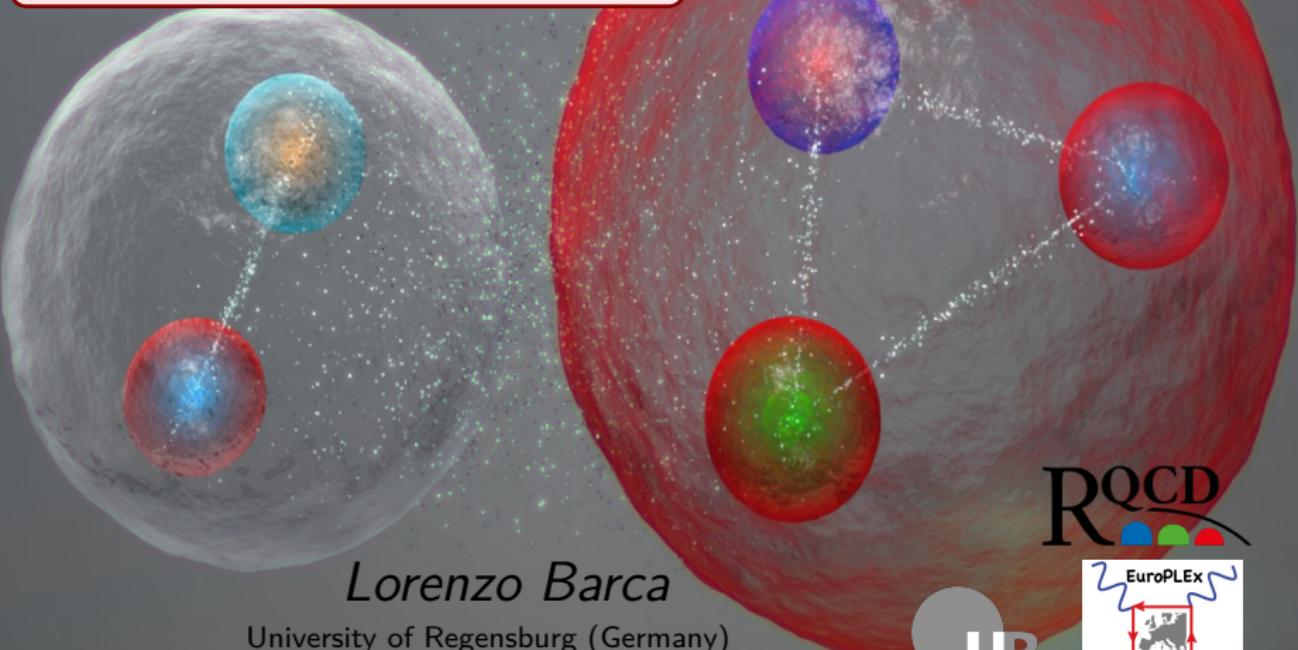


*Removing the  $N\pi$  contamination  
from axial matrix elements  
at non-vanishing momentum*

*The 39th International Symposium  
on Lattice Field Theory,  
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RQCD

UR



Nucleon form factors  $G_A$ ,  $G_P$ ,  $G_{\tilde{P}}$  parameterize the nucleon matrix elements (NME) and can be computed non-perturbatively through LQCD simulations.

$$O_N(x) = \epsilon_{abc} \left[ d_\alpha^a(x) C \gamma_5 u_\beta^b(x) \right] \psi_\gamma^c(x) \quad \mathcal{J}^a(z) = \bar{\psi}(z) \frac{\tau^a}{2} \Gamma \psi(z) \quad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$C_{2pt}(\mathbf{p}', t) = P^+ \langle O_N(\mathbf{p}', t) \bar{O}_N(\mathbf{p}', 0) \rangle \quad P^+, P_i^+ \text{ spin-parity projectors}$$

$$C_{3pt}^{\mathcal{J}, P_i}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_N(\mathbf{p}', t) \mathcal{J}^a(\mathbf{q}, \tau) \bar{O}_N(\mathbf{p}, 0) \rangle \quad \text{Ground state dominance (GSD)}$$

$$R_{P_i^+}^{\mathcal{J}}(\mathbf{p}', t; \mathbf{q}, \tau) = \frac{C_{3pt}^{\mathcal{J}, P_i}(\mathbf{p}', t; \mathbf{q}, \tau)}{C_{2pt}(\mathbf{p}', t)} \sqrt{\frac{C_{2pt}(\mathbf{p}', t) C_{2pt}(\mathbf{p}', \tau) C_{2pt}(\mathbf{p}, t-\tau)}{C_{2pt}(\mathbf{p}, t) C_{2pt}(\mathbf{p}, \tau) C_{2pt}(\mathbf{p}', t-\tau)}} \xrightarrow{0 \gg \tau \gg t} \propto \langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle \quad (\text{NME of current } \mathcal{J})$$

Smearing techniques enhance overlap with physical states

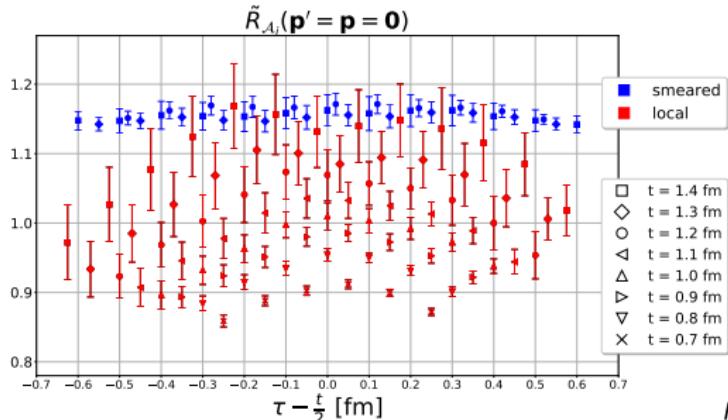
$$\langle N(\mathbf{p}') | \mathcal{A}_\mu^a(\mathbf{q}) | N(\mathbf{p}) \rangle = \bar{u}_N(\mathbf{p}') \left[ \gamma_\mu \gamma_5 G_A(Q^2) - i \frac{q_\mu}{2m} \gamma_5 G_{\tilde{P}}(Q^2) \right] \frac{\tau^a}{2} u_N(\mathbf{p})$$

$$\langle N(\mathbf{p}') | \mathcal{P}^a(\mathbf{q}) | N(\mathbf{p}) \rangle = \bar{u}_N(\mathbf{p}') \gamma_5 G_P(Q^2) \frac{\tau^a}{2} u_N(\mathbf{p}) \quad g_A \equiv G_A(Q^2 = 0) \text{ aka "axial charge"}$$

$G_A$ ,  $G_P$ ,  $G_{\tilde{P}}$  usually extracted from  $R_{P_i}^{\mathcal{A}_i}(q_{j \neq i})$ ,  $R_{P_i}^{\mathcal{P}}(q_i)$ ,  $R_{P_i}^{\mathcal{A}_i/\mathcal{A}_4}(q_i)$  with  $\mathbf{p}' = \mathbf{0}$

