Hadronic Parity Violation from Twisted Mass Lattice QCD

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in cooperation with

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Why Hadronic Parity Violation (PV) Studies?

- Hadronic **PV processes** in low energy regime **poorly understood** yet

- **Suggestion** [Desplanques et al. 1980]:

  Hadronic PV processes describable by one-meson exchange

- **Test** of this **weak meson exchange model**

  Determination of **parity-violating pion-nucleon coupling** h_π^1

  $\rightarrow$ **Experiment:** NPDGamma (2018)

  $\rightarrow$ **1st Lattice simulation:** Wasem (2012)

Methods: h_π^1 from PV Lagrangian

Effective PV Lagrangian for long-range single π contribution
[Desplanques et al. 1980]

$$\mathcal{L}^w_{PV} = -\frac{h^1_\pi}{\sqrt{2}}\,\bar{N}\,(\vec{\tau}\times\vec{\pi})_3\,N + \ldots$$

P-odd πN coupling

$$h^1_\pi = -\frac{i}{2m_N}\lim_{p_\pi\to 0}\langle n\,\pi^+|\mathcal{L}^w_{PV}|p\rangle$$

Methods: h_π^1 from PV Lagrangian

Matrix elements: $h_\pi^1 \overset{p_\pi \to 0}{\propto} \langle n\, \pi^+ | \mathcal{L}_{PV}^w | p \rangle$

$\Delta S = 0,\ \Delta I = 1$ channel [Kaplan, Savage 1993]

$$\mathcal{L}_{PV}^w = -\frac{G_F}{\sqrt{2}} \frac{\sin^2\theta_W}{3} \left(\sum_{i=1}^{3} C_i^{(1)}\, \theta_i^{(\ell)} + \sum_{i=1}^{4} S_i^{(1)}\, \theta_i^{(s)} \right)$$

$\theta^{(\ell)},\ \theta^{(s)} \mathrel{\widehat{=}}$ four quark interpolators (only u, d, s)

$C^{(1)}(\Lambda_\chi),\ S^{(1)}(\Lambda_\chi) \mathrel{\widehat{=}}$ Wilson coefficients

$\mathcal{L}_{PV}^w$ **approach** [Wasem, 2012] $\to$ **difficulties**

Methods: h_π^1 from PC Lagrangian

New approach: partially-conserved axial current relation (PCAC)

Construct effective PC Lagrangian [Feng, Guo, Seng 2018]

$$\mathcal{L}_{PC}^{w} = -\frac{G_F}{\sqrt{2}} \frac{\sin^2\theta_W}{3} \sum_i \left(C_i^{(1)}\, \theta_i^{(\ell)\prime} + S_i^{(1)}\, \theta_i^{(s)\prime} \right)$$

πN coupling [Feng, Guo, Seng 2018]

$$h_\pi^1 \approx -\frac{(\delta m_N)_{4q}}{\sqrt{2}\, F_\pi}$$

Induced **neutron-proton mass shift** $(\delta m_N)_{4q} = (m_n - m_p)_{4q} = \pm \frac{\langle p/n \,|\, \mathcal{L}_{PC}^{w} \,|\, p/n \rangle}{m_N}$

Methods: h_π^1 from PC Lagrangian

Quark bilinears $O_j^{2q} = \bar{q}\,\Gamma_j\,q$

current type j	S	P	V	A	T
Γ_j	$\mathbb{1}$	γ_5	γ_μ	$\gamma_\mu\gamma_5$	$\frac{1}{2}[\gamma_\mu, \gamma_\nu]$

