The de Sitter Instanton from Euclidean Dynamical Triangulations

Lattice 2022

August 10, 2022

Marc Schiffer, Perimeter Institute

In collaboration with

S. Bassler, J. Laiho, and J. Unmuth-Yockey: **Phys.Rev.D 103 (2021) 114504**

M. Dai, W. Freeman, J. Laiho, K. Ratliff, and J. Unmuth-Yockey: **WIP**

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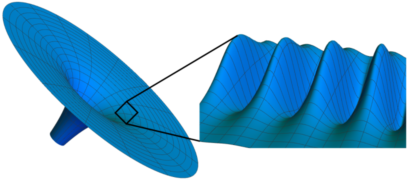
Asymptotically Safe Quantum Gravity



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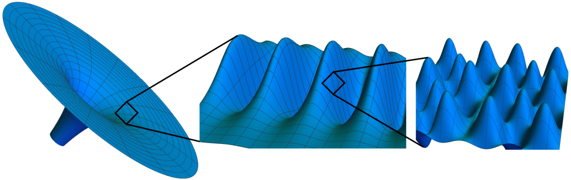
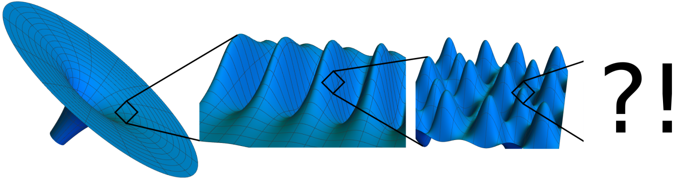


- Perturbative quantum gravity:

Asymptotically Safe Quantum Gravity



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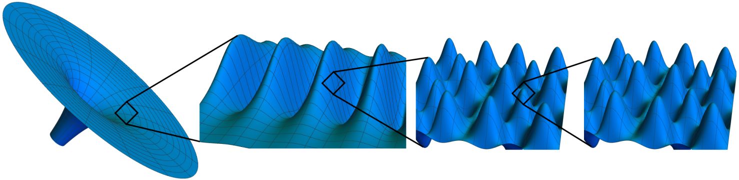
Asymptotically Safe Quantum Gravity



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- Key idea of asymptotic safety:
  Quantum realization of
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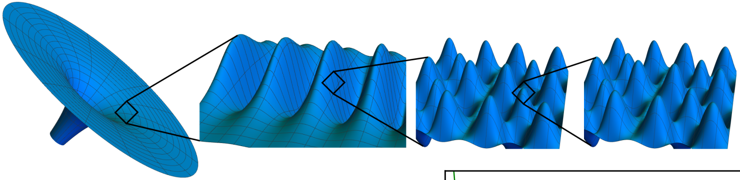
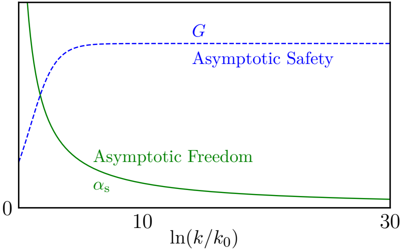


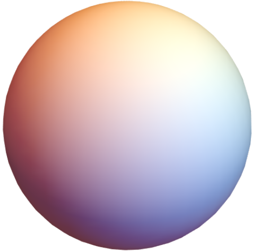
- Perturbative quantum gravity:
  loss of predictivity
- Key idea of asymptotic safety:
  Quantum realization of scale symmetry
  - imposes infinitely many conditions on theory space
  - relevant directions: need measurement
  - irrelevant directions: predictions of theory



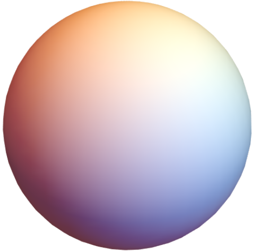
$$k\partial_k \alpha_s = -\frac{11}{2\pi}\alpha_s^2 + \mathcal{O}(\alpha_s^4)$$

$$k\partial_k G = \epsilon G - \frac{50}{3}G^2 + \mathcal{O}(G^3)$$

Dynamical triangulations



- Discretization of spacetime in terms of triangulations



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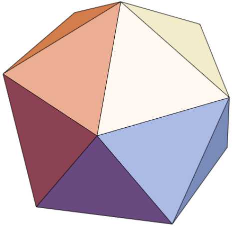

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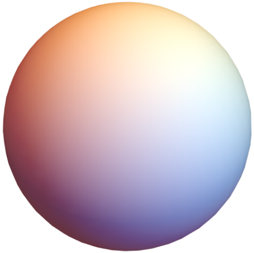