# Forward limit ( $Q^2 = 0$ ): the axial charge $g_A$

$m_\pi \approx 420$  MeV,  $a \approx 0.1$  fm



$$\tilde{R}_{A_i}(\mathbf{p}') = \frac{Z_A C_{3pt}^{A_i, P_i}(\mathbf{p}', t; \mathbf{q}=\mathbf{0}, \tau)}{C_{2pt}(\mathbf{p}', t)}$$

$$\longrightarrow g_A + \dots (ESC)$$

← Average with  $\mathbf{p}' = \mathbf{0}$  gives:  
 $g_A = 1.166 \pm 0.013$

Smearing enables GSD at small  $t$

Using smeared operators with  $\mathbf{q} = \mathbf{0}$ :  
 $\mathbf{p}' = \mathbf{p} = \hat{\mathbf{e}}_i = \frac{2\pi}{L} \hat{n}_i$ ,  $|\mathbf{p}'| \approx 525$  MeV

$$\tilde{R}_{A_i}(\mathbf{p}' = \hat{\mathbf{e}}_i) \longrightarrow g_A + \dots$$

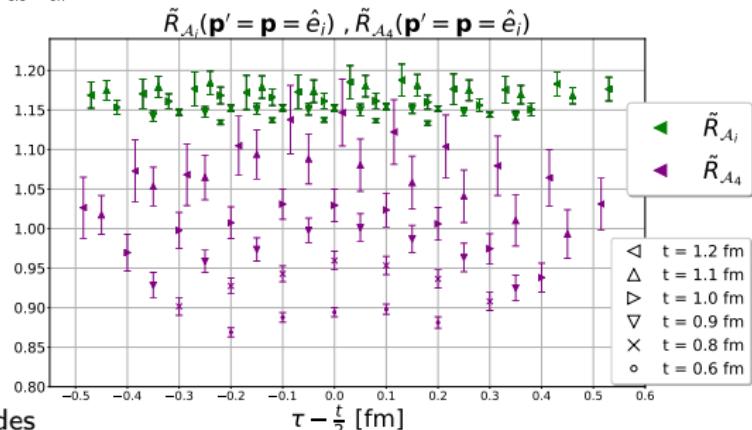
$$\tilde{R}_{A_4}(\mathbf{p}' = \hat{\mathbf{e}}_i) = -\frac{E}{p'} \frac{Z_A C_{3pt}^{A_4, P_i}(\hat{\mathbf{e}}_i, t; \mathbf{q}=\mathbf{0}, \tau)}{C_{2pt}(\hat{\mathbf{e}}_i, t)}$$

$$\longrightarrow g_A + \dots$$

$\tilde{R}_{A_i}(\hat{\mathbf{e}}_i)$  gives consistent results with

$\tilde{R}_{A_i}(\mathbf{0})$ , but larger errors.  $\tilde{R}_{A_4}(\hat{\mathbf{e}}_i)$  provides

10% – 25% smaller results in  $0.6 \text{ fm} \leq t \leq 1.1 \text{ fm}$ , observed also in [arXiv:1612.04388 ( $\chi$ QCD)]



# An unwanted signal in the forward limit

We investigate  $\mathcal{J} = \mathcal{P}$ ,  $\mathbf{q} = \mathbf{0}$ ,  $\mathbf{p}' = \hat{\mathbf{e}}_i$ :

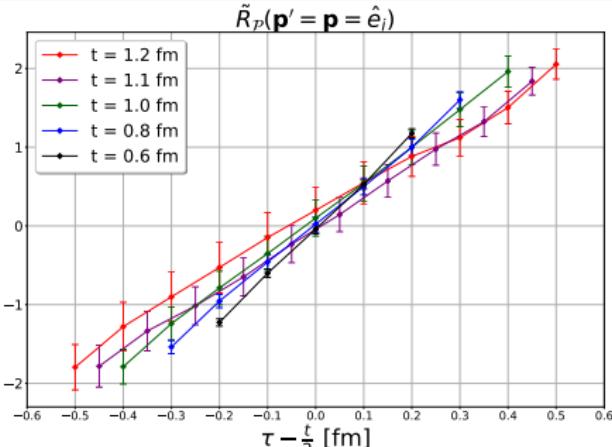
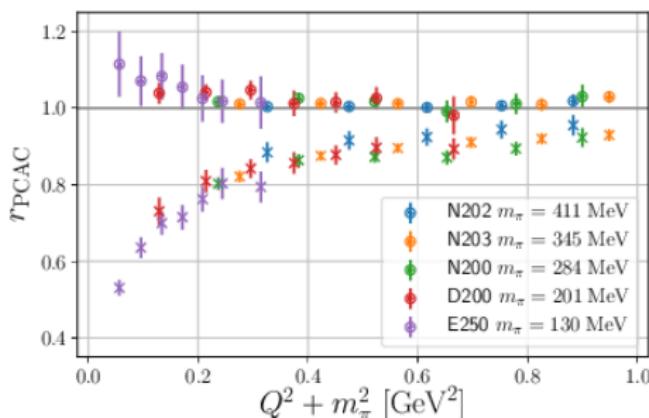
$$\tilde{R}_{\mathcal{P}} \equiv \frac{E}{\mathbf{p}'} \frac{C_{3pt}^{\mathcal{P}, P_i}(\mathbf{p}', t; \mathbf{q}=\mathbf{0}, \tau)}{C_{2pt}(\mathbf{p}', t)} \rightarrow 0 + \dots (\text{ESC})$$

The signal is purely ESC! [RQCD:1911.13150]

In particular,  $N\pi$  correction on 3pts at LO- $\chi$ PT:

$$\delta_{\text{LO-}\chi\text{PT}}^{P_i^+ \mathcal{P}} \propto \frac{E'}{E_\pi} e^{-(E' - m_\pi/2)t} \sinh(m_\pi(\tau - t/2))$$

[1906.03652, 1812.09191 (O. Bär), ...]



At  $Q^2 \neq 0$ ,  $G_A$ ,  $G_P$ ,  $G_{\bar{P}}$  can be extracted and they must satisfy PCAC/GT relation

$$m_N G_A(Q^2) = m_\ell G_P(Q^2) + \frac{Q^2}{4m_N} G_{\bar{P}}(Q^2)$$

that we check with

$$r_{\text{PCAC}} = \frac{4m_N m_\ell G_P(Q^2) + Q^2 G_{\bar{P}}(Q^2)}{4m_N^2 G_A(Q^2)} = 1 ?$$

if  $N\pi$  contamination is not taken into account [1911.13150 (RQCD), 1905.06470 (Los Alamos)]

## Multiparticle operator approach

$|N\pi\rangle$  are often<sup>1</sup> the dominant ESC in the nucleon 3pts with  $\mathcal{J} = \mathcal{A}_\mu, \mathcal{P}$ .

$$\begin{aligned} C_{3pt}^{\mathcal{J}, P_i}(\mathbf{p}', t; \mathbf{q}, \tau) &\approx \frac{e^{-E'_N(t-\tau)} e^{-E_N t}}{4E'_N E_N} P_i^+ \langle \Omega | O_n(\mathbf{p}', t) | N \rangle \langle N | \mathcal{J}^-(\mathbf{q}, \tau) | N \rangle \langle N | \bar{O}_p(\mathbf{p}, 0) | \Omega \rangle \\ &+ \frac{e^{-E'_{N\pi}(t-\tau)} e^{-E_N t}}{4E'_{N\pi} E_N} P_i^+ \langle \Omega | O_n(\mathbf{p}', t) | N\pi \rangle \langle N\pi | \mathcal{J}^-(\mathbf{q}, \tau) | N \rangle \langle N | \bar{O}_p(\mathbf{p}, 0) | \Omega \rangle \\ &+ \frac{e^{-E'_N(t-\tau)} e^{-E_{N\pi} t}}{4E'_N E_{N\pi}} P_i^+ \langle \Omega | O_n(\mathbf{p}', t) | N \rangle \langle N | \mathcal{J}^-(\mathbf{q}, \tau) | N\pi \rangle \langle N\pi | \bar{O}_p(\mathbf{p}, 0) | \Omega \rangle \end{aligned}$$

For this project, we compute  $C_{3pt}^{\mathcal{J}, N\pi}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_{N\pi}(\mathbf{p}', t) \mathcal{J}^-(\mathbf{q}, \tau) \bar{O}_p(\mathbf{p}, 0) \rangle$

$O_{N\pi}$  must be projected<sup>2</sup> to have same quantum numbers as neutron, i.e.

