HVP with C* boundary conditions

Part I

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August 10, 2022

Intro / Overview

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- **Part I**
  - **Strategy of calculation**
  - **Sources**
  - **Vector masses**
  - **Introduction to C* boundary conditions**
- Part II: from Anian Altherr (directly following this talk)
  - Implications of C* boundary conditions
  - HVP in dynamical QCD+QED simulations
  - Outlook
- Other related talks/posters
  - Sofie Martins: Finite-Size Effects of the Hadronic Vacuum Polarization Contribution to the Muon ($g-2$) with C* Boundary Conditions (**talk in this session at 09:40 AM**)
  - Jens Lücke: An update on QCD+QED simulations with C* boundary conditions (**talk in session "Hadron Spectroscopy and Interactions" on Fri 4:40 PM**)
  - Paola Tavella: Strange and charm contribution to the HVP from C* boundary conditions (**poster on Tue 8:00 PM**)
  - Alessandro Cotellucci: Tuning of QCD+QED simulations with C* boundary conditions (**poster on Tue 8:00 PM**)

Hadronic vacuum polarization contribution to the muon g-2

Status on calculation of the muon $g-2$

current values of anomalous magnetic moment a_μ and leading order hadronic vacuum polarization a_μ^{HVP}:

	source	value(total error)	relative error
a_μ	exp [1]	$11\,659\,206.1(4.1) \times 10^{-10}$	0.54 ppm
a_μ	theory [2]	$11\,659\,181.0(4.3) \times 10^{-10}$	0.36 ppm
a_μ^{HVP}	R-ratio [2]	$693.1(4.0) \times 10^{-10}$	0.6%
a_μ^{HVP}	lattice [2]	$711.6(18.4) \times 10^{-10}$	2.5%
a_μ^{HVP}	lattice [3]	$707.5(5.5) \times 10^{-10}$	0.8%

need to include isospin breaking effects: $\frac{m_u - m_d}{\Lambda_{\mathrm{QCD}}} \sim 0.01$, $\alpha = \frac{1}{137} \sim 0.01$

[1] Fermilab Muon $g-2$ [1, 2]
[2] Muon $g-2$ Theory Initiative [3]
[3] BMW [4]

g-2: Strategy of calculation

- full dynamical simulation of QCD + QED
- Wilson clover fermions (unoptimized c_{SW} term)
- C* boundary conditions in three spatial directions
- periodic boundary conditions in time

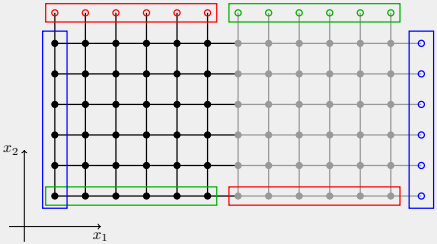

Figure: C* boundary conditions

g-2: Strategy of calculation

- time-momentum representation with kernel function $\tilde{K}(t; m_\mu)$ and current $j_k(t, \vec{x})$.
- conserved-local current
- use single-exponential fit where noise dominates signal

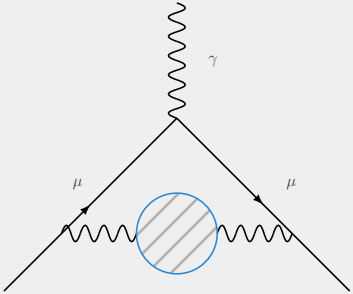

Figure: Hadronic vacuum polarization

$$a_\mu \sim \int_0^\infty \mathrm{d}t \underbrace{\int_{\mathbb{R}^3} \mathrm{d}^3x \left\langle j_k^{ps}(t, \vec{x}) j_k^{loc}(0, \vec{0}) \right\rangle}_{G_k(t)} \tilde{K}(t; m_\mu)$$

m_μ is the muon mass, k is a spatial index $1, 2, 3$, for details see Bernecker et al. [5] and Della Morte et al. [6]

