Constraining Beyond The Standard Model Nucleon Isovector Charges

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Motivation

- Precision studies of neutron decay observables are susceptible to beyond the Standard Model tensor and scalar interactions
- Importantly the Fierz interference term b

$$b^{\mathrm{BSM}} \approx 0.34 g_S \epsilon_S - 5.22 g_T \epsilon_T$$
$$b_\nu^{\mathrm{BSM}} \approx 0.44 g_S \epsilon_S - 4.85 g_T \epsilon_T$$

- Extracting bounds on new-physics effective couplings ϵ_T and ϵ_S requires knowledge of $g_T,\ g_S$ at the $\approx$ 10% level
- To fully utilise the potential of future experimental neutron physics programs, the scalar and tensor charge must be precisely calculated

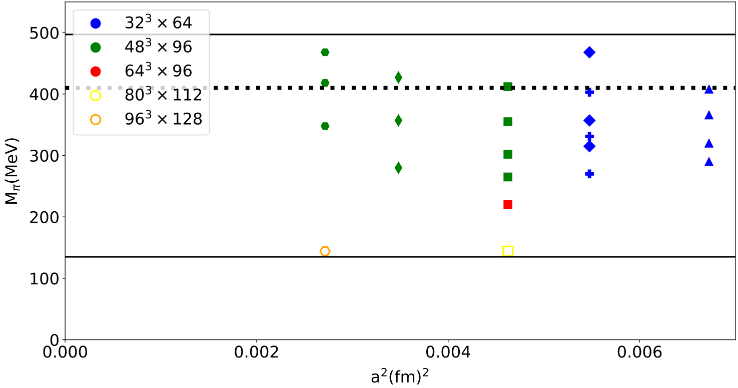
Motivation

- The neutron electric dipole moment (EDM) is sensitive to CP violation, hence is an excellent probe in the search for physics beyond the Standard Model
- The quark EDM contributions to the neutron EDM, d_n, are given by:

$$d_n = d_u g_T^d + d_d g_T^u + d_s g_T^s$$

- g_T^s consistent with 0
- d_u, d_d, d_s contain new CP violating interactions at the TeV scale
- knowing g_T^q put bound on d_n, constrain d_q and BSM theories

Analysis



Clover action comprises the tree-level Symanzik improved gluon action together with a stout smeared fermion action, modified for the use of the FH technique.

Calculating Matrix Elements using The Feynman-Hellmann Theorem

- Relates the derivative of the total energy to the expectation value of the derivative of the action

$$\frac{\partial E_{X,\lambda}(\vec{k})}{\partial \lambda} = \frac{1}{2E_{X,\lambda}(\vec{k})} \langle X, \vec{k}| \frac{\partial S}{\partial \lambda} |X, \vec{k}\rangle_{\lambda} \,,$$

- In lattice calculations, modify the action

$$S \to S + \lambda \mathcal{O}$$

- Examine the behaviour of hadron energies as λ changes
- Extract the matrix element
- Hadron energies are extracted from two-point functions
- Control of excited state contamination more simple than standard three-point analyses

Calculating Matrix Elements using The Feynman- Hellmann Theorem

- To calculate the tensor charge we have the modified action:

$$S \to S + \lambda \int d^4 x \bar{q}(x) \gamma_5 \sigma_{\mu\nu} q(x),$$

- Applying the Feynman- Hellmann Theorem we get:

Spin-up

$$\frac{\partial E_\lambda}{\partial \lambda}\bigg|_{\lambda=0} = g_T^q$$

Spin-down

$$\frac{\partial E_\lambda}{\partial \lambda}\bigg|_{\lambda=0} = -g_T^q$$

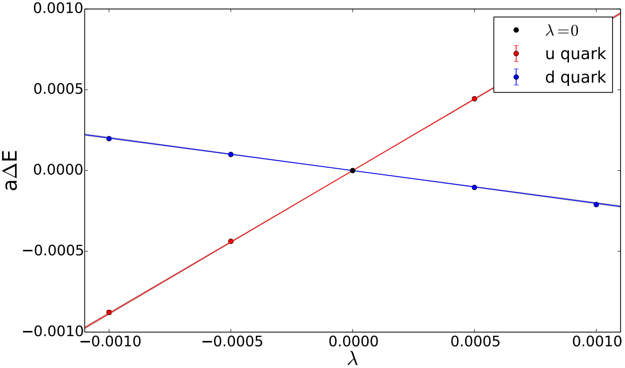
- The energy as a function of λ is therefore given by