- $O_{N\pi}$  must be projected on  $I = 1/2, I_z = -1/2 \rightarrow O_{N\pi}^{(n)} = \frac{1}{\sqrt{3}} O_{n\pi^0} - \sqrt{\frac{2}{3}} O_{p\pi^-}$  ;
- $O_{N\pi}(\mathbf{p}')$  must be projected on the lattice irreps  $G_1$  ( $S = 1/2, 7/2\dots$ ) ;

$$C_{3pt}^{\mathcal{J}, p\pi^-}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_{p\pi^-}(\mathbf{p}', t) \mathcal{J}^-(\mathbf{q}, \tau) \bar{O}_p(\mathbf{p}, 0) \rangle \propto 12 \text{ Wick contractions}$$

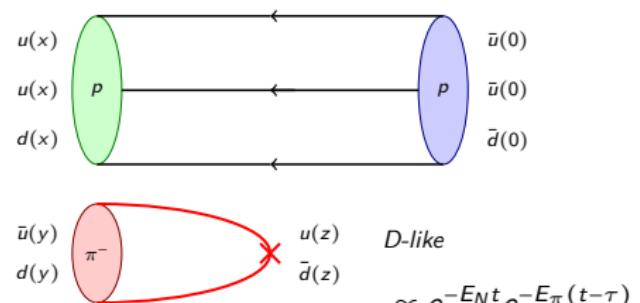
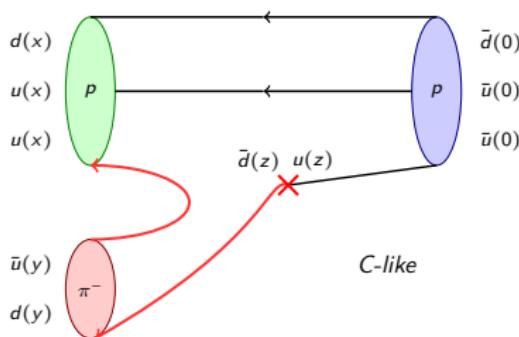
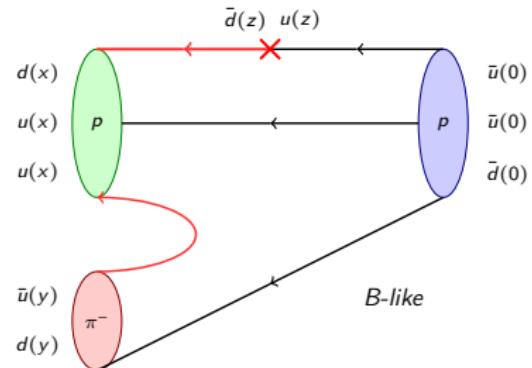
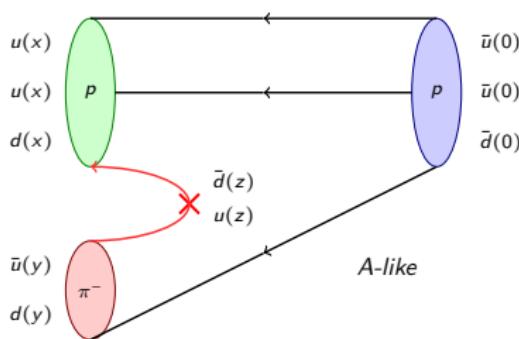
$$C_{3pt}^{\mathcal{J}, n\pi^0}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_{n\pi^0}(\mathbf{p}', t) \mathcal{J}^-(\mathbf{q}, \tau) \bar{O}_p(\mathbf{p}, 0) \rangle \propto 16 \text{ Wick contractions}$$

---

<sup>1</sup>For some ensembles  $N\pi\pi$  S-wave have smaller non-interacting energies than  $N\pi$  P-wave, see arXiv:1812.10574 (J. Green)]

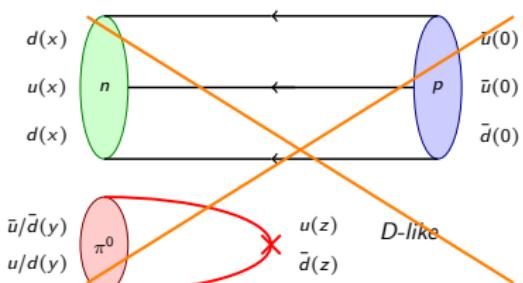
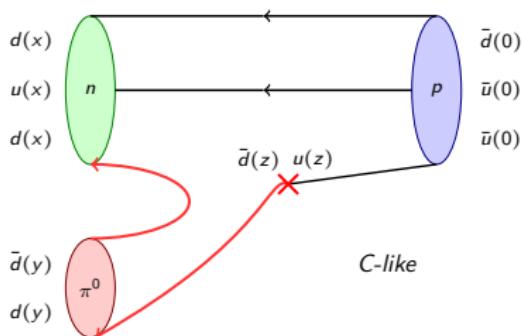
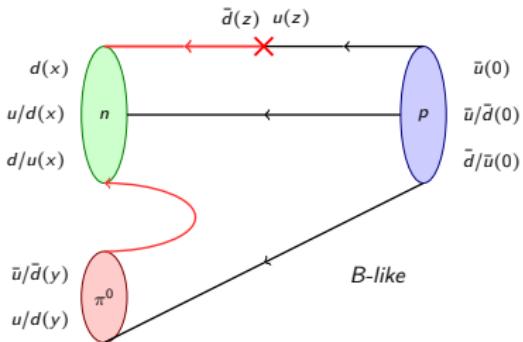
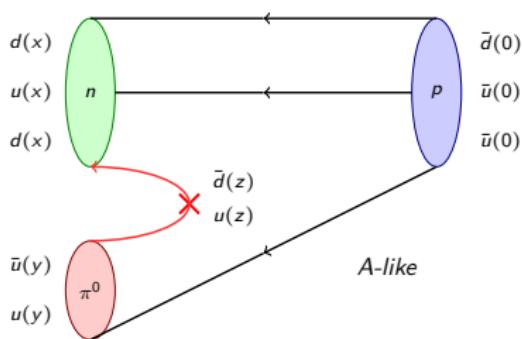
<sup>2</sup> $N\pi$  scattering and more: [arXiv:1610.01422 (C. Lang et al.), 1212.5055 (M. Göckeler et al.), 1607.06738 (S. Prelovsek et al.)]