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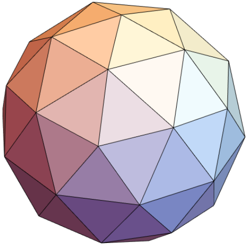

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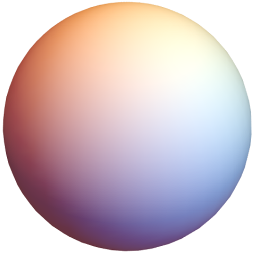

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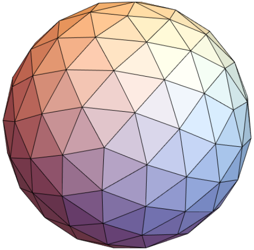

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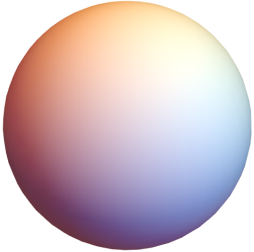

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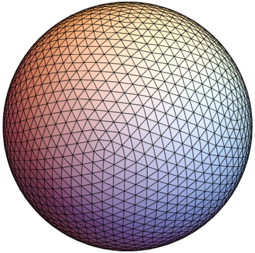

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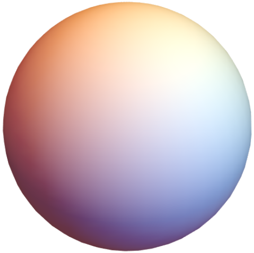

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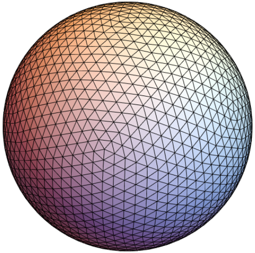

- Discretization of spacetime in terms of triangulations
  [Ambjørn and Jurkiewicz, 1992], [Agishtein and Migdal, 1992], . . .

$$\int \mathcal{D}g\, e^{-S[g]} \to \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}}\, e^{-S_{\mathrm{ER}}}$$

with Euclidean Einstein-Regge action $S_{\mathrm{ER}} = -\kappa_2 N_2 + \kappa_4 N_4$ [Regge, 1961]

Lattice quantum gravity in $d = 4$

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- in $d = 4$: no physical phase, no indications for higher-order transition in κ_2-κ_4 - space

[Ambjørn, Jain, Jurkiewicz, Kristjansen, 1993], [Bakker, Smit, 1994]
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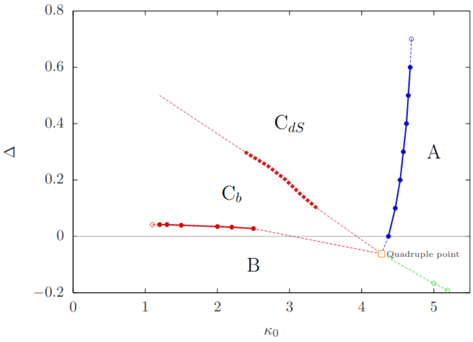

Taken from [Loll, 2020]

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$$\int \mathcal{D}g \, e^{-S[g]} \rightarrow \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}} \left[\prod_{j=1}^{N_2} \mathcal{O}(t_j)^{\beta} \right] e^{-S_{\mathrm{ER}}}$$

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  [Laiho, Coumbe, 2011], [Ambjørn, Glaser, Goerlich, Jurkiewicz, 2013]
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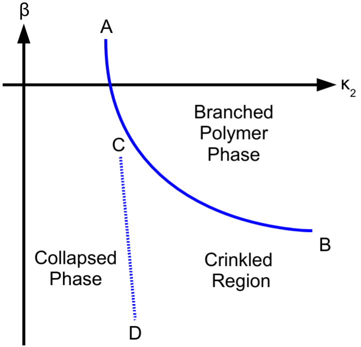

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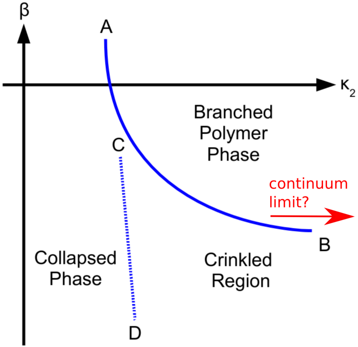