Four-quark interpolators: express **all** $\theta_i^{(\ell)\prime}$ in terms of

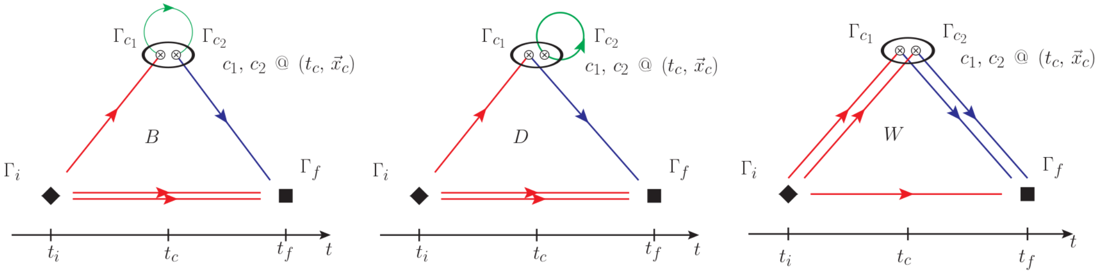
$$O_1^{(\ell)} = \bar{q}_a\,\mathbb{1}\otimes\mathbb{1}\,q_a\,\bar{q}_b\,\mathbb{1}\otimes\tau_3\,q_b, \quad O_2^{(\ell)} = \bar{q}_a\,\gamma_5\otimes\mathbb{1}\,q_a\,\bar{q}_b\,\gamma_5\otimes\tau_3\,q_b$$

$$O_3^{(\ell)} = \bar{q}_a\,\gamma_\mu\otimes\mathbb{1}\,q_a\,\bar{q}_b\,\gamma_\mu\otimes\tau_3\,q_b, \quad O_4^{(\ell)} = \bar{q}_a\,\gamma_\mu\,\gamma_5\otimes\mathbb{1}\,q_a\,\bar{q}_b\,\gamma_\mu\,\gamma_5\otimes\tau_3\,q_b$$

strange quark sector: $\theta_i^{(s)\prime}$ analog

Methods: Quark flow diagrams

Inserting **4-quark operators** makes 3pt correlator decompose into **3 diagram types**:



- Nucleon interpolators at **source** of form $\bar{u}_a\,\Gamma_i\,\bar{d}_b^T\,\bar{u}_c$
- Nucleon interpolators at **sink** of form $u_a^T\,\Gamma_f\,d_b\,u_c$
- 4-quark operators at **insertion point** of form $\bar{q}(t_c,\vec{x}_c)\,\Gamma_{c_1}\,q(t_c,\vec{x}_c)\,\bar{q}(t_c,\vec{x}_c)\,\Gamma_{c_2}\,q(t_c,\vec{x}_c)$

Methods: Feynman-Hellmann Theorem (FHT)

FHT:

- Weak interaction as perturbation $S \to S_{LQCD} + \lambda \sum_x \mathcal{L}^w_{PC,\lambda}(x)$
- Relates **matrix elements** & **energy spectrum variations** [Bouchard et al. 2017]:

$$\langle N \,|\, \mathcal{L}^w_{PC,\lambda} \,|\, N \rangle = \frac{\partial m_N}{\partial \lambda}$$

Recap: Parity-odd $N\pi$ coupling

$$h^1_\pi \propto \langle N \,|\, \mathcal{L}^w_{PC} \,|\, N \rangle$$

Methods: Feynman-Hellmann Theorem (FHT)

Origin of FHT ratio: symmetrized **effective mass**

FHT ratio

$$R(t,\tau) = \frac{1}{\tau}\frac{z}{\sqrt{z^2-1}}\left[\frac{C^{\text{3pt}}(t+\tau)+C^{\text{3pt}}(t-\tau)}{C^{\text{2pt}}(t+\tau)+C^{\text{2pt}}(t-\tau)} - \frac{C^{\text{3pt}}(t)}{C^{\text{2pt}}(t)}\right]$$

$$z := \frac{C^{\text{2pt}}(t+\tau)+C^{\text{2pt}}(t-\tau)}{2C^{\text{2pt}}(t)},$$

off-set τ

Target matrix elements: $R^{(j)}_{k,X}(t,\tau) \xrightarrow{t\to\infty} \frac{\langle N|\theta^{(j)\prime}_{k,X}|N\rangle}{2m_N}$ $\forall \tau \geq 1$

