Measurement of hadron masses in 2-color finite density QCD

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in collaboration with

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Contents

- Introduction
- Review of 2-color QCD
- Simulation details
- Results
  - Pion and ρ meson
  - $I = 0$, $0^{\pm}$ channel
- Conclusion/Future work

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Hadron spectrum in medium

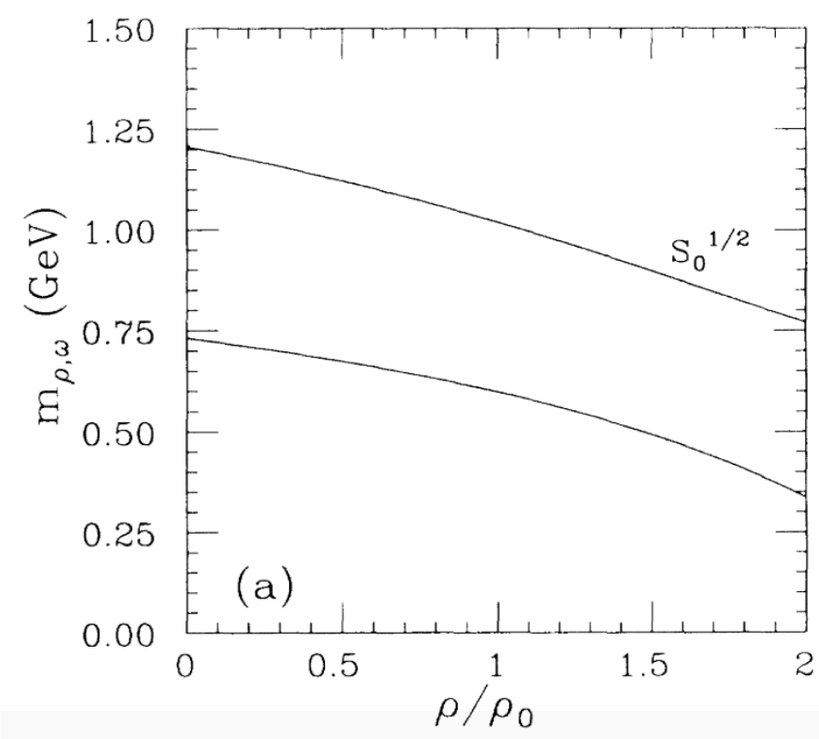
- studies of hadron spectrum in medium play an important role (e.g. hadron interactions in a dense matter)

- zero density: QCD inequality guarantees
  pion is the lightest meson

  for $^\forall\Gamma$, $C_\Gamma(t) \equiv \langle \bar{\psi}\Gamma\psi(t)\ (\bar{\psi}\Gamma\psi(0))^\dagger \rangle \leq C_{\gamma_5}(t)$ ➡ $m_{meson} \geq m_\pi$ for all $(I = 1)$ mesons

- finite density: **QCD inequality does not hold**
  - chiral EFT: scaling law
    $m_\sigma^*/m_\sigma \approx m_N^*/m_N \approx m_\rho^*/m_\rho \approx m_\omega^*/m_\omega \approx f_\pi^*/f_\pi$
    [Brown, Rho, 1991]
  - QCD sum rules: **decrease of** $m_{\rho,\omega}^*$
    [Hatsuda, Lee, 1992]
  - **lattice QCD: difficult due to sign problem**
- we study hadron spectrum in **2-color QCD**

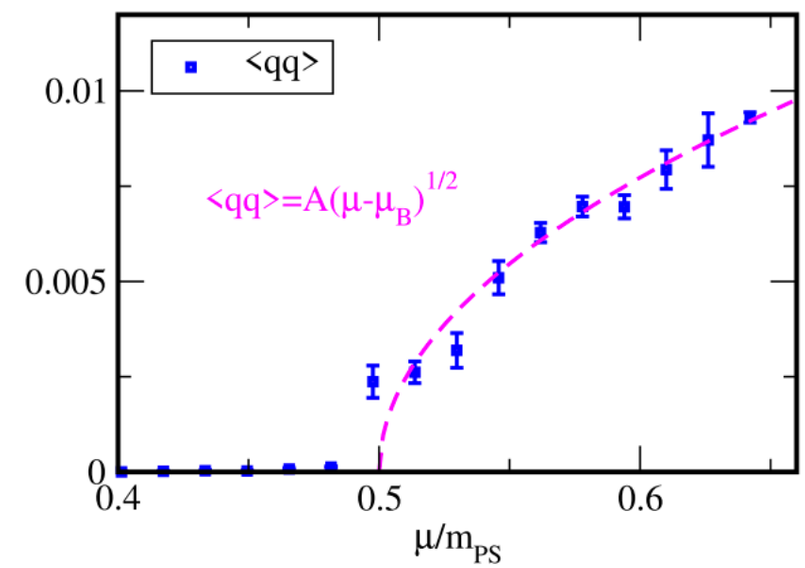
(T. Hatsuda and S. H. Lee, Phys. Rev. C **46**, no.1, R34 (1992))



2-color QCD

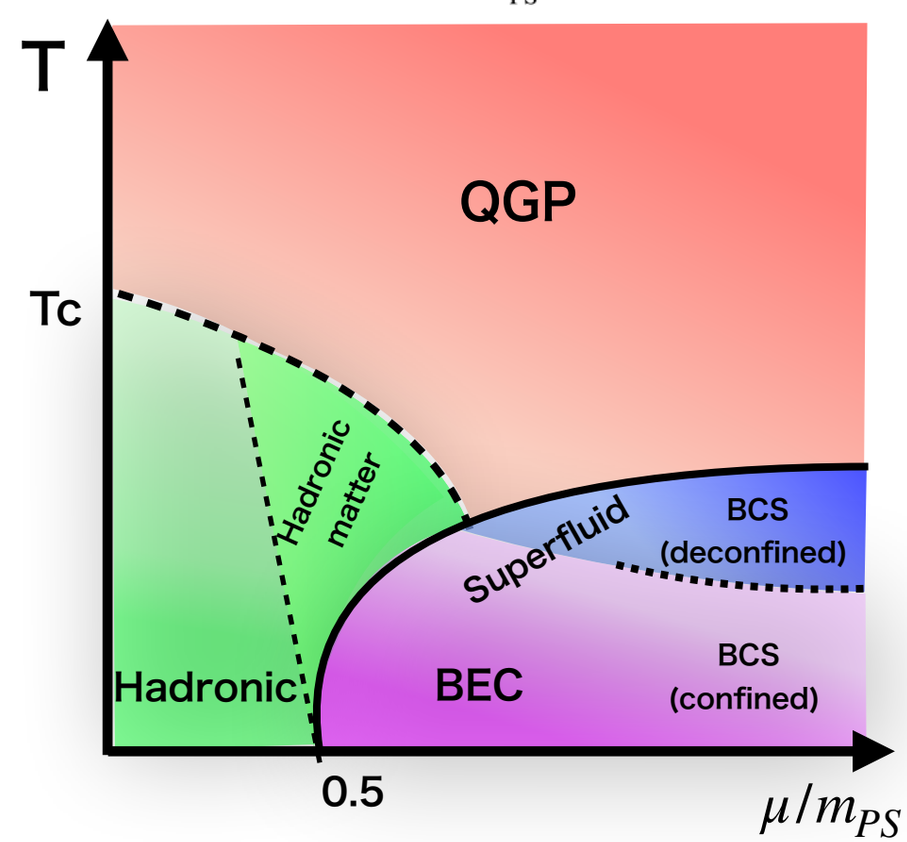
- 2-color QCD: QCD-like theory in which lattice simulation is available in finite density

(K. Iida, E. Itou and T. -G. Lee, JHEP **01**, 181 (2020))



- phase transition between **hadronic phase** and **superfluid phase** at $\mu \approx m_\pi/2$ in low temperature

- Hadron spectrum
  - 2-color QCD has an alternative inequality
    ➡ $I = 0$, 0^+ **diquark is the lightest for any density**
  - previous lattice studies: [Muroya et al., 2003] **flips π/ρ spectral ordering**



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2-color QCD with diquark source term

- Quark action (we consider $N_f = 2$)

$$S_F = \int d^4x \; \bar{\psi}_f(x)(\gamma_\mu D_\mu + m)\psi_f(x) + \mu\hat{N} + j\hat{D} \qquad (f = 1,2)$$

$$\hat{N} = \int d^4x \; \bar{\psi}_f(x)\gamma_0\psi_f(x)$$: number operator

$$\hat{D} = -\frac{1}{2}\int d^4x \; [\bar{\psi}_1(x)(C\gamma_5)\tau_2\bar{\psi}_2^T(x) - \psi_2^T(x)(C\gamma_5)\tau_2\psi_1(x)]$$

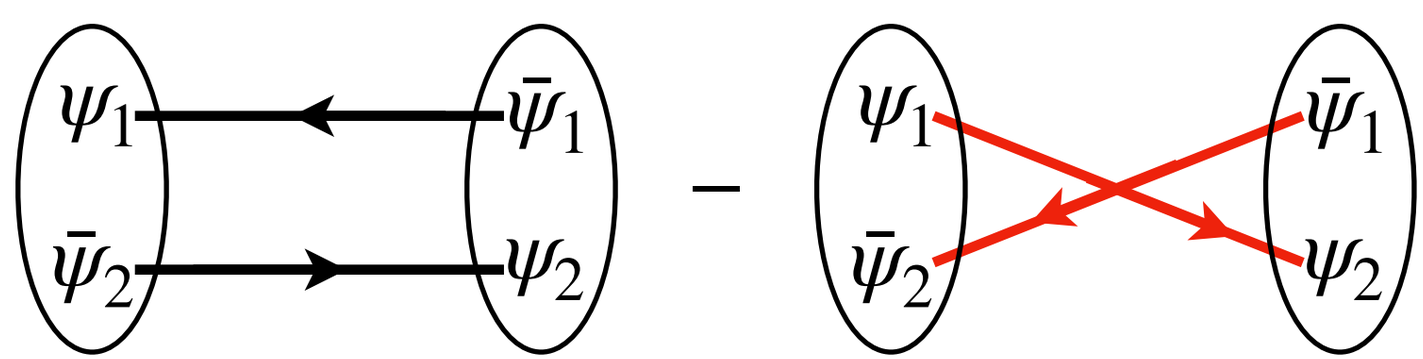
:diquark source term

→ 3 types of quark propagators

- $\psi_f\bar{\psi}_f$: normal propagator
- $\psi_2\psi_1^T$: anomalous propagator (quark → antiquark) ($\propto j$)
- $\bar{\psi}_2^T\bar{\psi}_1$: anomalous propagator (antiquark → quark) ($\propto j$)

- e.g. $I = 1$ meson 2pt function

$$\langle\bar{\psi}_2\Gamma\psi_1(t) \; (\bar{\psi}_2\Gamma\psi_1)^\dagger(0)\rangle =$$



Global symmetry in 2-color QCD $(N_f = 2,\ j = 0)$

$m = 0,\ \mu = 0$

$SU(4)$ enhanced symmetry because of pseudo-reality of $SU(N_c = 2)$

$m \neq 0,$

$\langle \bar{\psi}\psi \rangle \neq 0$

$$Sp(4) \xrightarrow[\mu \neq 0]{} U(1)_B \times SU(2)_V \xrightarrow[\langle \psi\psi \rangle \neq 0]{} Sp(2)_V$$

$\sim SO(3)$

5 pseudo-NG bosons

$\pi^{\pm}, \pi^0, qq, \bar{q}\bar{q}$

mass splitting of

$qq, \bar{q}\bar{q}$

$\sim SU(2)_V$

isospin symmetry

- in superfluid phase, the states are labeled by **spin**, **parity** and **isospin** (no baryon number)

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Simulation details

- Conf.: Iwasaki gauge action + 2-flavor Wilson quark action in QC2D in finite μ and j (gauge fixing, 400 confs.)

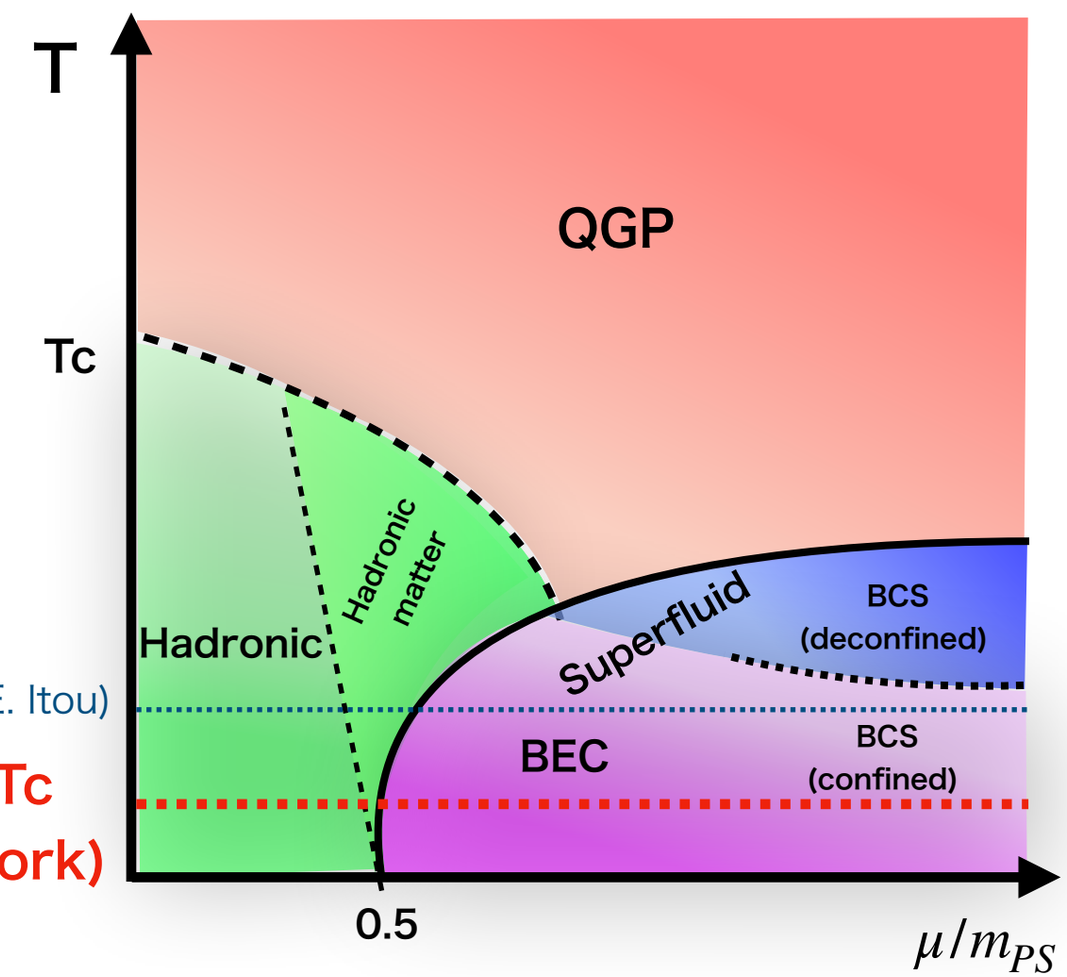
- $\beta = 0.8$, 32^4 lattices, $m_\pi/m_\rho \approx 0.81$, $T \approx 0.19T_c$