# Topologies for $p \xrightarrow{\mathcal{I}^-} p + \pi^-$



like our LO- $\chi$ PT prediction! [[arXiv:1911.13150 \(RQCD\)](https://arxiv.org/abs/1911.13150)]

# Topologies for $p \xrightarrow{\mathcal{I}^-} n + \pi^0$



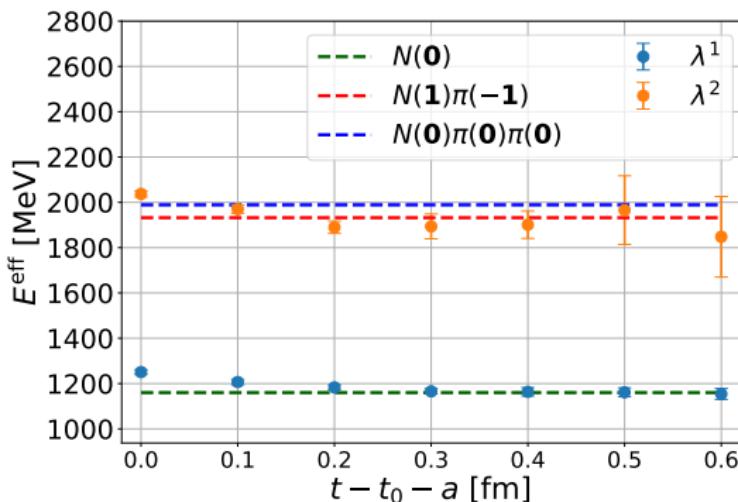
## Solving the GEVP: eigenvalues in the rest frame

We diagonalise them by constructing a matrix of  $C_{ij}^{(2)}(\mathbf{p}, t) = \langle O_i(\mathbf{p}, t) | \bar{O}_j(\mathbf{p}, 0) \rangle$  with  $O_i \in \{O_N, \Phi O_N, O_{N\pi}, \Phi O_{N\pi}\}$  and we solve the GEVP with  $B_1, B_2, B_3, B_4$ :

$$C^{(2)}(t)V(t, t_0) = C^{(2)}(t_0)V(t, t_0)\Lambda(t, t_0)$$

$$\begin{aligned} (B_1 &= \{O_N, \Phi O_N\}, & B_2 &= \{O_N, O_{N\pi}\}, \\ B_3 &= \{\Phi O_N, O_{N\pi}\}, & B_4 &= \{\Phi O_N, \Phi O_{N\pi}\}) \\ \Phi &= \text{Smearing Operator} \end{aligned}$$

$$\Lambda(t, t_0) = \text{diag}(\lambda_1(t, t_0), \lambda_2(t, t_0)), \quad \lambda_n(t, t_0) \approx d_n(t_0)e^{-E_n t}, \quad V(t, t_0) = (v^1(t, t_0), v^2(t, t_0))$$



← Best results presented here are for  $B_4 = \{\Phi O_N, \Phi O_{N\pi}\}$ ,  $t_0 = 2a$ ,  $\mathbf{p} = \mathbf{0}$

$$E_{\lambda_n}^{\text{eff}} = \ln\left(\frac{\lambda_n(t)}{\lambda_n(t+a)}\right)$$

← •  $E_{\lambda_2}^{\text{eff}}$  is very close to  $N\pi$  P-wave (blue dashed line) or  $N\pi\pi$  S-wave (orange dashed line) non-interacting energies

← •  $E_{\lambda_1}^{\text{eff}}$  very close to nucleon mass (green dashed line)

## Solving the GEVP: eigenvectors in the rest frame

$$C^{(2)}(t)V(t, t_0) = C^{(2)}(t_0)V(t, t_0)\Lambda(t, t_0)$$

$$(B_1 = \{O_N, \Phi O_N\}, \quad B_2 = \{O_N, O_{N\pi}\}, \\ B_3 = \{\Phi O_N, O_{N\pi}\}, \quad B_4 = \{\Phi O_N, \Phi O_{N\pi}\})$$

$\Phi$  = Smearing Operator

$$V(t, t_0) = (v^1(t, t_0), v^2(t, t_0))$$

$$\Lambda(t, t_0) = \text{diag}(\lambda_1(t, t_0), \lambda_2(t, t_0)),$$

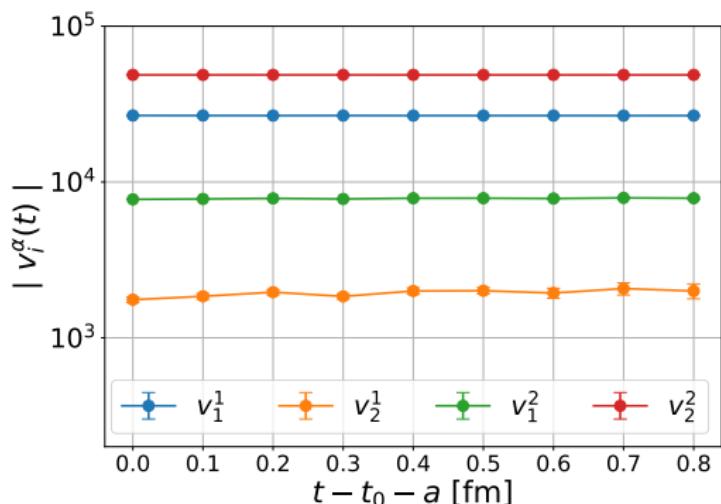
$$\lambda_n(t, t_0) \approx d_n(t_0)e^{-E_n t}$$

Best results presented here →  
 $B_4 = \{\Phi O_N, \Phi O_{N\pi}\}, t_0 = 2a, \mathbf{p} = \mathbf{0}$

Eigenvectors normalised s.t.

$$v_i^\alpha(t; t_0) C_{ij}(t_0) v_j^\beta(t; t_0) = \delta_{\alpha\beta}$$

Eigenvectors pretty constant with  $t$



We solve GEVP also for moving frames  $\mathbf{p} = \pm \hat{\mathbf{e}}_z = \frac{2\pi}{L}(0, 0, \pm 1)$  with

- $O_{N\pi,2}(\mathbf{p}) = O_N(\mathbf{p})O_\pi(\mathbf{0})$       Relevant in the forward limit
- $O_{N\pi,1}(\mathbf{p}) = O_N(\mathbf{0})O_\pi(\mathbf{p})$       Relevant for  $Q^2 \neq 0$

## GEVP-projected correlation functions

What are GEVP results useful for?

[arXiv:1108.3774, 0902.1265 (ALPHA collaboration), ...]

Similarly to  $\langle \Omega | O_N | N \rangle \not\propto \langle \Omega | O_N | N\pi \rangle$ , we also find that  $\langle \Omega | O_{N\pi} | N\pi \rangle \not\propto \langle \Omega | O_{N\pi} | N \rangle$ . We thus diagonalise the system by solving

$$C^{(2)}(\mathbf{p}, t) v^\alpha(\mathbf{p}, t; t_0) = C^{(2)}(\mathbf{p}, t_0) v^\alpha(\mathbf{p}, t; t_0) \lambda^\alpha(\mathbf{p}, t; t_0)$$

Improve construction of correlation functions projected on  $\alpha, \beta = N, N\pi$ :

$$C_{2pt}^\alpha(\mathbf{p}, t) = v_i^\alpha(\mathbf{p}, t; t_0) C_{ij}^{(2)}(\mathbf{p}, t) v_j^\alpha(\mathbf{p}, t; t_0)$$

talk by Ferenc Pittler  
(ETMC), 14:30

$$C_{3pt}^{\alpha\beta}(\mathbf{p}, t, \mathbf{q}, \tau; P_k; \mathcal{J}) = v_i^\alpha(\mathbf{p}', t; t_0) C_{ij}^{(3)}(\mathbf{p}', t, \mathbf{q}, \tau; P_k; \mathcal{J}) v_j^\beta(\mathbf{p}, t; t_0)$$

thus, GEVP-improved extraction of matrix elements:  $(C_{ij}^{(3)}(P_k, \mathcal{J}) = P_k \langle O_i \mathcal{J} \bar{O}_j \rangle)$

