Nuclear Transition in the Strong Coupling Limit

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Strong-interaction matter under extreme conditions

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Outline

Motivation

- The transition between hadrons and nuclear matter is like a liquid-gas phase transition (first order transition with a critical end point because of the resemblance between the nuclear force and a Van der Waals interaction with a long range attractive potential and a short range repulsive core). (Epelbaum, Hammer, and Meissner 2009)
- This phase transition plays an important role during the collapse of supernovae to a neutron star. Therefore, it is interesting to study the dependence of the nuclear force and the transition on different parameters.
- The nuclear force between Baryons is due to the residual strong interactions between quarks and gluons and hence should be accurately described by QCD. (Ishii, Aoki, and Hatsuda 2007)
- However, at finite density lattice QCD has the infamous sign problem. Methods overcoming this, work only for small lattice volumes and chemical potentials. So we need an alternative approach to study this.

Phase diagram of nuclear matter (from experiment)

At low temperatures:
low μ_B - dilute hadrons
high μ_B - nuclear matter

Motivation

- A different approach is to study lattice QCD at the strong coupling limit (infinite bare gauge coupling)
- In this approach, the gauge (link) integration is performed first followed by the grassmann integration (Rossi and Wolff 1984)
- Benefits of this procedure:
  - The sign problem is mild and the sign fluctuations are taken into account exactly (rather than probabilistically)
  - The degrees of freedom are color-singlet mesons and baryons and thus we can see their contributions to observables separately (This makes the interpretation better)
  - Chiral symmetry breaking and confinement properties of QCD are still present in this model
  - The complete phase diagram can be calculated in this limit
- The strong coupling limit leads to a Dual Formulation of lattice QCD.

3D effective theory

Finite density lattice QCD can also be analysed using Polyakov loop effective theory based on the hopping parameter.(Talk by Christoph Konrad, 12.08(Non-zero temperature session))
This is performed at large quark masses. To make contact with this, we need to use the Dual Formulation at large quark masses.

Dual formulation of the QCD partition function

- At strong coupling ($\beta = 0$ implies no e^{-S_G} factor in partition function), the link integral of the QCD partition function factorizes and we can rewrite it as a product of one-link integrals.

$$Z = \int \prod_x \left(\mathrm{d}\chi_x \ \mathrm{d}\chi_x \mathrm{e}^{2am_q \bar{\chi}_x \chi_x} \prod_{\hat{\mu}} z(x, \hat{\mu}) \right)$$

  $z(x, \hat{\mu})$ is the gauge integral of individual links.

- The one-link group integral $z(x, \hat{\mu})$ can be solved and rewritten in terms of new variables

$$M(z) = \bar{\chi}_z \chi_z \quad \text{and} \quad B(z) = \frac{1}{N!} \varepsilon_{i_1 \ldots i_N} \chi_{z,i_1} \cdots \chi_{z,i_N}$$

  These are the mesonic(a quark-antiquark pair) and baryonic(N_c quarks/anti-quarks) color singlet states. The solution of the one-link integral has terms like $(M(x)M(y))^k$, $\bar{B}(y)B(x)$ and $\bar{B}(x)B(y)$ in terms of the new variables.

- After the gauge integration, we can perform the remaining Grassmann integral which gives us terms of the form $(2am_q M(x))^{n_x}$.

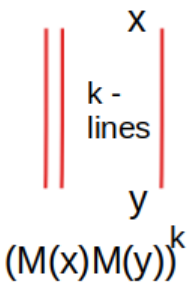
Pictorial representation of dual degrees of freedom

To get non-vanishing results in the Grassmann integration, we need the constraint that every degree of freedom has to be paired.

Since, $B(x)$ saturates all quark degrees of freedom at site x, there should be a term $\bar{B}(x)$ from another baryon link. Therefore, baryons must form self-avoiding oriented loops ℓ.

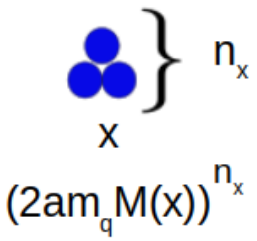
(n_x,k,ℓ) are the dual degrees of freedom.

Dimer



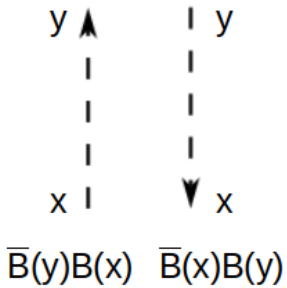
$M(x)M(y) \rightarrow$ has a quark anti-quark pair at site x and another pair at site y. This can be represented as a non-oriented link.
The power k denotes the number of such links.