Used ensembles / parameters

Ensemble	A400	A360	A380
flavors	3(u/d/s) + 1(c)	1(u) + 2(d/s) + 1(c)	1(u) + 2(d/s) +1(c)
β	3.24	3.24	3.24
α	0.0	0.04063(6)	0.00708(2) $\approx 1/137$
m_π[MeV]	399(3)	359(3)	380
a [fm]	0.05393(24)	0.05054(27)	0.05323(28)
size	32x32x32x64	32x32x32x64	32x32x32x64
#configs	200	181	200

Sources

Sources

Correlator of interest:

$$G_k(x_0) \sim \frac{1}{|\Omega|} \sum_{y \in \Omega} \sum_{\vec{x} \in \Lambda_3(\vec{y})} \mathrm{tr}_{CD} \left[\gamma^5 \gamma_k D^{-1}(x_0 + y_0, \vec{x}|y) \gamma^5 \gamma_k D^{-1}(x_0 + y_0, \vec{x}|y)^\dagger \right]$$

$$G_k(x_0) \sim e^{-E_0 x_0} \left(1 + \mathcal{O}(e^{-\Delta E x_0})\right)$$

where [1] [2]

- Ω set of source points ($|\Omega| \approx 10$)
- $\Lambda_3(\vec{y})$ set of spatial lattice points symmetric around $\vec{y}$ [3]
- $k = 1, 2, 3$, spatial Lorentz-indices
- $D^{-1}(x|y)$ inverse Dirac-operator, i.e. propagator from lattice point y to x

[1] we define the **effective mass** as $m_{eff} = E_0 (= a m_{phys})$
[2] ΔE is the energy difference to the first excited state
[3] See talk by Sofie Martins

Sources

The propagator $D^{-1}(x|y)$ needs inversions of the Dirac operator using source fields located at source point y:

1. Point sources
   - a point source η at point (y, β, b) obeys $\eta(x)_{\alpha a} = \delta_{xy}\delta_{\alpha\beta}\delta_{ab}$
   - **12 inversions per point source** y
2. Spin-diluted stochastic wall sources
   - located at a time-slice y_0
   - diluted in the 4 spin-indices $\implies$ **4 inversions per wall source**
   - $(\mathbb{Z}_2 + i\mathbb{Z}_2)$-noise
   - gives an average over spatial lattice: $\Omega = \Lambda_3$
3. Smeared sources
   - Give better overlap with ground state $\implies$ for mass extraction
   - HVP requires unsmeared sources

Charm: Point sources ↔ spin-diluted wall sources

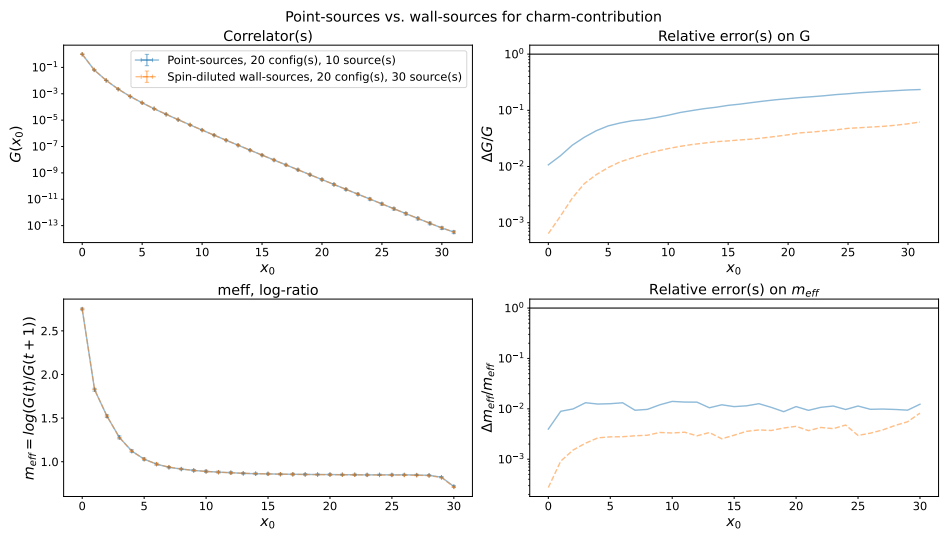

Figure: Low-statistics study: Sources for **charm**-contribution ($q_c = 2/3$). $G(x_0)$ is the vector-vector correlator integrated over the spatial lattice. The effective mass m_{eff} is fitted in a certain range, where the correlator G exhibits single-exponential behavior.