Numerical simulation: Gauge field ensemble

- $N_f = 2+1+1$ **gauge field ensemble** **cA211.30.32** (Extended Twisted Mass Collaboration)

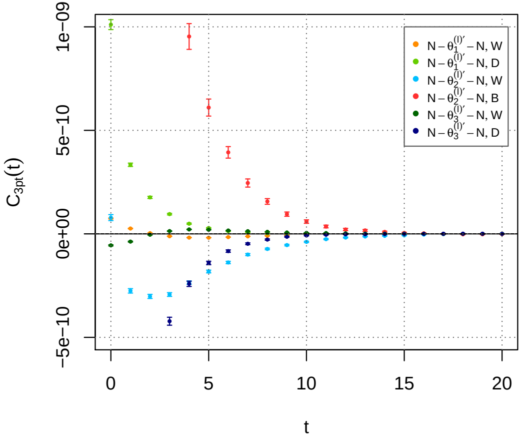
$L^3 \times T$	a [fm]	L [fm]	m_π [MeV]	$m_\pi L$	m_N [MeV]
$32^3 \times 64$	0.097	3.1	$261.1(1.1)$	4.01	$1028(4)$

- **# gauge configurations** $= 1262$

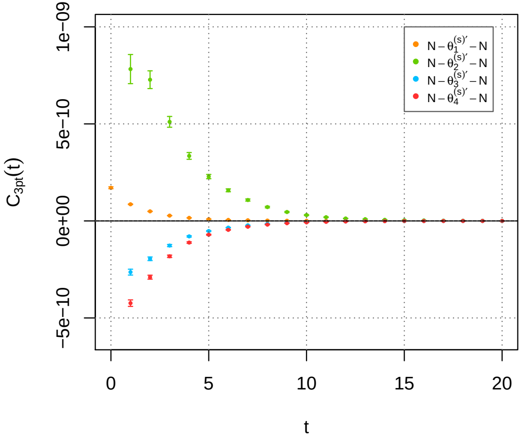
diagram	# stochastic samples	# source coordinates
B, D	1	8
W	8	2

- **Smearing techniques**
  - **gauge field:** APE
  - **fermion field:** Wuppertal

Results: 3pt Correlation functions (cfs)



cfs for light 4-quark operators



cfs for strange 4-quark operators

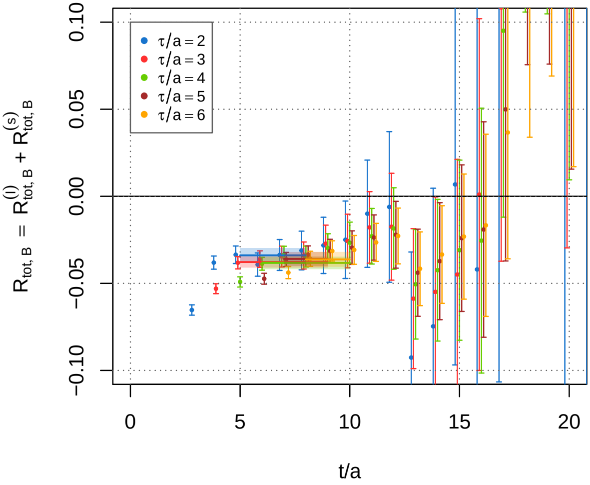
PRELIMINARY Results: Matrix elements

k	$\frac{\langle N \mid \theta^{(\ell)\prime}_{B+D,k} \mid N \rangle}{2m_N}$	χ^2/ndof	p-value	$\frac{\langle N \mid \theta^{(s)\prime}_{B/D,k} \mid N \rangle}{2m_N}$	χ^2/ndof	p-value
1	1.302(93)	0.69	0.56	$0.002(7) \times 10^{-2}$	0.95	0.45
2	3.911(279)	0.69	0.56	$-1.115(142) \times 10^{-2}$	0.75	0.56
3	−1.316(94)	0.70	0.55	$0.186(16) \times 10^{-2}$	0.79	0.50
4		—		$0.187(17) \times 10^{-2}$	0.81	0.56

k	$\frac{\langle N \mid \theta^{(\ell)\prime}_{W,k} \mid N \rangle}{2m_N}$	χ^2/ndof	p-value
1	$2.13(14) \times 10^{-3}$	0.55	0.58
2	$1.40(14) \times 10^{-2}$	0.86	0.49
3	$-2.11(17) \times 10^{-3}$	0.88	0.48

