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Radiative corrections to neutron decay: the $\gamma - W$ box diagram using Lattice QCD

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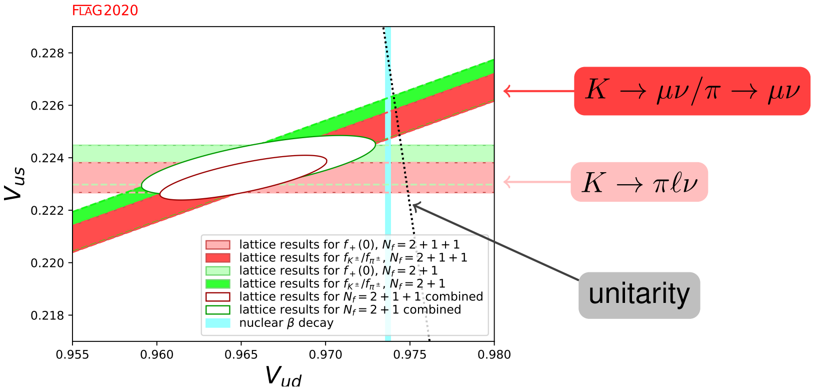
Status

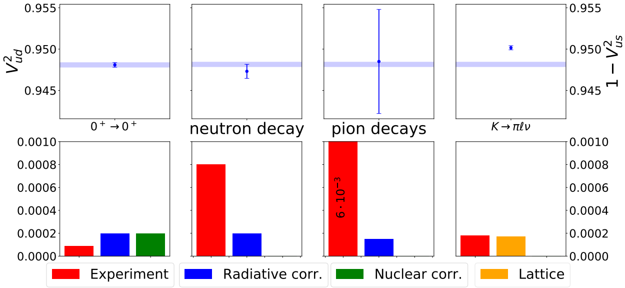


Two anomalies (deviations from black line $\implies \Delta_{\mathrm{CKM}} = 0$):

1. kaon decays and LQCD point to $\sim 2\sigma$ tension with unitarity
2. including superallowed β decays brings the discrepancy to $\sim 4\sigma$

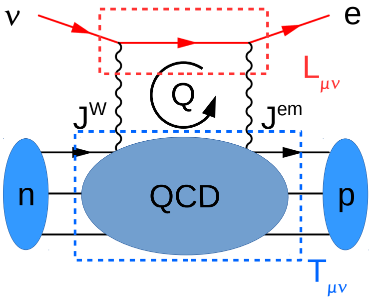
Is this a hint of BSM physics?

$|V_{ud}|$ and $|V_{us}|$ are the targets of our calculation



- $|V_{ud}|$ extracted from β decays of pions, neutrons, and nuclei
  Current: $|V_{ud}|^2 = 0.94809(27)$ dominated by $0^+ \to 0^+$ β decays
- $|V_{us}|^2 = 0.05040(36)$ from kaon decays ($K \to \pi e \bar{\nu}_e$, $K \to \mu \bar{\nu}_\mu$)
- $|V_{ub}|^2 \approx (2 \pm 0.4) \times 10^{-5}$ is tiny, no impact on the unitarity test
- The uncertainty in $\Delta_{\rm CKM}$ receives comparable contributions from $|V_{ud}|$ and $|V_{us}|$

The calculation: $\gamma - W$ Box Diagram



Radiative Correction in $|V_{ud}|$ is dominated by $\gamma - W$ box diagram!

- $\Delta_{\rm np}$ given by the product of leptonic, $L^{\mu\nu}$, and hadronic, $T_{\mu\nu}$, parts

$$\Delta_{\rm np} = \int_0^{+\infty} dQ^2 \int_{-Q}^{Q} dQ_0 \frac{1}{Q^4} \frac{1}{Q^2 + m_W^2} L^{\mu\nu}(Q, Q_0) T_{\mu\nu}^{VA}(Q, Q_0)$$

