Isosinglet and Isovector Nucleon Charges

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Definitions

The nucleon charges are (forward limit) matrix elements $\langle p|\overline{u}\Gamma d|n\rangle$, where $\Gamma \in \{\mathbb{I}, \gamma^5\gamma^\mu, \gamma^\mu, \gamma^\mu\gamma^\nu\}$ defines the charge. In the isospin limit the charges can be written as

$$\langle p|\overline{u}\Gamma d|n\rangle = \langle n|(\overline{d}\Gamma d - \overline{u}\Gamma u)|n\rangle = \langle p|(\overline{u}\Gamma u - \overline{d}\Gamma d)|p\rangle = g_\Gamma^u - g_\Gamma^d = g_\Gamma$$

These charges are relevant for:

- Scalar g_S^q ($\Gamma = \mathbb{I}$) and sigma terms $\sigma_q = m_q \langle N|\overline{q}q|N\rangle$.
  - Decomposition of the nucleon mass.
  - Calculation of dark matter-nucleon scattering cross sections within dark matter models.
- Axial g_A^q ($\Gamma = \gamma^5\gamma^\mu$).
  - Intrinsic quark spin contribution to nucleon spin.
- Tensor g_T^q ($\Gamma = \gamma^\mu\gamma^\nu$).
  - Can be used to constrain fits to transversity PDFs [Huey-Wen Lin et. al.: arXiv:1710.09858].
- Vector g_V^q ($\Gamma = \gamma^\mu$).
  - Isovector $g_V = 1$ provides a good sanity check.

The isovector g_A and g_V appear in the β-decay rate within the standard model at tree level, however, there may be BSM scalar and tensor interactions too.

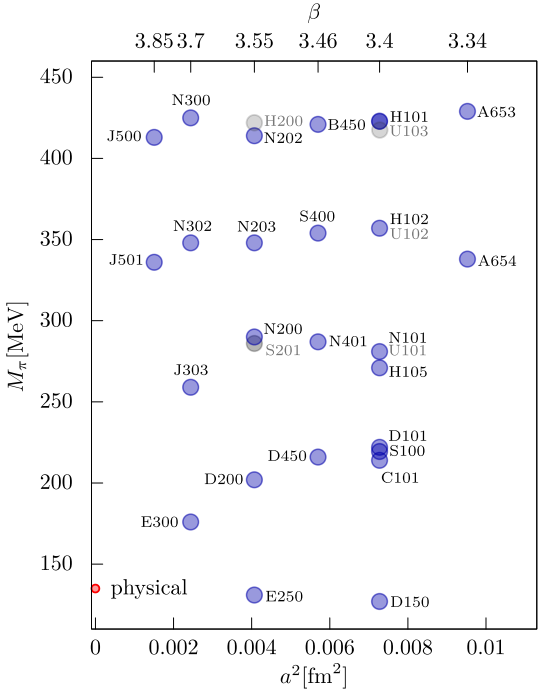
Actions

The action used by CLS [M. Bruno: arXiv:1411.3982] consists of the Lüscher-Weisz gluonic action, and the $N_f = 2 + 1$ Sheikholeslami-Wohlert fermionic action.

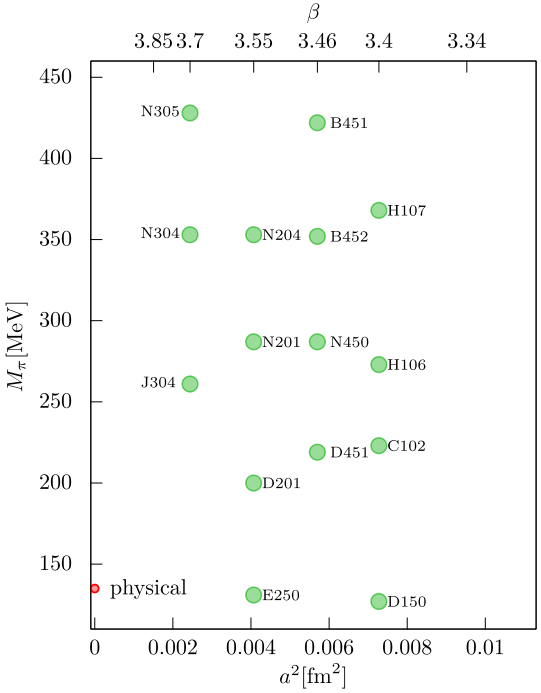
Aspects of note:

- Chiral symmetry breaking by the Wilson Fermion Term $a\nabla_\mu^* \nabla_\mu$.
- $\mathcal{O}(a)$ improvement via the Clover term $\widehat{F}_{\mu\nu}$, where the c_{SW} coefficient is determined non-perturbatively [J. Bulava: arXiv:1304.7093].
- Some ensembles have open boundary conditions in time.