1$^{\text{st}}$ LQCD estimates of matrix elements with $\theta^{(\ell/s)\prime}_k$ for B, D

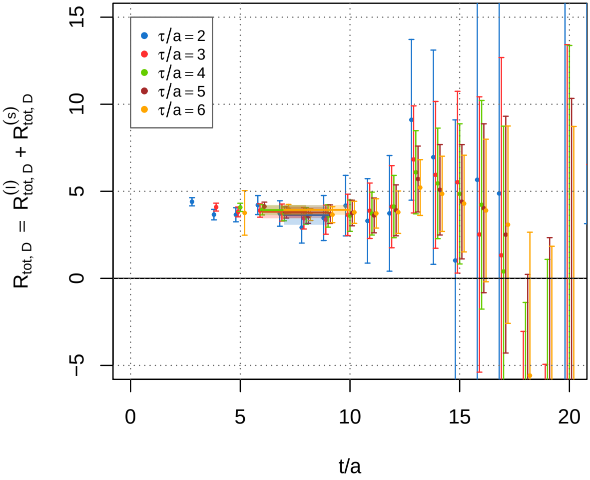
PRELIMINARY Results: Combined FHT ratios for B



Constant fit results

τ	$\mathcal{M}_{\text{tot},B}/10^{-2}$	χ^2/ndof	p-value
2	$-3.39(42)$	0.42	0.74
3	$-3.78(32)$	0.48	0.75
4	$-$**3.83(36)**	0.83	0.51
5	$-3.60(40)$	0.41	0.66
6	$-3.62(43)$	0.75	0.47

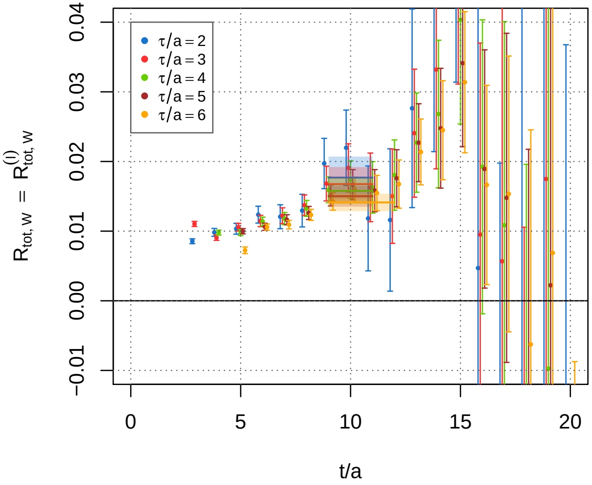
PRELIMINARY Results: Combined FHT ratios for D



Constant fit results

τ	$\mathcal{M}_{\mathrm{tot},D}$	χ^2/ndof	p-value
2	3.64(56)	0.54	0.58
3	3.82(37)	0.29	0.75
4	3.92(29)	0.50	0.61
5	3.78(32)	0.44	0.65
6	**3.93(28)**	0.70	0.55

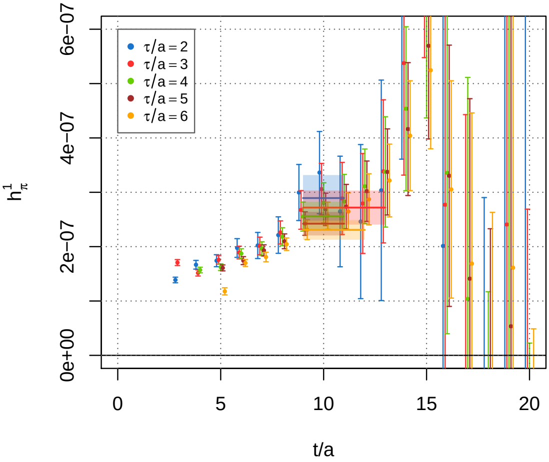
PRELIMINARY Results: Combined FHT ratios for W