- Lattice QCD needed for $T^{\mu\nu}$ in $0.1 < Q^2 < 2{\rm GeV}^2$

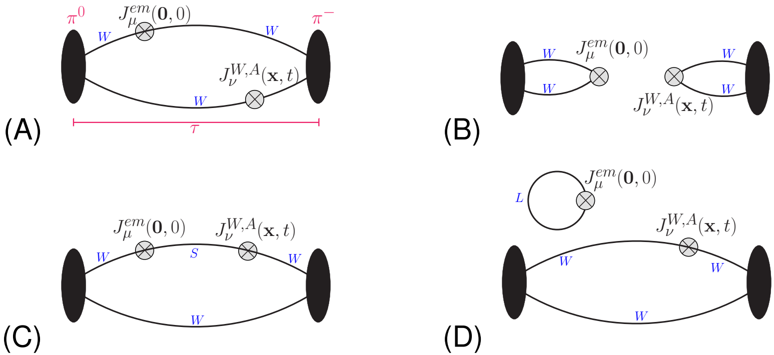
$$T_{\mu\nu}^{VA} = \frac{1}{2} \int d^4x \, e^{iQ\cdot x} \langle N_f(p)| T \left[J_\mu^{em}(0,0) J_\nu^{W,A}(\vec{x}, t) \right] |N_i(p)\rangle$$

$\gamma - W$ Box Diagram – Contd.

- The uncertainty in the integral over Q^2 is dominated by $T^{\mu\nu}$, especially in the low $0 < Q^2 \lesssim 2$ GeV^2 regime where QCD gives large corrections
- The best estimates in the literature combine robust theoretical information on the behavior of the integrand at $Q^2 \sim 0$, where it is determined by the nucleon elastic form factors, and at large $Q^2 \gtrsim 2\,\mathrm{GeV}^2$, where operator product expansion and perturbation theory are reliable
- Proposed lattice QCD calculations aim to reduce the uncertainty in the problematic intermediate region, $0.1 < Q^2 \lesssim 2$ GeV^2, which is currently being $approximated$ using models

The proposed calculation will
determine the RC in the neutron/pion/kaon decay.

RC to pion decay:



- Signal does not degrade with source-sink separation $\longrightarrow$ No excited state artifacts
- Much simpler contractions to construct correlation functions
- Future: do kaon and nucleon at same time

Steps in the calculation

- Coulomb gauge fixing
- Wall Source, Sink propagators ($\boldsymbol{p} = 0$) (labeled W)
- Diagram (A): Calculate W from sources and sinks to the current insertion and multiply appropriate Gamma matrix
- Diagram (C): Compute additional propagator S from random points where $J_\mu^{\rm em}$ sits and shift to $(\mathbf{0}, 0)$. Appropriate contractions of W and S propagators at site $(\boldsymbol{x}, t)$ with insertion of $J_\nu^{W,A}$ (or $J_\mu^{\rm em}$)
- Diagram (D): Calculate the disconnected quark loop L with the insertion of the electromagnetic current shown in the upper part of diagram (D). A stochastic method is used to estimate L at all points of the lattice. Calculate the correlation of loop L with the bottom part of the diagram (D), a 3-point function with insertion of $J_\nu^{W,A}$ at point $(\boldsymbol{x}, t)$.

- $\mathcal{M}_H(Q^2)$ can be calculated from $H^{VA}_{\mu\nu} = \langle \pi^0(p)|T\left[J^{em}_\mu(0,0)J^{W,A}_\nu(\vec{x},t)\right]|\pi^-(p)\rangle$
- The relevant hadronic tensor is:

$$T^{VA}_{\mu\nu} = \frac{1}{2}\int d^4x\, e^{iQ\cdot x}\langle \pi^0(p)|T\left[J^{em}_\mu(0,0)J^{W,A}_\nu(\vec{x},t)\right]|\pi^-(p)\rangle$$

- The spin-independent part of $T^{VA}_{\mu\nu}$ has only one term

$$T^{VA}_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta T_3 + \dots$$

- Knowing T_3 as a function of Q^2, the γW-box correction is given by

$$\Box^{VA}_{\gamma W} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}\mathcal{M}_H(Q^2)$$

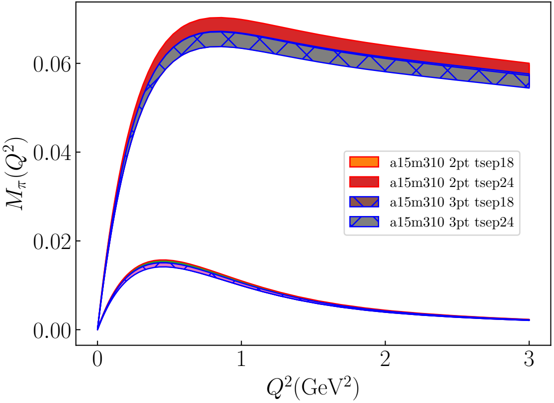
Lattice Setup

- Carried out pion γW box measurement on 7 different HISQ-Clover ensembles
- Different lattice spacing can be used for continuum extrapolation ($a \to 0$)
- Results from various values of m_π are interpolated to the physical pion mass