Monomer



$2am_qM(x) \rightarrow$ is the mass term at a site. This can be represented by a point.
The power n_x denotes the number of such terms at a site.

Baryon links



$\bar{B}(y)B(x) \rightarrow$ has N_c quarks at site x and N_c anti-quarks at site y. This can be represented as an oriented link from x to y.

QCD partition function

- The partition function in terms of the dual variables (k, n_x, l) is

$$Z = \sum_{\{k,n,\ell\}} \prod_{b=(x,\hat{\mu})} \frac{(3-k_b)!}{3!k_b!} \gamma^{2k_b\delta_{0,\hat{\mu}}} \prod_x \frac{3!}{n_x!} \left(2am_q\right)^{n_x} \prod_\ell w(\ell)$$

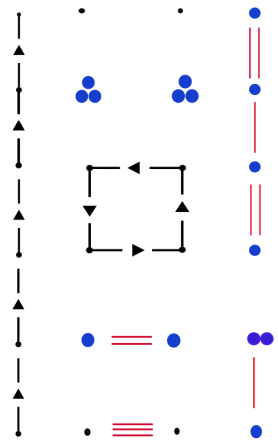
$$w(\ell) = \frac{1}{\prod_{x\in\ell} 3!} \sigma(\ell) \gamma^{3N_{\hat{0}}} \exp\left(3N_t r_\ell a_t \mu\right)$$

$$\sigma(\ell) = (-1)^{r+N_-(\ell)+1} \prod_{b=(x,\hat{\mu})\in\ell} \eta_{\hat{\mu}}(x)$$

- The sign can be explicitly calculated for every configuration. The Dual formulation has a +1 or -1 sign, therefore sign reweighting techniques can be applied.
- Observables in terms of the new degrees of freedom can be calculated.
- Energy density :

$$a^4\epsilon = \frac{\xi}{\gamma}\frac{\partial\gamma}{\partial\xi}\langle 2N_{Dt} + 3N_{Bt}\rangle - \langle N_M\rangle$$

N_{Dt} is the number of temporal dimers, N_{Bt} is the number of temporal baryon loops and N_M is the number of monomers in a configuration.



A sample configuration. The degrees of freedom are saturated at each lattice site.

Finding the Baryon mass

- A static baryon in the dual formulation is a closed loop in the $\hat{0}$-direction. The loop is built of links $\bar{B}(x')B(x)$ which is a baryon hopping (annihilation at x and creation at x').
- At low temperature $T = \frac{1}{N_t}$, the probability for this hopping is proportional to $\exp(-m_B)$. So for the whole loop ℓ, the probability is

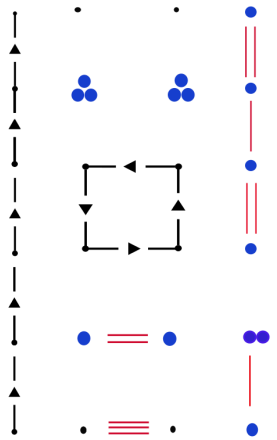
$$p_l \approx \exp(-m_B N_t) = \exp(-m_B/T)$$

- The probability can also be expressed as $\exp(-\Delta F/T)$ where ΔF is the difference in free energy between a configuration having a static baryon world line and a configuration without.

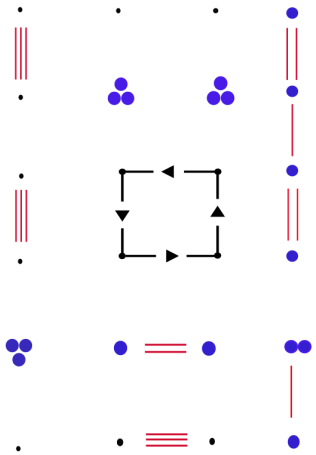
$$m_B = \Delta F$$

- At low temperatures, the free energy is approximately equal to the energy. So baryon mass can be calculated by ΔE which is the difference in energy density between a configuration having a static baryon world line and a configuration without.

$$m_B = \Delta E$$



Sample with a static baryon.