Constant fit results

τ	$\mathcal{M}_{\text{tot},W}/10^{-2}$	χ^2/ndof	p-value
2	1.77(30)	0.92	0.40
3	1.68(24)	0.61	0.54
4	1.58(18)	0.38	0.68
5	1.50(15)	0.64	0.53
6	**1.41(13)**	0.73	0.53

PRELIMINARY Results: h^1_π(only W) from matrix elements



Constant fit results

τ	$h^1_\pi/10^{-7}$	χ^2/ndof	p-value
2	2.89(42)	0.33	0.72
3	2.72(31)	0.55	0.70
4	2.56(27)	0.50	0.61
5	2.42(22)	0.86	0.42
6	**2.31(18)**	0.98	0.40

$$h^1_\pi(\text{only } W) \approx \left(\frac{G_F \sin^2(\theta_W)}{3\, a\, F_\pi}\right) \left[C_1^{(1)} R^{(\ell)}_{1,W} + C_2^{(1)} R^{(\ell)}_{2,W} + C_3^{(1)} R^{(\ell)}_{3,W}\right] \qquad \forall \tau \geq 1$$

Comparison

group (year)	method	only W	R*	m_π/MeV	$h_\pi^1/10^{-7}$
Page et al. (1986)	experiment	–	–	140	$0.4^{+1.4}_{-0.4}$
NPDGamma (2018)	experiment	–	–	140	$2.6(1.2)$
Wasem (2012)	$\mathcal{L}_{PV}$, "direct"	✓	✗	390	$1.10(51)$
our work (2022)	$\mathcal{L}_{PC}$, SPT + FHT	✓	✗	260	**2.31(18)**

* R $\widehat{=}$ renormalization

Main limitations (our work): No renormalization yet, **only** W

Progress and Outlook

Achieved:

- Add **strange** quark sector ✓
- **Non-zero signal** for all combined FHT ratios ✓
- Best fit matrix elements have $\lesssim 10\%$ **statistical uncertainties** ✓
- Get **estimate** for h_π^1 using only W ✓

Next Steps:

- **Renormalization**
- Towards **physical pion mass** & **continuum limit** $(a \to 0)$ & **infinite volume** $(L \to \infty)$

Thank you for your attention

Backup: Four-quark operators

Four quark operators with quark isospin doublet $q = (u\, d)^T$:

$$\theta_1^{(\ell)\prime} = \bar{q}_a\,\gamma_\mu\,\mathbb{1}\,q_a\,\bar{q}_b\,\gamma^\mu\,\tau^3\,q_b, \quad \theta_2^{(\ell)\prime} = \bar{q}_a\,\gamma_\mu\,\mathbb{1}\,q_b\,\bar{q}_b\,\gamma^\mu\,\tau^3\,q_a, \quad \theta_3^{(\ell)\prime} = \bar{q}_a\,\gamma_\mu\,\gamma_5\,\mathbb{1}\,q_a\,\bar{q}_b\,\gamma^\mu\,\gamma_5\,\tau^3\,q_b$$

$$\begin{aligned} \theta_1^{(s)\prime} &= \bar{s}_a\,\gamma_\mu\,s_a\,\bar{q}_b\,\gamma^\mu\,\tau^3\,q_b, \quad \theta_3^{(s)\prime} = \bar{s}_a\,\gamma_\mu\,\gamma_5\,s_a\,\bar{q}_b\,\gamma^\mu\,\gamma_5\,\tau^3\,q_b, \\ \theta_2^{(s)\prime} &= \bar{s}_a\,\gamma_\mu\,s_b\,\bar{q}_b\,\gamma^\mu\,\tau^3\,q_a, \quad \theta_4^{(s)\prime} = \bar{s}_a\,\gamma_\mu\,\gamma_5\,s_b\,\bar{q}_b\,\gamma^\mu\,\gamma_5\,\tau^3\,q_a \end{aligned}$$