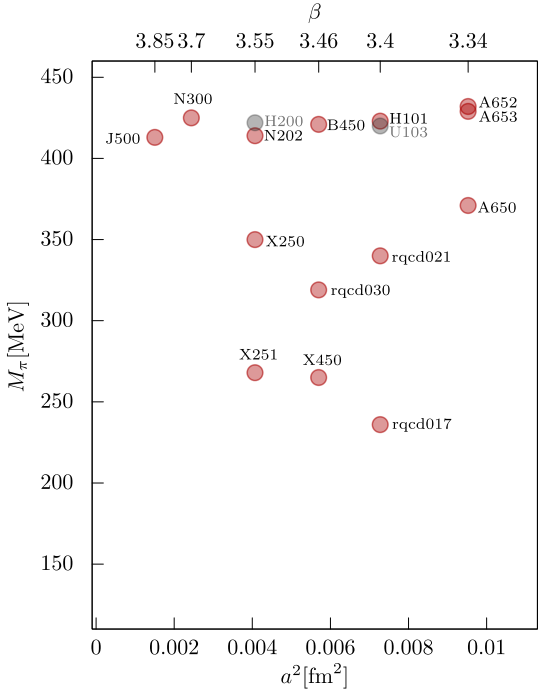
Ensembles



Constant average quark mass



Constant strange quark mass

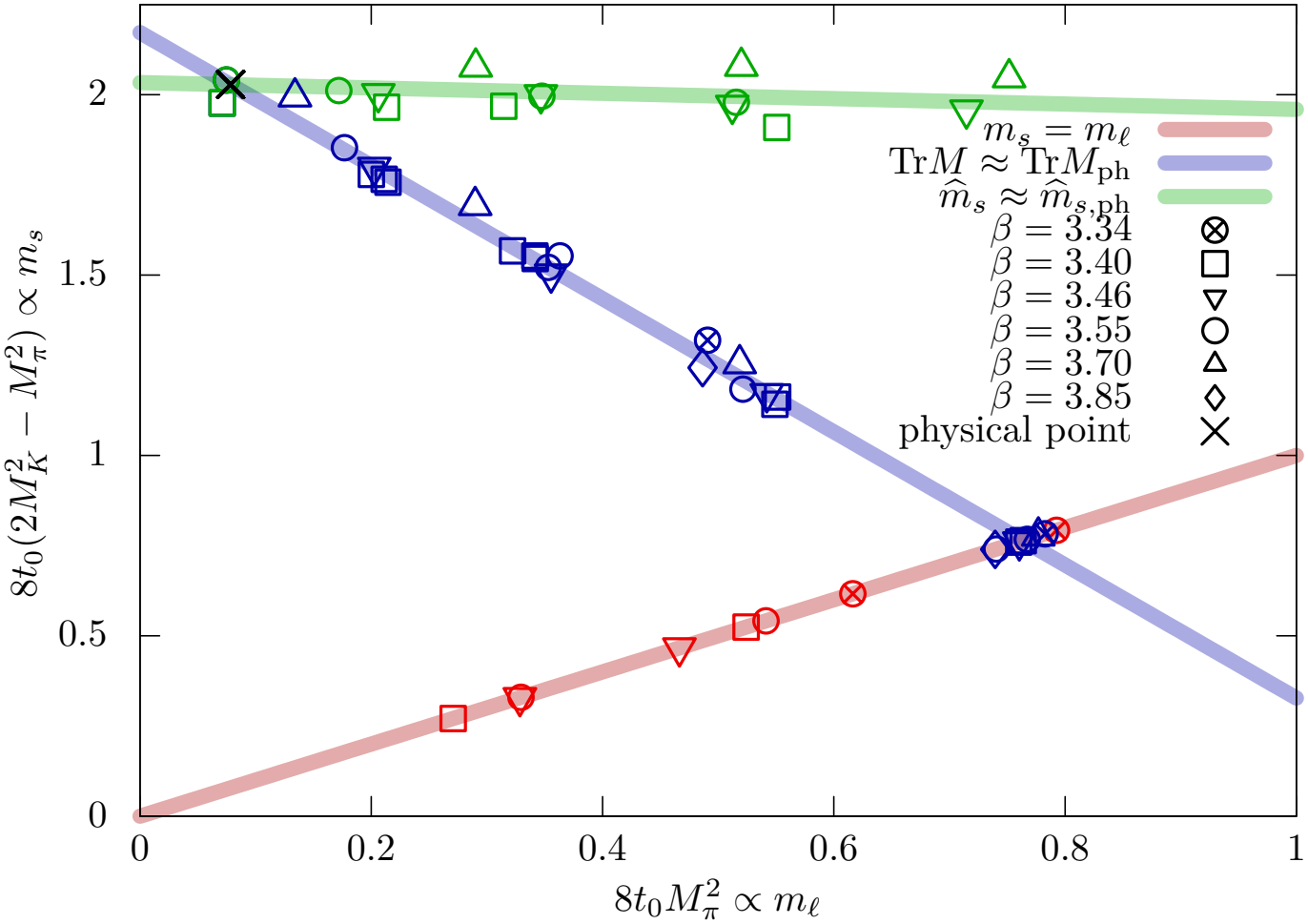


Equal quark mass

Geometries A:$(24^3, 48)$ B:$(32^3, 64)$ C:$(48^3, 96)$ D:$(64^3, 128)$ E:$(96^3, 192)$ H:$(32^3, 96)$ J:$(64^3, 192)$ N:$(48^3, 128)$ S:$(32^3, 128)$ U:$(24^3, 128)$ X:$(48^3, 64)$.

Six lattice spacings with $0.039\,\text{fm} < a < 0.098\,\text{fm}$. Pion masses between 130 and 420 MeV. Typically between 1000 and 2000 configs each. $Lm_\pi \gtrsim 4$, with some smaller L for volume studies.

CLS Lattice Landscape



These three trajectories tightly constrain the quark mass dependence.

Correlation Functions

The correlation functions in the forward limit that we use are:

$$C_{2pt}(t_f, t_i) = \left\langle \mathcal{N}(t_f)\overline{\mathcal{N}}(t_i)\right\rangle$$
$$C_{3pt}(t_f, t, t_i) = \left\langle \mathcal{N}(t_f)J(t)\overline{\mathcal{N}}(t_i)\right\rangle - \langle J(t)\rangle \left\langle \mathcal{N}(t_f)\overline{\mathcal{N}}(t_i)\right\rangle$$

- $\mathcal{N}$ (resp. $\overline{\mathcal{N}}$) is the interpolation operator that destroys (creates) a nucleon.
- $J(t) = \overline{q}(t)\Gamma q(t)$ is the current.

Related to the matrix elements via the spectral decomposition (with the first excitation).

$$C_{2pt}(t_f, t_i = 0) \sim Z_1^2 \mathrm{e}^{-t_f m}\left[1 + \frac{Z_2^2}{Z_1^2}\mathrm{e}^{-\Delta m t_f}\right]$$

$$C_{3pt}(t_f, t, t_i = 0) \sim Z_1^2 \mathrm{e}^{-t_f m}\left[\langle 1|J|1\rangle + \frac{Z_2}{Z_1}\langle 2|J|1\rangle\left(\mathrm{e}^{-\Delta m(t_f - t)} + \mathrm{e}^{-\Delta m t}\right) + \frac{Z_2^2}{Z_1^2}\langle 2|J|2\rangle \mathrm{e}^{-\Delta m t_f}\right]$$

where $Z_j = \langle j|\overline{\mathcal{N}}(0)|0\rangle = Z_j^*$, and $|0\rangle$, $|1\rangle$, and $|2\rangle$ are the vacuum, ground, and first excited state respectively, where Δm is the energy difference between the latter of these two states.

Correlation Functions

Taking the ratio we get

$$R(t_f, t, 0) = \frac{C_{3pt}(t_f, t, 0)}{C_{2pt}(t_f, 0)} \sim A + B(\mathrm{e}^{-\Delta m(t_f - t)} + \mathrm{e}^{-\Delta m t}) + K\mathrm{e}^{-\Delta m t_f}.$$

One can also use the summation method.

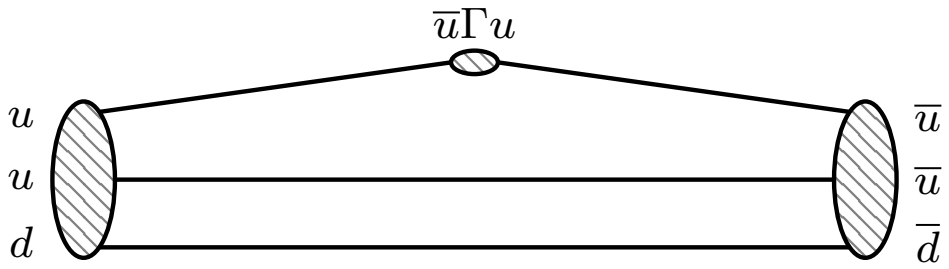
$$\sum_{t=c}^{t_f - c} R\left(t_f, t, 0\right) \sim A(t_f - 2c + 1) + K(t_f - 2c + 1)\mathrm{e}^{-\Delta m t_f} + \frac{2B}{1 - \mathrm{e}^{\Delta m}}\left(\mathrm{e}^{\Delta m(c - t_f)} - \mathrm{e}^{\Delta m(1-c)}\right)$$

We also perform linear fits ($K = B = 0$).

A dense spectrum of excited (multi-particle) states is expected as m_π is reduced towards the physical point and the spatial extent is increased to maintain $Lm_\pi \geq 4$.

We also investigate two excited state fits, fixing one state using a prior to the smallest of $N(-\vec{p})\pi(\vec{p})$ or $N(0)\pi(0)\pi(0)$.