(K. Iida, E. Itou and T. -G. Lee, PTEP **2021**, no.1, 013B05 (2021))

- for now,
  - use finite j (not so small value in superfluid phase)
  - neglect contribution from disconnected diagrams

0.39Tc
(EoS analysis)(talk by E. Itou)

0.19Tc
(this work)



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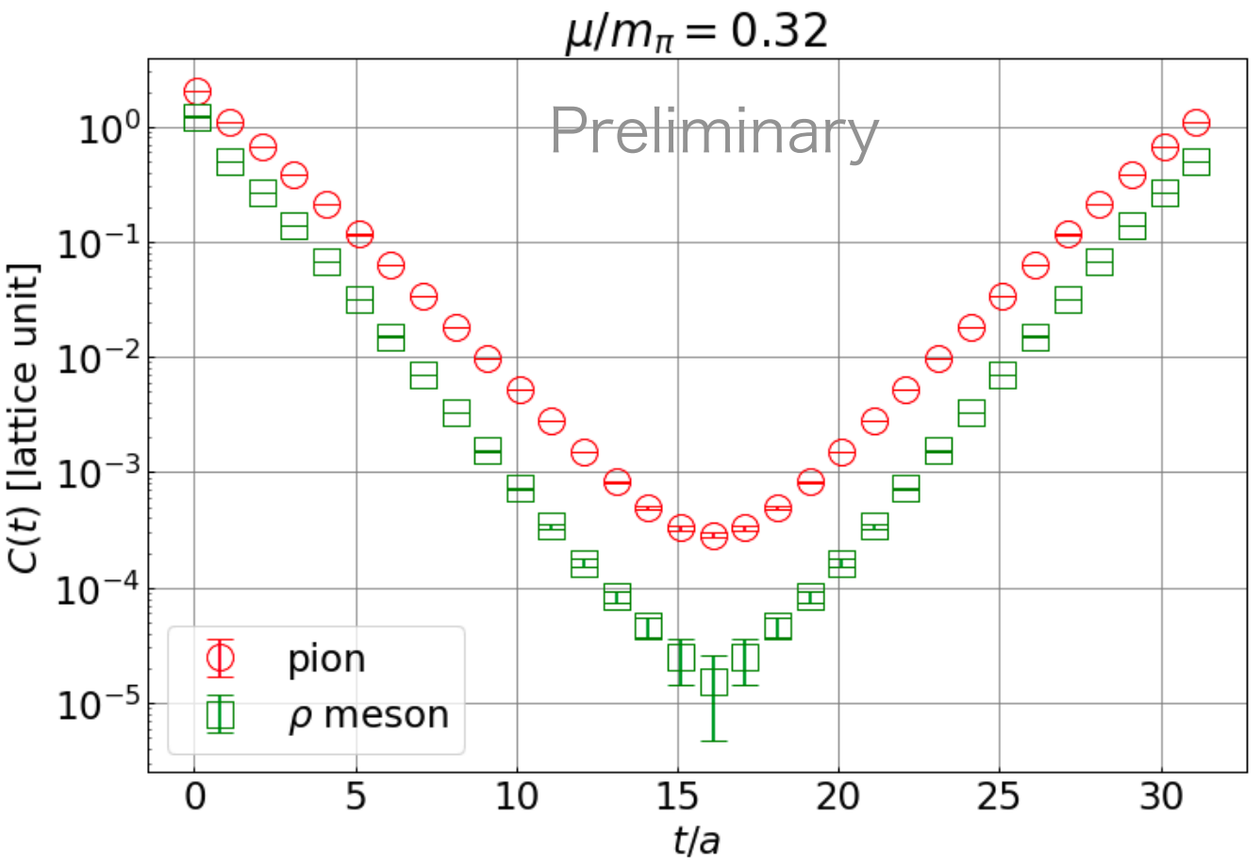
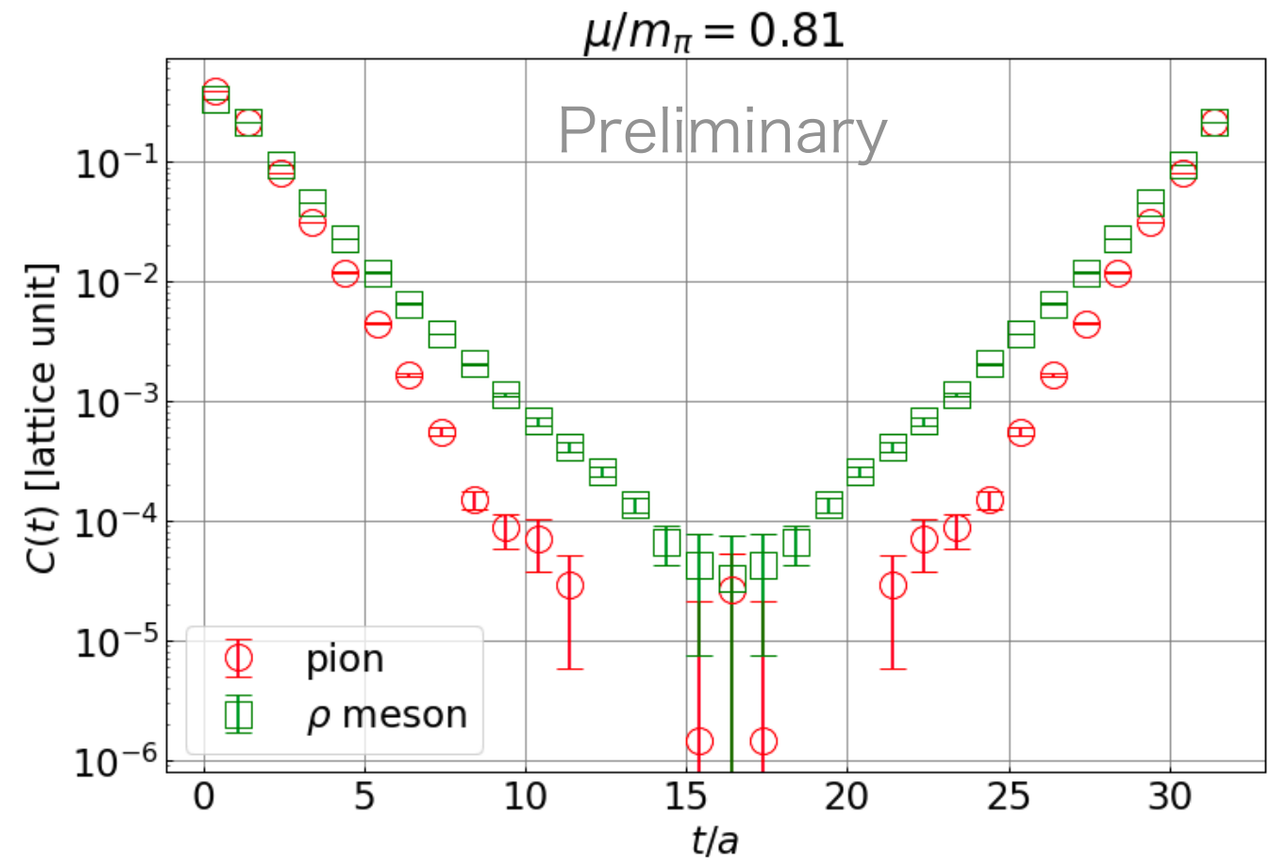
2pt function of pion and ρ meson

in hadronic phase



in superfluid phase



- $C^\rho(t) < C^\pi(t)$ holds
- both signals are clear

- the hierarchy breaks down
- pion becomes noisy

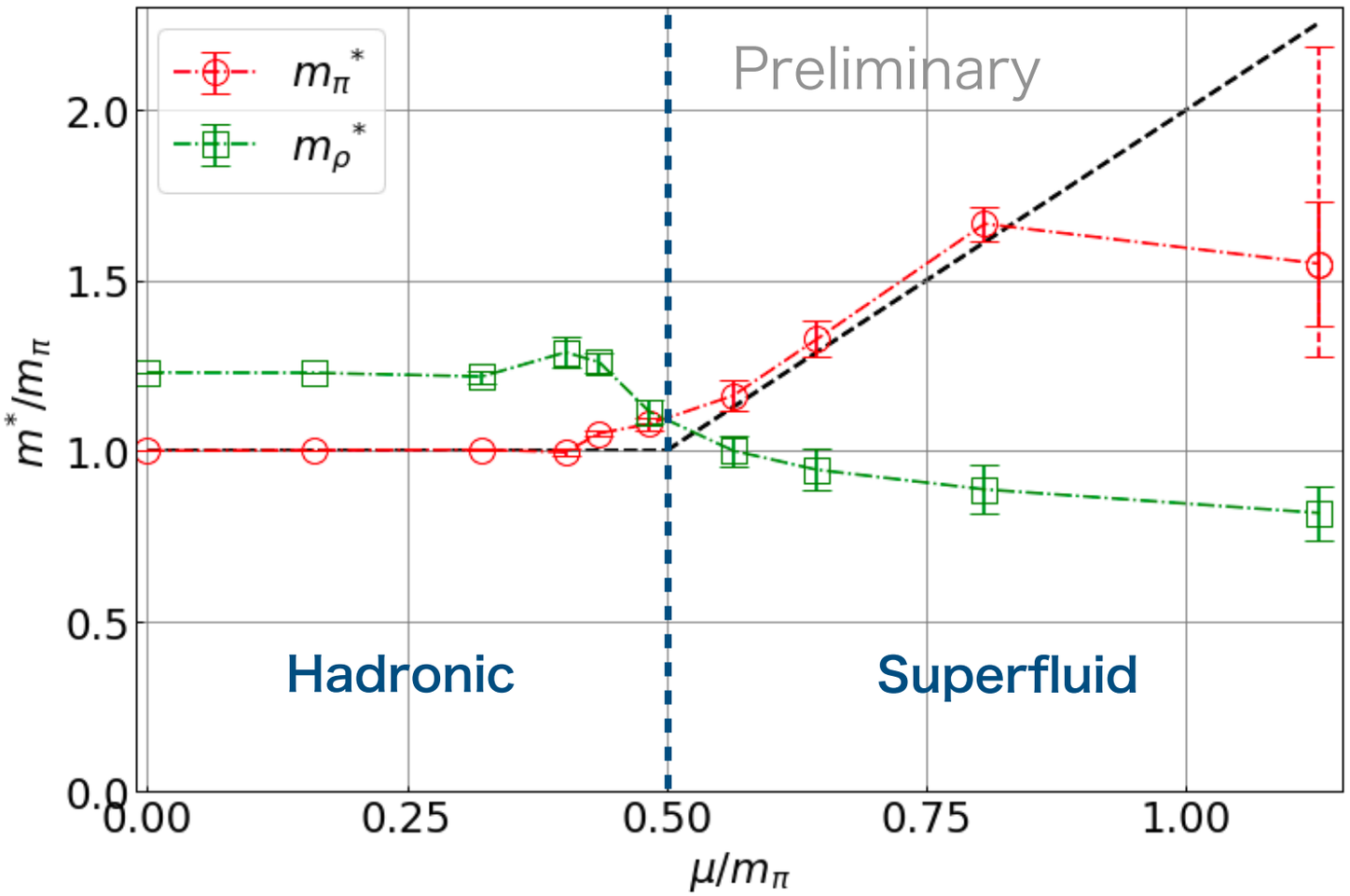
← 2-body decay channel?

Masses of π and ρ meson

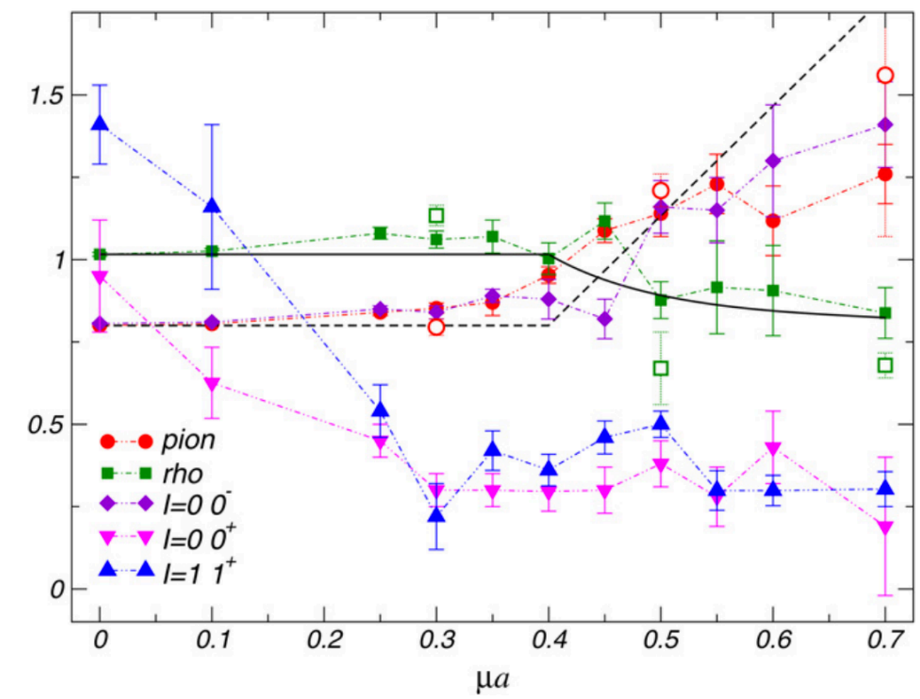
(dashed errorbar: systematic error estimated by fitrange dependence)

------ : m_π^* from χPT in 2-color QCD
[Kogut et al., 2000]

Our results



(S. Hands, P. Sitch and J. I. Skullerud, Phys. Lett. B 662 (2008), 405-412)



- ordering of π and ρ flips at $\mu \approx m_\pi/2$

- consistent with the results in the previous lattice studies and the argument in other studies

[Hatsuda, Lee,1992;
Kogut et al., 2000]

[Muroya et al., 2003; Hands et al., 2008;
Wilhelm et al., 2019]