$$E(\lambda) = E(0) \pm \lambda g_T^q + \mathcal{O}(\lambda^2).$$

- Nucleon Isovector Charges are then:

$$g_{\mathcal{O}}^{u-d} = g_{\mathcal{O}}^u - g_{\mathcal{O}}^d$$

Results



Calculated at $a = 0.082 fm$, $(\kappa_l, \kappa_s) = (0.119930, 0.119930)$

$$g_T^u = 0.8529(53),\ g_T^d = -0.1947(42)$$

Weighted Averaging Method

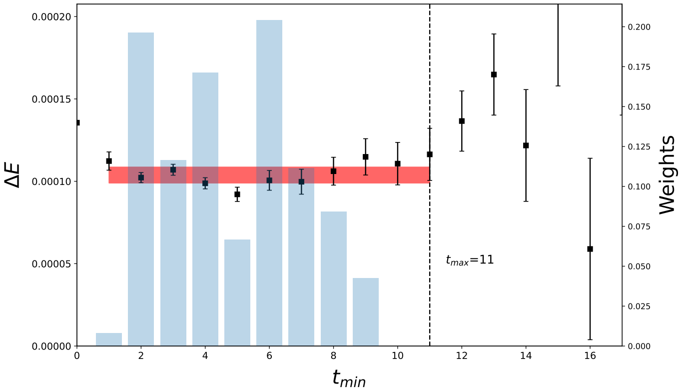
- The dependency of the fits on the time ranges used causes systematic uncertainties
- Time ranges are systematically sampled and each result is assigned a weight:

$$\tilde{w}^f = \frac{p_f (\delta E^f)^{-2}}{\sum_{f'=1}^{N} p_{f'} (\delta E^{f'})^{-2}},$$

- f labels choice of fit range specified by t_{min}
- $p_f = \Gamma(N_{\text{dof}}/2,\ \chi^2/2)/\Gamma(N_{\text{dof}}/2)$ is the p-value of the fit
- δE^f is the uncertainty in the value of the change in energy of the hadron state.
- Taking a weighted average of the N fit findings provides the final unbiased estimate of the energy shift

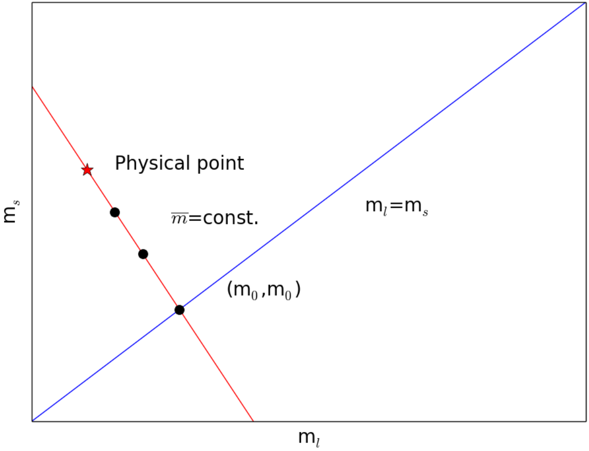
(NPLQCD/QCDSF, Phys. Rev. D, 2021)

Weighted Averaging Method



Energy shift for the down quark at $\lambda = 0.0005$, spin-up. Calculated at $a = 0.082fm$, $(\kappa_l, \kappa_s) = (0.119930, 0.119930)$

Flavour Symmetry Breaking



Flavour Symmetry Breaking

- $SU(3)$ unbroken $\Rightarrow$ operator octet expressed as two couplings (f,d)
- $SU(3)$ broken $\Rightarrow$ construct quantities (D_i, F_i) Example:

