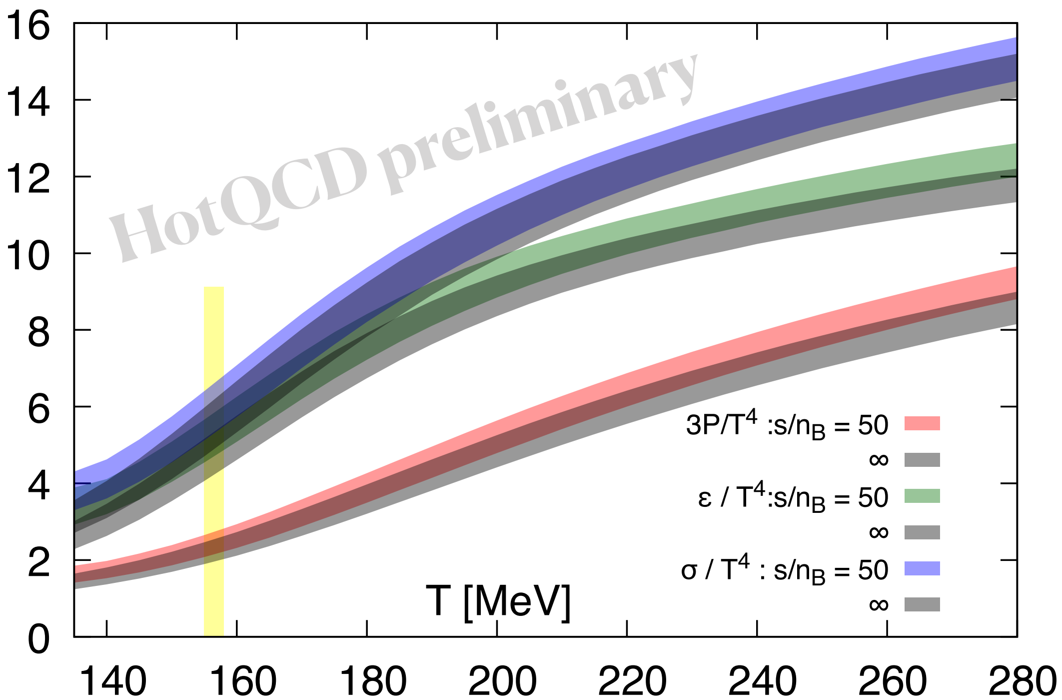
$$\hat{\mu}_B = \sum_{n=1}^{\infty} h_{2n-1} \left(\frac{\hat{s}}{\hat{n}_B} \right)^{1-2n}$$

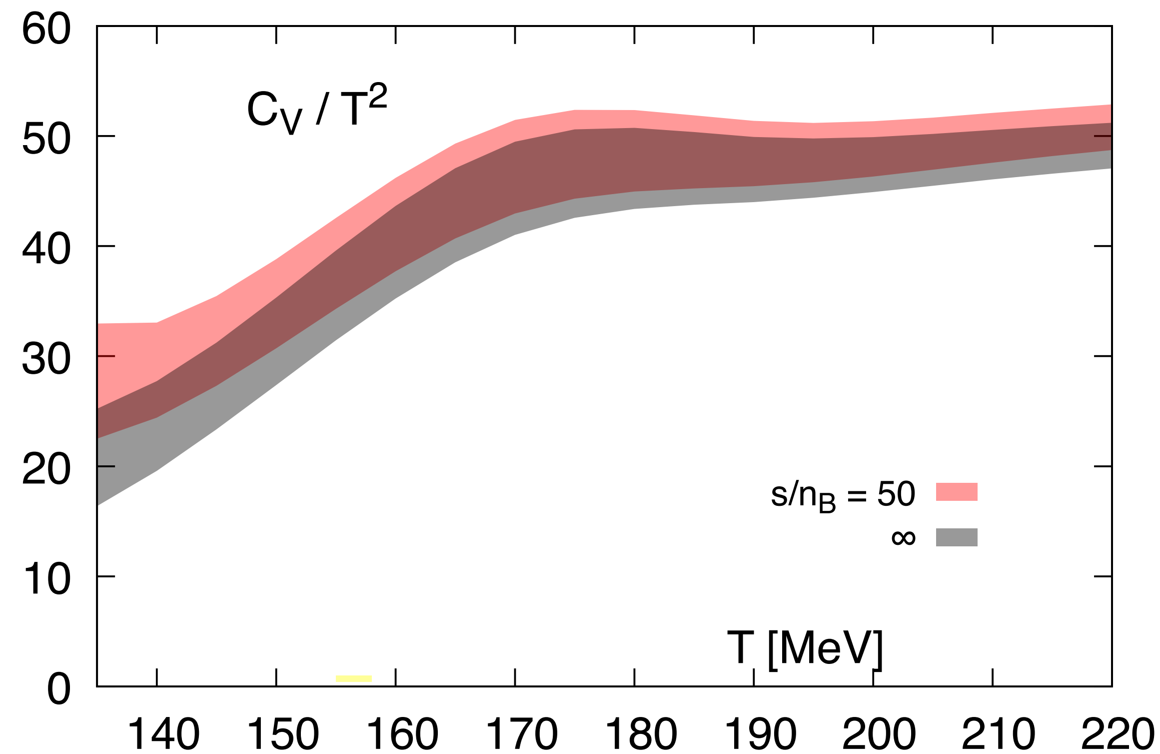
Series expansion in μ_B/T is used for the determination of the lines of constant entropy per baryon number $\hat{s}/\hat{n}_B$, which could be used for characterizing the expansion of dense matter created in heavy ion collisions