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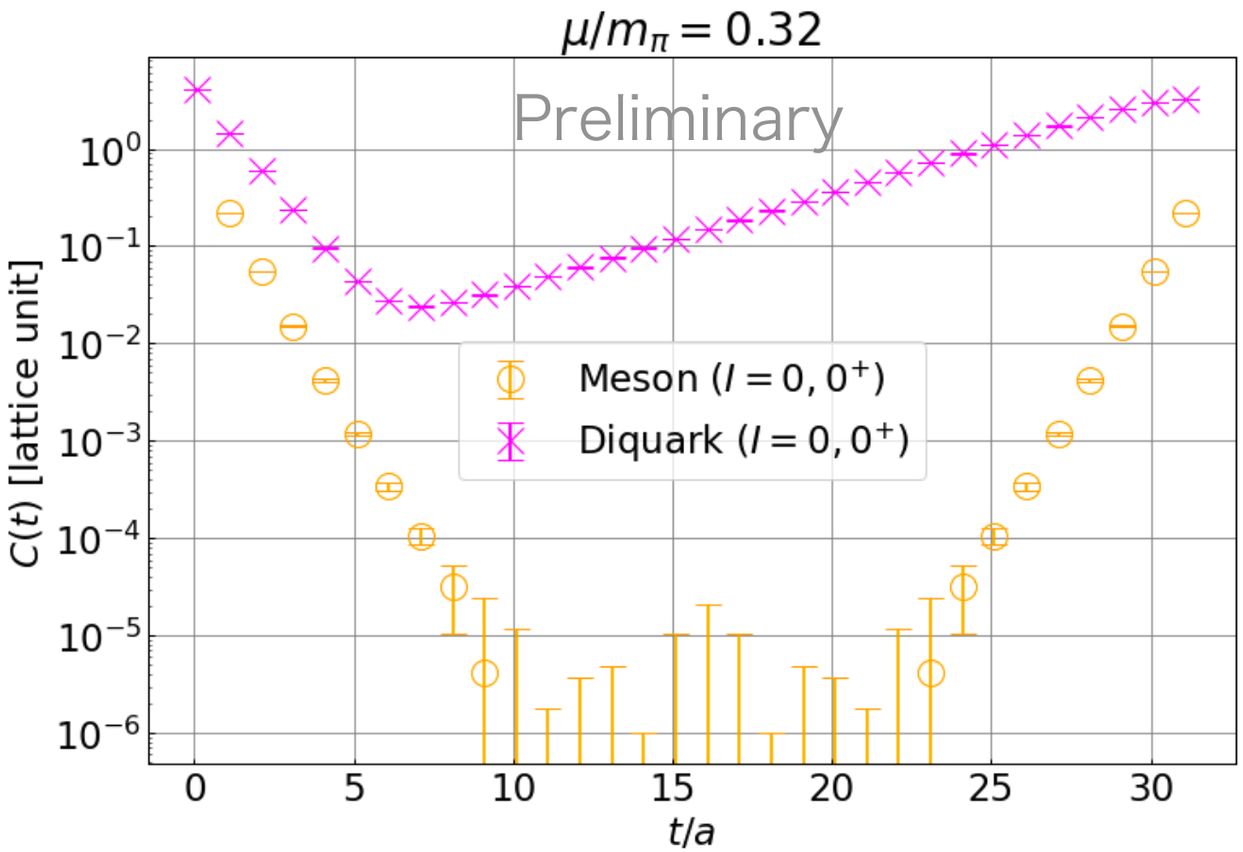
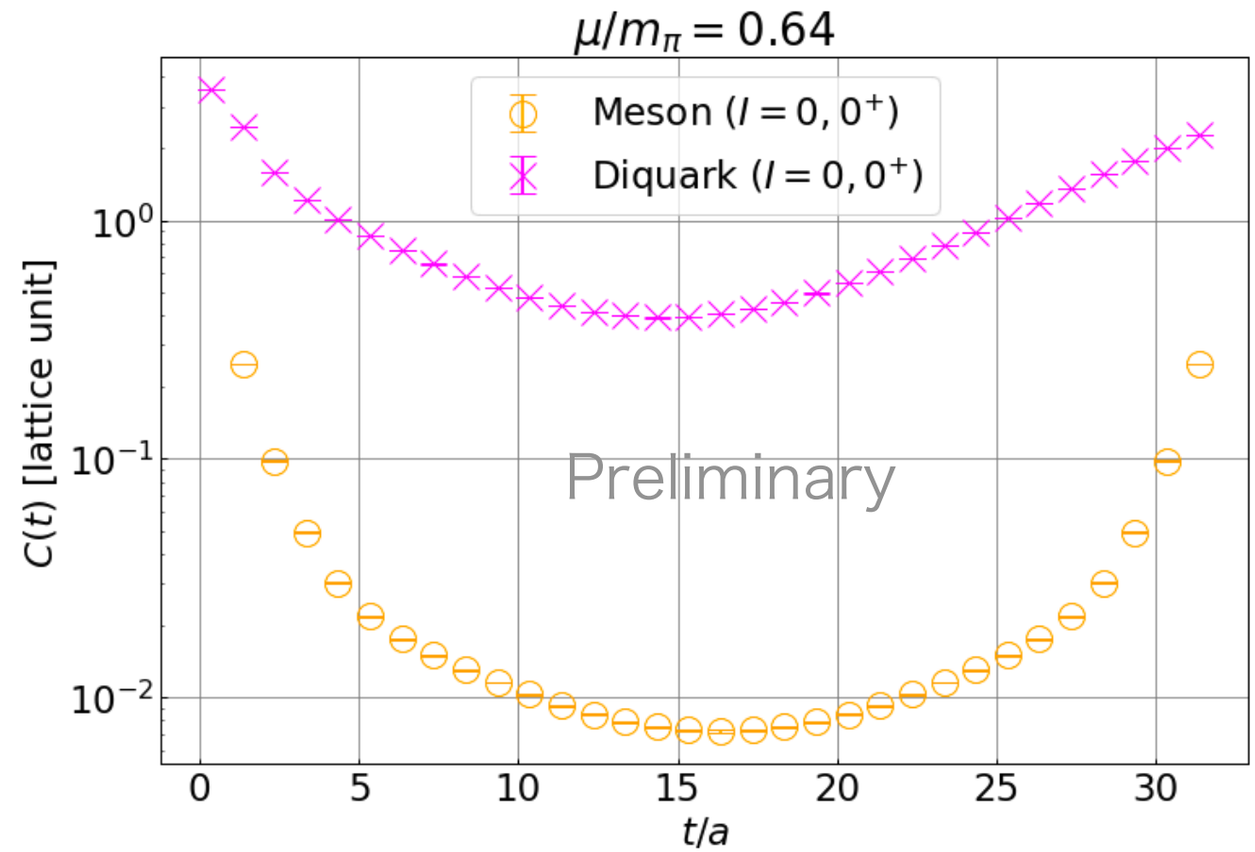
2pt function in $I = 0,\ 0^+$ channel

in hadronic phase



in superfluid phase

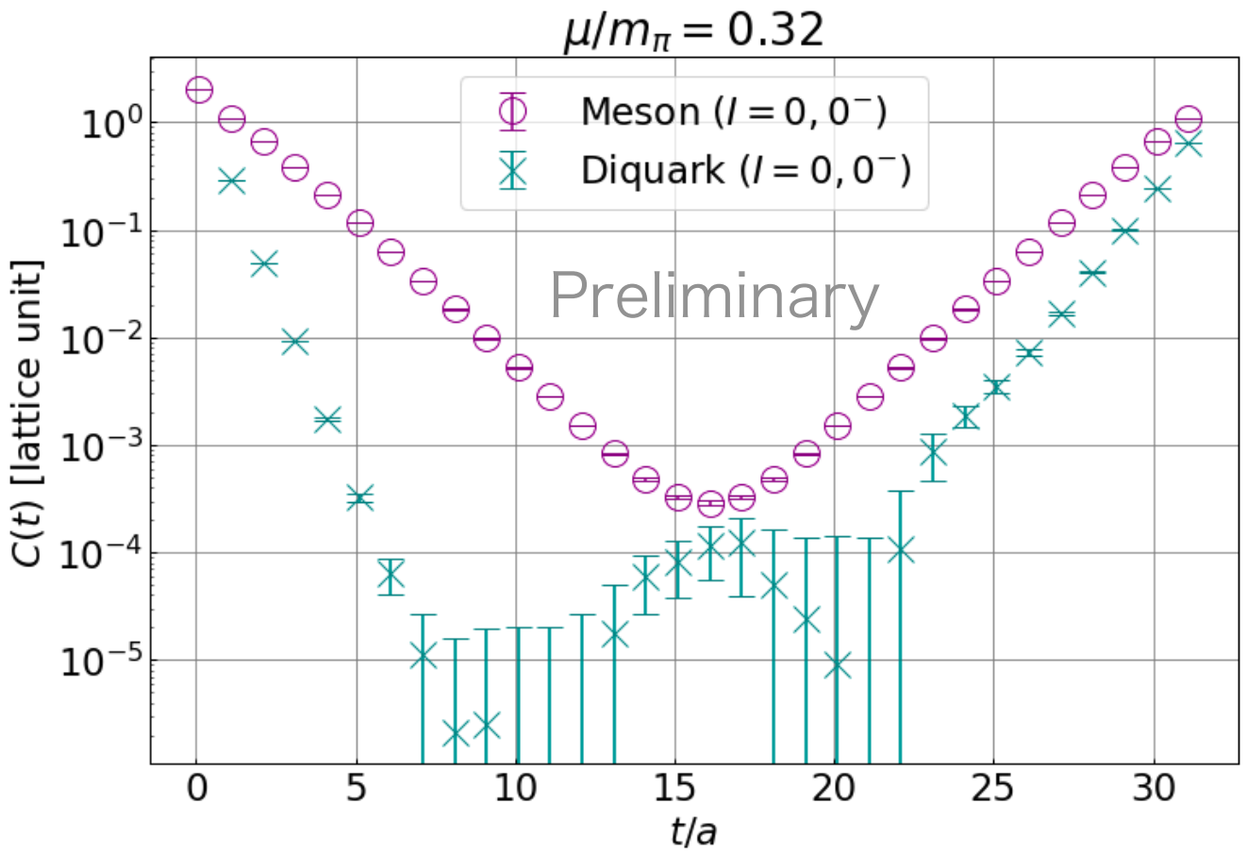


- meson: symmetric
- diquark: **asymmetric**
- we define
  - diquark mass: fit from $C(T - t)$
  - antidiquark mass: fit from $C(t)$

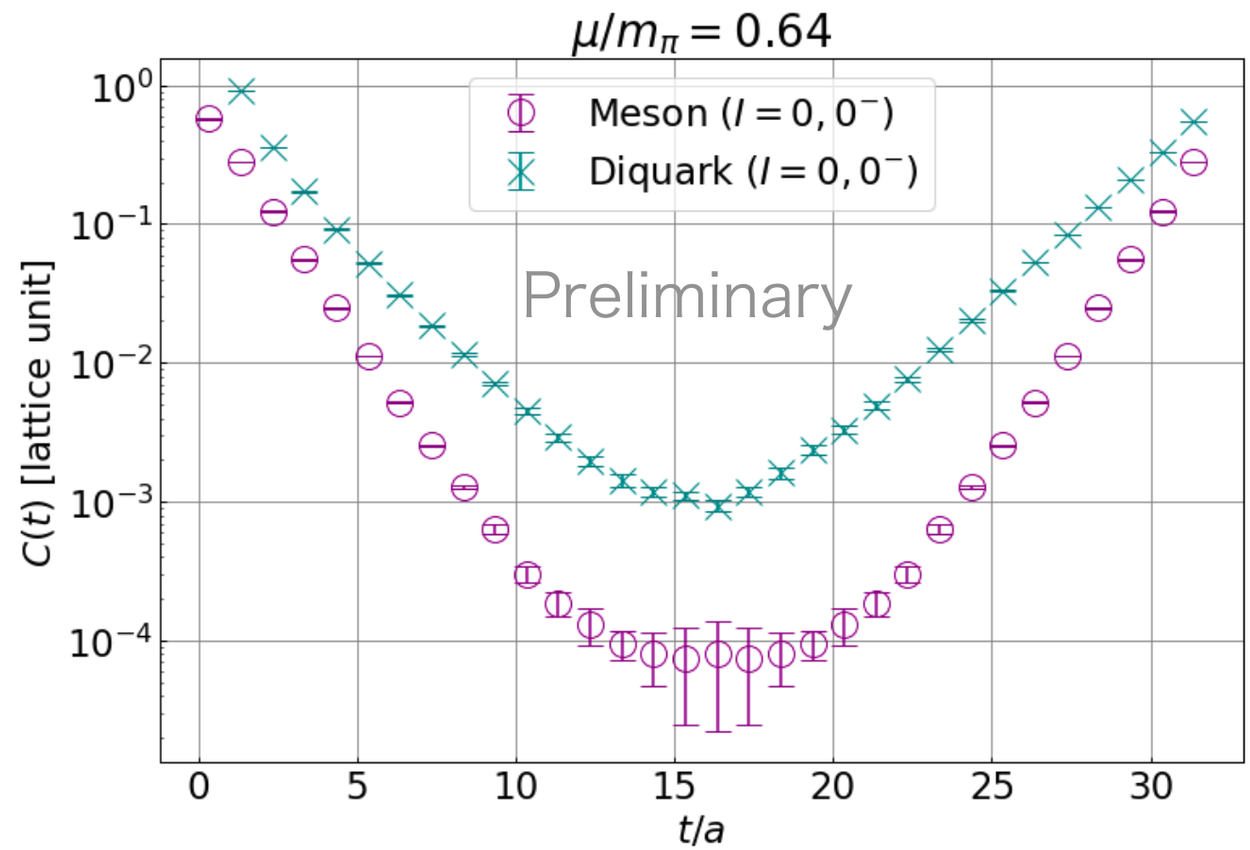
- meson: signal gets clear
- diquark: **getting symmetric**

2pt function in $I = 0,\ 0^-$ channel

in hadronic phase



in superfluid phase



- meson: symmetric
- diquark: asymmetric
- we define
  - diquark mass: fit from $C(T-t)$
  - antidiquark mass: fit from $C(t)$