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Phase diagram of EDT

- Idea:
  follow AB-line towards large κ_2
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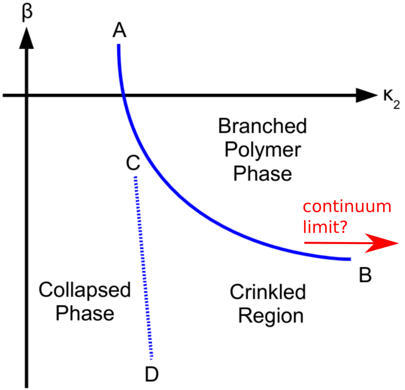

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- Challenge:
  acceptance rate p drops:

$$\kappa_2 = 1.7\,,\ \ p \sim 10^{-3};$$
$$\kappa_2 = 2.45\,, p \sim 10^{-4};$$
$$\kappa_2 = 3.0\,,\ \ p \sim 3 \cdot 10^{-5};$$
$$\kappa_2 = 3.8\,,\ \ p \sim 8 \cdot 10^{-7};$$
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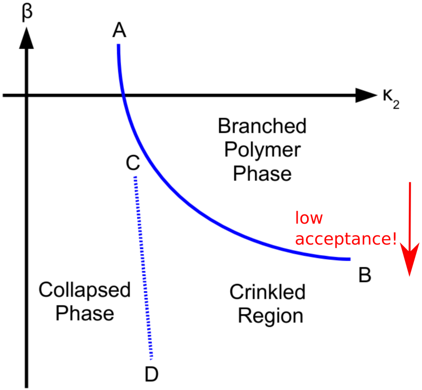

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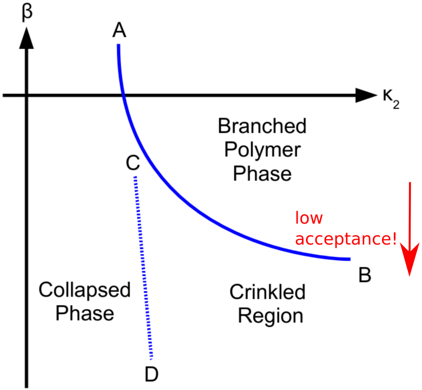

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Need efficient algorithm for low acceptance rates p.

Progress I: Rejection free algorithm for EDT

- Adapt algorithms used in studies of dynamical systems (e.g., growth of crystals)
  [Norman, Cannon, 1972], [Bortz, Kalos, Lebowitz, 1975], [Gillespie, 1976], [Schulze, 2004], . . .

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- Proof of principle: 2d Ising model
- EDT implementation:

$$\kappa_2 = 3.0, \text{ speedup} \approx 30 \quad ;$$

$$\kappa_2 = 3.8, \text{ speedup} \approx 300 \quad ;$$

$$\kappa_2 = 4.5, \text{ speedup} \approx 500 \quad ;$$

Key object: the shelling function

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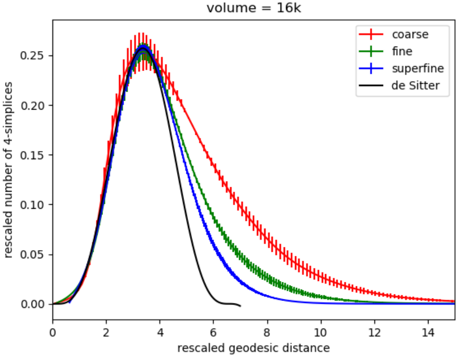
- Peak-height: order parameter of AB-transition
- Lattice volume profiles: approximate de Sitter profile better for finer lattices



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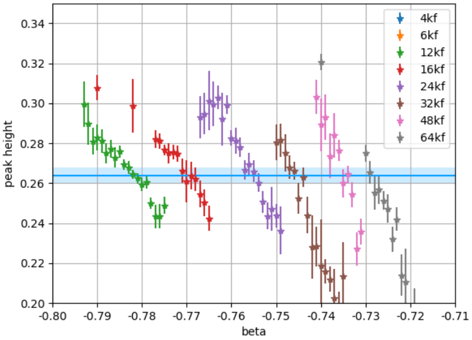
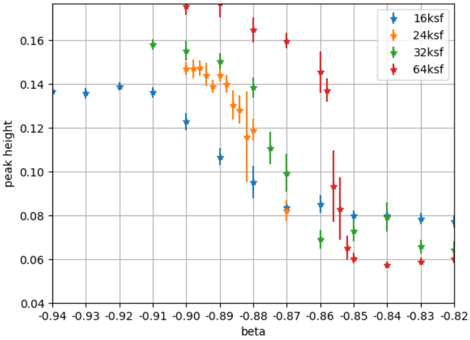
Progress II: More and finer ensembles

- Generate fine ($\kappa_2 = 3.0$), super-fine ($\kappa_2 = 3.8$) and ultra-fine ($\kappa_2 = 4.5$) ensembles
- Only feasible with rejection-free algorithm