David Clarke, 11 Aug 2022, 09:20

At T_{pc}, this covers the STAR beam energy range, $200 - 20$ GeV

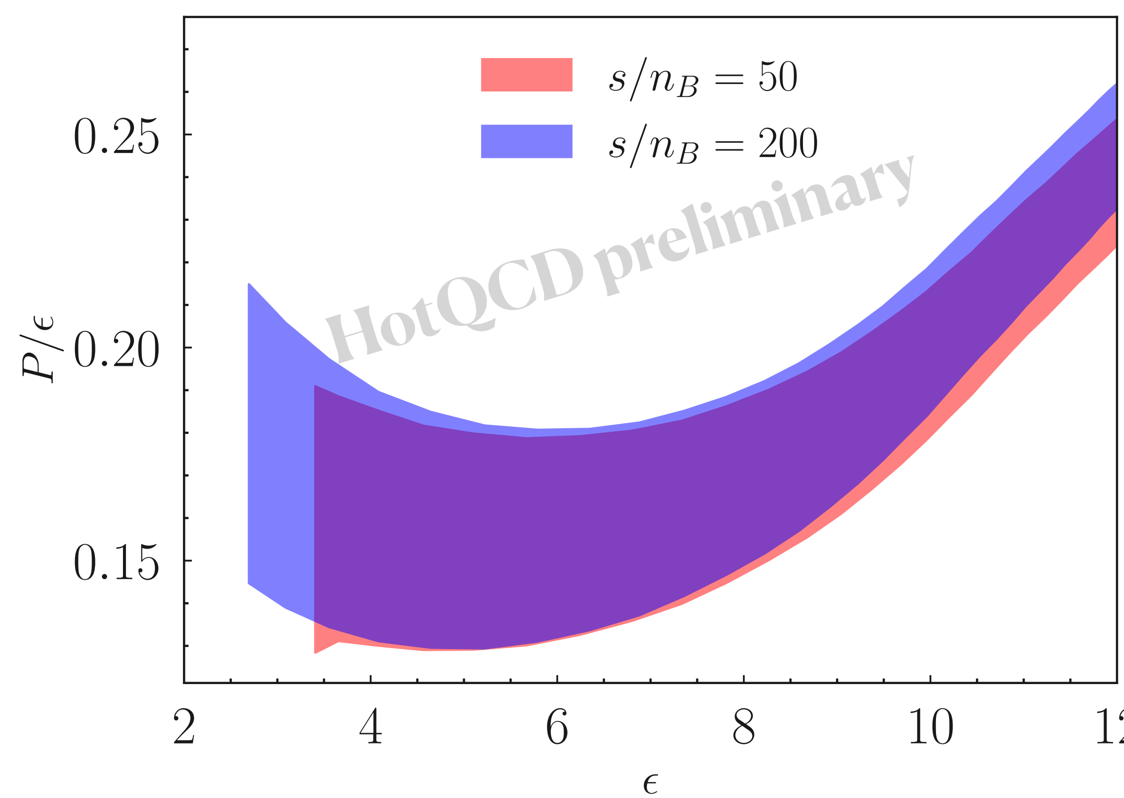
Isentropic EoS of (2+1)-flavor QCD : relevant for HIC