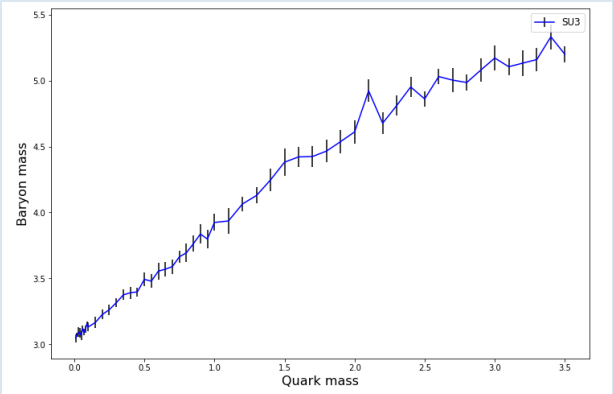


Sample without a static baryon.

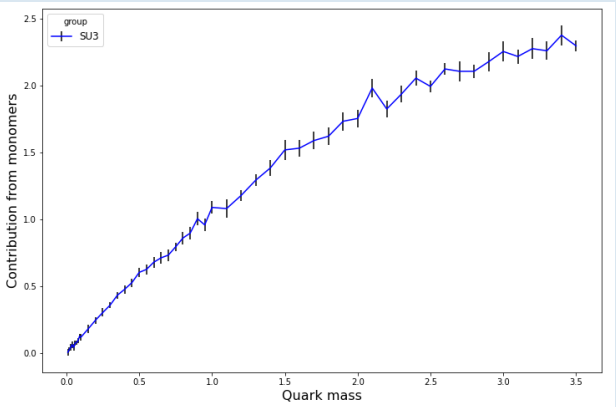
Baryon mass at different Quark masses

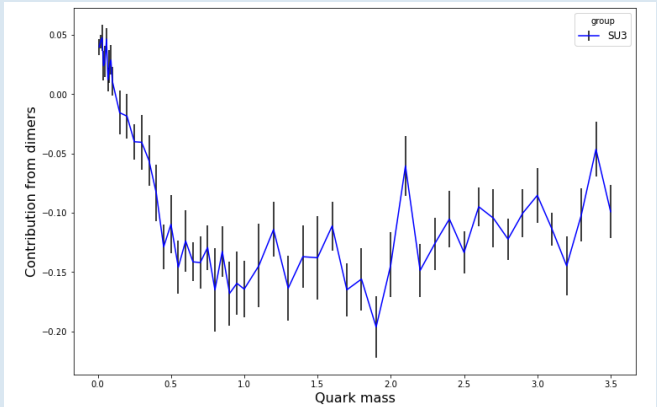
Baryon mass

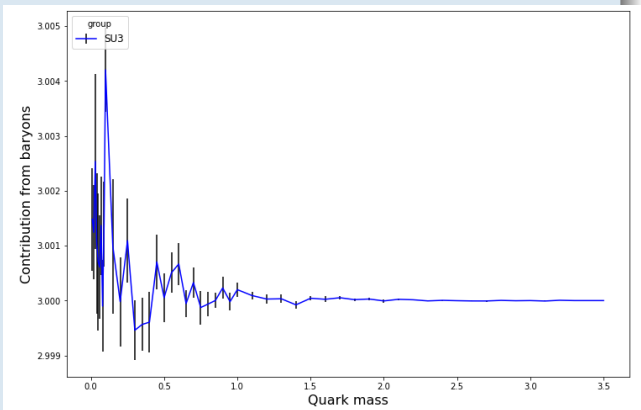
- We can use $m_B = \Delta E$ to calculate the baryon mass for different quark masses
- Parameters used : Lattice size - $8 \times 8 \times 8 \times 4$, Temperature - 0.25, Quark mass $\in$ [0.01,3.5]