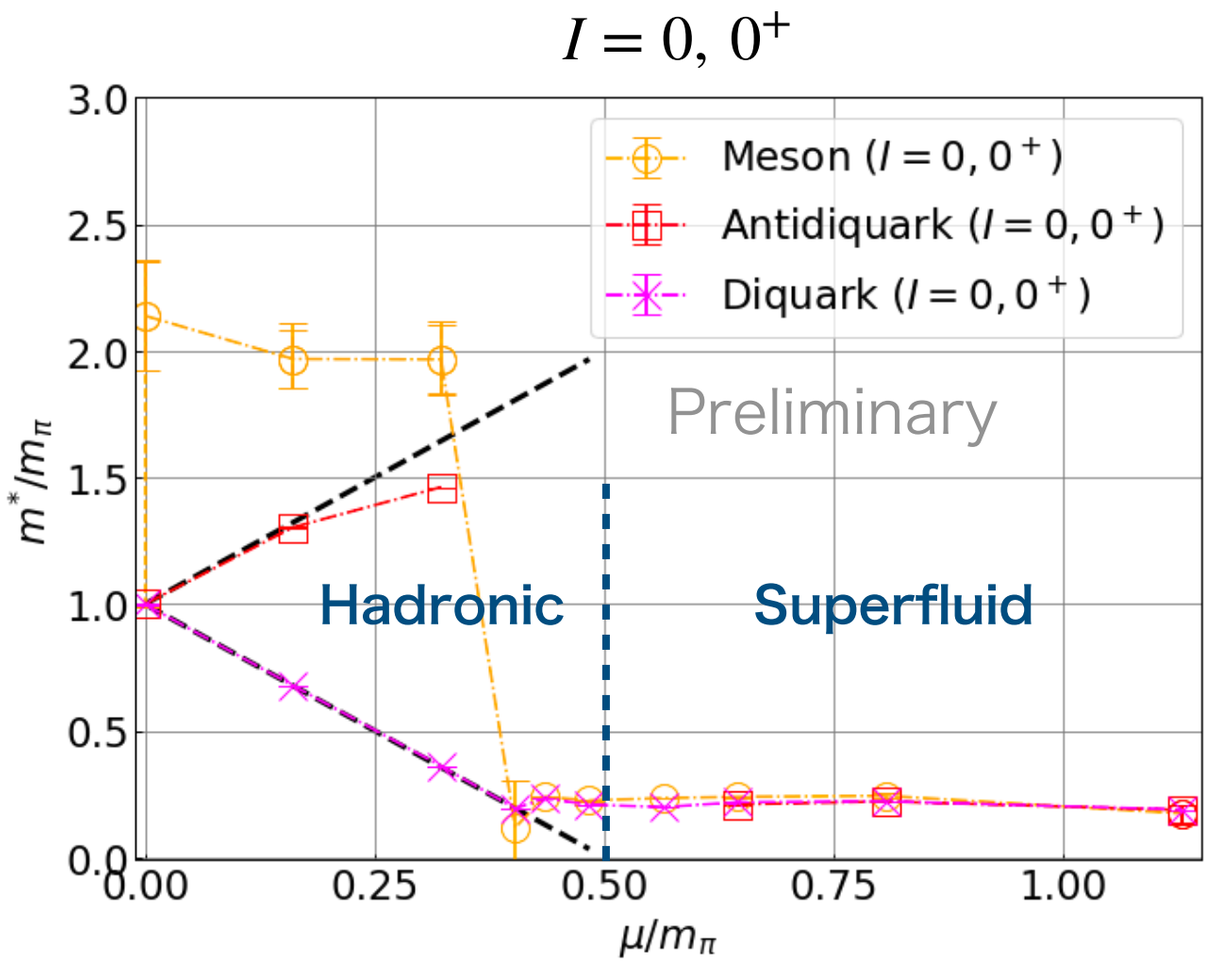
- diquark: getting symmetric
- meson: **different magnitude of slopes in early and late t**

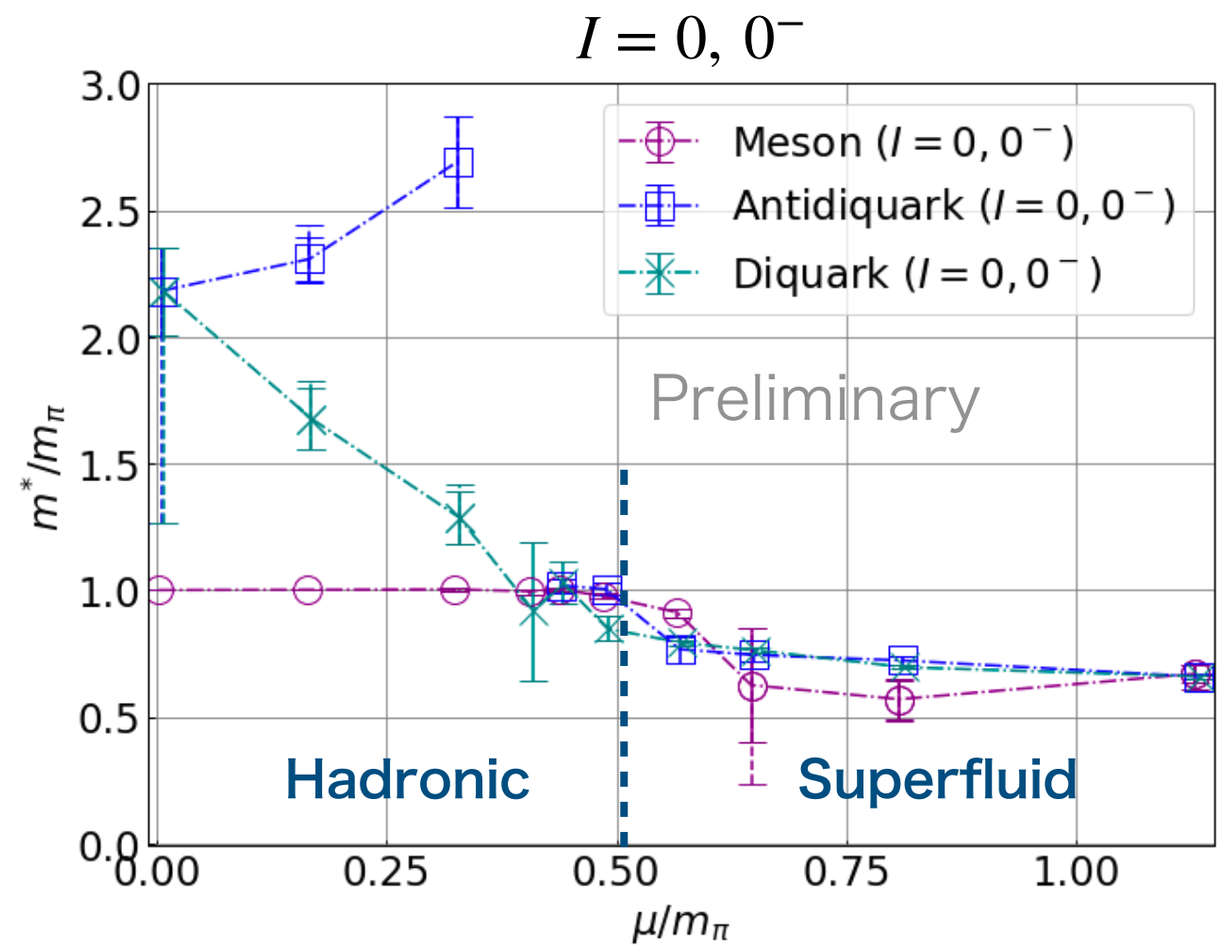
  ➡ use double-exp (cosh) fitting

Masses in $I = 0,\ J^P = 0^{\pm}$ channels

(dashed errorbar: systematic error estimated by fitrange dependence)

------ : $m_\pi \pm 2\mu$ (from χPT in 2-color QCD) [Kogut et al., 2000]





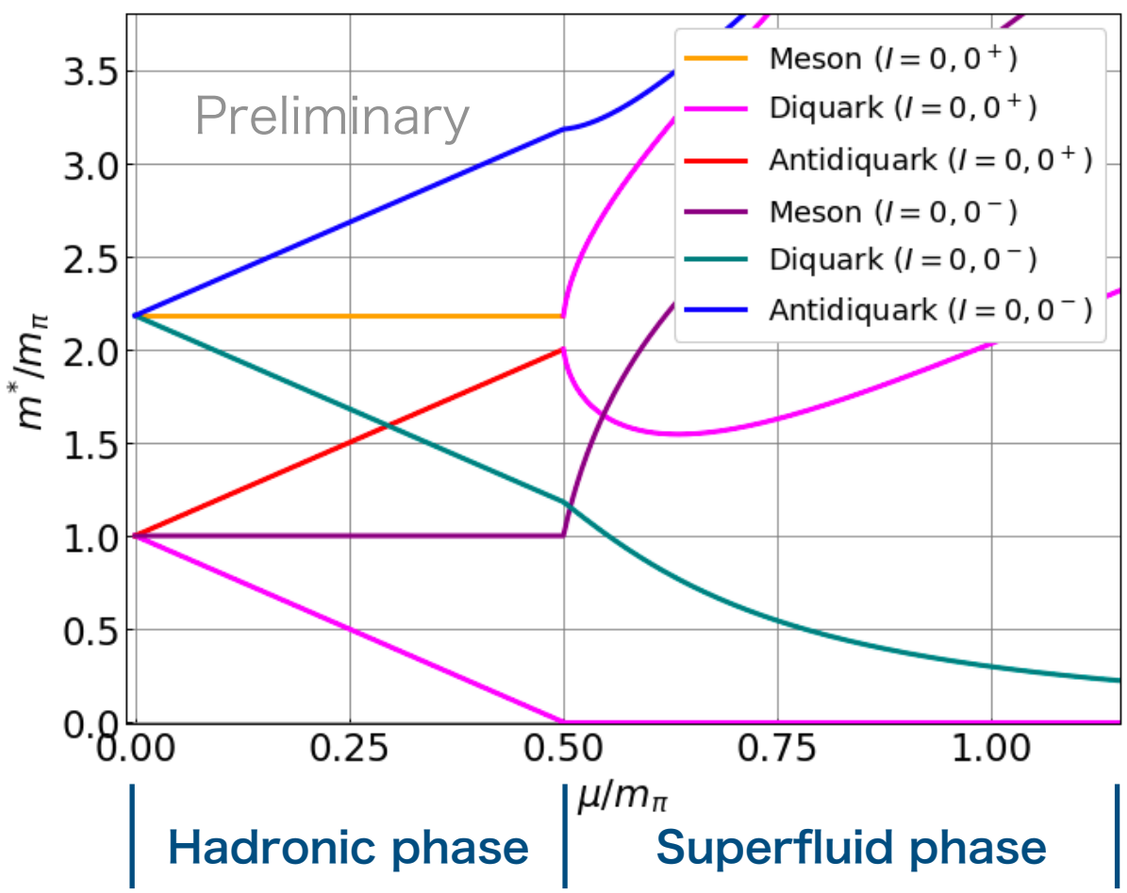
- hadronic phase: diquark/antidiquark behaves as $m(\mu) = m \pm 2\mu$
- diquark is the lightest for all μ
- superfluid phase: all three masses almost have the same values

⟵ meson-diquark-antidiquark mixing?

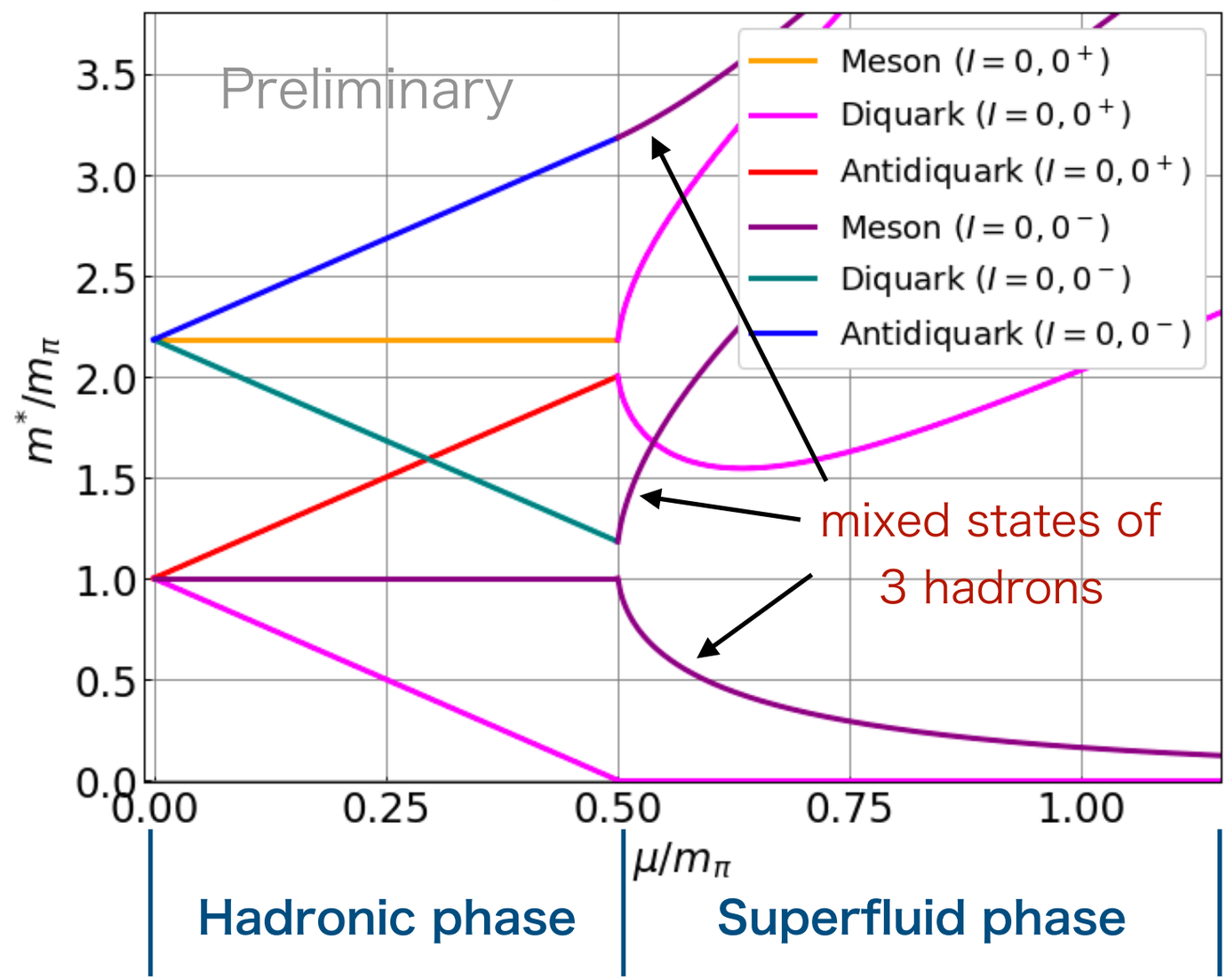
Linear sigma model with diquark gap

- **Linear sigma model with diquark gap** explains the meson-diquark-antidiquark mixing [Suenaga et al., in preparation]