ID	a(fm)	$m_\pi(\mathrm{MeV})$	latt. size	$m_\pi L$	# configs.	# src	# /src
a06m310	0.0582(04)	319.6(2.2)	$48^3 \times 144$	4.52	48	8	256,64
a09m130	0.0871(06)	139.1(1.0)	$64^3 \times 96$	3.90	46	8	256,64
a09m310	0.0888(08)	313.0(2.8)	$32^3 \times 96$	4.51	60	8	256,64
a12m220	0.1184(10)	227.9(1.9)	$32^3 \times 64$	4.38	98	8	256,64
a12m220L	0.1189(09)	227.6(1.7)	$40^3 \times 64$	5.49	50	8	256,64
a12m310	0.1207(11)	310.2(2.8)	$24^3 \times 64$	4.55	99	8	256,64
a15m310	0.1510(20)	320.6(4.3)	$16^3 \times 48$	3.93	80	8	256,64

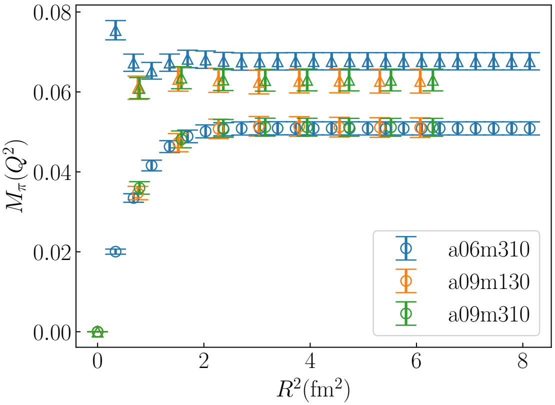
Table: Ensembles used for RC to the charged pion β decay

$\mathcal{M}_H(Q^2)$ using 2-, 3-, 4-pt correlation functions



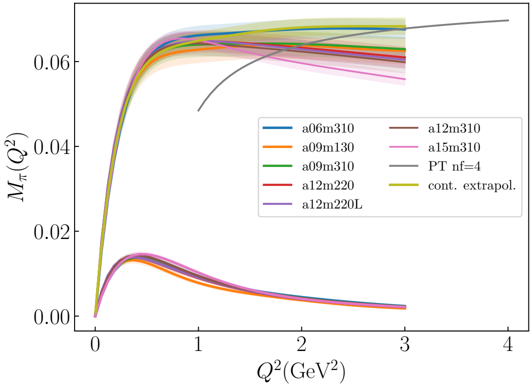
- $\mathcal{M}_H\left(Q^2\right) =$
  $-\frac{1}{6}\frac{1}{F_+^H}\frac{\sqrt{Q^2}}{m_H}\int d^4x\omega(t,\vec{x})\epsilon_{\mu\nu\alpha 0}x_\alpha\mathcal{H}_{\mu\nu}^{VA}(t,\vec{x})$
- $\mathcal{H}_{\mu\nu} = \frac{2m_\pi C_{4pt}}{C_{2pt}}$
- Combine with prefactor $F_+^\pi = \frac{C_{3pt}}{C_{2pt}}$
- ratio $\mathcal{H}_{\mu\nu}/F_+^\pi = \frac{2m_\pi C_{4pt}}{C_{3pt}}$ has better S2N due to cancellation of correlated fluctuations.