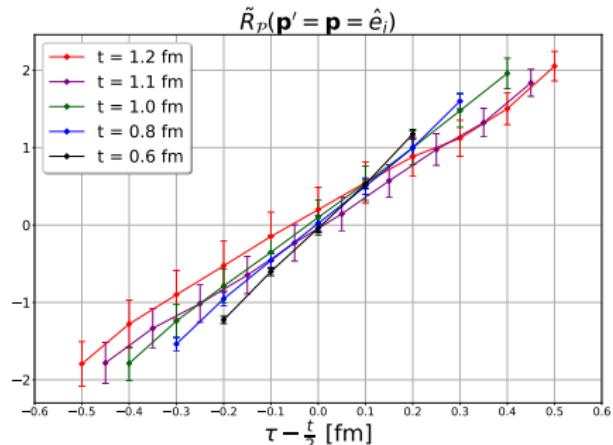
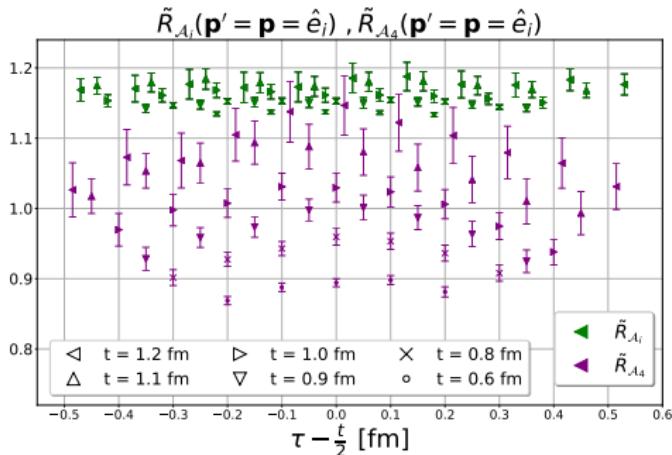
$\langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle, \langle N\pi(\mathbf{p}') | J(\mathbf{q}) | N(\mathbf{p}) \rangle$  (!)  $(\langle O_{N\pi} \mathcal{J} \bar{O}_{N\pi} \rangle \text{ not considered})$

like for instance

$$\left[ \langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle \right] = \frac{C_{3pt}^{NN}(\mathbf{p}', t, \mathbf{q}, \tau; P_k; \mathcal{J})}{C_{2pt}^N(\mathbf{p}', t)} \sqrt{\frac{C_{2pt}^N(\mathbf{p}', \tau) C_{2pt}^N(\mathbf{p}', t) C_{2pt}^N(\mathbf{p}, t-\tau)}{C_{2pt}^N(\mathbf{p}, \tau) C_{2pt}^N(\mathbf{p}, t) C_{2pt}^N(\mathbf{p}', t-\tau)}}$$

## Results in the forward limit

There was a problem in the forward limit:



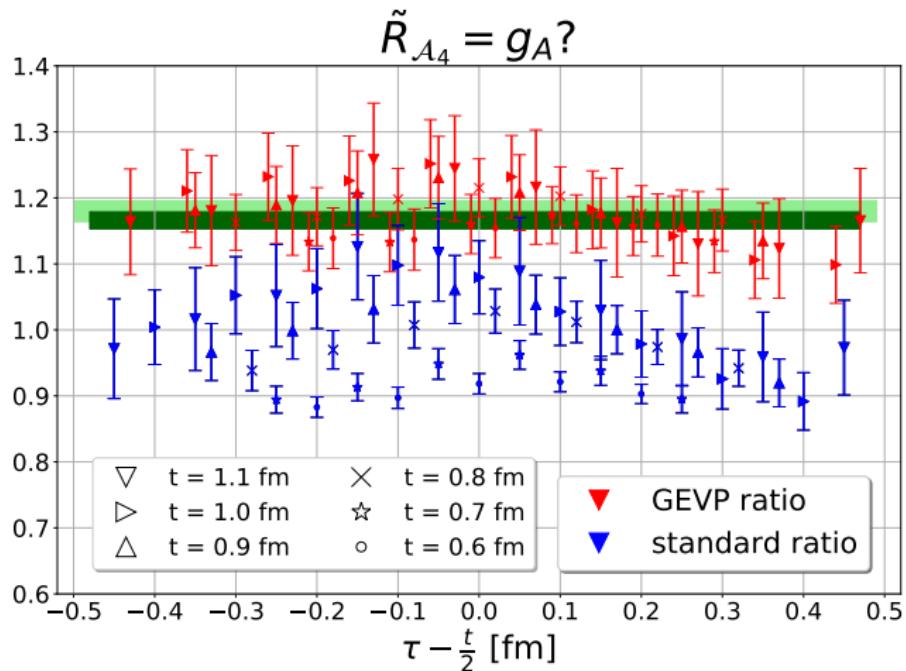
### Results in the forward limit ( $Q^2 = 0 \text{ GeV}^2$ )

Does the GEVP approach provide

- $\left[ \langle N(\hat{\mathbf{e}}_z) | \mathcal{A}_4(\mathbf{0}) | N(\hat{\mathbf{e}}_z) \rangle \right]_{GEVP} = g_A ?$
- $\left[ \langle N(\hat{\mathbf{e}}_z) | \mathcal{P}(\mathbf{0}) | N(\hat{\mathbf{e}}_z) \rangle \right]_{GEVP} = 0 ?$

GEVP-improved ratios:  $\mathcal{J} = \mathcal{A}_4$ ,  $\mathbf{p}' = \mathbf{p} = \hat{\mathbf{e}}_z$  ( $\mathbf{q} = \mathbf{0}$ )

$$\tilde{R}_{\mathcal{A}_4} \equiv \frac{C_{3pt}^{NN}(\hat{\mathbf{e}}_z, t, \mathbf{0}, \tau; P_z; \mathcal{A}_4))}{C_{2pt}^N(\hat{\mathbf{e}}_z, t)} \frac{-E_N}{P_z} = \left[ \langle N(\hat{\mathbf{e}}_z) | \mathcal{A}_4(\mathbf{0}) | N(\hat{\mathbf{e}}_z) \rangle \right] = g_A + ESC$$



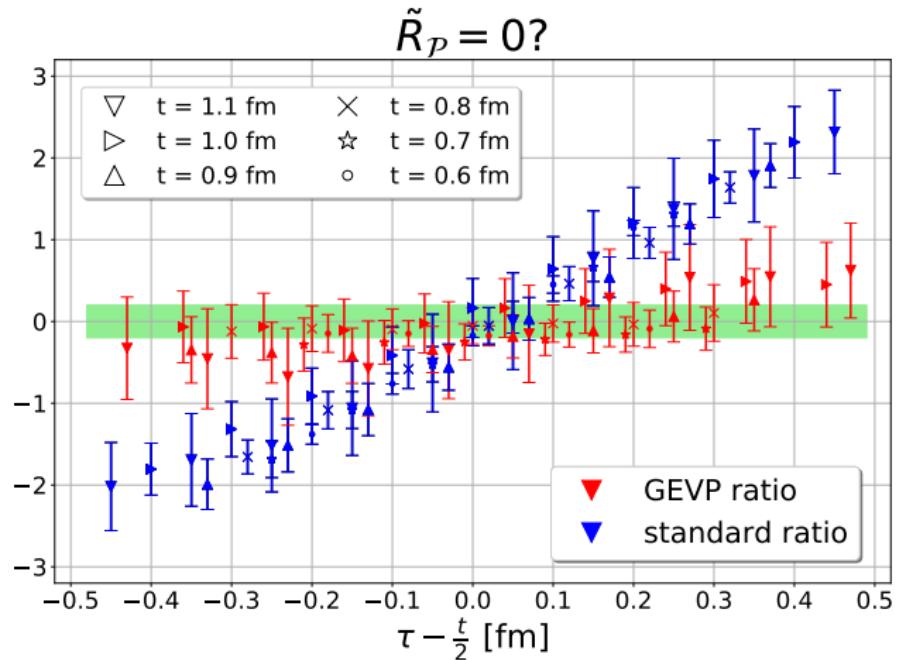
D-like contribution:  $\langle N(\hat{\mathbf{e}}_z)\pi(\mathbf{0}) | \mathcal{A}_4(\mathbf{0}) | N(\hat{\mathbf{e}}_z) \rangle$ ,  $\langle N(\hat{\mathbf{e}}_z) | \mathcal{A}_4(\mathbf{0}) | N(\hat{\mathbf{e}}_z)\pi(\mathbf{0}) \rangle$