w/o mixing in $I = 0,\ 0^-$ channel



w/ mixing in $I = 0,\ 0^-$ channel



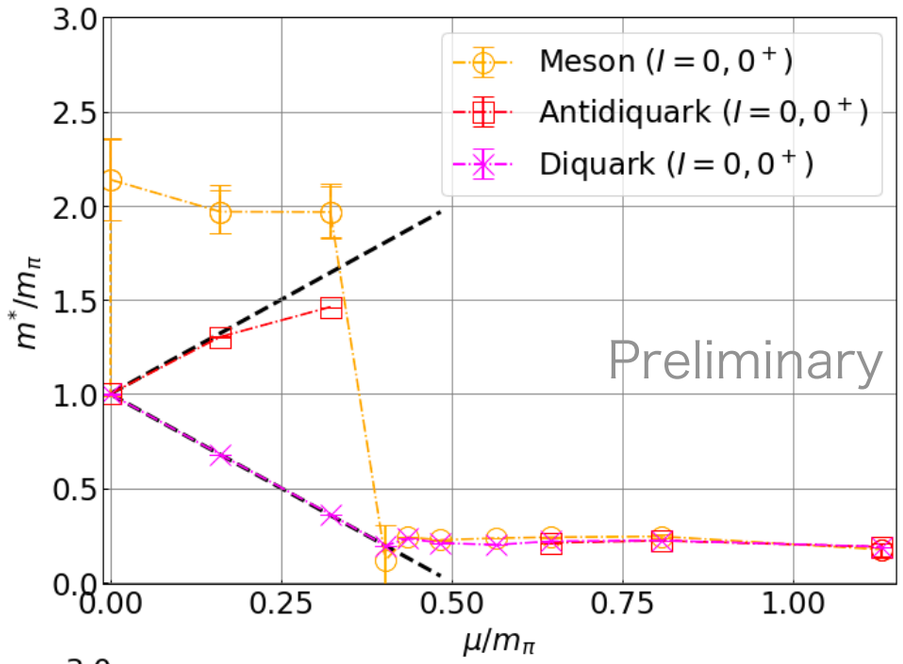
Comparison with results in LSM

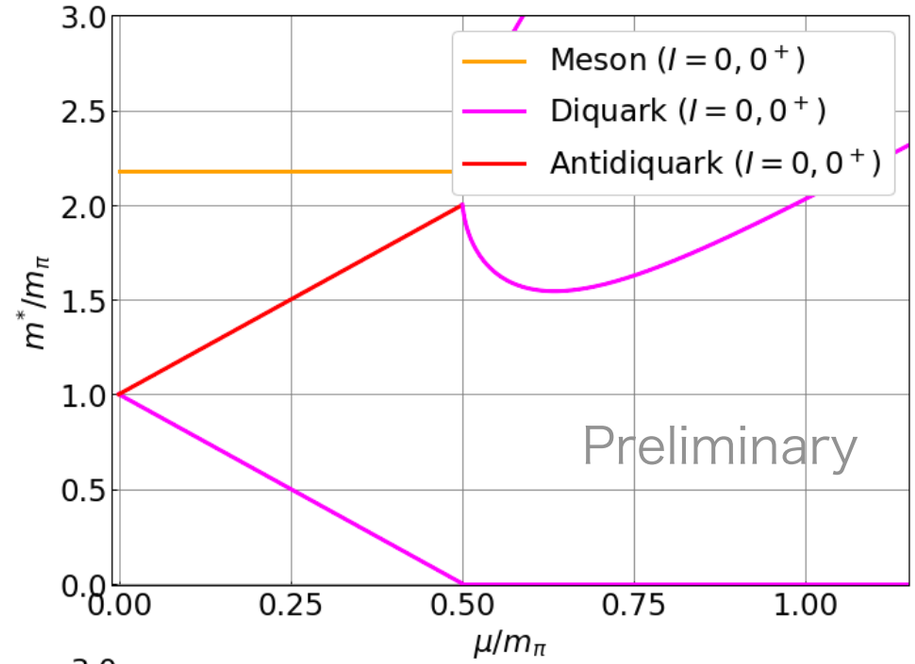
- input of LSM: masses of pion and diquark in $I = 0,\ 0^-$, approximate $m_\sigma \approx m_{a_0}$

lattice results

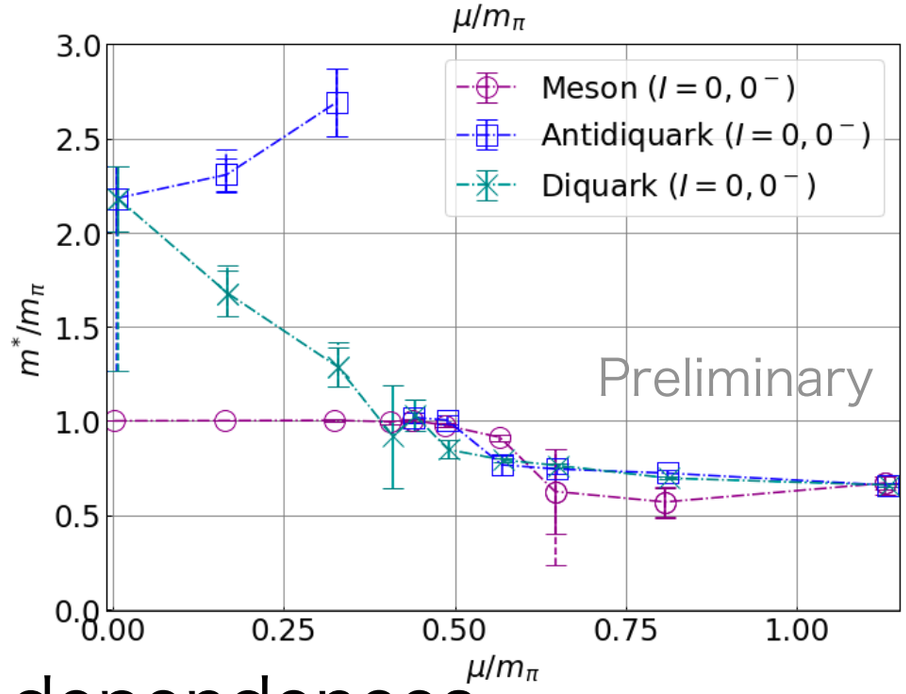
LSM [Suenaga et al., in preparation]

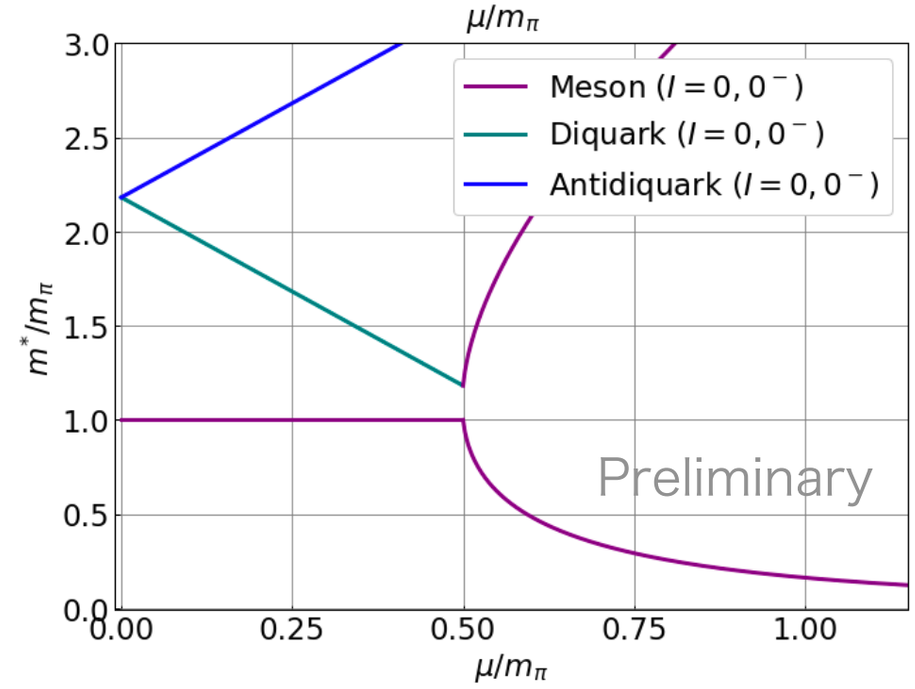
$I = 0,\ 0^+$





$I = 0,\ 0^-$





- similar μ dependences

- some discrepancies ← $j \neq 0,\ T \neq 0$ effect?

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Conclusion

- we study hadron spectrum in low temperature and finite density
- order of π and ρ meson spectrum flips at $\mu \approx m_\pi/2$
- signal of meson-diquark-antidiquark mixing in $I = 0,\ 0^{\pm}$ channels in superfluid phase

Future work

- investigate other channels
- $j \to 0$ extrapolation, include disconnected diagrams
- estimate mixing angle quantitatively
  ← variational method?

Backups

Quark propagators in detail

- 4 types of quark propagators
  - $\Delta(\mu)$: (usual) Dirac operator at chemical potential μ
  - $J = \kappa j,\ K = (C\gamma_5)\tau_2$

$(f = 1,2)$

- $\overbracket{\psi_f \bar{\psi}_f} = S_N = (\Delta^\dagger(\mu)\Delta(\mu) + J^2)^{-1}\Delta^\dagger(\mu)$: normal propagator

  $(S_N(\mu, j = 0) = \Delta^{-1}(\mu))$

- $\overbracket{\bar{\psi}_f^T \psi_f^T} = \bar{S}_N = K\gamma_5(\Delta^\dagger(-\mu)\Delta(-\mu) + J^2)^{-1}\Delta^\dagger(-\mu)K\gamma_5$: back propagator

  $(\bar{S}_N = -S_N^T)$

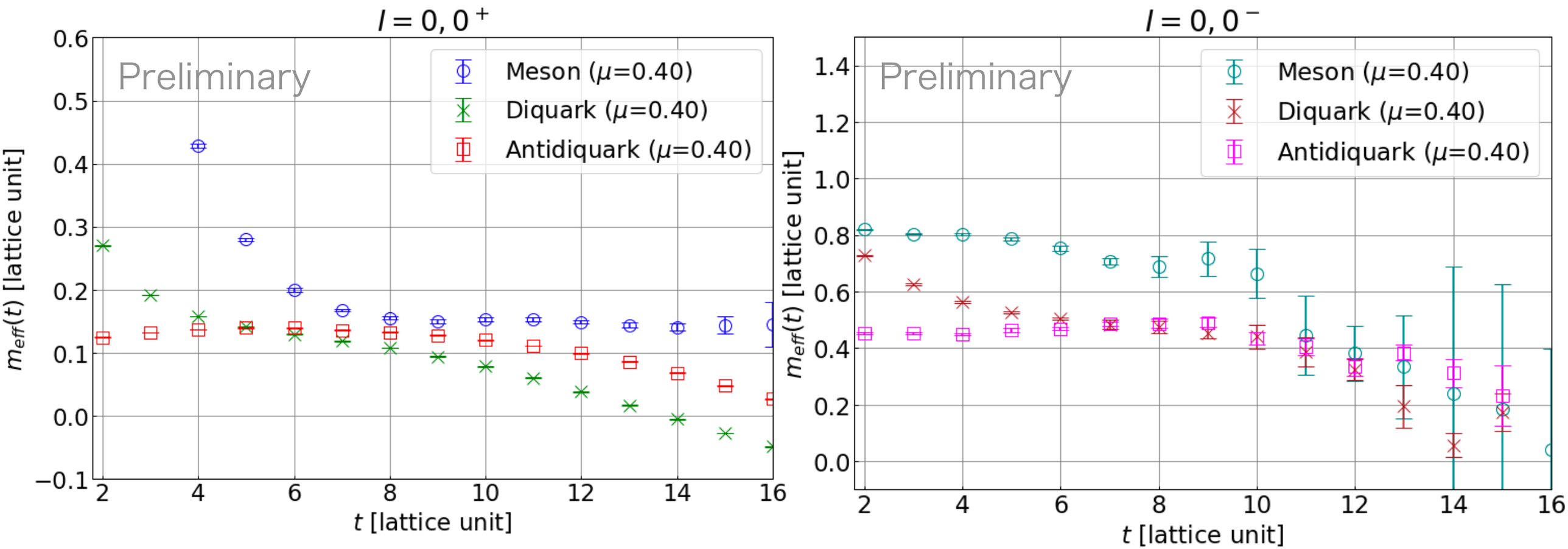
- $\overbracket{\psi_2 \psi_1^T} = S_A = J(\Delta^\dagger(\mu)\Delta(\mu) + J^2)^{-1}K$: anomalous propagator

  (quark -> antiquark)

- $\overbracket{\bar{\psi}_2^T \bar{\psi}_1} = \bar{S}_A = JK\gamma_5(\Delta^\dagger(-\mu)\Delta(-\mu) + J^2)^{-1}\gamma_5$: anomalous propagator

  (antiquark -> quark)

Effective mass in $I = 0$, $J^P = 0^{\pm}$ channels in superfluid phase



- position of plateaux of all three hadrons looks the same

  ← **meson-diquark-antidiquark mixing?**

- 0^- meson: decrease slowly ← contribution from excited state?

LSM with diquark gap in a nutshell

[Suenaga et al., in preparation]

- QC2D ($N_f = 2$)
  - quark field

$$\Psi_i = \begin{pmatrix} \psi_R \\ \sigma^2\tau^2\psi_L^* \end{pmatrix}$$

$$\Psi \to g\Psi \qquad g \in SU(4)$$

  - meson, diquark op

$$\Phi_{ij} \sim \Psi_j^T \sigma^2 \tau^2 \Psi_i$$

$$\Phi \to g\Phi g^T$$

- LSM($N_f = 2$)
  - meson, diquark field

$$\Sigma = \frac{1}{2}\begin{pmatrix} 0 & -B' + iB & \frac{\sigma - i\eta + a^0 - i\pi^0}{\sqrt{2}} & a^+ - i\pi^+ \\ B' - iB & 0 & a^- - i\pi^- & \frac{\sigma - i\eta - a^0 + i\pi^0}{\sqrt{2}} \\ -\frac{\sigma - i\eta + a^0 - i\pi^0}{\sqrt{2}} & -a^- + i\pi^- & 0 & -\bar{B}' + i\bar{B} \\ -a^+ + i\pi^+ & -\frac{\sigma - i\eta - a^0 + i\pi^0}{\sqrt{2}} & \bar{B}' - i\bar{B} & 0 \end{pmatrix}$$

$$\Sigma \to g\Sigma g^T$$

  - chiral and diquark gap

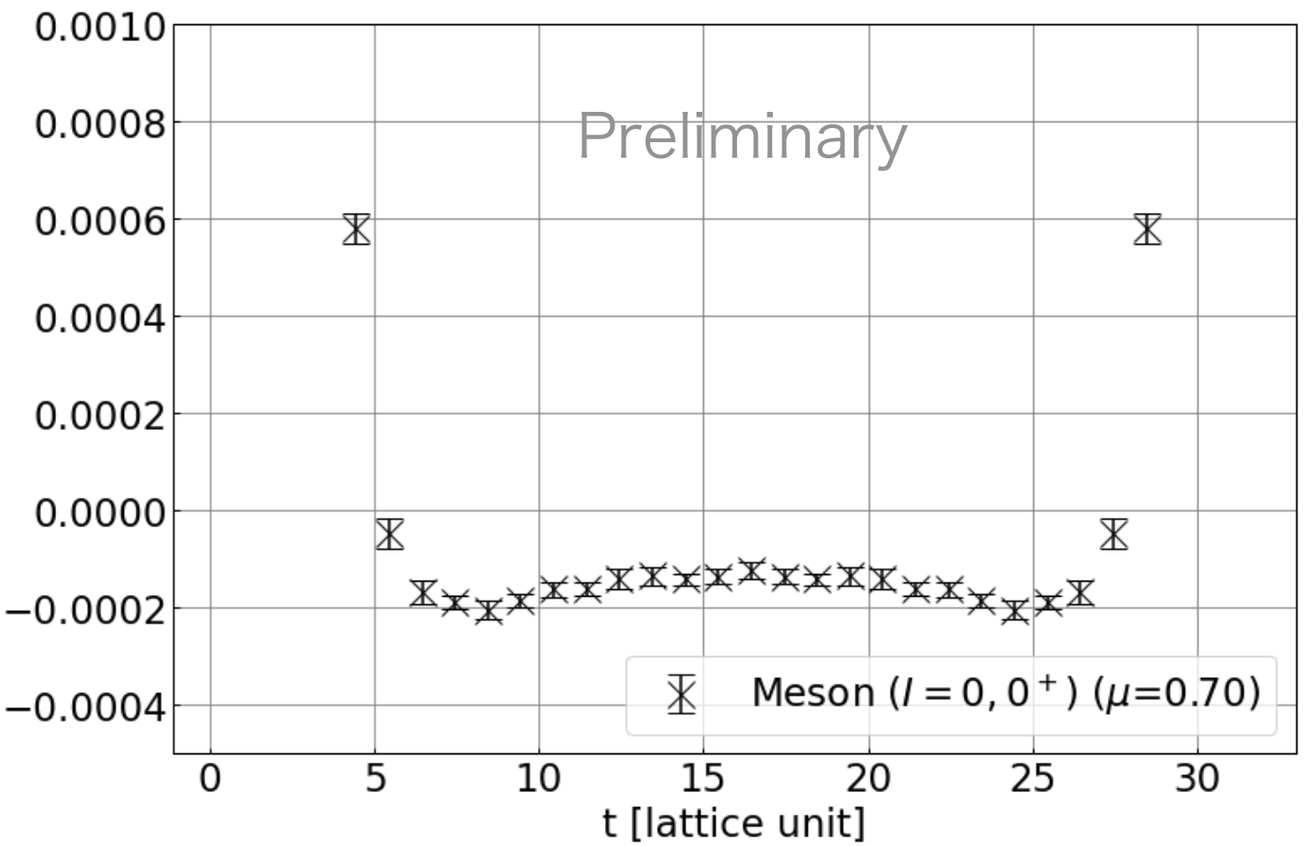
$$\sigma \to \sigma + \sigma_0$$

$$\frac{B + \bar{B}}{\sqrt{2}} \to \Delta + \frac{B + \bar{B}}{\sqrt{2}}$$

Contribution from excited states

- for some channels **in large** μ, we see an explicit sign of contribution from excited states

