Non-perturbative heavy quark action tuning using machine learning

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Motivation(s)

- plan is to do heavy-light spectroscopy and scattering $D\pi, D\eta$ ... etc

  D. Mohler talk Tuesday

- want a charm action with small discretisation effects

$$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{(p^2)^2}{8M_4^3} - \frac{a^3 W_4}{6}\sum_i p_i^4 + \ldots,$$

- ideally relativistic dispersion relation $c^2 = 1$ $(M_2 \approx M_1)$ for charm really $aE = \sqrt{(am)^2 + c^2(ap)^2}$
- investigate tuning's impact on states of interest (tetraquarks, b-mesons... etc)

Our Solution:

- fully nonperturbatively tune heavy quark action (relativistic or NRQCD) using neural networks (NNs)

$$D_{xy} = \delta_{xy} - \kappa_c\left[\sum_i (r_s - \nu\gamma_i)U_i(x)\delta_{x+i,y} + (r_s + \nu\gamma_i)U_i^\dagger(x)\delta_{x,y+i}\right] - \kappa_c\left[(r_t - \gamma_t)U_t(x)\delta_{x+t,y} + (r_t + \gamma_t)U_t^\dagger(x)\delta_{x,y+t}\right]$$
$$- \kappa_c\left[c_B\sum_{i,j}\sigma_{ij}F_{ij}(x) + c_E\sum_i \sigma_{it}F_{it}(x)\right]\delta_{xy}\,.$$

- use the "Tsukuba" heavy-quark action
- has 5 tuneable parameters $\kappa_C, r_s, \nu, c_E, c_B, r_t = 1$
- randomly draw parameters and measure simple charmonia spectrum and (spin-averaged) c^2 of η_c and J/ψ
- train a Neural Network to classify patterns between parameters and states
- give network continuum states to predict best parameters

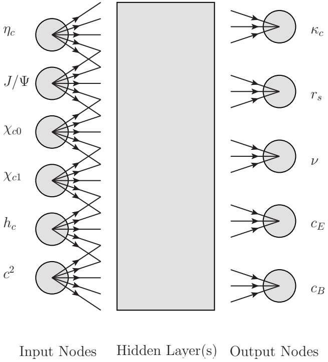


Figure: Neural network tuning picture

Charm operators

- use simple quark-line connected local meson operators

$$O(x) = (\bar{c}\Gamma c)(x)$$

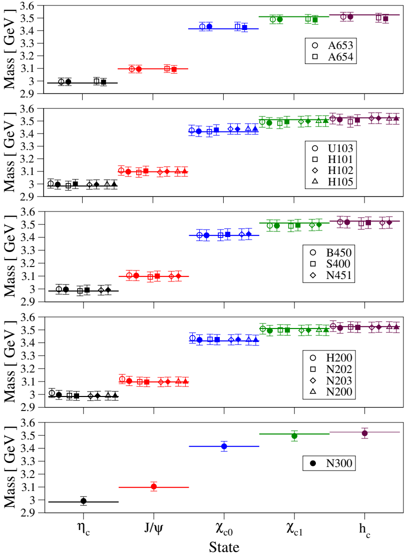
- partially twisted bcs along (θ, θ, θ) for c^2

State	η_c	J/ψ	χ_{c0}	χ_{c1}	h_c
Γ	γ_5	γ_i	I	$\gamma_i\gamma_t$	$\gamma_i\gamma_j$
J^{PC}	0^{-+}	1^{--}	0^{++}	1^{++}	1^{+-}
Experiment [GeV]	2.9839	3.096916	3.41471	3.51072	3.52549

Table: List of operators used in our measurement of charmonium, their expected quantum number equivalents to continuum states and the experimental masses of these states [1]

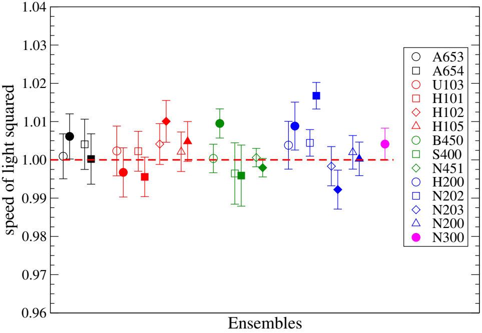
β	Name	$L^3 \times L_T$	T-Boundary	a^{-1} [GeV]	m_π [GeV]
3.34	**A653**	$24^3 \times 48$	**Periodic**	**1.987(20)**	**0.422(4)**
3.34	A654	$24^3 \times 48$	Periodic	1.987(20)	0.331(3)
3.40	**U103**	$24^3 \times 128$	**Open**	**2.285(28)**	**0.419(5)**
3.40	H101	$32^3 \times 96$	Open	2.285(28)	0.416(5)
3.40	H102	$32^3 \times 96$	Open	2.285(28)	0.354(5)
3.40	H105	$32^3 \times 96$	Open	2.285(28)	0.284(4)
3.46	**B450**	$32^3 \times 64$	**Periodic**	**2.585(33)**	**0.416(4)**
3.46	S400	$32^3 \times 128$	Open	2.585(33)	0.351(4)
3.46	N451	$48^3 \times 128$	Periodic	2.585(33)	0.287(4)
3.55	**H200**	$32^3 \times 96$	**Open**	**3.071(36)**	**0.419(5)**
3.55	N202	$48^3 \times 128$	Open	3.071(36)	0.410(5)
3.55	N203	$48^3 \times 128$	Open	3.071(36)	0.345(4)
3.55	N200	$48^3 \times 128$	Open	3.071(36)	0.282(3)
3.70	N300	$48^3 \times 128$	Open	3.962(45)	0.421(4)