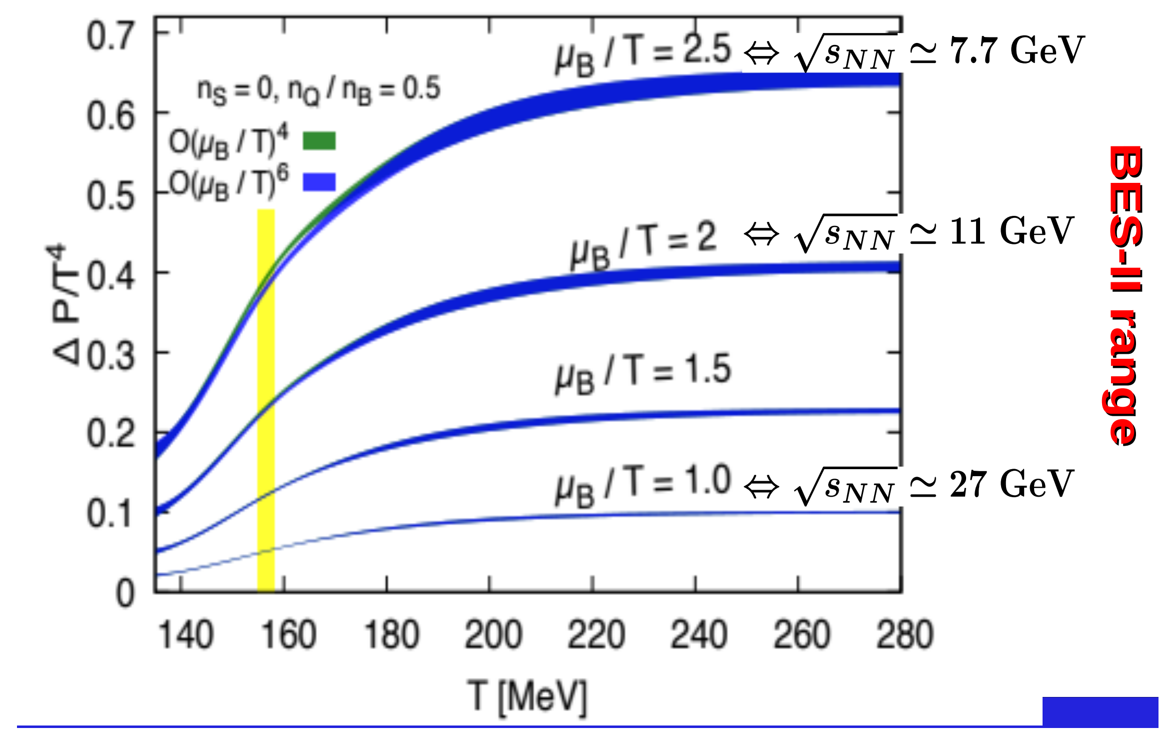
The observables doesn't show any s/n_B dependence.

$$(p/\epsilon)_{min} = 0.16(4)$$



Conclusions

- Good convergence of Taylor series of energy density, entropy and specific heat for $\hat{\mu}_B/T \sim 2, 2$ and 1.5 respectively.
- Using the Padé resummed pressure series we construct the Padé approximants of other thermodynamic observables and show that they agree with each other $|\hat{\mu}_B| \leq 2$ at $T > 130$ MeV.



Outlook:
Comparison of other resummation schemes Of Thermodynamic quantities for a reliable estimation at finite chemical potential beyond the estimated radius of convergence from Taylor series.

Thank you

References

Direct Method

Imaginary chemical potential : M. D'Elia and M. P. Lombardo[hep-lat/0209146], J. Guenther et al [arXiv:1607.02493]

Taylor expansion : Bielefeld-Swansea collaboration [hep-lat/0501030], HotQCD collaboration [1701.04325 [hep-lat]] , Datta, Gavai and Gupta[1612.06673 [hep-lat]]...

Resummation Method

Padé approximant : Gavai and Gupta[arXiv:hep-lat/0412035, arXiv:0806.2233] , Bielefeld Parma collaboration [2110.15933 [hep-lat]] , HotQCD collaboration [arXiv:2202.09184:This talk]

There is more,

S. Borsanyi et al arXiv:2102.06660 arXiv:2202.05574.

S. Mondal et al arXiv:2106.03165,

S. Mitra , arXiv: 2205.08517, Lattice2022

Conformal maps: V. Skokov arXiv:1008.4549,M. Giordano et al., arXiv:2004.10800 , G. Basar, arXiv:2105.08080

Thermodynamics using Lattice QCD

Expansion of the QCD pressure:

$$\frac{P(T,\vec{\mu})}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k \;,\; \hat{\mu} = \mu/T,$$

$$\mu_Q = 0$$

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + \ldots..$$

$$\chi_0^B(T,\hat{\mu}_B) \equiv \frac{P(T,\mu_B) - P(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\hat{\mu}_B^{2k}$$

$$\chi_1^B(T,\hat{\mu}_B) \equiv \frac{n_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} N_{2k-1}^B(T)\hat{\mu}_B^{2k-1}$$

$$\chi_2^B(T,\mu_B) = \sum_{k=0}^{\infty} \tilde{\chi}_2^{B,k}(T)\hat{\mu}_B^{2k}$$

$$P_{2k} \equiv \bar{\chi}_0^{B,2k}/(2k)!$$

$$N_{2k-1} \equiv \bar{\chi}_0^{B,2k}/(2k-1)!$$

Same set of coefficients

$$\tilde{\chi}_2^{B,k} \equiv \bar{\chi}_2^{B,2k}/(2k)!$$