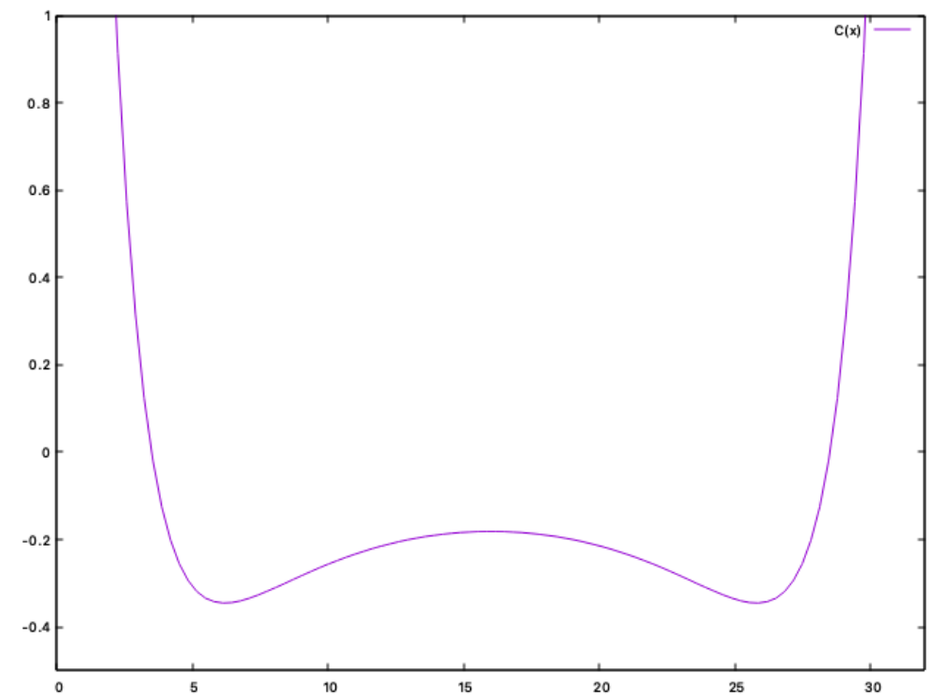
2pt function of meson ($I = 0,\ 0^+,\ a\mu = 0.7$)



plot of

$$f(t) = A_1(e^{-m_1 t} + e^{-m_1(32-t)}) + A_2(e^{-m_2 t} + e^{-m_2(32-t)})$$

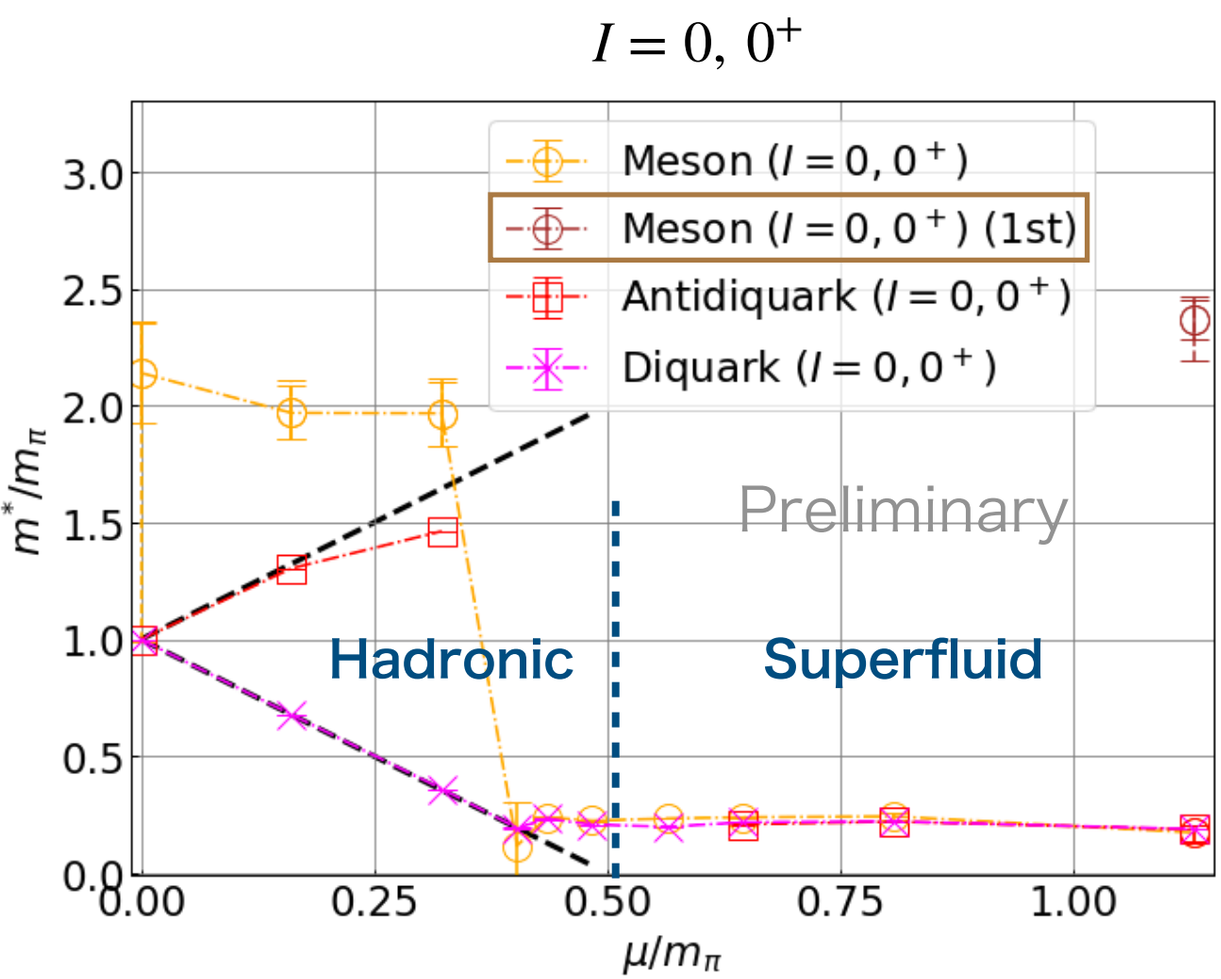
with $A_1 = -1.0,\ m_1 = 0.15,\ A_2 = 10.0,\ m_2 = 0.8$

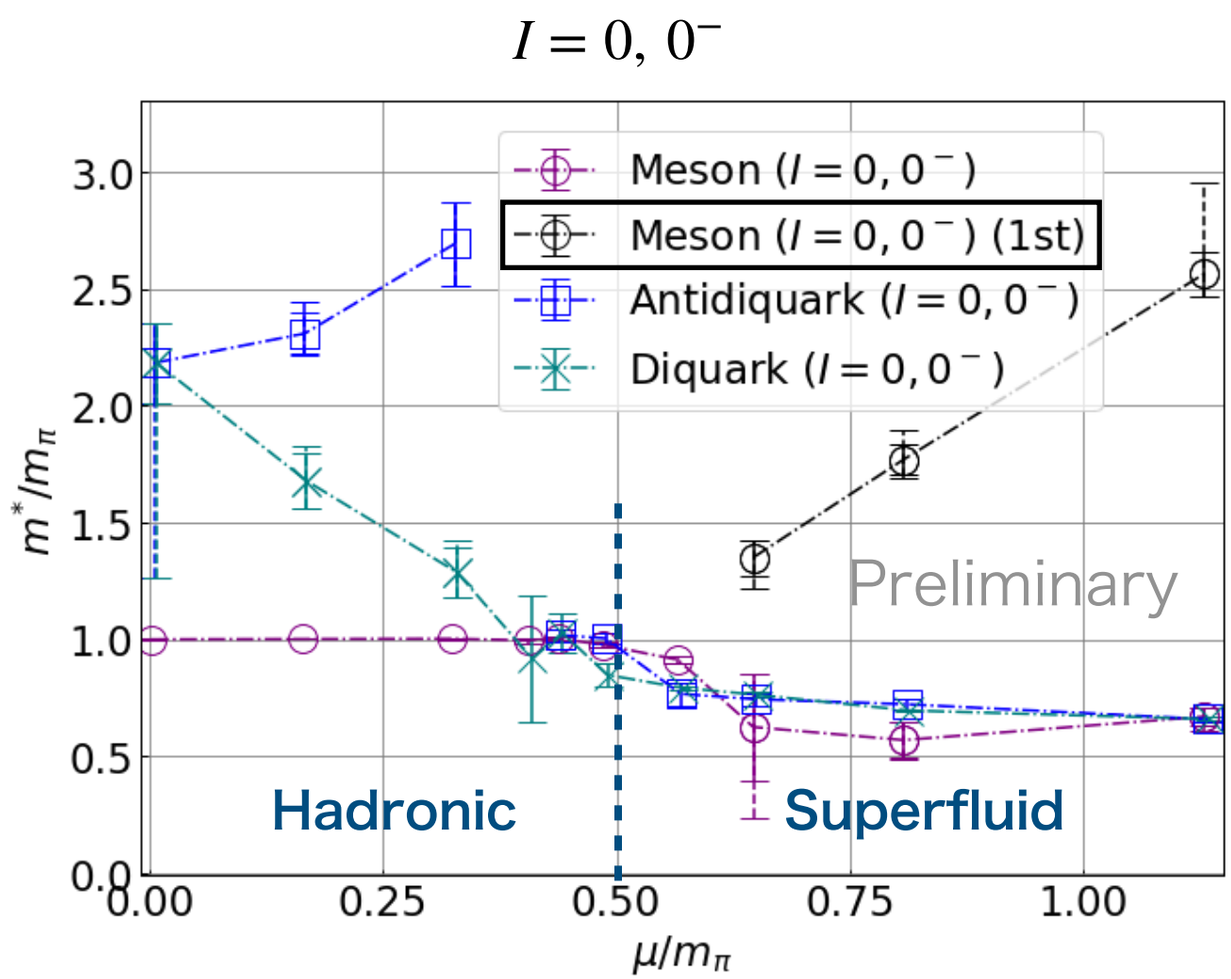


- imply the comparable overlap with excited states

- we use **double cosh function** in fitting for **meson in** $I = 0,\ 0^+$ **at** $a\mu = 0.7$ **and** $I = 0,\ 0^-$ **at** $a\mu = 0.4 - 0.7$

Results of excited states



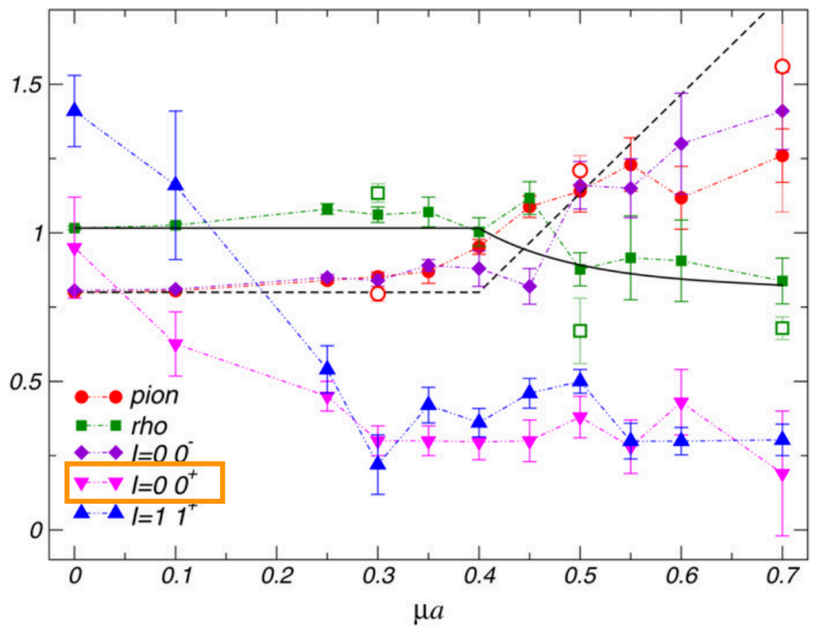


- massive excited states
- level repulsion in 0^- channel?