Table: Table of ($n_f = 2 + 1$, CLS [2], clover-Wilson) ensembles used in the two studies here, ensembles in **bold** are used to tune NRQCD



- Top: coarse ($a \approx 0.1$ fm)
  Bottom: fine ($a \approx 0.05$ fm)
- Left: heaviest pion
  Right: lightest pion
- Open: individual
  Filled: same β (grouped)

Dispersion relations



- relativistic dispersion relation achieved at the 1% level
- individual ensemble tuning slightly better than grouped tuning for the dispersion relation

NRQCD action

Typical [3] tadpole-improved NRQCD action (here we will use n=4)

$$
\begin{aligned}
H_0 &= -\frac{1}{2aM_0}\Delta^2, \\
H_I &= \left(-c_1\frac{1}{8(aM_0)^2} - c_6\frac{1}{16n(aM_0)^2}\right)\left(\Delta^2\right)^2 + c_2\frac{i}{8(aM_0)^2}(\tilde{\Delta}\cdot\tilde{E} - \tilde{E}\cdot\tilde{\Delta}) + c_5\frac{\Delta^4}{24(aM_0)} \\
H_D &= -c_3\frac{1}{8(aM_0)^2}\sigma\cdot\left(\tilde{\Delta}\times\tilde{E} - \tilde{E}\times\tilde{\Delta}\right) - c_4\frac{1}{8(aM_0)}\sigma\cdot\tilde{B} \\
\delta H &= H_I + H_D.
\end{aligned}
$$

Propagators are generated through applications of the symmetric evolution equation

$$
G(x, t+1) = \left(1 - \frac{\delta H}{2}\right)\left(1 - \frac{H_0}{2n}\right)^n \tilde{U}_t(x, t_0)^\dagger \left(1 - \frac{H_0}{2n}\right)^n \left(1 - \frac{\delta H}{2}\right) G(x, t).
$$

NRQCD operators

Consider (again) only quark-line connected parts of simple meson operators

$$O(x) = (\bar{b}\Gamma(x)b)(x),$$

State	PDG mass [GeV] [1]	$\Gamma(x)$
$\eta_b(1S)$	9.3987(20)	γ_5
$\Upsilon(1S)$	9.4603(3)	γ_i
$\chi_{b0}(1P)$	9.8594(5)	$\sigma \cdot \Delta$
$\chi_{b1}(1P)$	9.8928(4)	$\sigma_j\Delta_i - \sigma_i\Delta_j \; (i \neq j)$
$\chi_{b2}(1P)$	9.9122(4)	$\sigma_j\Delta_i + \sigma_i\Delta_j \; (i \neq j)$
$h_b(1P)$	9.8993(8)	Δ_i

Table: Table of lattice operators used and their continuum analogs

- due to additive mass we must only consider splittings
- we subtract the η_B from all states
- perform tuning at $\mathrm{SU}(3)_f$-symmetric point
- gauge-fixed wall sources
- tuning can absorb different tadpole factors
- tuning precision is about 1%
- excited states from 4×4 symmetric GEVP