## GEVP-improved ratios: $\mathcal{J} = \mathcal{P}$ , $\mathbf{p}' = \mathbf{p} = \hat{\mathbf{e}}_z$ ( $\mathbf{q} = \mathbf{0}$ )

$$\tilde{R}_{\mathcal{P}} \equiv \frac{C_{3pt}^{NN}(\hat{\mathbf{e}}_z, t, \mathbf{0}, \tau; P_z; \mathcal{P}))}{C_{2pt}^N(\hat{\mathbf{e}}_z, t)} \frac{E_N}{P_z} = \left[ \langle N(\hat{\mathbf{e}}_z) | \mathcal{P}(\mathbf{0}) | N(\hat{\mathbf{e}}_z) \rangle \right] = 0 + ESC$$

green band is NGS  
 (Nucleon Ground State)  
 expectation value (zero)

The GEVP-projection  
 removes at each  $t$   
 the  $N\pi$  contamination!



D-like contribution:  $\langle N(\hat{\mathbf{e}}_z) \pi(\mathbf{0}) | \mathcal{P}(\mathbf{0}) | N(\hat{\mathbf{e}}_z) \rangle, \langle N(\hat{\mathbf{e}}_z) | \mathcal{P}(\mathbf{0}) | N(\hat{\mathbf{e}}_z) \pi(\mathbf{0}) \rangle$

## Results with $Q^2 \neq 0$

Results with  $\tilde{Q}^2(\min(p) \neq 0 = \frac{2\pi}{L}) = 0.297 \text{ GeV}^2$

Does the GEVP approach for the extraction of

- $\left[ \langle N(\mathbf{0}) | \mathcal{A}_4(\mathbf{q}) | N(\mathbf{q}) \rangle \right]_{GEVP} \propto G_A(\tilde{Q}^2), G_{\tilde{P}}(\tilde{Q}^2) ;$
- $\left[ \langle N(\mathbf{0}) | \mathcal{A}_z(\mathbf{q}) | N(\mathbf{q}) \rangle \right]_{GEVP} \propto G_A(\tilde{Q}^2), G_{\tilde{P}}(\tilde{Q}^2) ;$
- $\left[ \langle N(\mathbf{0}) | \mathcal{P}(\mathbf{q}) | N(\mathbf{q}) \rangle \right]_{GEVP} \propto G_P(\tilde{Q}^2) .$

provide  $G_A$ ,  $G_{\tilde{P}}$  and  $G_P$  that satisfy PCAC/GT?

## GEVP-improved ratios: $\mathcal{J} = \mathcal{P}$ , $\mathbf{p}' = \mathbf{0}$ ( $\mathbf{q} = -\mathbf{p} = \hat{\mathbf{e}}_z$ )

$$R_{\mathcal{P}} \equiv \frac{C_{3pt}^{NN}(\mathbf{p}'=\mathbf{0}, t, \hat{\mathbf{e}}_z, \tau; P_z; \mathcal{P})}{C_{2pt}^N(\mathbf{0}, t)} \sqrt{\frac{C_{2pt}^N(\mathbf{0}, t) C_{2pt}^N(\mathbf{0}, \tau) C_{2pt}^N(\hat{\mathbf{e}}_z, t-\tau)}{C_{2pt}^N(\hat{\mathbf{e}}_z, t) C_{2pt}^N(\hat{\mathbf{e}}_z, \tau) C_{2pt}^N(\mathbf{0}, t-\tau)}} \propto G_P(\tilde{Q}^2)$$

band is NGS determined a posteriori

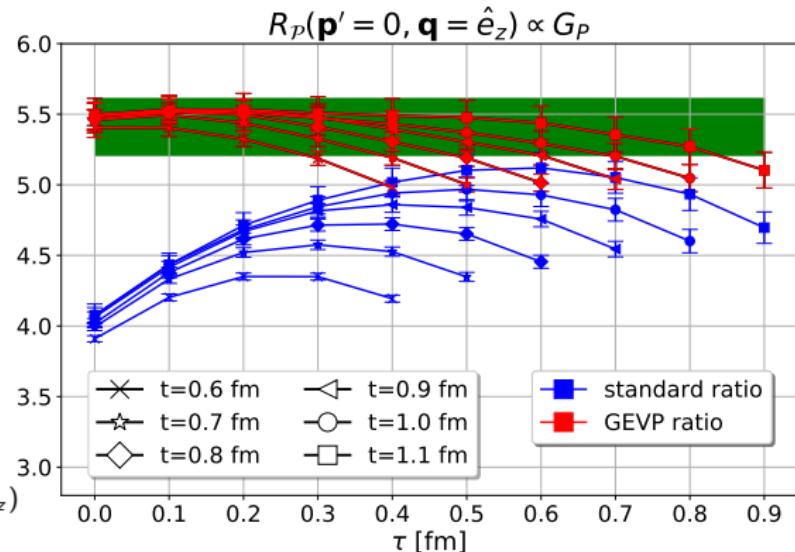
- simultaneous fit to standard ratio with Los Alamos approach gives  $G_P = 25.96 \pm 0.96$
- simultaneous fit to standard ratio with  $\chi$ PT-based ansatz gives  $G_P = 27.11 \pm 0.96$
- fit to GEVP-projected ratio with fit formula  $N(E)G_P + Ze^{-E(t-\tau)}$  gives  $G_P = 26.53 \pm 0.57$   
( $N(E)$  is a known kinematic factor)

The GEVP approach removes most of the contamination at  $\tau = 0.0$  fm ( $\mathbf{p} = \hat{\mathbf{e}}_z$ )

Culprits:

$$O_{N\pi}(-\hat{\mathbf{e}}_z) = O_N(\mathbf{0})O_\pi(-\hat{\mathbf{e}}_z) \quad (\tau = 0 \text{ fm})$$

$$O_{N\pi}(\mathbf{0}) = O_N(-\hat{\mathbf{e}}_z)O_\pi(\hat{\mathbf{e}}_z) \quad (\tau = t)$$