$\mathcal{M}_H(Q^2)$ as a function of integration radius



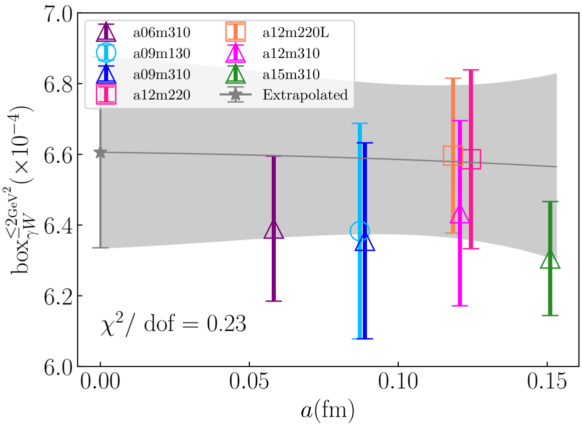
- $\mathcal{M}_H\left(Q^2\right) = -\frac{1}{6}\frac{1}{F_+^H}\frac{\sqrt{Q^2}}{m_H}\int d^4x\omega(t,\vec{x})\epsilon_{\mu\nu\alpha 0}x_\alpha \mathcal{H}_{\mu\nu}^{VA}(t,\vec{x})$
- $\int d^4x$ is done within a finite radius R on lattice
- Saturates at some radius
- triangle marker $Q^2 = .317\mathrm{GeV}^2$ and circle marker $Q^2 = 3.0\mathrm{GeV}^2$

Q^2-dependence of $\mathcal{M}_H(Q^2)$ for the pion



- In $Q^2 \gtrsim 2\text{GeV}^2$ regime, results from coarse (large a) ensembles were farther away from the PT result
- Taking the continuum limit where $\alpha_s(a^{-1})a \to 0$, the PT result and continuum extrapolation overlaps in $2\text{GeV}^2 < Q^2 < 3\text{GeV}^2$
- The surplus in the low-Q^2 regime compensates for deficiency in high-Q^2 for coarse lattice
  $\to$ small a-dependence of the integral

Integrated box contribution below $Q^2 \le 2\text{GeV}^2$



Leading a- and m_π-dependence (in χPT) is expected to be

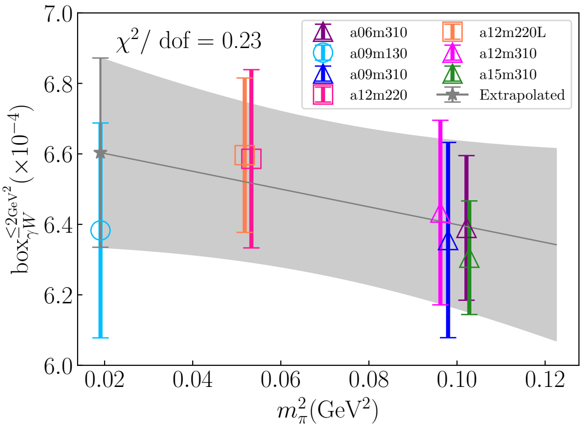
$$\square_{\gamma W}^{VA}|_\pi^{Q^2 \le 2\text{GeV}^2}(m_\pi, a) = c_0 + c_1 m_\pi^2 + c_2 a \alpha_s(a^{-1}) \quad (1)$$

Extrapolated result at physical point

$$\square_{\gamma W}^{VA}|_\pi^{Q^2 \le 2\text{GeV}^2} = 6.61(27) \times 10^{-4} \quad (2)$$

Mild dependence on a and m_π

Integrated box contribution below $Q^2 \leq 2\mathrm{GeV}^2$



Our lattice result at physical point

$$\Box_{\gamma W}^{VA}|_\pi^{Q^2 \leq 2\mathrm{GeV}^2} = 0.661(27) \times 10^{-3}$$

can be combined with pQCD result

$$\Box_{\gamma W}^{VA}|_\pi^{Q^2 > 2\mathrm{GeV}^2} = 2.159(6)(7) \times 10^{-3}$$

$$\Rightarrow \quad \Box_{\gamma W}^{VA}|_\pi = 2.820(28) \times 10^{-3}$$

[Our Result]

$$\mathrm{cf.)} \quad \Box_{\gamma W}^{VA}|_\pi = 2.830(11)(26) \times 10^{-3}$$

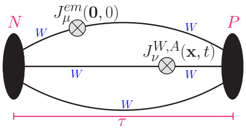
[Xu Feng, et al., PRL (2020)]