Different contributions to the baryon mass

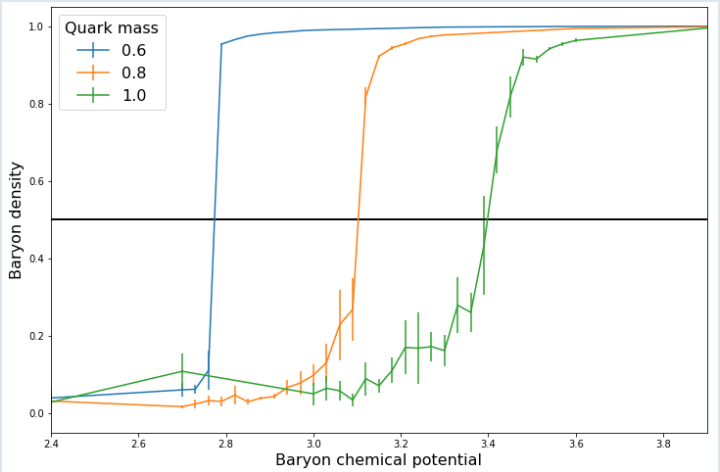


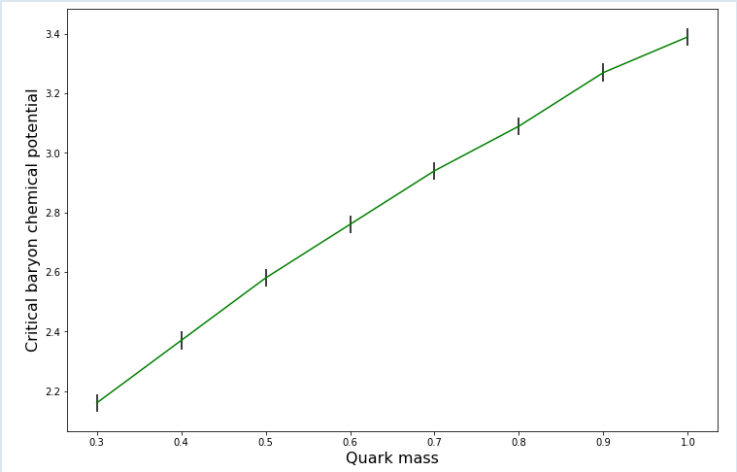




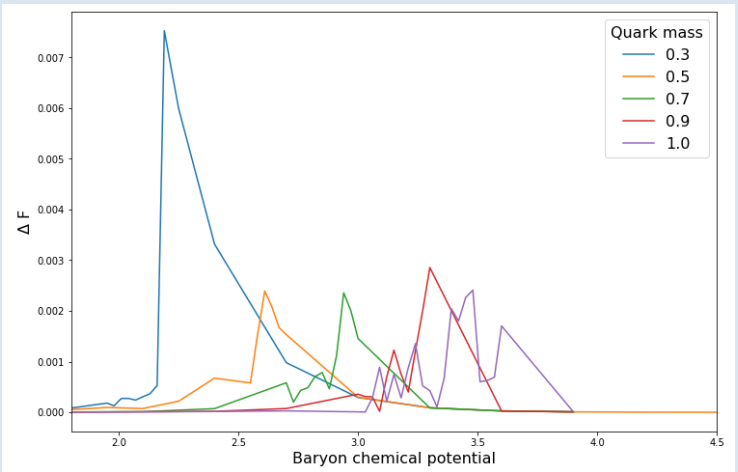
Nuclear transition

Nuclear transition with the Dual Formulation





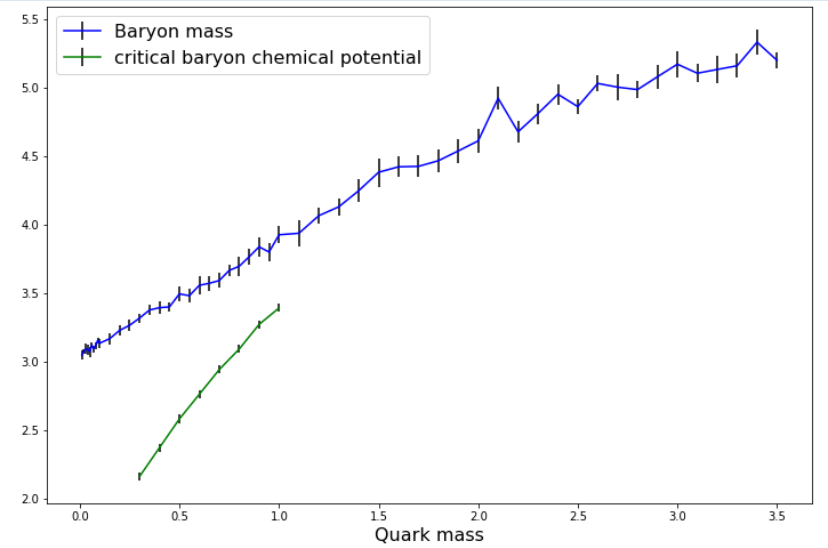
Sign problem at different chemical potential