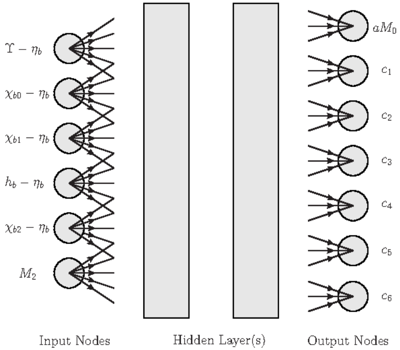


Figure: Schematic picture of our NRQCD setup

Tree-level parameters

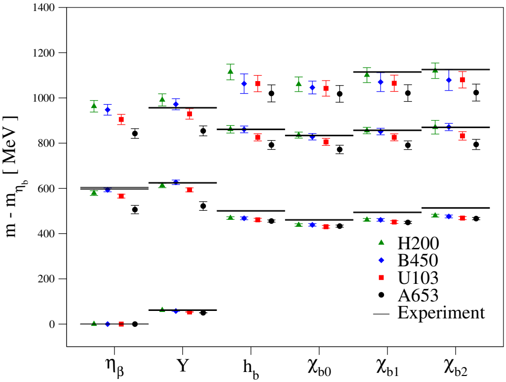


Figure: Preliminary excited states for the tree-level NRQCD tuning

NN parameters

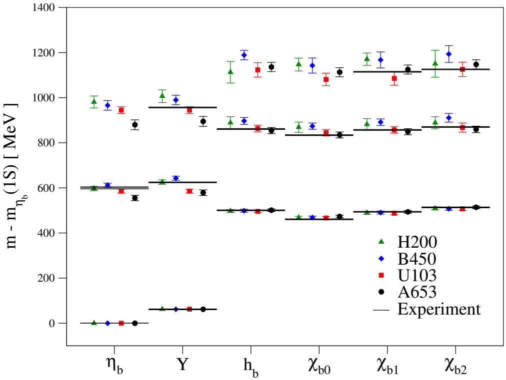


Figure: Preliminary excited states for the NN NRQCD tuning

$ud\bar{b}\bar{b}$ tetraquark - operators

talks: S. Aoki, M. Pflaumer, Padmanath, S. Prelovsek
posters: M. Wagner

- follow [4] and create 4×4 GEVP of operators:

$$\begin{aligned} D(x) &= ({u_a}^T C\gamma_5 d_b)(\bar{b}_a C\gamma_i \bar{b}_b^T)(x), \\ E(x) &= ({u_a}^T C\gamma_t\gamma_5 d_b)(\bar{b}_a C\gamma_i\gamma_t \bar{b}_b^T)(x), \\ M(x) &= (\bar{b}_a\gamma_5 u_a)(\bar{b}_b\gamma_i d_b)(x) - [u \leftrightarrow d], \\ N(x) &= (\bar{b}_a I u_a)(\bar{b}_b\gamma_5\gamma_i d_b)(x) - [u \leftrightarrow d]. \end{aligned}$$

- use convolved smeared sinks [5] and gauge-fixed wall sources
- measure tetraquark mass and subtract BB^* threshold to obtain binding

$ud\bar{b}\bar{b}$ tetraquark - results

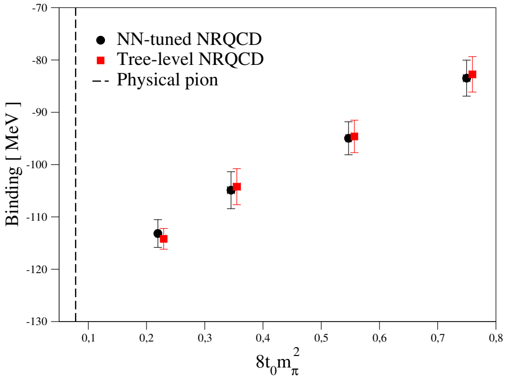


Figure: Preliminary $ud\bar{b}\bar{b}$ tetraquark with NN-tuned and tree level NRQCD tunings. Tree level results have been shifted for clarity

Positive-parity B_s mesons

posters: L. Gayer

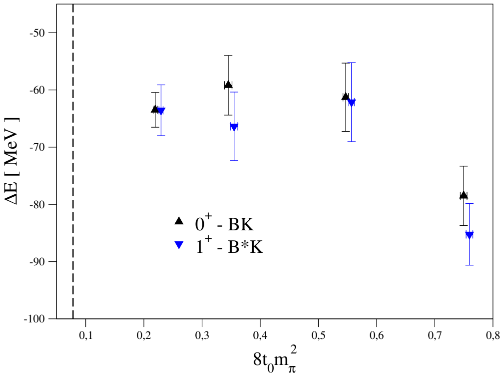


Figure: Incredibly Preliminary scalar and axial B_s mesons with their expected 2-meson thresholds subtracted, from the NN NRQCD tuning

Conclusions

- illustrated the feasibility of tuning heavy quark actions using neural networks
- successfully obtained a relativistic dispersion relation for charm quark action
- shown that a fully non-perturbative tuning of NRQCD is possible and can cure badly-behaved spin-orbit splittings
- shown that NRQCD discretisation/truncation effects play little rôle in $ud\bar{b}\bar{b}$ tetraquark binding

- PARTICLE DATA GROUP collaboration, P. Zyla et al., *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01.
- M. Bruno et al., *Simulation of QCD with $N_f = 2 + 1$ flavors of non-perturbatively improved Wilson fermions*, *JHEP* **02** (2015) 043, [1411.3982].
- G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea and K. Hornbostel, *Improved nonrelativistic QCD for heavy quark physics*, *Phys. Rev. D* **46** (1992) 4052–4067, [hep-lat/9205007].
- R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, *A lattice investigation of exotic tetraquark channels*, *Phys. Rev. D* **102** (2020) 114506, [2006.14294].
- R. J. Hudspith and D. Mohler, *A fully non-perturbative charm-quark tuning using machine learning*, *Arxiv* (12, 2021) , [2112.01997].