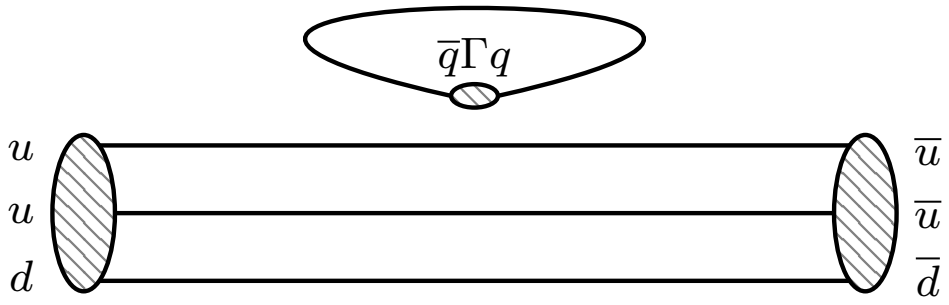
Computational Methods - Connected



Connected Quark Line Diagram

- Sequential Source Method and (on some ensembles) Coherent Sink Method [LHPC Collab: arXiv:1001.3620].
- Typically 4 source-sink separations (typically 10 measurements) $0.7\,\text{fm}$ (1), $0.9\,\text{fm}$ (2), $1.0\,\text{fm}$ (3), $1.2\,\text{fm}$ (4).
- Wuppertal Smearing for the source and sink to improve overlap with the ground state.
  - Quark rms radius between 0.6 (at 420 MeV) and 0.85 fm (at physical pion mass).

Computational Methods - Disconnected

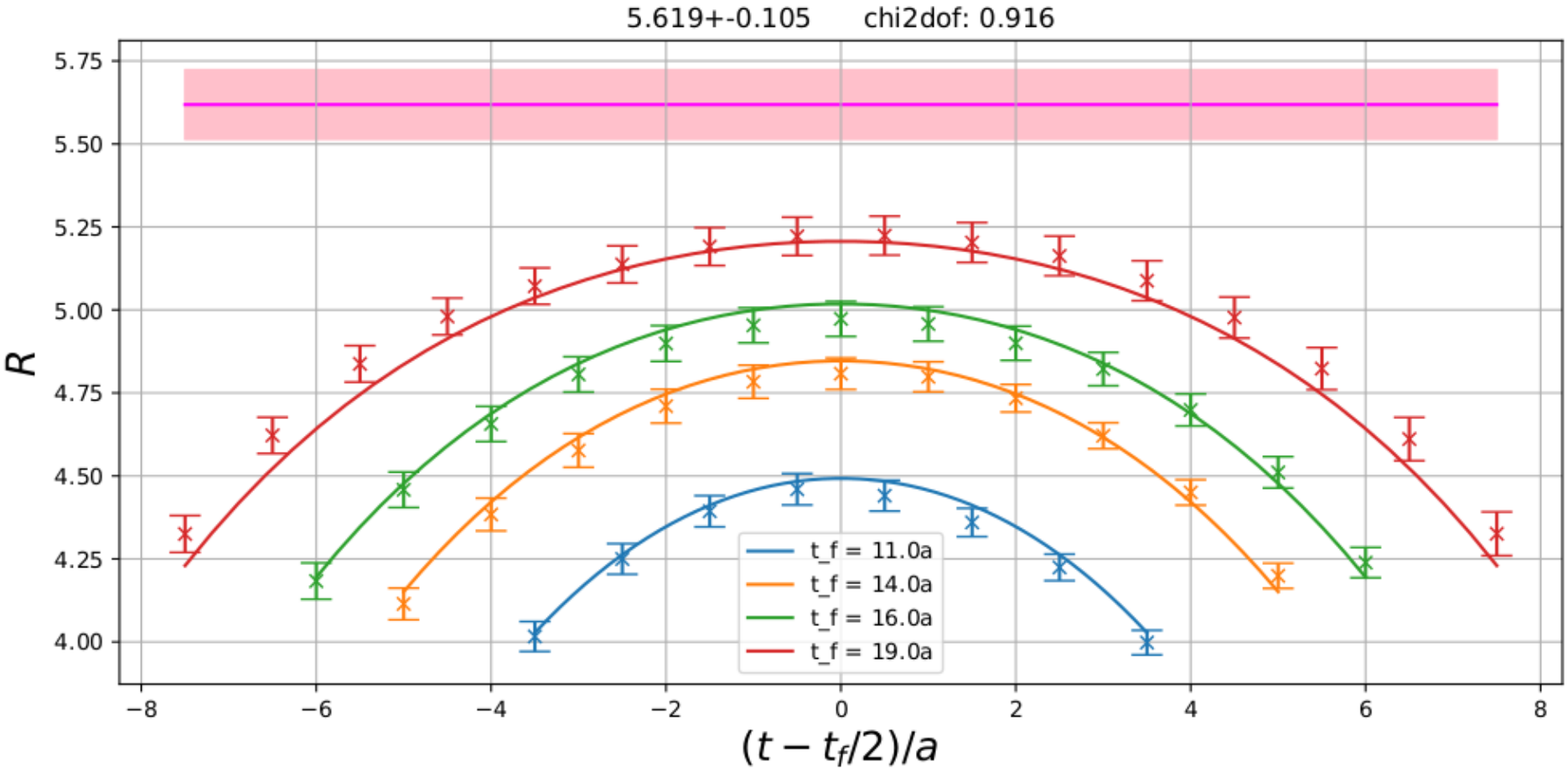


Disconnected Quark Line Diagram

- Stochastic Loop Estimation
  - Truncated Solver Method [G. Bali et. al.: arXiv:0910.3970], Hopping Parameter Expansion [C. Thron et. al.: arXiv:hep-lat/9707001], Time Partitioning [S. Bernardson et. al. 1993].
- Solvers
  - IDFLS [M. Lüscher et. al.: arxiv:0710.5417] or DD-αAMG[A. Frommer et. al.: arxiv:1303.1377].
- Typically 20 measurements of the disconnected three-point function.