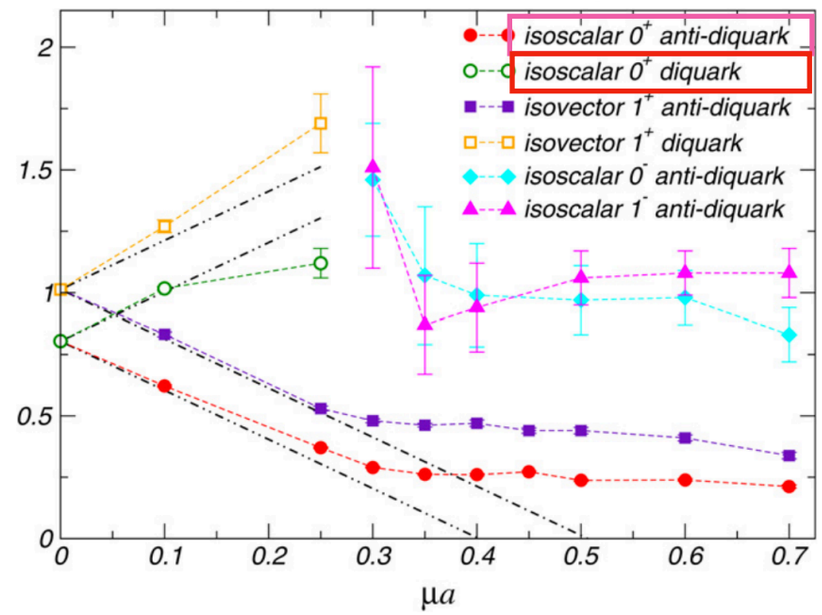
Consistency with the previous studies for $I = 0$, 0^+ channel

(S. Hands, P. Sitch and J. I. Skullerud, Phys. Lett. B 662 (2008), 405-412)

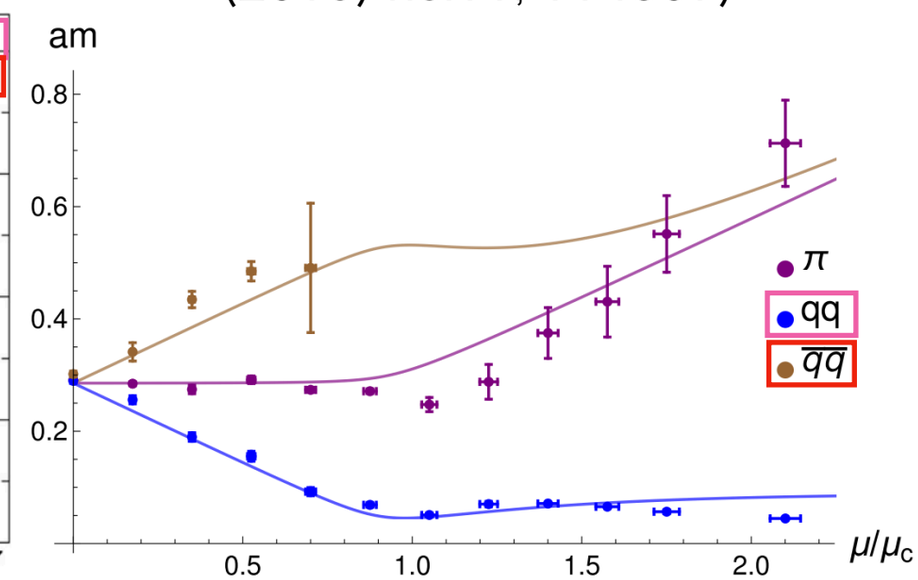
Meson



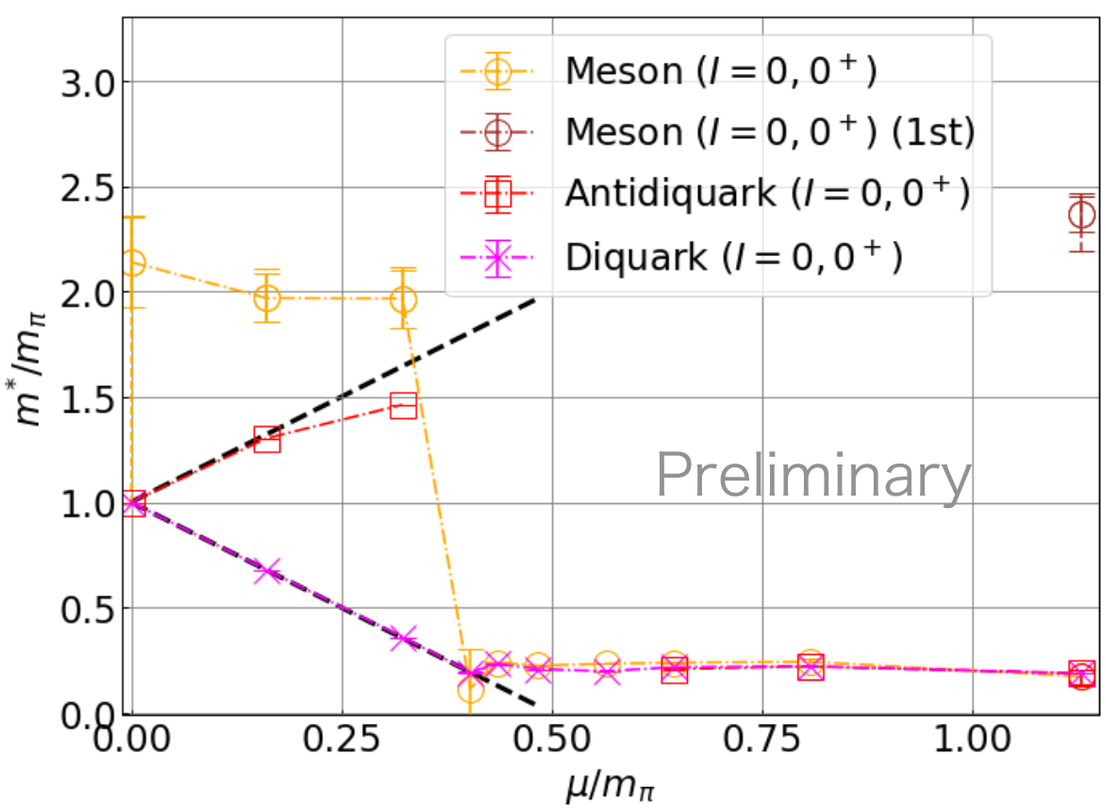
Diquark/Antidiquark



(J. Wilhelm et al., Phys. Rev. D 100 (2019) no.11, 114507)



Our results

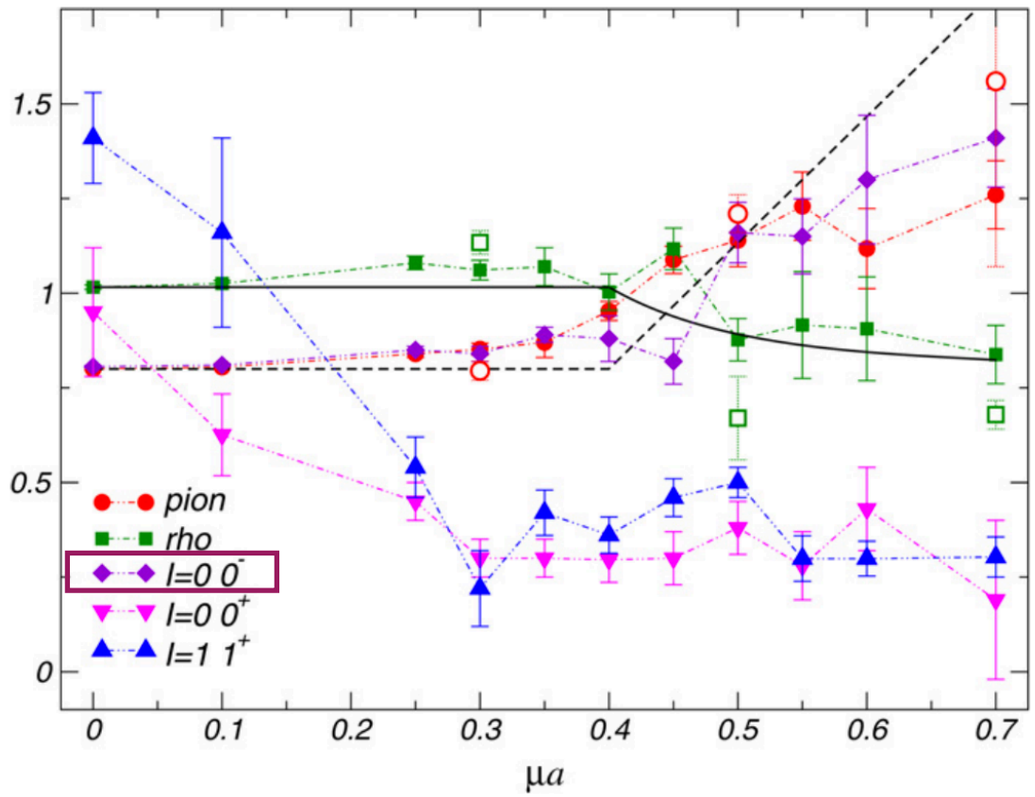


- diquark/antidiquark: consistent
- meson: different behavior in small μ ⟵ finite j effect?

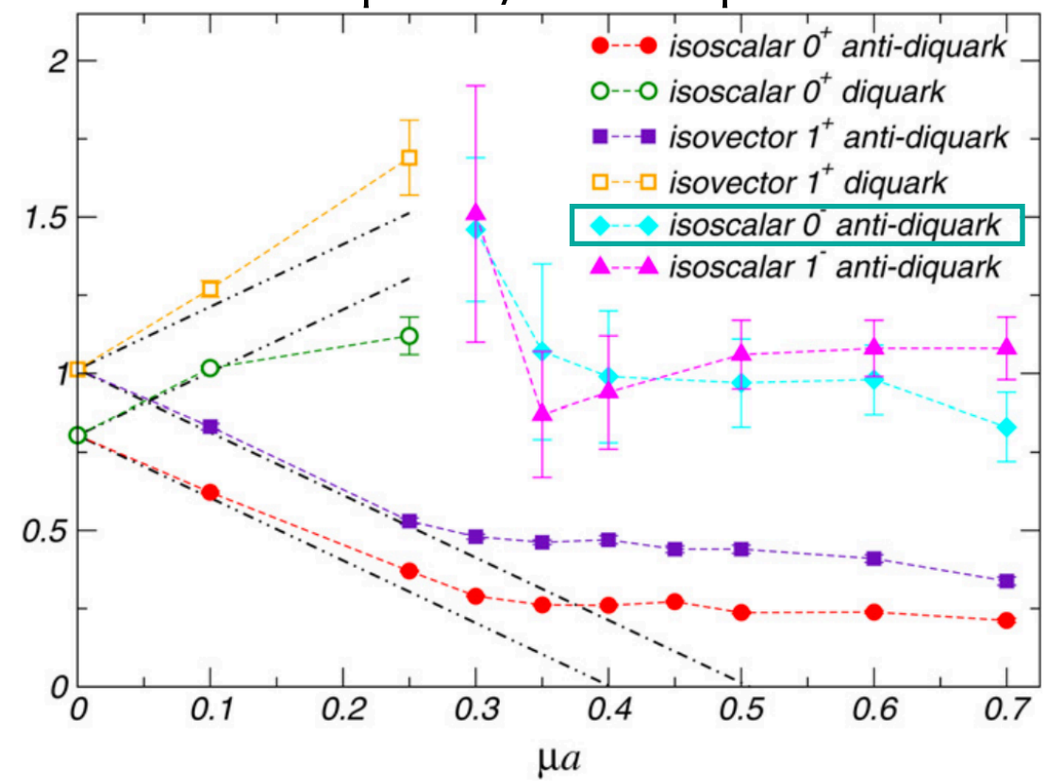
Consistency with the previous studies for $I = 0$, 0^- channel

(S. Hands, P. Sitch and J. I. Skullerud, Phys. Lett. B 662 (2008), 405-412)

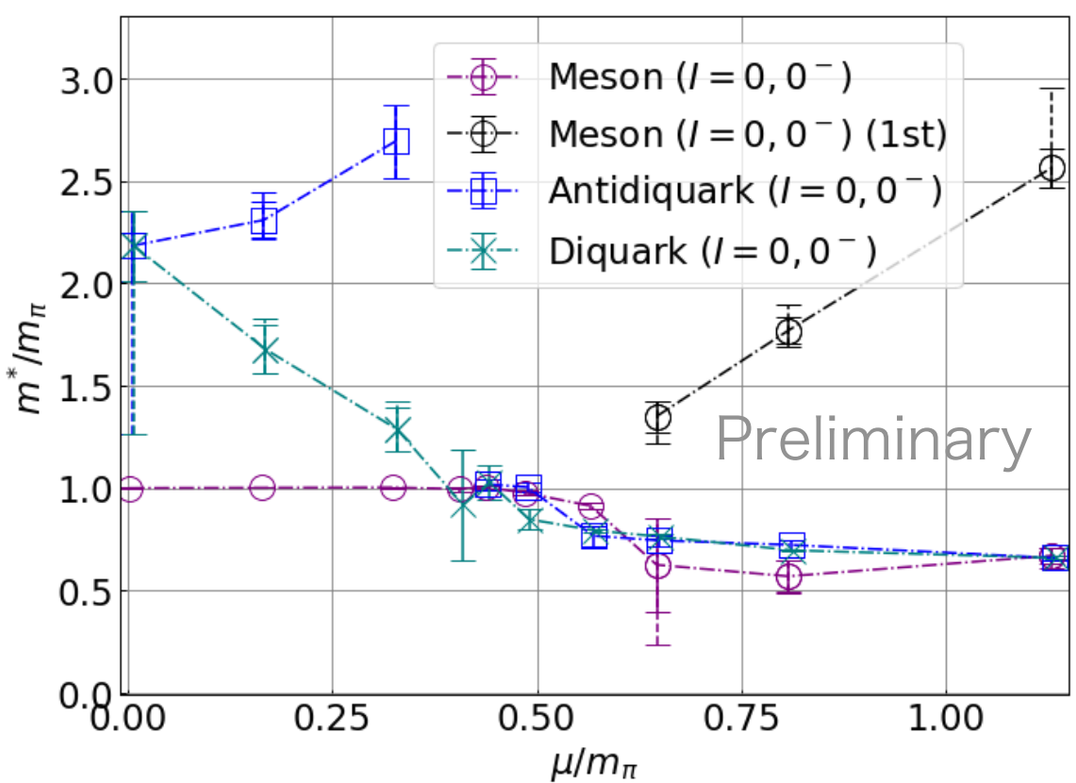
Meson



Diquark/Antidiquark



Our results



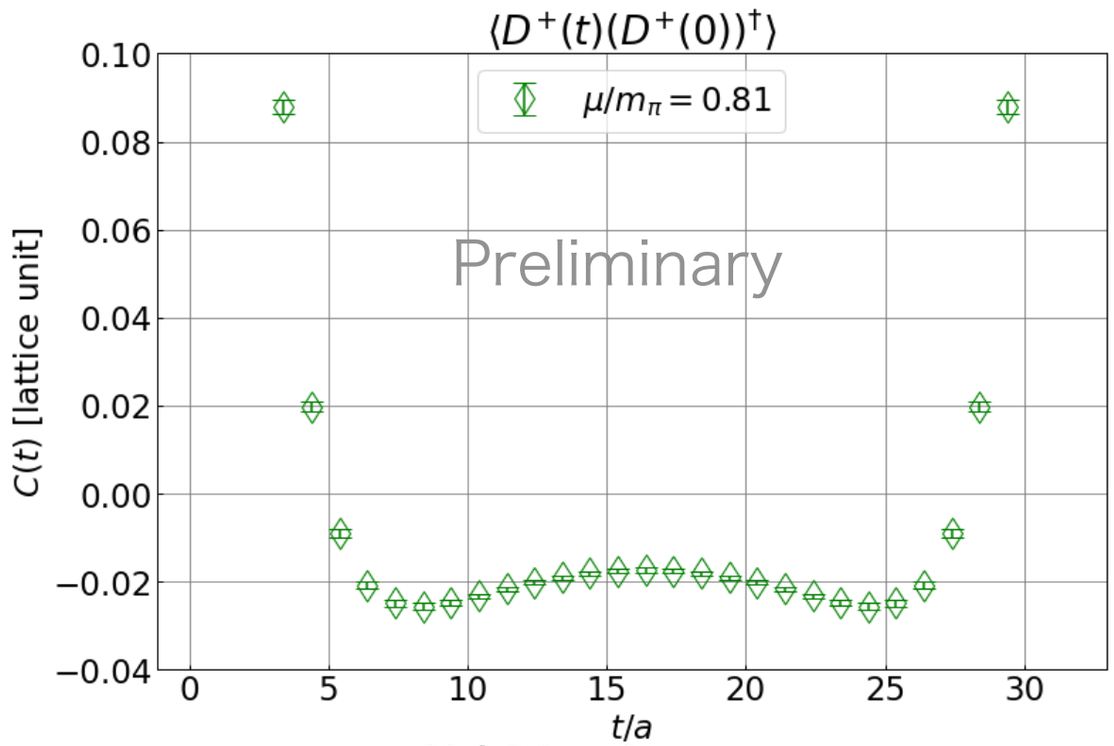
- diquark/antidiquark: consistent (in large μ)
- meson: different in large μ