Backup: Correlators

- **Nucleon-nucleon 2pt correlator:** $C^{\text{2pt}}(t) = \lim_{\lambda\to 0} C_\lambda(t)$

  where $C_\lambda(t) = \langle \lambda | N(t)\, \bar{N}(0) | \lambda \rangle$

- **3pt correlator:** $\partial_\lambda C_\lambda(t)\big|_{\lambda=0} = -\sum_{x_c} \langle N | \mathcal{L}^w_{PC}(x_c) | \bar{N} \rangle_{\lambda=0} \propto \sum_{t_c} C^{\text{3pt}}(t, t_c)$

  $$x_c = (t_c, \vec{x})$$

Effective nucleon state mass

$$C_\lambda(t) \xrightarrow{t\to\infty} \frac{1}{2\, m_N} \langle \lambda | N | p \rangle \langle p | \bar{N} | \lambda \rangle \; e^{-t\, m_N(\lambda)} \quad \curvearrowright \quad m_{\text{eff}}(t, \tau; \lambda) = \frac{\log(C_\lambda(t)/C_\lambda(t+\tau))}{\tau}$$

$$\xrightarrow{t\to\infty} m_N(t, \tau; \lambda)$$

Backup: Energy shift

Leading order perturbation theory, large t limit

$$\left.\frac{\partial m_N(t,\tau;\lambda)}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left(\frac{\partial_\lambda C_\lambda(t)}{C^{\text{2pt}}(t)} - \frac{\partial_\lambda C_\lambda(t+\tau)}{C^{\text{2pt}}(t+\tau)}\right)_{\lambda=0}$$

Determine πN coupling: $\langle N|\mathcal{L}^w_{PC}|N\rangle \overset{\text{FHT}}{=} \partial_\lambda m_N|_{\lambda=0} \longrightarrow h^1_\pi \propto \langle N|\mathcal{L}^w_{PC}|N\rangle$

Backup: Detailed FHT ratio

FHT ratio

$$R_{k,X}^{(j)}(t,\tau) = \frac{1}{\tau}\frac{z^{(j)}}{\sqrt{\left(z^{(j)}\right)^2 - 1}}\left[\frac{C_{k,X}^{\text{3pt}(j)}(t+\tau) + C_{k,X}^{\text{3pt}(j)}(t-\tau)}{C^{\text{2pt}(j)}(t+\tau) + C^{\text{2pt}(j)}(t-\tau)} - \frac{C_{k,X}^{\text{3pt}(j)}(t)}{C^{\text{2pt}(j)}(t)}\right]$$

$$z^{(j)} := \frac{C^{\text{2pt}(j)}(t+\tau) + C^{\text{2pt}(j)}(t-\tau)}{2C^{\text{2pt}(j)}(t)}$$

off-set τ, operator $k = 1,\, 2,\, 3,\, \ldots,$ $X = B,\, D,\, W,$ $j = \ell,\, s$

Target matrix elements: $R_{k,X}^{(j)}(t,\tau) \xrightarrow{t\to\infty} \frac{\langle N|\theta_{k,X}^{(j)\prime}|N\rangle}{2m_N} \quad \forall \tau \geq 1$

Backup: Combined FHT ratios

combine FHT ratios:

$$R^{(\ell)}_{\text{tot},X} = R^{(\ell)}_{1,X} + R^{(\ell)}_{2,X} + R^{(\ell)}_{3,X} \qquad X = B,\, D,\, W$$

$$R^{(s)}_{\text{tot},D/B} = R^{(s)}_{4/5,D/B} + R^{(s)}_{6/7,D/B}$$

Combined FHT ratio for B, D

$$R_{\text{tot},X} = R^{(\ell)}_{\text{tot},X} + R^{(s)}_{\text{tot},X} \qquad X = B,\, D$$

No Wilson coefficients!