Fit to connected C_{3pt}/C_{2pt} for g_S^u

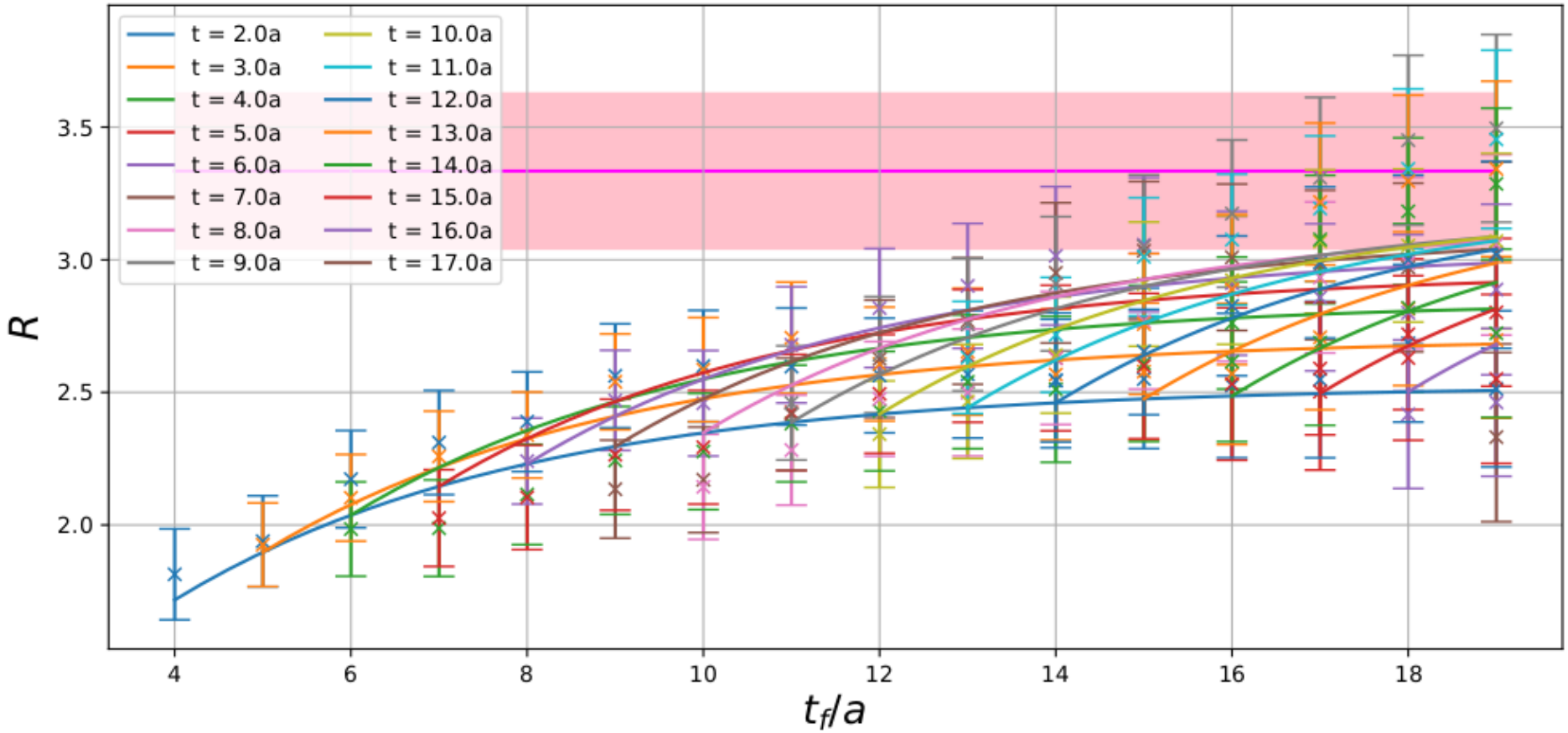
Preliminary Results



The connected and disconnected contributions for several charges are fitted simultaneously. The fit above is to ensemble N202; $V = (48^3, 128) \rightarrow (3.08^3,\ 8.22)$ fm; $m_\pi = 411$ MeV; $\beta = 3.55$; $a = 0.0642$ fm.

Disconnected Contribution to g_S^u

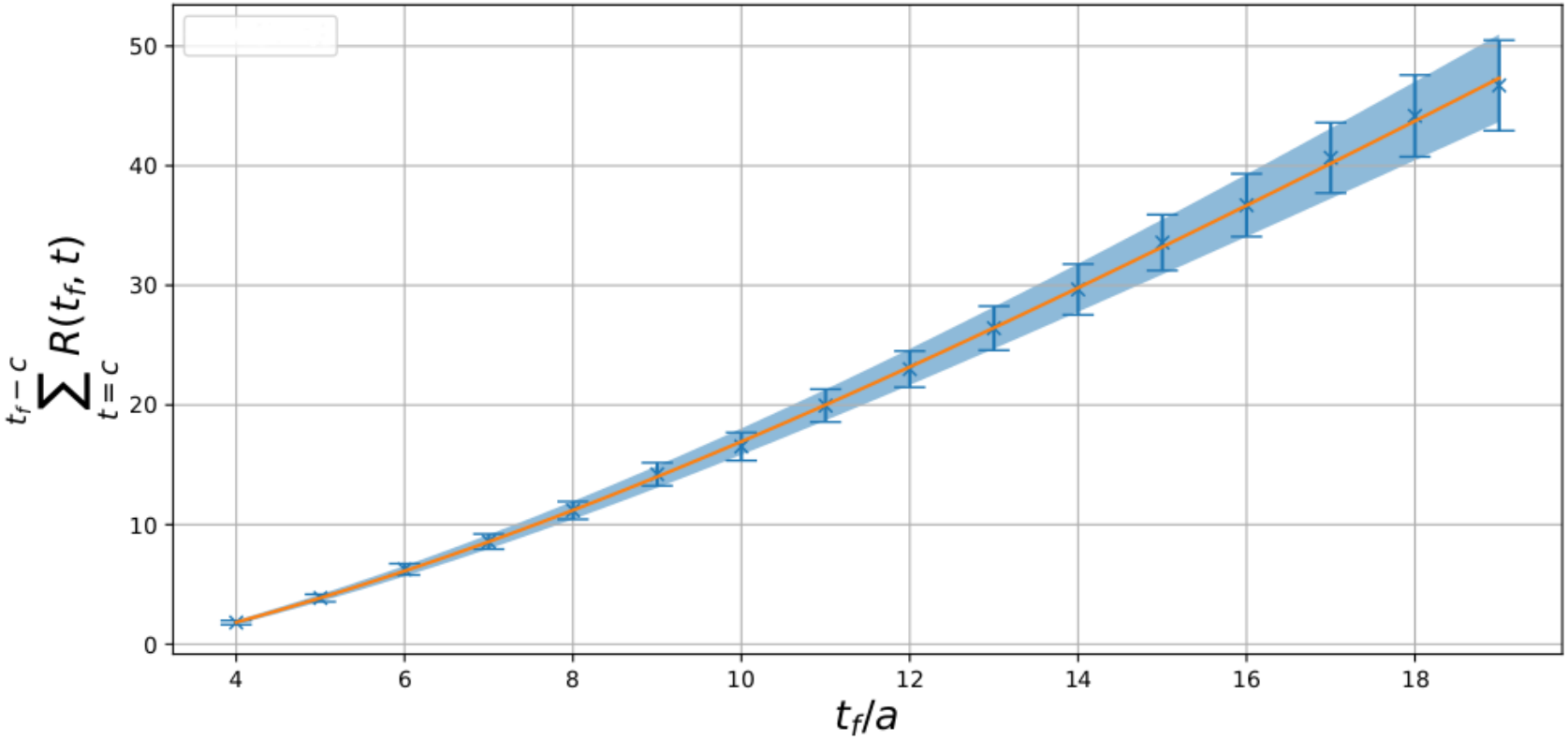
Preliminary Results



Simultaneous fit including the connected contribution (for several charges).
The fit above is to ensemble N202; $V = (48^3, 128) \rightarrow (3.08^3,\ 8.22)$ fm; $m_\pi = 411$ MeV; $\beta = 3.55$; $a = 0.0642$ fm.