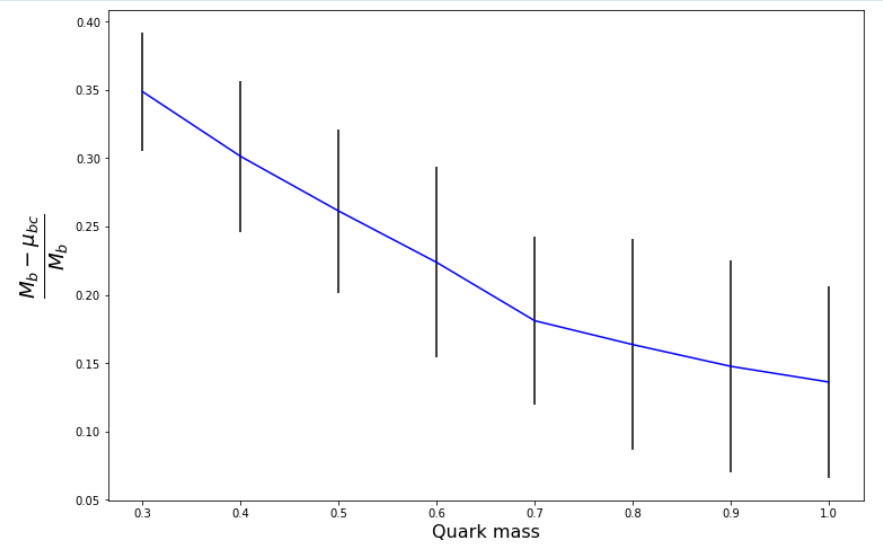


- Δ F tells us about the severity of the sign problem.
- We see that Δ F is close to 0 for all the chemical potentials for each quark mass except near the critical chemical potential
- The sign problem is also milder as the quark masses increase.

Nuclear Interaction

- The change in free energy to have a non-interacting baryon in a dilute bath of mesons (which happens at the transition to get a non-zero baryon density) is given its mass. Therefore, the transition should occur when $\mu_{B,c} = m_B$.
- However, it is seen that there is a difference between $\mu_{B,c}$ and m_B. This difference should be due to the nuclear interaction (which is due to the residual strong interaction between quarks).(Bili ć, Karsch, and Redlich 1992)
- We can quantify the nuclear interaction with the Dual formulation.





Mean Field theory

- We can compare the Dual Formulation results with the mean field approach of LQCD (1/d expansion of Z followed by Hubbard-Stratonovich transformation).
- The free energy can be obtained after this procedure as (Nishida 2004)

$$F_{\text{eff}}\ [\sigma, T, \mu_{\text{B}}] = \frac{N_{\text{c}} d}{4}\sigma^2 - T \log\left\{2\cosh\left[\mu_{\text{B}}/T\right] + \frac{\sinh\left[(N_{\text{c}}+1)\,E/T\right]}{\sinh[E/T]}\right\}$$

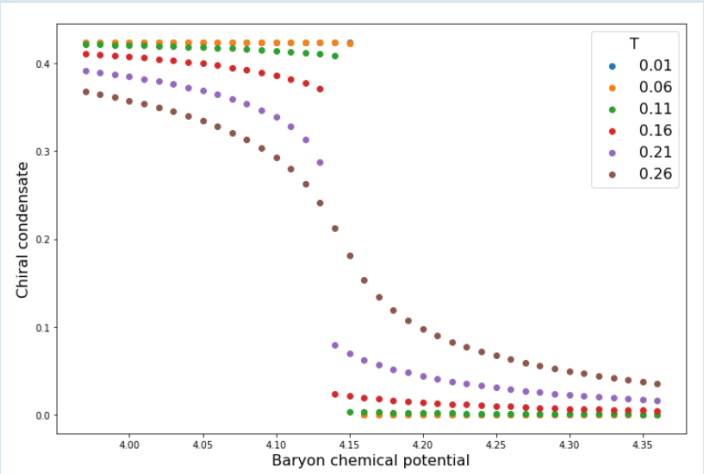
$$E[M] = \text{arcsinh}\left[\sqrt{M^2 + (d/2)^2}\right]$$

where E[M] is the one-dimensional quark excitation energy.

- The global minima of the free energy with respect to the chiral condensate gives us the chiral condensate as a $f(T, \mu, m_q)$.
- At given T and m_q, σ can be plotted as a function of μ.
- The chemical potential where there is a discontinuity in the chiral condensate indicates the critical chemical potential.