Up: Point sources ↔ spin-diluted wall sources

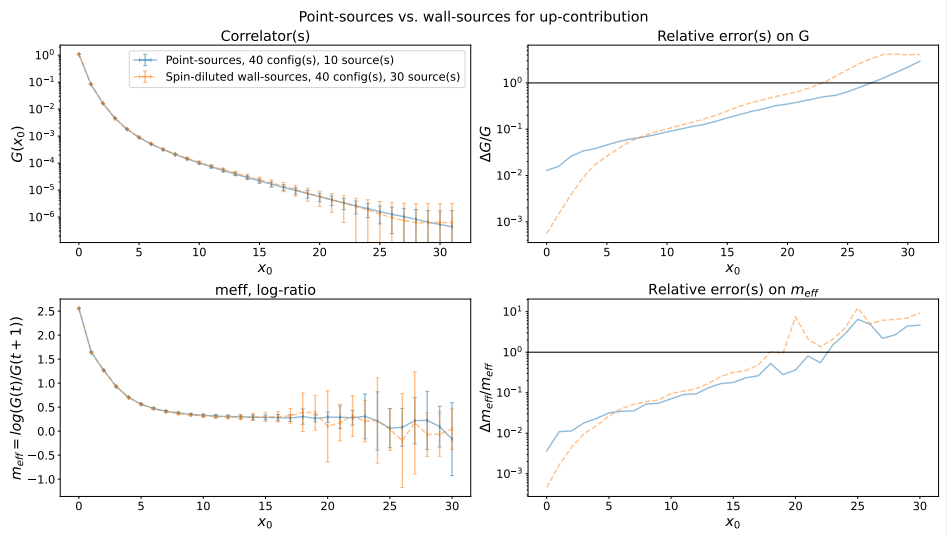

Figure: Low-statistics study: Sources for **up**-contribution ($q_u = 2/3$)

Down/strange: Point sources ↔ spin-diluted wall sources

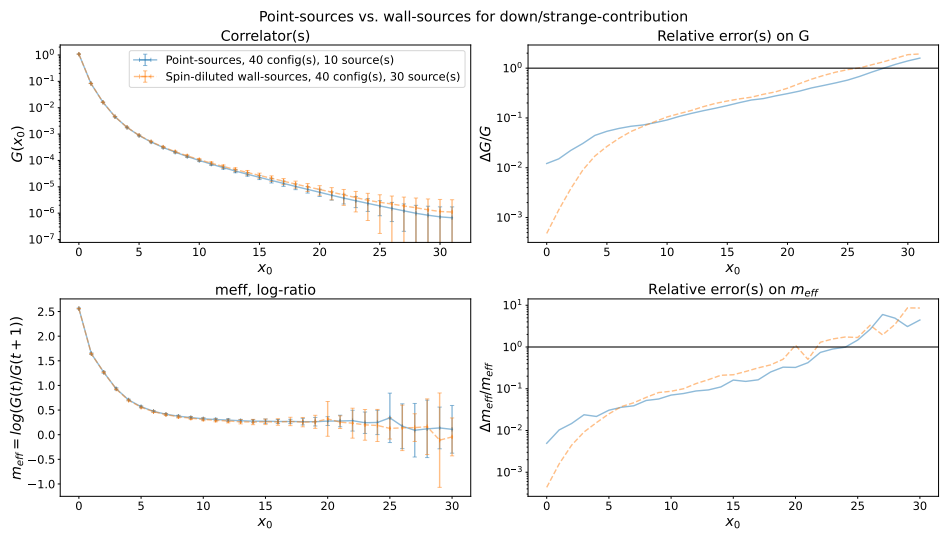

Figure: Low-statistics study: Sources for **down/strange**-contribution ($q_{d/s} = -1/3$)