Summation Method for Disconnected Contribution to g_S^u

Preliminary Results



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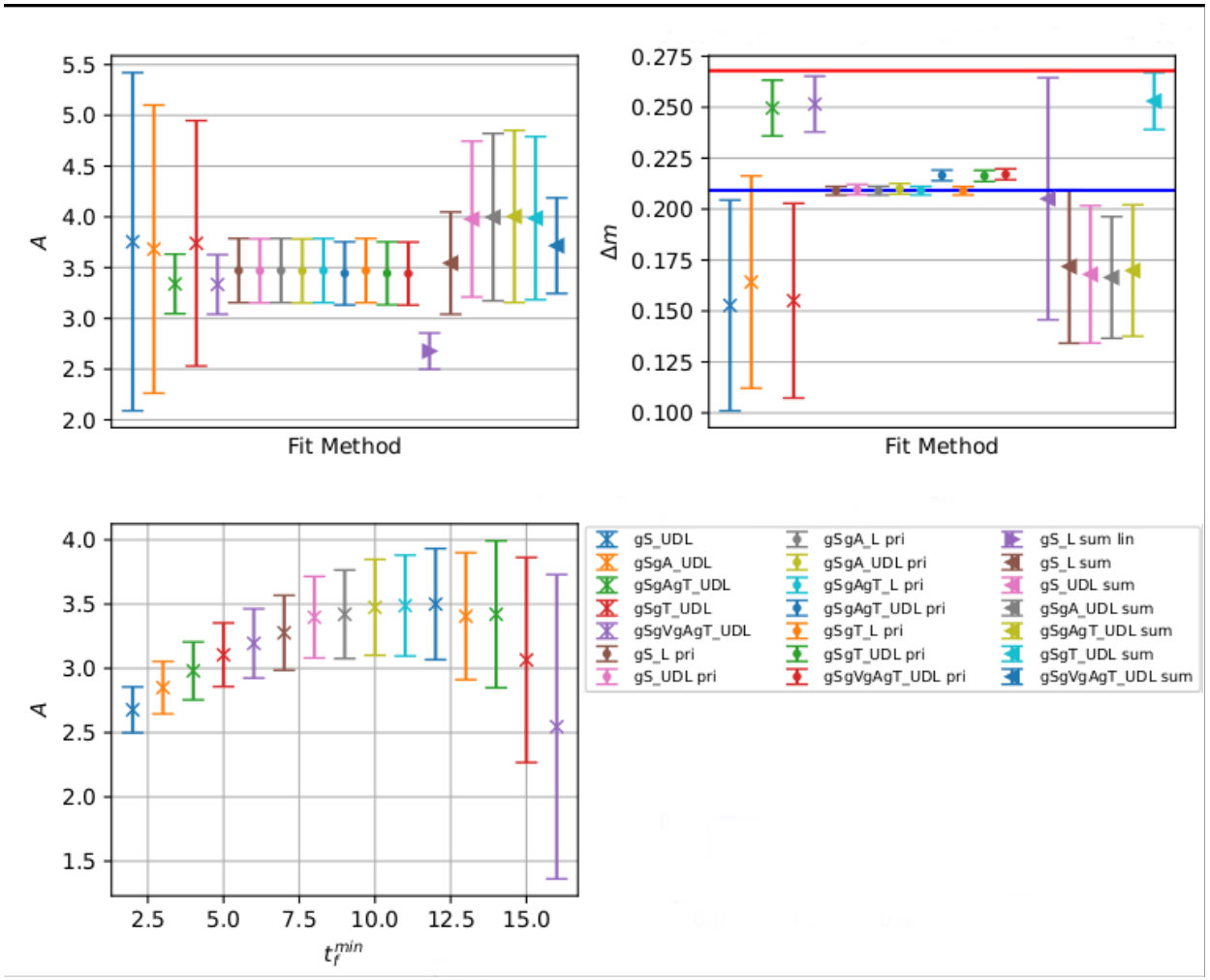
Simultaneous Fits to Multiple Charges



Top left: bare matrix element for disconnected g_S^u against fitting method. Top right: excited state mass gap against fitting method, with $N\pi$ (blue line) and $N\pi\pi$ (red line) masses indicated. Bottom left: fit range variation for linear summation method. For smaller pion masses there is some variation with the fit range and charges included.

Renormalisation, Mixing, and Improvement

Renormalisation and $\mathcal{O}(a)$ improvement for isovector charges:

$$g_X = Z_X(1 + am_l b_X + 3a\overline{m}\tilde{b}_X)g_X^{\text{bare}}$$

Due to chiral symmetry breaking, non-singlet charge combinations renormalise with $Z_{\mathcal{O}}^{ns}$ and singlet charge combinations with $Z_{\mathcal{O}}^{s}$, with the ratio

$$r_{\mathcal{O}} = Z_{\mathcal{O}}^{s}/Z_{\mathcal{O}}^{ns} = 1 + \mathcal{O}(\alpha_s^n), \qquad n \geq 2$$

The deviation from 1 is small for the axial and tensor, but significant for the scalar.

The results that are presented for the isosinglet axial and tensor only have the Z^{ns} employed.

Continuum, Quark Mass and Finite Volume Extrapolation

Consider the following fit form for the axial, tensor and scalar charges:

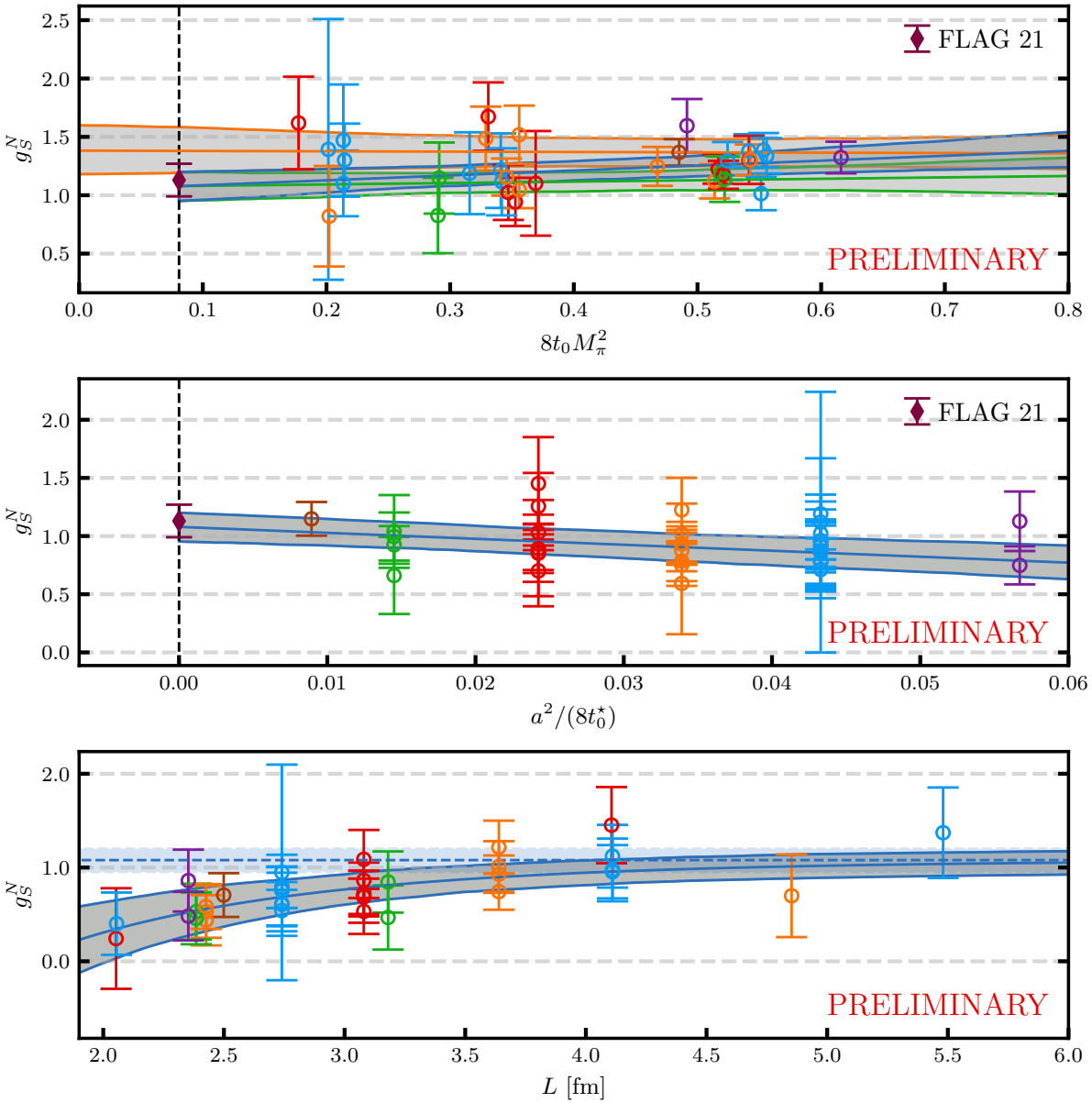
$$\begin{aligned} f(\ldots) =& c_0 + c_\pi m_\pi^2 + c_K m_K^2 + \\ & c_a a^2 + c_{a\pi} a^2 m_\pi^2 + c_{aK} a^2 m_K^2 \\ & c_{v\pi} \frac{m_\pi^2}{\sqrt{L m_\pi}} \mathrm{e}^{-L m_\pi} + c_{vK} \frac{m_K^2}{\sqrt{L m_K}} \mathrm{e}^{-L m_K} + c_{v\eta} \frac{m_\eta^2}{\sqrt{L m_\eta}} \mathrm{e}^{-L m_\eta} \end{aligned}$$