Determination of $|V_{ud}|$

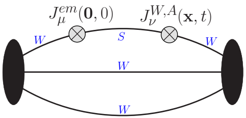
- $\Gamma_{\pi\ell 3} = \frac{G_\mu^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3}(1+\delta)I_\pi$
- χPT result: $\delta = 0.0334(10)_{\rm LEC}(3)_{\rm HO}$
- $\delta = \frac{\alpha_e}{2\pi}\left[\bar{g} + 3\ln\frac{m_Z}{m_p} + \ln\frac{m_Z}{m_W} + \tilde{a}_g\right] + \delta_{HO}^{QED} + 2\Box_{\gamma W}^{VA}$
- Earlier result on lattice: $\delta = 0.0332(1)_{\gamma W}(3)_{HO}$ [Xu Feng, et al., PRL (2020)]
- Our preliminary result: $\delta = 0.0332(1)_{\gamma W}(3)_{HO}$

Nucleon Box diagram Status

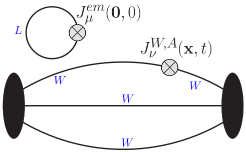
- No diagram B
- S2N is an issue
- Investigate and find cost-effective and good spin projection
- Diagrams share the same Wall (W), point (S) and disconnected loop (L) propagators with the pion calculation



(A)



(C)



(D)

Summary

- Reduce uncertainty in radiative correction to neutron decay using LQCD
- Computed electroweak γW-box corrections to the semileptonic pion decay on 7 ensembles and obtained $\Box_{\gamma W} = 2.811(33) \times 10^{-3}$ (paper in preparation)
- Nucleon box diagrams are under investigation

- Goal: precision results for RC to pion, kaon, neutron decays

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Appendix

Two ways to probe BSM physics:

Two ways to probe Beyond the Standard Model (BSM) physics:

1. Directly produce new particles in high energy experiments
2. look for tiny deviations from the SM predictions =
   Confront precision measurements with accurate predictions of the SM

Improve the theoretical input to test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix

- The SM has only one-type of charged-current interactions

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^+ \overline{U}_i \gamma^\mu (1-\gamma_5) (V_{\rm CKM})_{ij} D_j + \text{h.c.} , \qquad \begin{array}{c} \overline{U} = (\bar{u}, \bar{c}, \bar{t}) \\ D^T = (d, s, b) \end{array}$$

- Unitarity of $V_{\rm CKM}$ would imply no BSM physics

$$\Delta_{\rm CKM} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

$|V_{ud}|^2$ from neutron decay: $\delta|V_{ud}|^2 \approx (8 \to 3) \times 10^{-4}$

- $|V_{ud}|^2$ from neutron decay is given by the master formula

$$|V_{ud}|^2 = \left(\frac{G_\mu^2 m_e^5}{2\pi^3} f\right)^{-1} \frac{1}{\tau_n(1+3g_A^2)(1+\mathrm{RC})} = \frac{5099.3(4)\mathrm{s}}{\tau_n(1+3g_A^2)(1+\mathrm{RC})}$$

τ_n: free neutron lifetime
g_A: axial coupling obtained from the neutron β decay (asymmetry parameter A)
G_μ: Fermi constant extracted from muon decays,
$f = 1.6887(1)$ is a phase space factor

- Experimental Proposal: $\delta\tau_n \approx$ 0.1 sec, $\delta A/A \approx 0.1$ % .
- Theory Proposal: reduce uncertainty in RC to the 10% level, ie, by a factor of two

This will test the SM up to scales of 15 TeV, which are inaccessible at the LHC

Radiative correction to $\beta-$decay

- The theoretical error in $|V_{ud}|$, is dominated by the uncertainty in the RC, which is expressed as the sum of three terms

$$\mathrm{RC} = \frac{\alpha_{\mathrm{em}}}{2\pi}\left\{\bar{g}(E_m) + 4\ln\frac{m_Z}{m_p} + \Delta_{\mathrm{np}}\right\}$$

- The first two terms dominate the RC but have very small uncertainties:
  - $\bar{g}(E_m)$, where E_m is the electron endpoint energy, arises from the emission of soft photons, integrated over the allowed phase space
  - $\ln(m_Z/m_p)$ encodes perturbative short-distance γ-Z_{boson} loop effects
  - Together they give 0.036, or about 95% of RC
- $\alpha_{\mathrm{em}}\Delta_{\mathrm{np}}/(2\pi) \sim 0.002$: This non-perturbative long distance effect is comparatively small, but its estimated uncertainty, $\sim 20\%$, dominates the theory error budget.

Lattice QCD is the only known *controlled* method to determine Δ_{np} at the 10% level and to reach $\delta|V_{ud}|^2 \leq 3\times 10^{-4}$