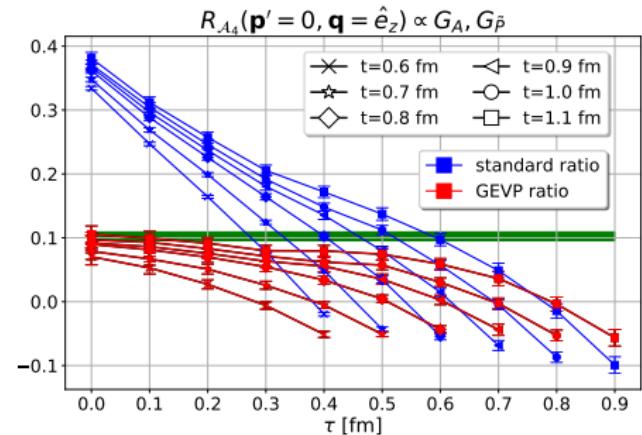
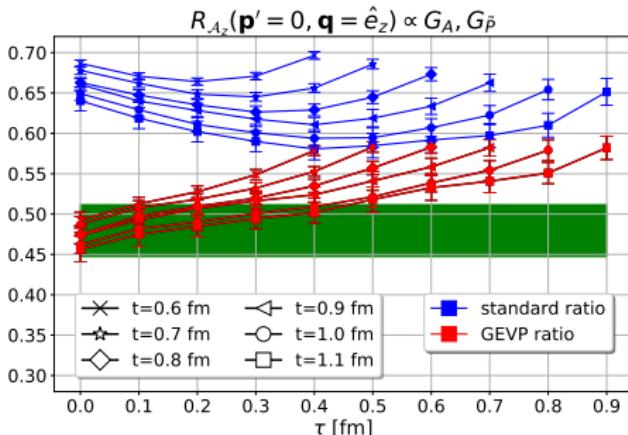


Consistent with other studies and  $\chi$ PT prediction  
[arXiv:1911.13150 (RQCD), 1905.06470 (Los Alamos)]

D-like contribution:  $\langle N(-\hat{\mathbf{e}}_z)\pi(\hat{\mathbf{e}}_z)|\mathcal{P}(\hat{\mathbf{e}}_z)|N(-\hat{\mathbf{e}}_z) \rangle, \langle N(\mathbf{0})|\mathcal{P}(\hat{\mathbf{e}}_z)|N(\mathbf{0})\pi(-\hat{\mathbf{e}}_z) \rangle$

## GEVP-improved ratios: $\mathcal{J} = \mathcal{A}_z, \mathcal{A}_4$ , $\mathbf{p}' = \mathbf{0}$ ( $\mathbf{q} = -\mathbf{p} = \hat{\mathbf{e}}_z$ )

$$R_{\mathcal{A}_z(\mathcal{A}_4)} \equiv \frac{C_{3pt}^{NN}(\mathbf{p}'=\mathbf{0}, t, \hat{\mathbf{e}}_z, \tau; P_z; \mathcal{A}_z(\mathcal{A}_4))}{C_{2pt}^N(\mathbf{0}, t)} \sqrt{\frac{C_{2pt}^N(\mathbf{0}, t) C_{2pt}^N(\mathbf{0}, \tau) C_{2pt}^N(\hat{\mathbf{e}}_z, t-\tau)}{C_{2pt}^N(\hat{\mathbf{e}}_z, t) C_{2pt}^N(\hat{\mathbf{e}}_z, \tau) C_{2pt}^N(\mathbf{0}, t-\tau)}} \propto G_A, G_{\bar{P}}$$



The theoretical NGS contribution to the double ratio  $R_{\mathcal{A}_z}/R_{\mathcal{A}_4} = \frac{E_N + m_N}{e_z} \approx 4.7$ .

$G_A$  extracted from  $R_{\mathcal{A}_i}^{P_i}(q_{j \neq i})$  and  $G_P, G_{\bar{P}}$  extracted from GEVP-improved ratio satisfy PCAC and PPD relation up to  $\mathcal{O}(a^2)$  effects, similarly as  $\chi$ PT-based ansatz and multiparticle state fit. Some trace of contamination left at the sink where  $\mathbf{p}' = \mathbf{0}$  suggests the fit formula  $G_X \pm |Z| e^{-\delta E(t-\tau)}$ .

## Summary & Conclusions

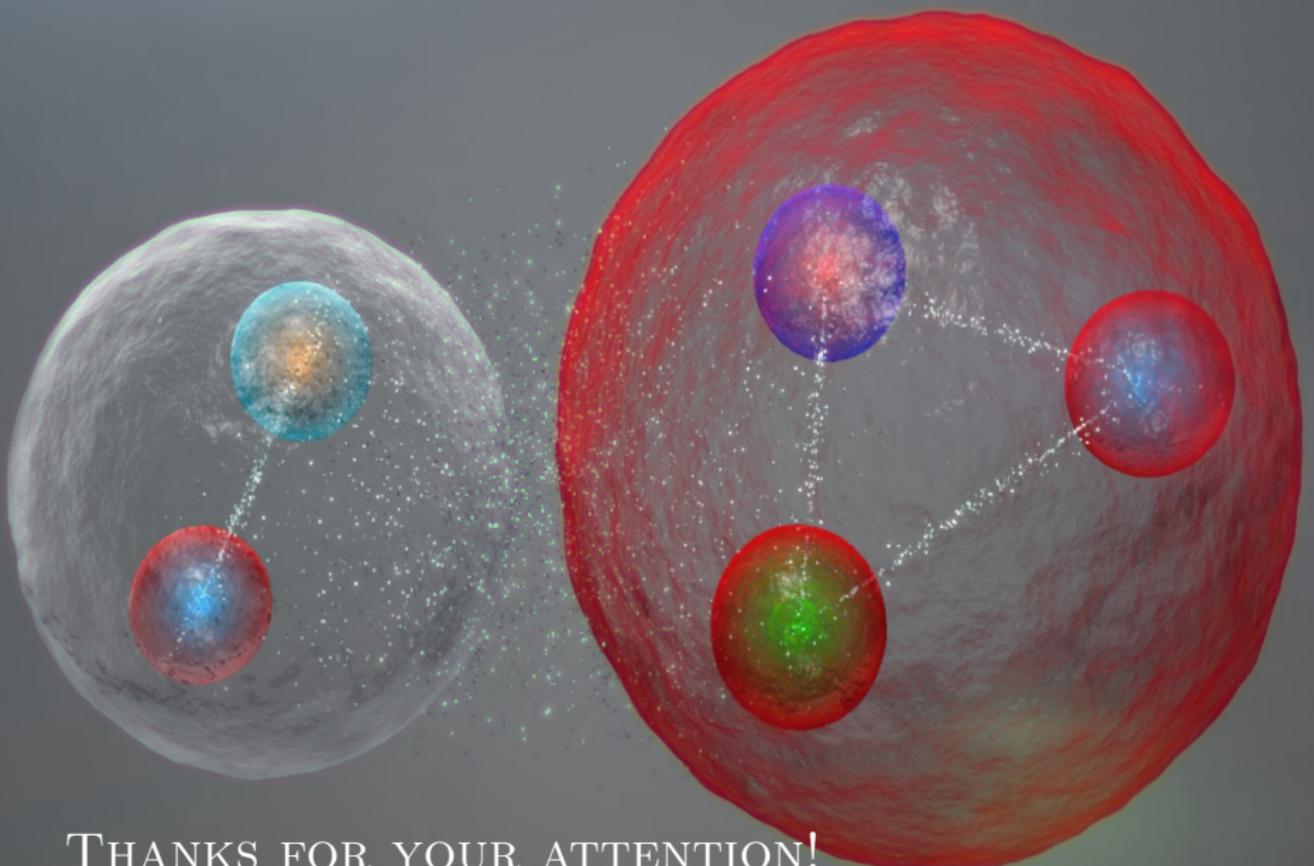
- Data-driven and  $\chi$ PT-based studies agree that  $N\pi$  contamination must be taken into account to extract reliable nucleon form factors;
- Multiparticle operator approach confirms these statements and GEVP improvement provides consistent results at  $Q^2 = 0 \text{ GeV}^2$  and  $Q^2 = 0.297 \text{ GeV}^2$ ;
- Our approach provides insights for new finite-volume matrix elements  $\langle N\pi(\mathbf{p}')|\mathcal{J}(\mathbf{q})|N(\mathbf{p})\rangle$ , which are relevant for neutrino oscillation experiments;
- With this approach we learned that the disconnected D-like diagram is the major source of  $N\pi$  contamination. This term follows the exponential behaviour predicted by  $\chi$ PT and it inherits the signal from  $N$ -2pts and  $\pi\mathcal{J}$ -2pts. Thus it is sensitive to  $\langle 0|\mathcal{J}(\mathbf{q})|\pi\rangle$ ;
- We will study  $\langle N\pi(\mathbf{p}')|\mathcal{J}(\mathbf{q})|N(\mathbf{p})\rangle$  and results for GEVP-improved matrix elements will appear (hopefully) soon in two separate publications;
- What is the remaining contamination?