With current set of ensembles, can only resolve c_0, c_π, c_K, c_a and $c_{v\pi}$.

For the vector we use

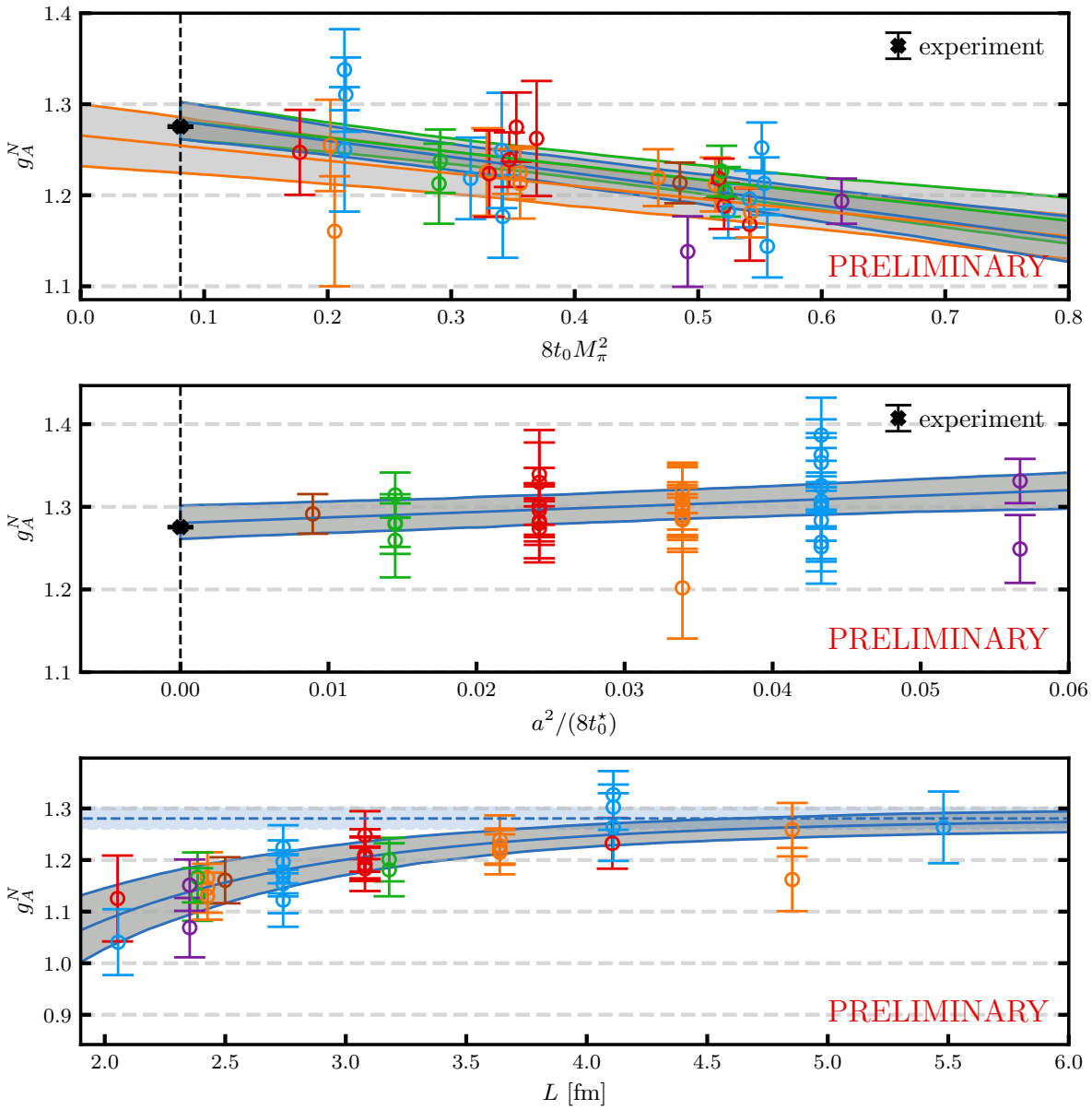
$$f(\ldots) = c_0 + c_a a^2 + c_{a\pi} a^2 m_\pi^2 + c_{aK} a^2 m_K^2 + c_{a3} a^3$$

