Pion condensation at lower than physical quark masses

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2+1 flavour QCD at non-zero isospin potential

Partition function of 2+1 flavour QCD after integration over fermion fields

$$\mathcal{Z} = \int \mathcal{D}U_\mu \, e^{-\beta S_G} \, (\mathcal{M}_{ud})^{1/4} \, (\mathcal{M}_s)^{1/4} \, ,$$

$$\mathcal{M}_{ud} = \begin{pmatrix} \not{D}(\mu_I) + m_{ud} & \lambda\eta_5 \\ -\lambda\eta_5 & \not{D}(-\mu_I) + m_{ud} \end{pmatrix} \, , \quad \mathcal{M}_s = \not{D}(0) + m_s \, ,$$

$$\eta_5 = (-1)^{x+y+z+t} \quad , \qquad \not{D}(\mu_I)^\dagger = \eta_5 \not{D}(-\mu_I)\eta_5 \; .$$

λ is an unphysical pionic source necessary to see a spontaneous symmetry breaking in a finite volume. To get physical results extrapolation $\lambda \to 0$ is necessary.

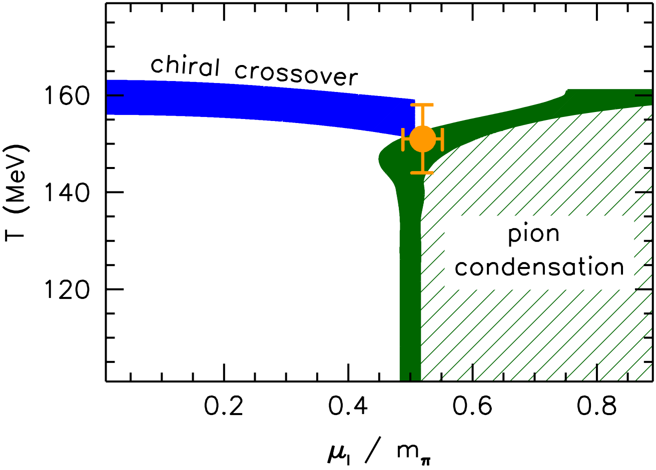
Absence of the sign problem

While $\det\left(\not{D}(\mu_I)+m_{ud}\right)$ can be complex for $\mu_I>0$, $\det\mathcal{M}_{ud}$ is real and positive.

$$B=\begin{pmatrix}1 & 0\\ 0 & \eta_5\end{pmatrix}\ ,\quad B\mathcal{M}_{ud}B=\begin{pmatrix}\not{D}(\mu_I)+m_{ud} & \lambda\\ -\lambda & \left(\not{D}(\mu_I)+m_{ud}\right)^\dagger\end{pmatrix}$$

$$\begin{aligned}\det\mathcal{M}_{ud}&=\det B\mathcal{M}_{ud}B=\\ &=\det\left(\left(\not{D}(\mu_I)+m_{ud}\right)^\dagger\left(\not{D}(\mu_I)+m_{ud}\right)+\lambda^2\right)>0\end{aligned}$$

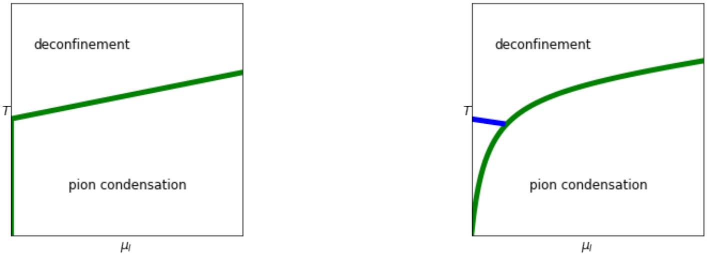
Phase structure at physical quark masses



Brandt, Endrődi, Schmalzbauer (2018)

At zero temerature pion condensation happens at $\mu_I = m_\pi/2$, and for small light quark masses $m_\pi \sim m_{ud}^{1/2}$

Possible (schematic) phase structures:



- NJL models He, Jin, Zhuang (2005)
- FRG study Svanes, Andersen (2011)

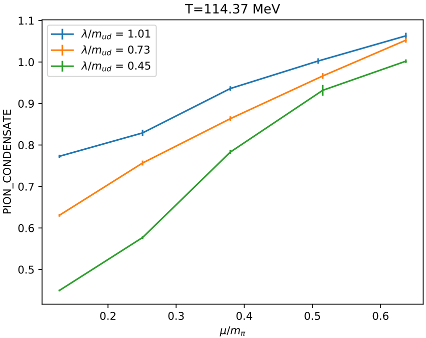
Simulation details

- Staggered fermions, tree-level Symanzik-improved gluon action
- $24^3 \times 8$ lattice
- $T = 114.37, 132.24, 141.96$ MeV
- 5 values of μ_I, $0.1 \leq \mu_I / m_{\pi,\text{phys}} \leq 0.7$
- 3 values of λ, $0.4 \leq \lambda / m_{ud} \leq 1.5$
- ~ 200 configurations per parameter set, 5 updates between configurations, 1000 thermalization updates.