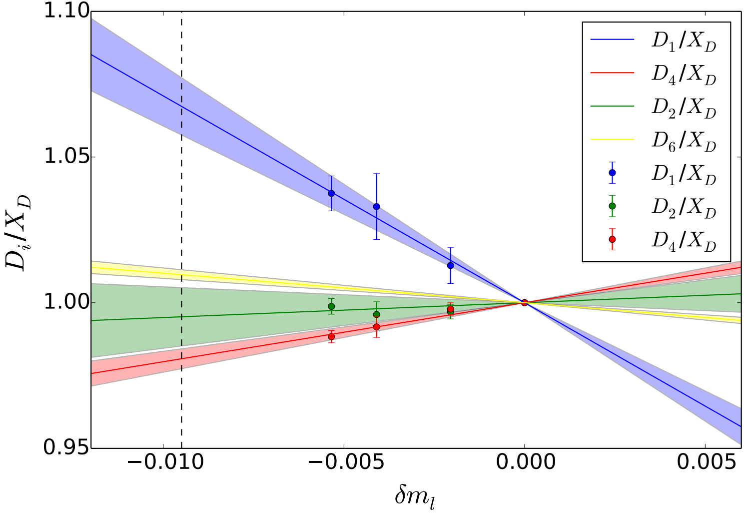
$$
\begin{aligned}
D_1 &= -(A_{\bar{N}\eta N} + A_{\bar{\Xi}\eta\Xi}) \\
&= -(\frac{1}{\sqrt{6}}g_p^u + g_p^d) + \frac{1}{\sqrt{6}}(g_{\Xi}^u - 2g_{\Xi}^s))
\end{aligned}
$$

- Fan plots with slope parameters (r_i, s_i) constraining them
- Calculate each quantity (D_i, F_i) for each quark mass calculated on the lattice

$$
\begin{aligned}
D_1 &= 2d - 2r_1\delta m_l \\
D_2 &= 2d + (r_1 + 2\sqrt{3}r_3)\delta m_l \\
\text{where } \delta m_l &= m_l - \bar{m}
\end{aligned}
$$

- An 'average D' can also be constructed from the diagonal amplitudes $X_D = \frac{1}{6}(D_1 + 2D_2 + 3D_4) = 2d + \mathcal{O}(\delta m_l^2)$

Fan plots



$a = 0.082fm$ using 4 pion masses

Flavour Symmetry Breaking

- Calculate Flavour diagonal matrix elements and hence the nucleon isovector charges through:

$$\langle p|\,\bar{u}\Gamma u\,|p\rangle = 2\sqrt{2}f + \left(\sqrt{\frac{3}{2}}r_1 - \sqrt{2}r_3 + \sqrt{2}s_1 - \sqrt{\frac{3}{2}}s_2\right)\delta m_l^*$$
$$\langle p|\,\bar{d}\Gamma d\,|p\rangle = \sqrt{2}(f - \sqrt{3}d) + \left(\sqrt{\frac{3}{2}}r_1 - \sqrt{2}r_3 - \sqrt{2}s_1 - \sqrt{\frac{3}{2}}s_2\right)\delta m_l^*$$

Where r_1, r_3 , s_1 and s_2 are the slopes of the fan plots, and:

$$g_{\mathcal{O}}^{u-d} = \langle p|\,\bar{u}\Gamma u\,|p\rangle - \langle p|\,\bar{d}\Gamma d\,|p\rangle\,.$$

and using δm_l^* at physical point

Systematics

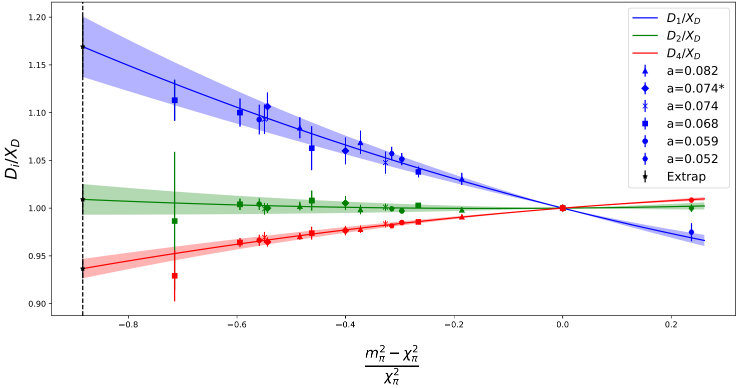
- Flavour breaking expansion method only accounts for the quark mass-dependence of the matrix elements
- In order to quantify systematic uncertainties, we extend this theory to also account for lattice spacing, a, finite volume effects, mL, and second order mass terms, δm_l^2
- Perform a combination of global fits over all ensembles

$$X'_{D,F} = X_{D,F}(1 + c_1 \frac{1}{3}[f_L(m_\pi) + 2f_L(m_K)]) + c_2 a^2 + c_3 \delta m_l^2,$$