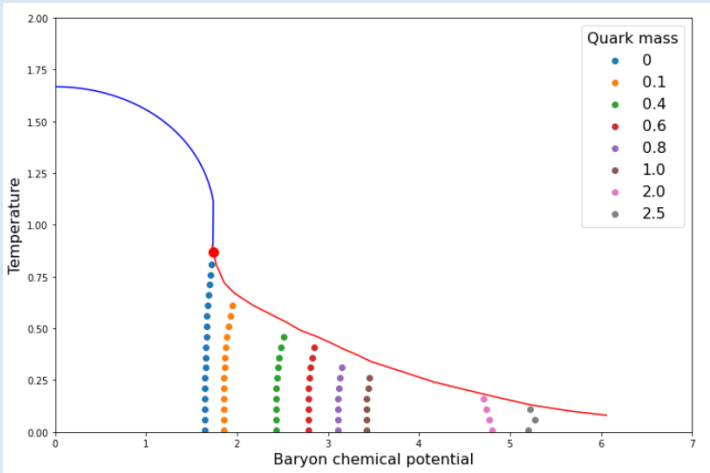
Mean Field theory

Chiral condensate for quark mass 1.5



There is a clear discontinuity at low temperatures - first order transition
At higher temperatures the chiral condensate changes continuously - crossover region
The temperature and chemical potential where the chiral condensate changes from a discontinuous variable to a continuous one can be assumed to be the critical end point.

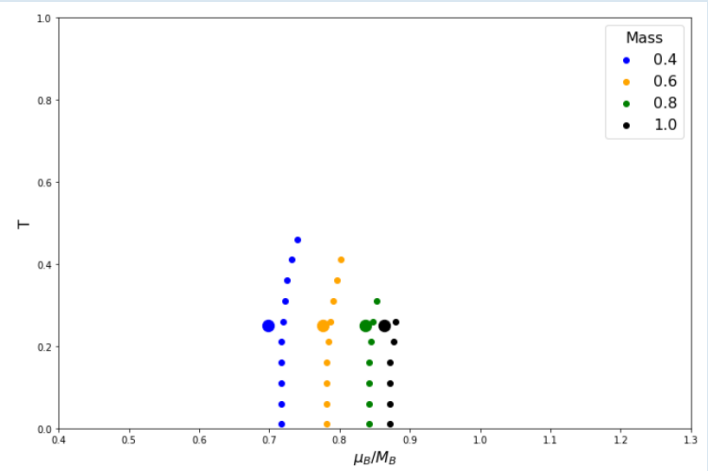
Phase transition of lattice QCD by the mean field approach



- The blue line shows the second order transition at the chiral limit.
- The phase transition for different quark masses obtained from the mean field approach are plotted.
- The red line shows the critical end point.

Comparision with mean field theory

Nuclear transition



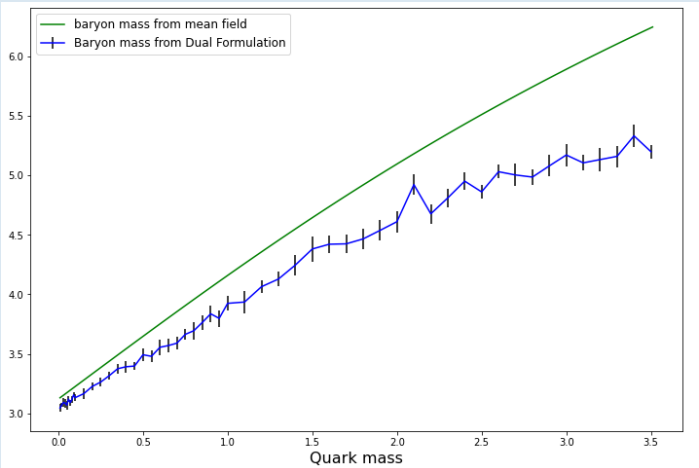
- The phase diagram in the plane $(\mu_B/M_B, T)$ is plotted for different quark masses.
- smaller dots - from the mean field approach
- bigger dots - from Dual formulation simulations
- The results from both the formalisms are similar.

Baryon mass

- The baryon mass computed from mean field theory is a function of the quark mass.(Kluberg-Stern et al. 1983)

$$\sinh am_B = \frac{1}{2}\left(m + \sqrt{m^2+1}\right)^N (2d)^{N/2}$$

- Results from Dual formulation are similar to mean field theory.



Conclusion and Outlook

Conclusion

- Dual formulation of lattice QCD has a mild sign problem, thus allowing for calculations at finite density.
- We can identify the individual contributions from monomers, dimers and baryon loops to different observables.
- μ_{Bc} and baryon mass found from the Dual formulation is similar to the predictions from the mean field approach.

Outlook

- We plan to perform simulations at higher quark masses and lower temperatures to make contact with the polyakov loop effective theory. This is challenging because the autocorrelations increase with higher quark mass.
- We plan to find the baryon mass after including the gauge corrections in the Dual formulation.

THANK YOU

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