Point sources $\leftrightarrow$ spin-diluted wall sources

Different measurement setup for different quark flavours:

- **heavy quarks** $\longrightarrow$ spin-diluted stochastic wall-sources (semwall)
- **light quarks** $\longrightarrow$ point-sources
- **mass spectra** $\longrightarrow$ smeared sources (see next slide)

Ensemble (α, m_π)	flavor	#sources	source type	#configs
A400 (0.0, 399 MeV)	u/d/s	10	point	200
	c	30	semwall	200
A360 (0.04, 359 MeV)	u	10	point	181
	d/s	10	point	181
	c	30	semwall	181
A380 ($\approx$ 1/137, 380 MeV)	u	10	point	200
	d/s	10	point	200
	c	30	semwall	200

Vector Masses

- Ground state masses extracted on a subset of configs using smeared sources
  - Gaussian smearing in sink and source quark fields
  - gradient-flow smearing for the gauge fields (preserves unitarity)
- This mass is an input parameter for the HVP determination

ensemble (α)	quark	m_{eff}[4]
A400 (0.0)	u/d/s	0.278(7)
A360 (0.04)	d/s	0.262(7)
	u	0.267(8)
A380 ($\approx 1/137$)	d/s	0.265(6)
	u	0.266(4)

- No mass spectroscopy for the charm

[4] Errors estimated using jackknife procedure, a dedicated study of autocorrelations to follow.

Introduction to C* boundary conditions

C* BOUNDARY CONDITIONS

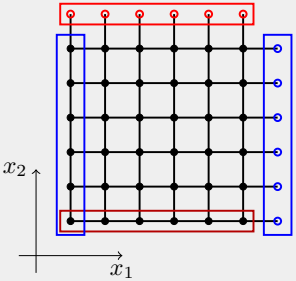

Figure: Lattice with periodic boundary conditions

C* BOUNDARY CONDITIONS

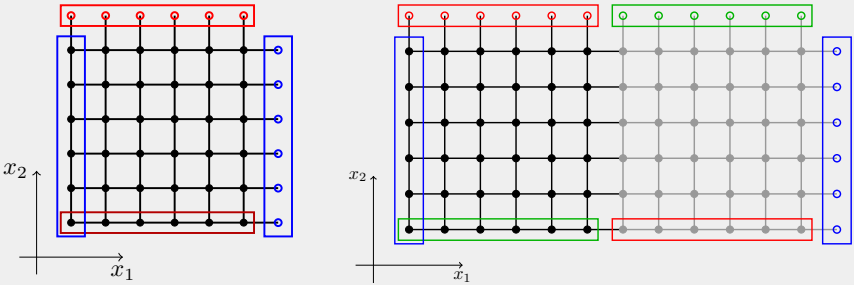

Figure: Lattice with periodic boundary conditions (left), with C* boundary conditions in direction x_1 (right)

- physical lattice: $x_1 < L_1$
- mirror lattice: $L_1 \leq x_1 \leq 2L_1$

C* BOUNDARY CONDITIONS

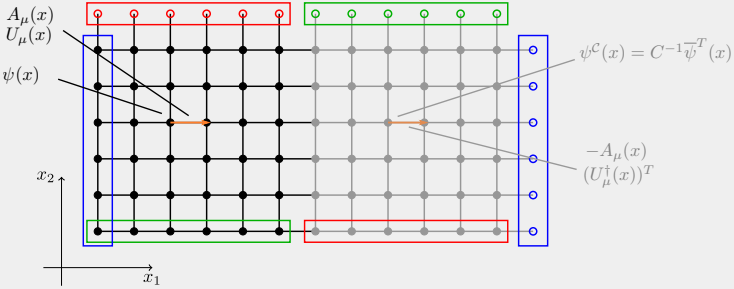

Figure: Fermions $\psi(x)$, QCD links $U_\mu(x)$, QED photon field $A_\mu(x)$.