EDT lattice spacings: a/ℓ

- a: lattice spacing; edge length
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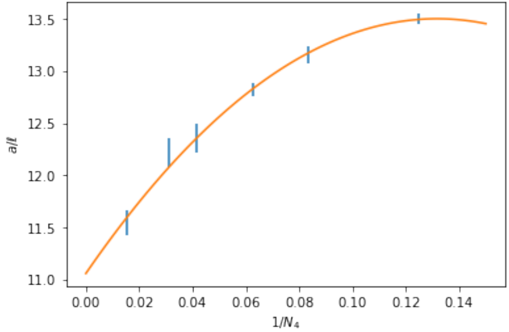
- fit $\frac{a}{\ell} = A + \frac{B}{V} + \frac{C}{V^2}$
- Example: fine lattices:
  $\frac{a}{\ell} = 11.157(161)$ at $\chi^2/\mathrm{dof} = 0.41$



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EDT lattice spacings: $\ell_{\rm rel}$

- From lattices extract
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  Results in $\Lambda_0(\sqrt{N_4})$
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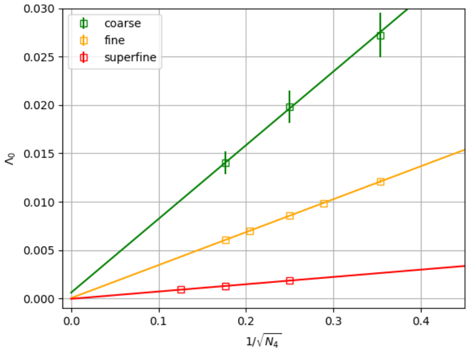

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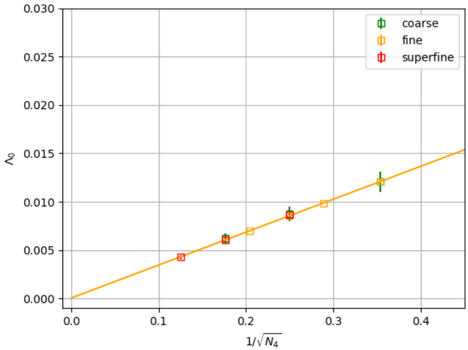
- Rescale $\Lambda_0(\sqrt{N_4})$ to match fine-slope
  Rescaling factor encodes ℓ_{rel}



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The Saddle point approximation in EDT

- Saddle point approximation:
  [Ambjorn, Goerlich, Jurkiewicz, Loll, 2012]

$$\langle N_4 \rangle \simeq \frac{k^2}{4(\kappa_4 - \kappa_4^c)^2} \Rightarrow k = |\kappa_4 - \kappa_4^c| \sqrt{N_4}\,.$$

Use finite-volume scaling of κ_4 to test recovery of semi-classical limit

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- Match lattice saddle point approximation with continuum calculation:

$$Z(\kappa_2,\, \kappa_4) \approx \exp\left(\frac{k^2(\kappa_2)}{4(\kappa_4 - \kappa_4^c)} \right) = \exp\left(\frac{3\pi}{G\,\Lambda} \right)$$

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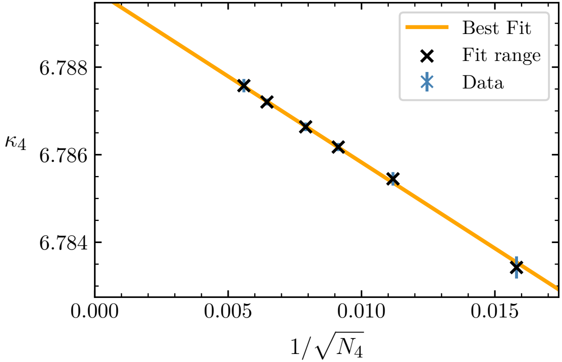
  Assumption: Continuum is dominated by de Sitter instanton [Hawking, Moss, 1987]
  Extract G from lattice data:

$$\frac{G}{\ell_{\text{fid}}^2} \sim \left(\frac{a}{\ell}\right)^2 \frac{\ell_{\text{rel}}^2}{|s|}\,,$$

Numerical result: finite volume scaling

Example at $\beta = -0.6$, $\kappa_2 = 2.245$:



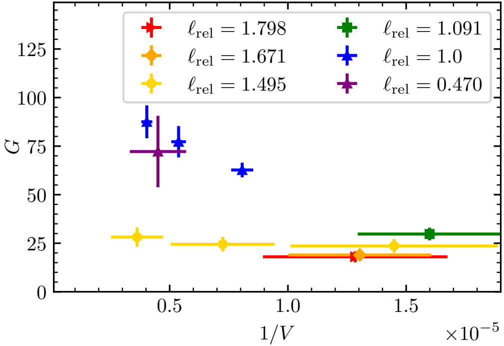
Extract slope s: $\kappa_4(N_4) = A + s\frac{1}{\sqrt{N_4}}$

Fit result: $s = -0.393 \pm 0.022$

$\chi^2/\mathrm{d.o.f} = 0.15$, p-value: 0.96.