$$\text{where: } f_L(m) = (am)^2 \frac{e^{-mL}}{X_N L},$$

$$D_1 = 2d - 2(r'_1 + b_1 a^2)\delta m_l + d_1 \delta m_l^2.$$

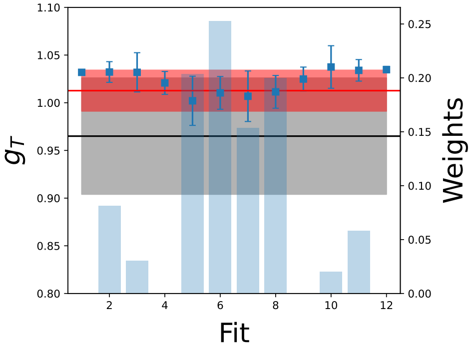
Systematics



a^2 and δm_l^2 correction included. All points shifted in the limit $a \to 0$

In order to combine all fits we extend our weighted averaging method

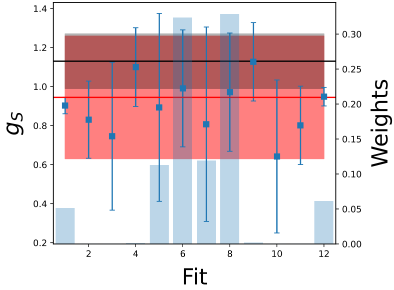
Results

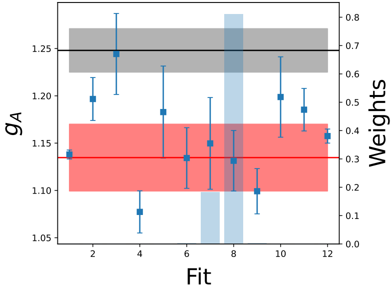


x-axis
1. No Corrections
2. a^2
3. a
4. δm_l^2
5. $a,\ \delta m_l^2$
6. $a^2,\ \delta m_l^2$
7. $a,\ \delta m_l^2,\ mL$
8. $a^2,\ \delta m_l^2,\ mL$
9. $\delta m_l^2,\ mL$
10. $a,\ mL$
11. $a^2,\ mL$
12. mL

- $g_T = 1.013(21)$
- Black band Flag Review 2021 (arXiv:2111.09849)

Results



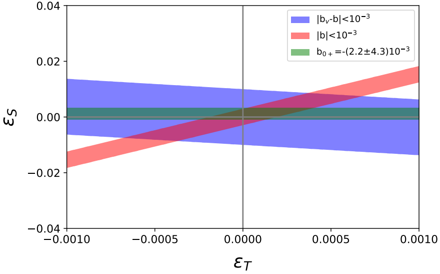


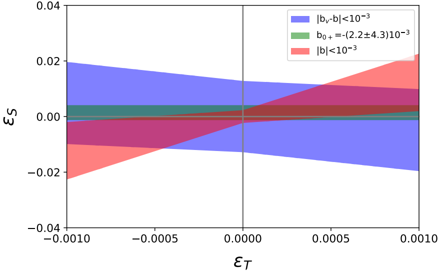
$$g_S \;=\; 0.94(31) \qquad\qquad g_A \;=\; 1.135(35)$$

Renormalised at $\mu = 2\text{GeV}$ in the $\overline{\text{MS}}$ scheme

Impact on Phenomenology

- Following the method presented in Bhattacharya et al., PRD, 2012
- Green band is the existing band from $0^+ \to 0^+$ nuclear beta decay
- $g_T = g_S = 1$
- No uncertainty





- Tensor and scalar charges using our results:
  $g_T = 1.013(21)$
  $\delta g_T / g_T \approx 2\%$
  $g_S = 0.94(31)$
  $\delta g_S / g_S \approx 30\%$

Conclusion

- Method presented we have:
  - Control of excited state contamination from FH method
  - Weighted Averaging Method- removing possible systematic uncertainties which arise from bias choices of fit windows
  - Quark mass- Flavour symmetry breaking to extrapolate towards the physical point
  - Extended flavour symmetry breaking method in account for lattice spacing, finite volume effects and second order mass terms, have control over these systematics
- Calculated g_T to $\approx 2\%$ level, successfully reaching the goal of understanding at the 10% level.
- Work is still needed on g_S and g_A to understand it at the same level.