- periodic boundaries on extended lattice, i.e. $\psi(x + 2L_1\hat{1}) = \psi(x)$

C* BOUNDARY CONDITIONS

- + $A_\mu(x + L_1\hat{x}_1) = -A_\mu(x)$ (C-odd) $\implies$ allowed momenta: $\frac{\pi}{L}(2\mathbb{Z}+1)$, zero modes eliminated by construction $\rightarrow$ no need for quenching zero modes (like in e.g: QED_L).
- + propagation of electrically charged states can be simulated on the lattice from first principles
- + suppressed finite volume QCD effects.[5]
- - lattice volume doubled by introducing a mirror lattice (**32**x32x32x64 $\rightarrow$ **64**x32x32x64)
- - flavour mixing $\rightarrow$ exponentially suppressed by volume [6]
- - global U(1) broken down to $\mathbb{Z}_2$ $\rightarrow$ charge conservation is partially violated

[5] See part II
[6] See Lucini et al., JHEP 2016 [7].

TO BE CONTINUED

Thank you for listening!
Please stay tuned for part II

References I

[1] G. W. Bennett et al. (Muon g-2 Collaboration), "Final report of the E821 muon anomalous magnetic moment measurement at BNL", Phys. Rev. D **73**, 072003 (2006),

[2] B. Abi et al. (Muon $g-2$ Collaboration), "Measurement of the positive muon anomalous magnetic moment to 0.46 ppm", Phys. Rev. Lett. **126**, 141801 (2021),

[3] G. Colangelo et al., "Prospects for precise predictions of a_μ in the Standard Model", arXiv:2203.15810.

[4] S. Borsanyi et al., "Leading hadronic contribution to the muon magnetic moment from lattice QCD", Nature 2021 593:7857 **593**, 51–55 (2021), arXiv:2002.12347,

[5] D. Bernecker and H. B. Meyer, "Vector correlators in lattice qcd: methods and applications", The European Physical Journal A 2011 47:11 **47**, 1–16 (2011),

[6] M. Della Morte et al., "The hadronic vacuum polarization contribution to the muon $g-2$ from lattice QCD", JHEP **2017**, 20 (2017), arXiv:1705.01775,

References II

[7] B. Lucini, A. Patella, A. Ramos, and N. Tantalo, “Charged hadrons in local finite-volume QED+QCD with C^* boundary conditions”, JHEP **02**, 076 (2016), arXiv:1509.01636.

Flavor violation

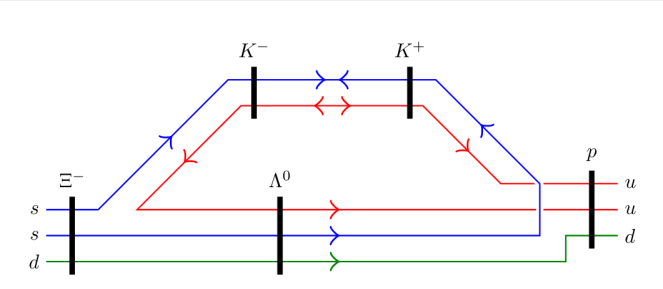

Figure: Ξ^-/p mixing: A $u\bar{u}$ pair is created, a $K^- = s\bar{u}$ travels around the torus and becomes $K^+ = \bar{s}u$, a $s\bar{s}$ pair annihilates. Compare Lucini et al. [7].

Charge violation

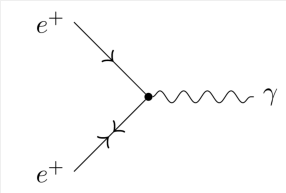

Figure: An e^+ travels around the torus and becomes e^-. Compare Lucini et al. [7].