(Aaron Meyer's  
plenary talk  
Wed 9:20)



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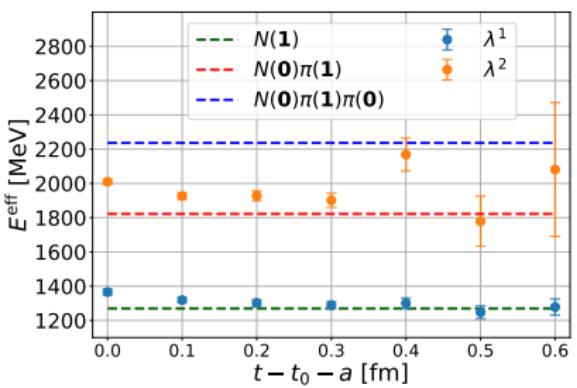
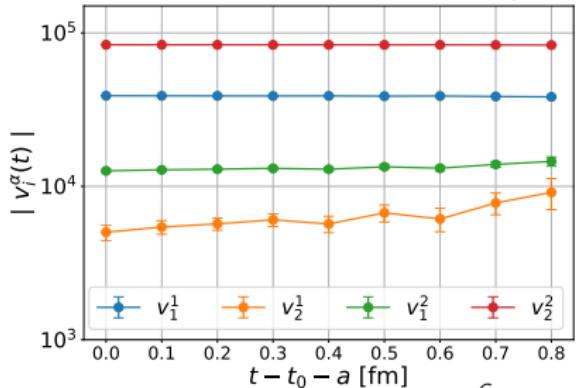


THANKS FOR YOUR ATTENTION!

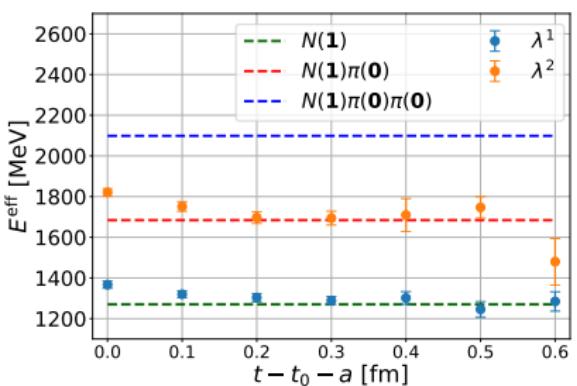
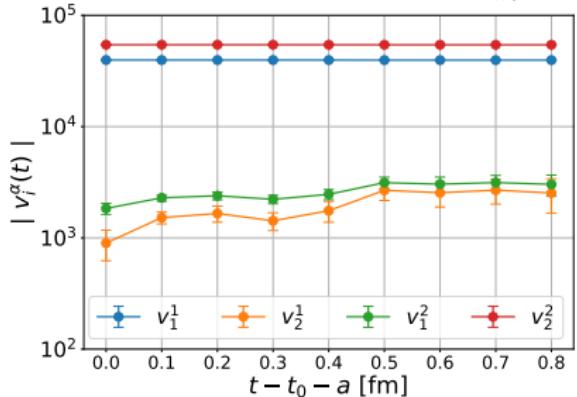
## BACKUP SLIDES

GEVP in moving frame ( $\mathbf{p} = \hat{\mathbf{e}}_z = \frac{2\pi}{L}\hat{n}_z$ ) for  $B_4 = \{\Phi O_N, \Phi O_{N\pi}\}$ ,  $t_0 = 2a$

$$O_{N\pi,1}^{G_1}(\hat{\mathbf{e}}_z) = O_N(\mathbf{0})O_\pi(\hat{\mathbf{e}}_z)$$



$$O_{N\pi,2}^{G_1}(\hat{\mathbf{e}}_z) = O_N(\hat{\mathbf{e}}_z)O_\pi(\mathbf{0})$$



# Projection and Wick contractions

see also [arXiv: 1610.01422 (C. Lang et al.), 1212.5055 (M. Göckeler et al.),  
1607.06738 (S. Prelovsek et al.)]

## Irrep Projection

$$O_{N\pi}(\mathbf{p}) \xrightarrow{\text{Irrep projection}} O_{N\pi}^{G_1, m_s}(\mathbf{p}) = \sum_{S_i \in G} T_{m_s m_s}^{G_1} T(S_i) O_N(\mathbf{p}_i) O_\pi(\mathbf{p}_i) T(S_i)^{-1}$$

Rest frame  $\mathbf{p} = \mathbf{0}$  ( $G = {}^2O_h$ )

$$\begin{aligned} O_{N\pi}^{G_1, \uparrow/\downarrow}(\mathbf{0}) = & + O_N^{\downarrow/\uparrow}(\mathbf{e}_x) O_\pi(-\mathbf{e}_x) - O_N^{\downarrow/\uparrow}(-\mathbf{e}_x) O_\pi(\mathbf{e}_x) + i O_N^{\downarrow/\uparrow}(\mathbf{e}_y) O_\pi(-\mathbf{e}_y) \\ & - i O_N^{\downarrow/\uparrow}(-\mathbf{e}_y) O_\pi(\mathbf{e}_y) + O_N^{\uparrow/\downarrow}(\mathbf{e}_z) O_\pi(-\mathbf{e}_z) - O_N^{\uparrow/\downarrow}(-\mathbf{e}_z) O_\pi(\mathbf{e}_z) \end{aligned}$$

Moving frame  $\mathbf{p} = \hat{\mathbf{e}}_i$  ( $G = {}^2C_{4v}$ )

$$O_{N\pi,1}^{G_1, \uparrow/\downarrow}(\hat{\mathbf{e}}_i) = O_N^{\downarrow/\uparrow}(\mathbf{0}) O_\pi(\hat{\mathbf{e}}_i), \quad O_{N\pi,2}^{G_1, \uparrow/\downarrow}(\hat{\mathbf{e}}_i) = O_N^{\downarrow/\uparrow}(\hat{\mathbf{e}}_i) O_\pi(\mathbf{0})$$

## Isospin Projection

$$O_{N\pi} \xrightarrow{\text{Isospin projection}} O_{N\pi}^{(n)} = \frac{1}{\sqrt{3}} O_{n\pi^0} - \sqrt{\frac{2}{3}} O_{p\pi^-}$$

$$C_{3pt}^{\mathcal{J}, p\pi^-}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_{p\pi^-}(\mathbf{p}', t) \mathcal{J}^-(\mathbf{q}, \tau) \bar{O}_p(\mathbf{p}, 0) \rangle \propto 12 \text{ Wick contractions}$$

$$C_{3pt}^{\mathcal{J}, n\pi^0}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_{n\pi^0}(\mathbf{p}', t) \mathcal{J}^-(\mathbf{q}, \tau) \bar{O}_p(\mathbf{p}, 0) \rangle \propto 16 \text{ Wick contractions}$$