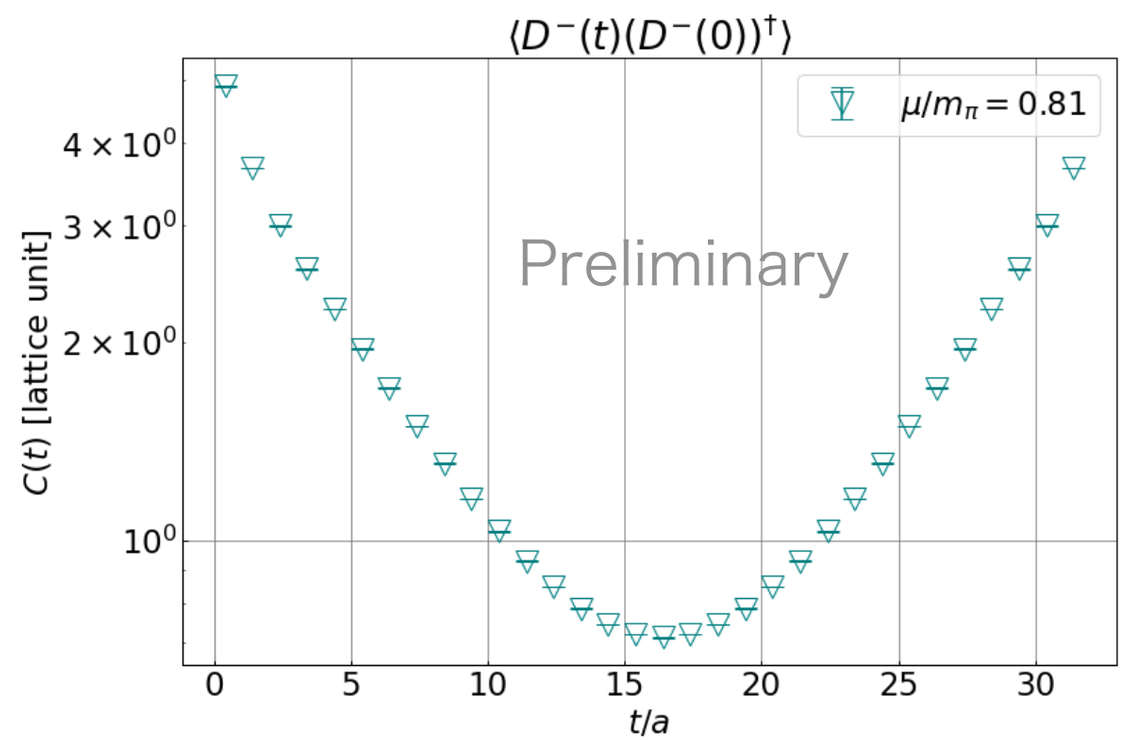
← detect excited state?

“Higgs” and “NG” 2pt functions

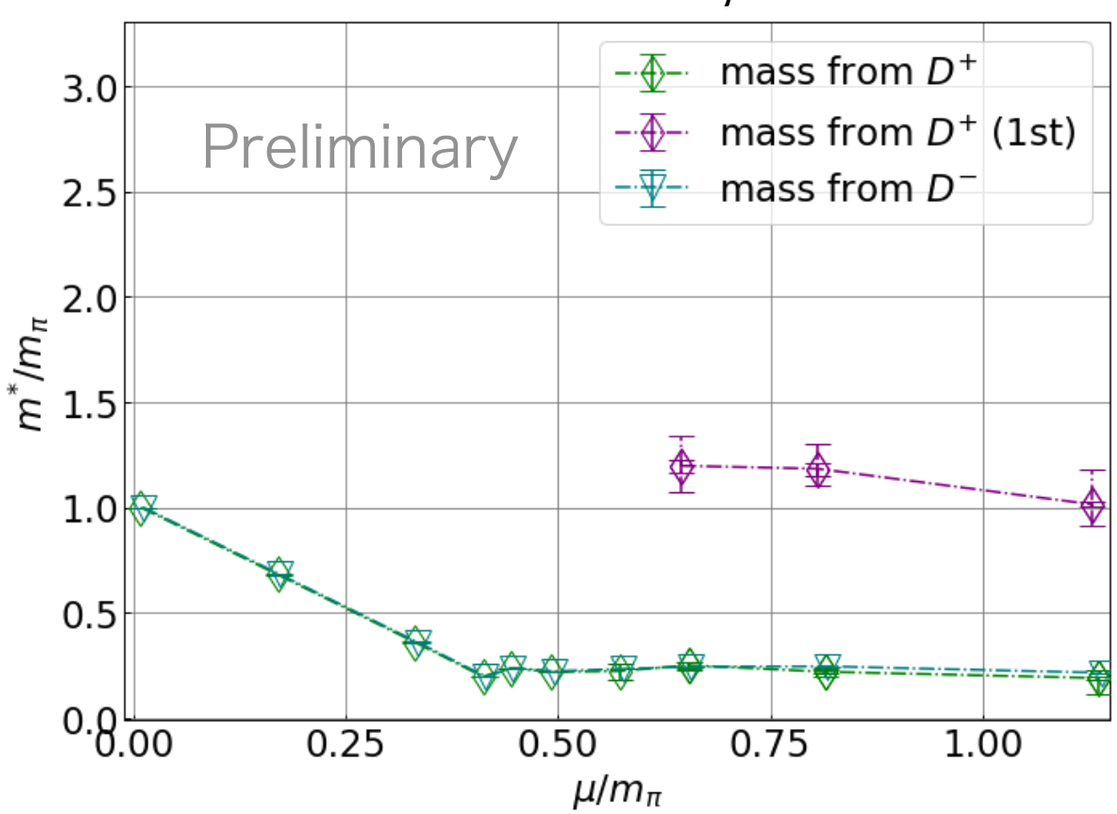
- $D^{\pm} = \bar{\psi}_1 C\gamma_5\tau_2\bar{\psi}_2^T \pm \psi_1^T C\gamma_5\tau_2\psi_2$: would-be Higgs(+)/NG(−) operators

[S. Hands, P. Sitch and J. I. Skullerud, 2008]





mass vs. μ



- D^-: large overlap with light mode

➡ almost NG mode 2pt func.

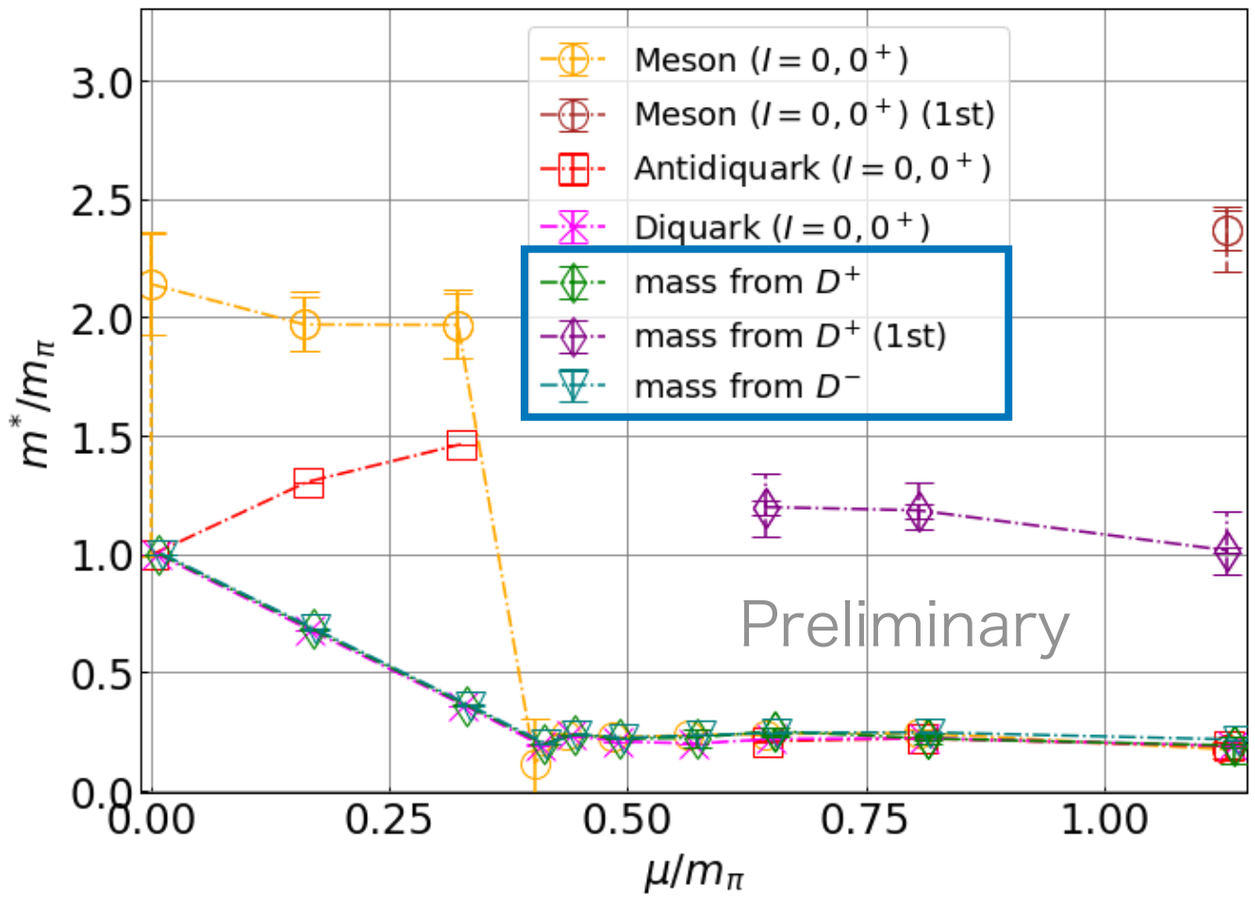
- D^+: heavy + light mode

➡ not pure Higgs

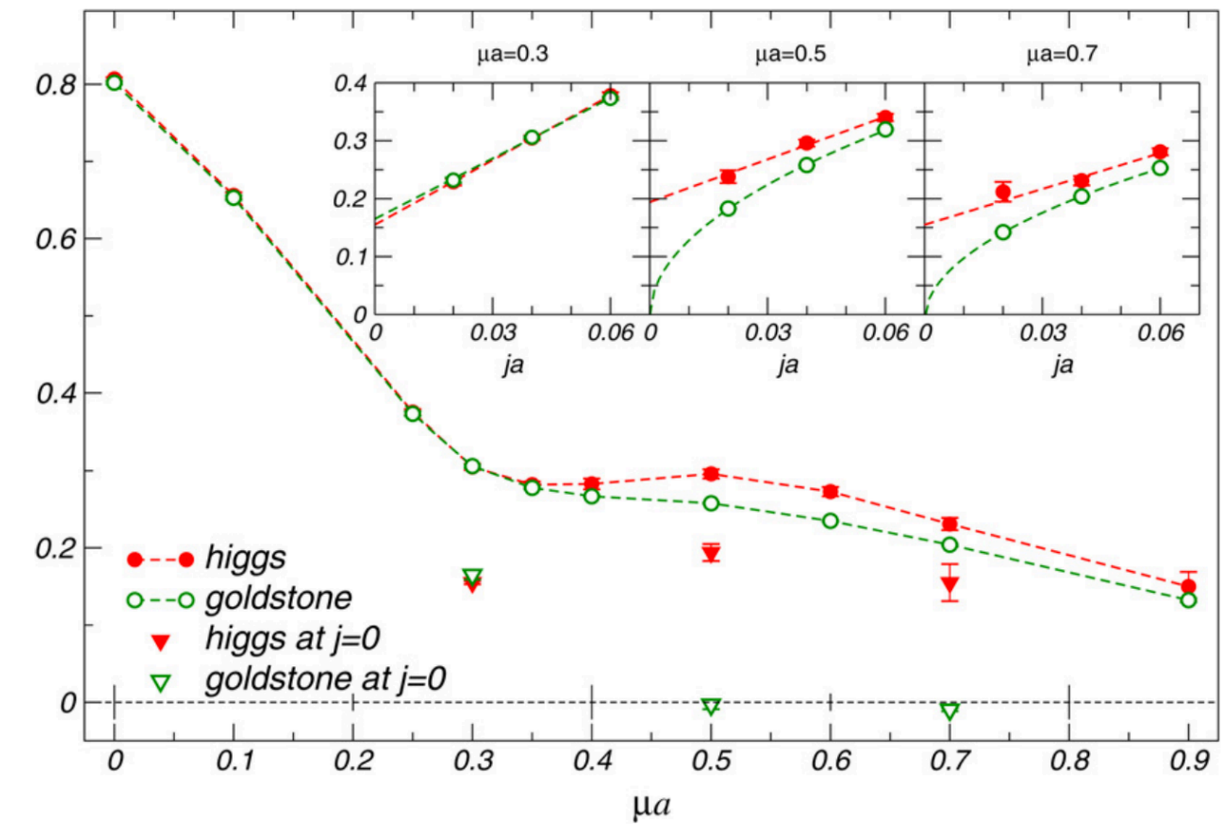
(mixing with meson?)

Comparison of $D^{\pm}$ with other results

Our results



(S. Hands, P. Sitch and J. I. Skullerud, Phys. Lett. B 662 (2008), 405-412)



- ground state of $D^{\pm}$: same as diquark mass, consistent behavior with other lattice results

- excited states of D^{+}: discrepancies among all results ⟵ systematics for fitting? lattice artifact?