Numerical result: the Newton coupling

- Extract slope for all ensembles
- Compute G for each of the ensembles

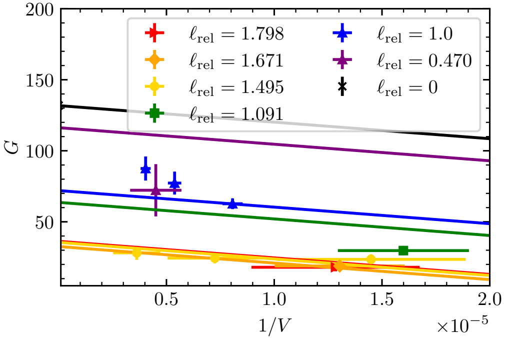


Numerical result: the Newton coupling

- Extract slope for all ensembles
- Compute G for each of the ensembles
- Perform continuum, infinite volume limit extrapolation:

$$G = \frac{H_G}{V} + J_G \ell_{\text{rel}}^2 + K_G \ell_{\text{rel}}^4 + G_0 \,,$$

$G_0 = 132 \pm 13\,;\ \ \chi^2/\text{d.o.f} = 3.9$



Numerical result: the Newton coupling

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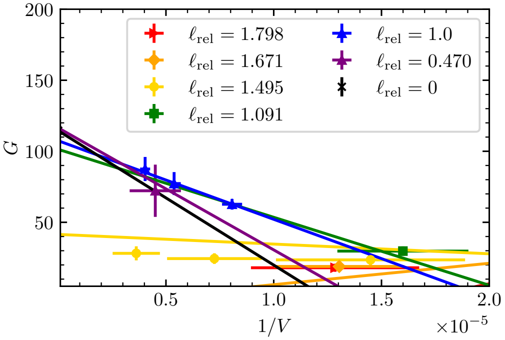
$$G = \frac{H_G}{V} + I_G \frac{\ell_{\text{rel}}^2}{V} + J_G \ell_{\text{rel}}^2 + K_G \ell_{\text{rel}}^4 + G_0\,,$$

$$G_0 = 114 \pm 28\,;\ \ \chi^2/\text{d.o.f} = 0.61$$



Numerical result: the Newton coupling

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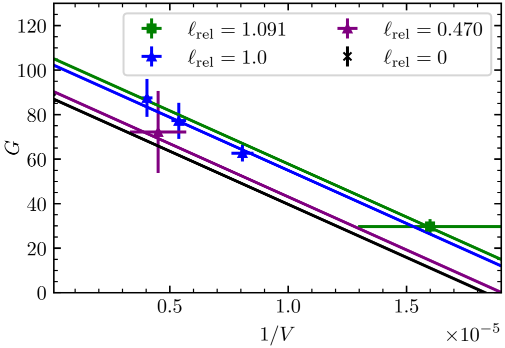
$$G = \frac{H_G}{V} + J_G \ell_{\text{rel}}^2 + G_0\,,$$

$$G_0 = 87 \pm 24\,;\ \ \chi^2/\text{d.o.f} = 0.21$$



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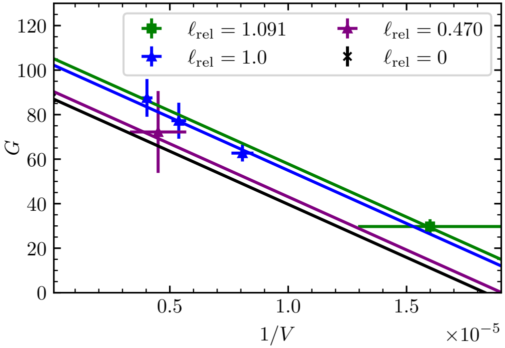

Continuum, infinite volume limit:

Requires more data-points for fine and super-fine lattices

Summary & Conclusion

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- New scheme to extract a/ℓ and ℓ_{rel}:
  based on de Sitter volume profile
- extraction of G_0:
  differs from previous scheme; different discretization and finite-size effects

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Thank you for your attention!