$$\left\langle \pi^{\pm} \right\rangle = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \lambda} = \frac{T}{2V}\left\langle \mathrm{Tr}\,\frac{\lambda}{|\not{D}(\mu_I) + m_{ud}|^2 + \lambda^2} \right\rangle$$

Renormalized condensate (additive divergence vanish at $\lambda \to 0$):

$$\Sigma_\pi = \frac{m_{ud}}{m_\pi^2 f_\pi^2}\left\langle \pi^{\pm} \right\rangle$$



Improved pion condensate

Banks-Casher type relation for the pion condensate

$$\left\langle \mathrm{Tr}\, \frac{\lambda}{|\not{D}(\mu_I) + m_{ud}|^2 + \lambda^2} \right\rangle = \left\langle \sum_n \frac{\lambda}{\xi_n^2 + \lambda^2} \right\rangle \;,$$

where ξ_n is the n-th singular value of the Dirac operator:

$$\left(\not{D}(\mu_I) + m_{ud}\right)^\dagger \left(\not{D}(\mu_I) + m_{ud}\right) \phi_n = \xi_n^2 \phi_n \;.$$

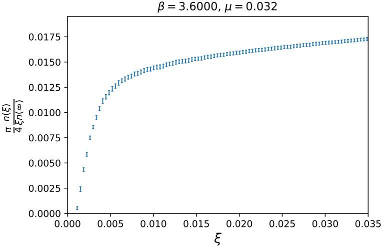
In the infinite volume limit summation can be replaced by integration

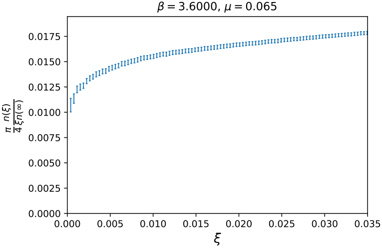
$$\left\langle \pi^\pm \right\rangle = \frac{\lambda}{2} \left\langle \int \mathrm{d}\xi\, \rho(\xi)(\xi^2 + \lambda^2)^{-1} \right\rangle \;,$$

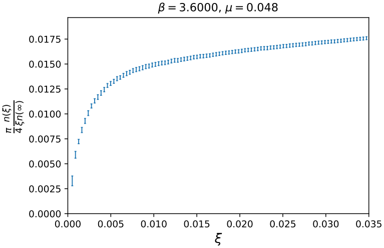
and taking the limit $\lambda \to 0$ we get

$$\left\langle \pi^\pm \right\rangle = \frac{\pi}{4} \left\langle \rho(0) \right\rangle \;.$$

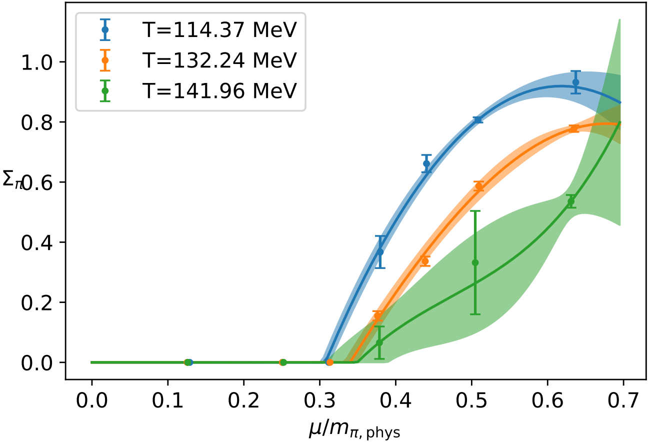
Improved pion condensate



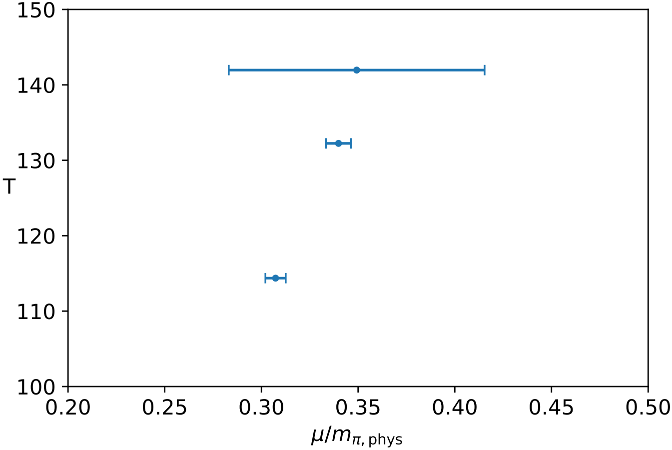




Improved pion condensate



Location of the phase transition



Summary

- Extrapolating $\lambda \to 0$ for the non-improved pion condensate is hard due to difficulty in numerical inversion of the Dirac operator for small λ,
- Improved pion condensate observable using the Banks-Casher type relation for the pion condensate has much less dependence on λ, and allows to use $\lambda \sim m_{ud}$.
- Results on coarse lattice suggest that the position of the transition at $m_{ud} = m_{ud,phys}/2$ show more temperature dependence than for the physical light quark mass.
- Pion condensation seems to happen at $\mu < m_\pi$ – comparison with results at finer lattices is needed.