Extrapolation of Isovector Charges - g_S



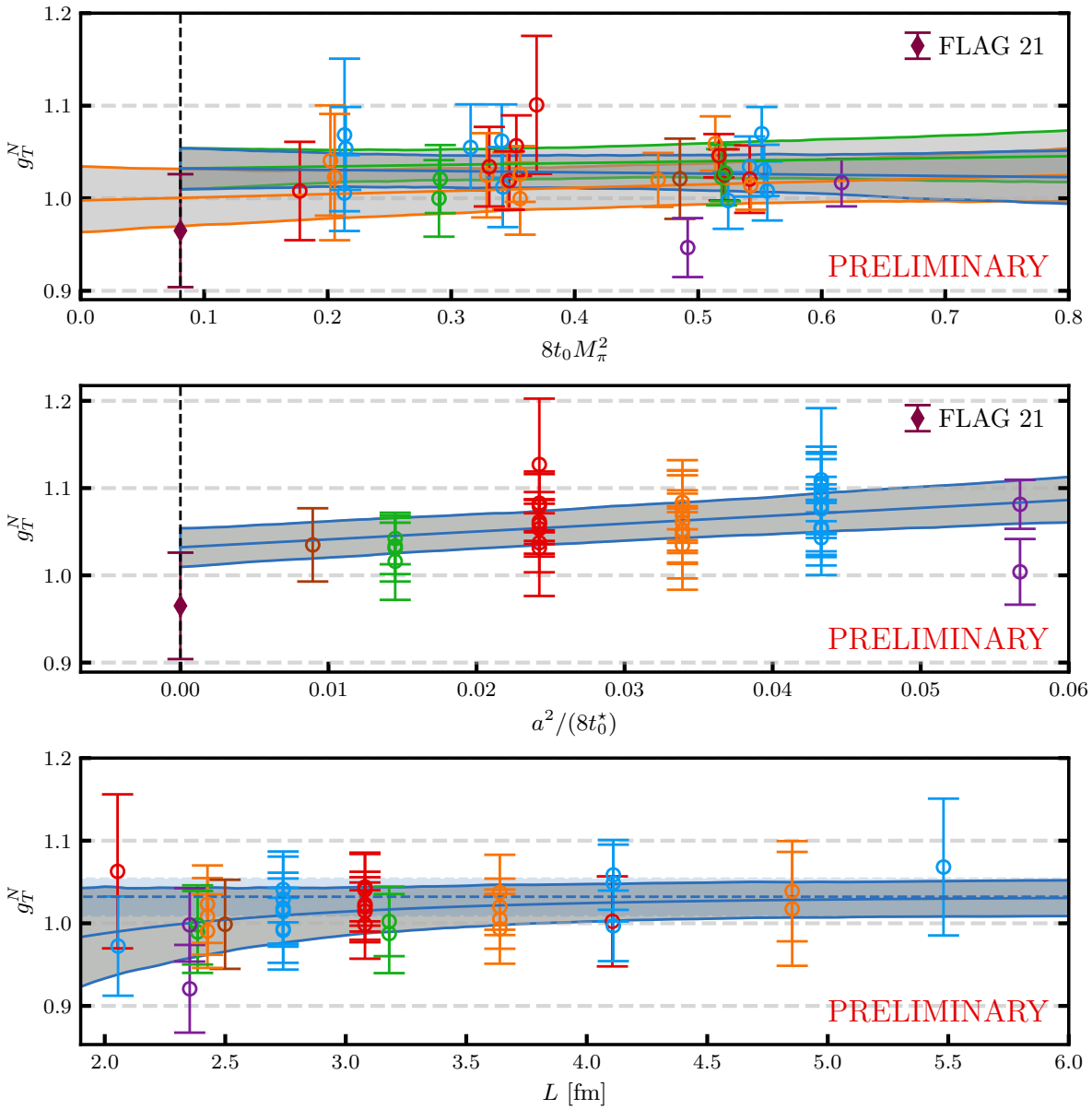
FLAG: Y. Aoki et. al.: arXiv:2111.09849

Extrapolation of Isovector Charges - g_A



Particle Data Group:
PTEP 2020 083C01

Extrapolation of Isovector Charges - g_T



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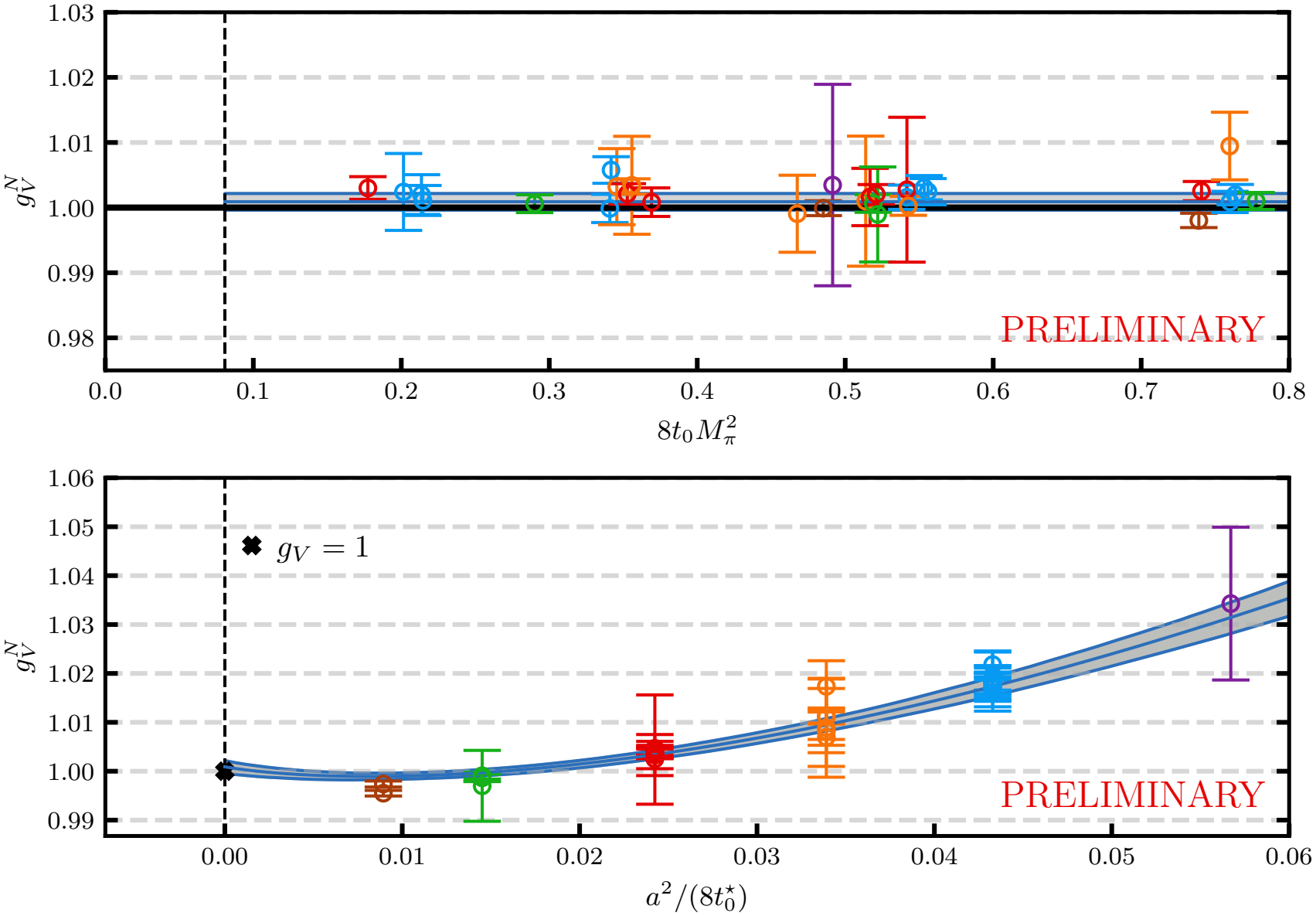
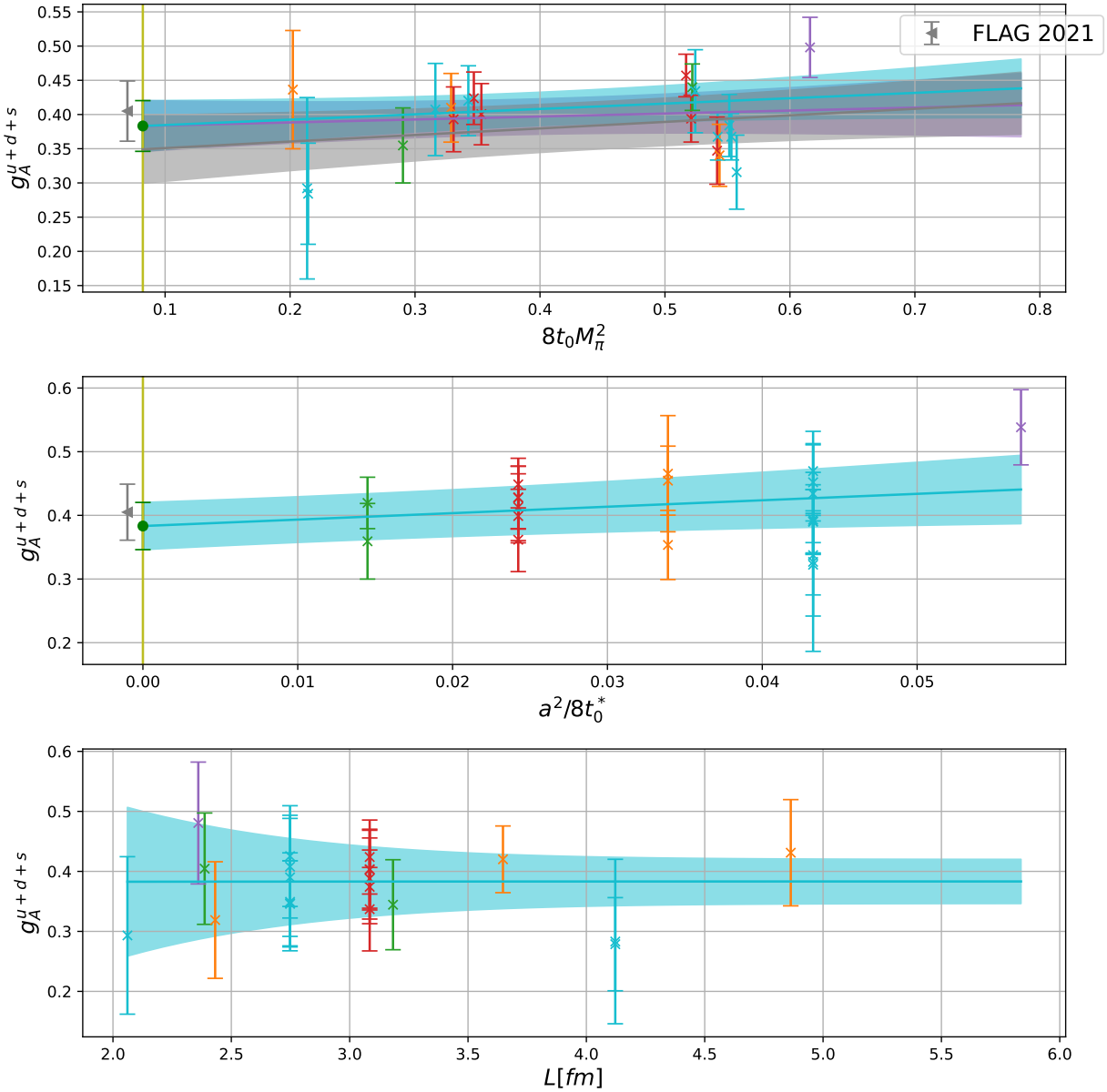
Extrapolation of Isovector Charges - g_V



Extrapolation of Isosinglet Charges - g_A^{u+d+s} (Preliminary)



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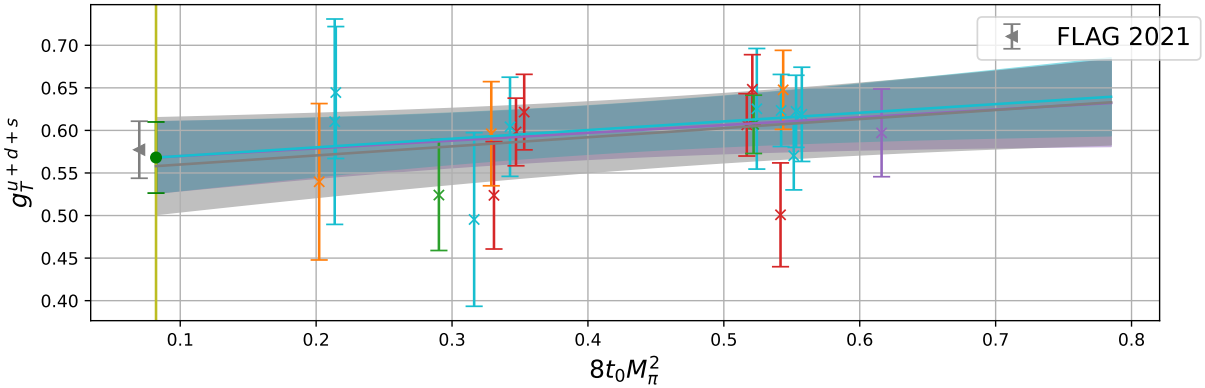
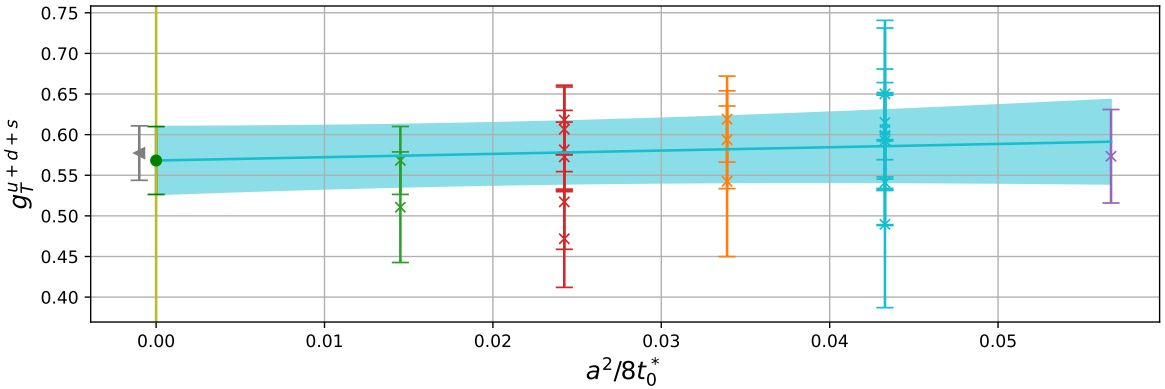
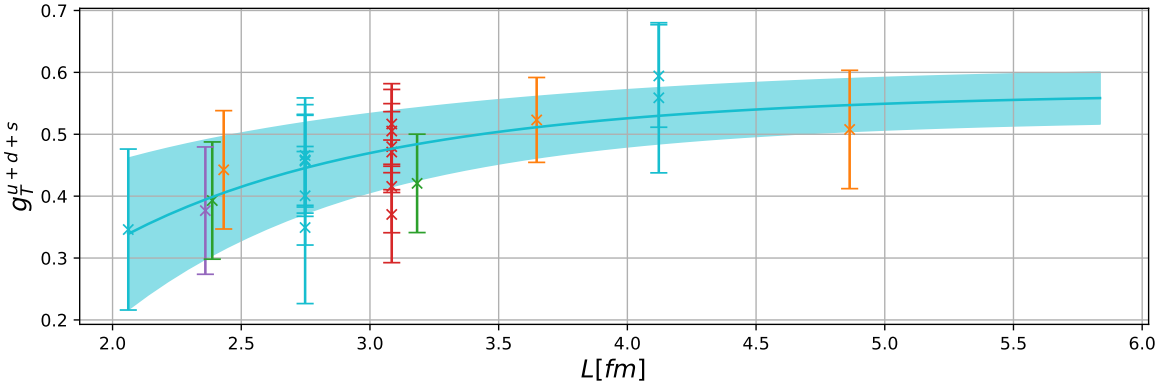
Extrapolation of Isosinglet Charges - g_T^{u+d+s} (Preliminary)







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Conclusions and Future Directions

- Add more ensembles, especially for the extrapolations of the isosinglets.
- Take systematics of multi-charge fits into account (including one and two excited state fits).
- Take the mixing of flavours under renormalisation into account.
- Perform extrapolations for sigma terms.
- See also the talk by Pia Leonie Jones Petrak on "Sigma terms of the baryon octet in $N_f = 2 + 1$ QCD with Wilson quarks" today at 18:10.