Swiss National Science Foundation

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Centro Svizzero di Calcolo Scientifico
Swiss National Supercomputing Centre

Pion pole contribution to HLbL at the physical point

Gurtej Kanwar (University of Bern)
In collaboration with Sebastian Burri, Marcus Petschlies, Urs Wenger + ETMC

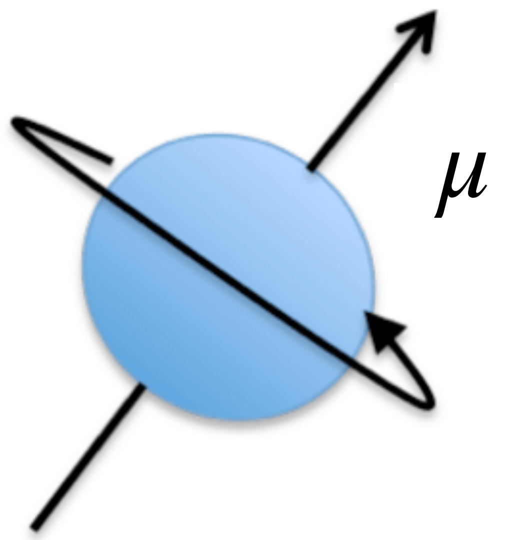
Extended Twisted Mass Collaboration

u^b

b UNIVERSITÄT BERN

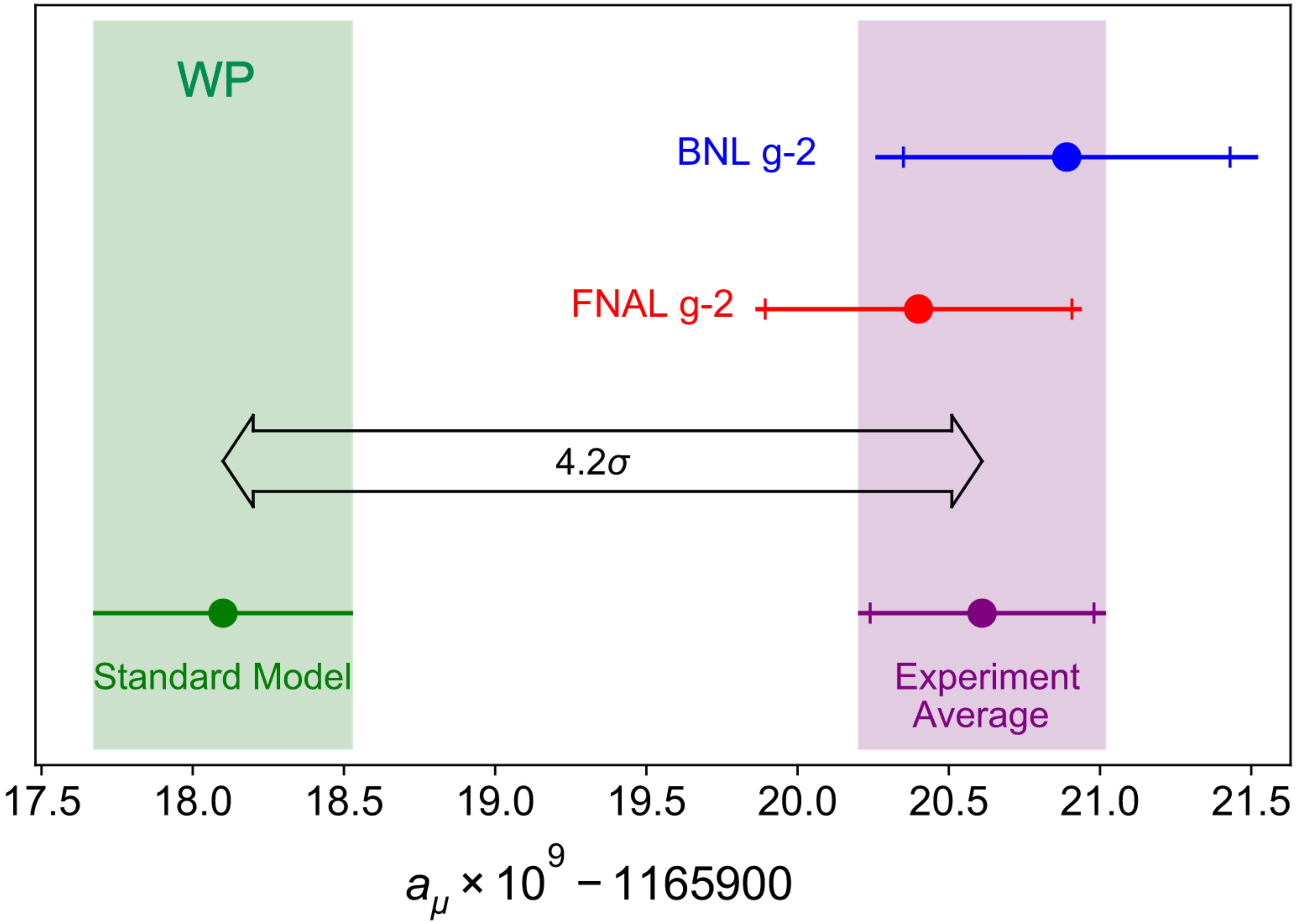
Lattice 2022 (Bonn, Germany)
Aug 11, 2022

Tensions in a_μ



Combined 4.2σ tension

$$\Delta a_\mu = 279(76) \times 10^{-11}$$



[g-2 WP, Phys. Rep. 887 (2020)]

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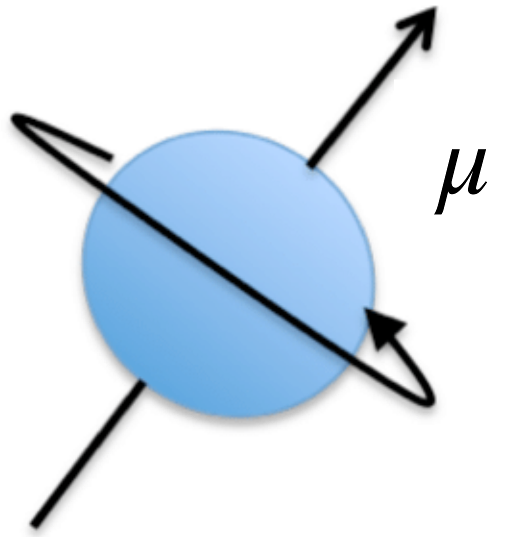

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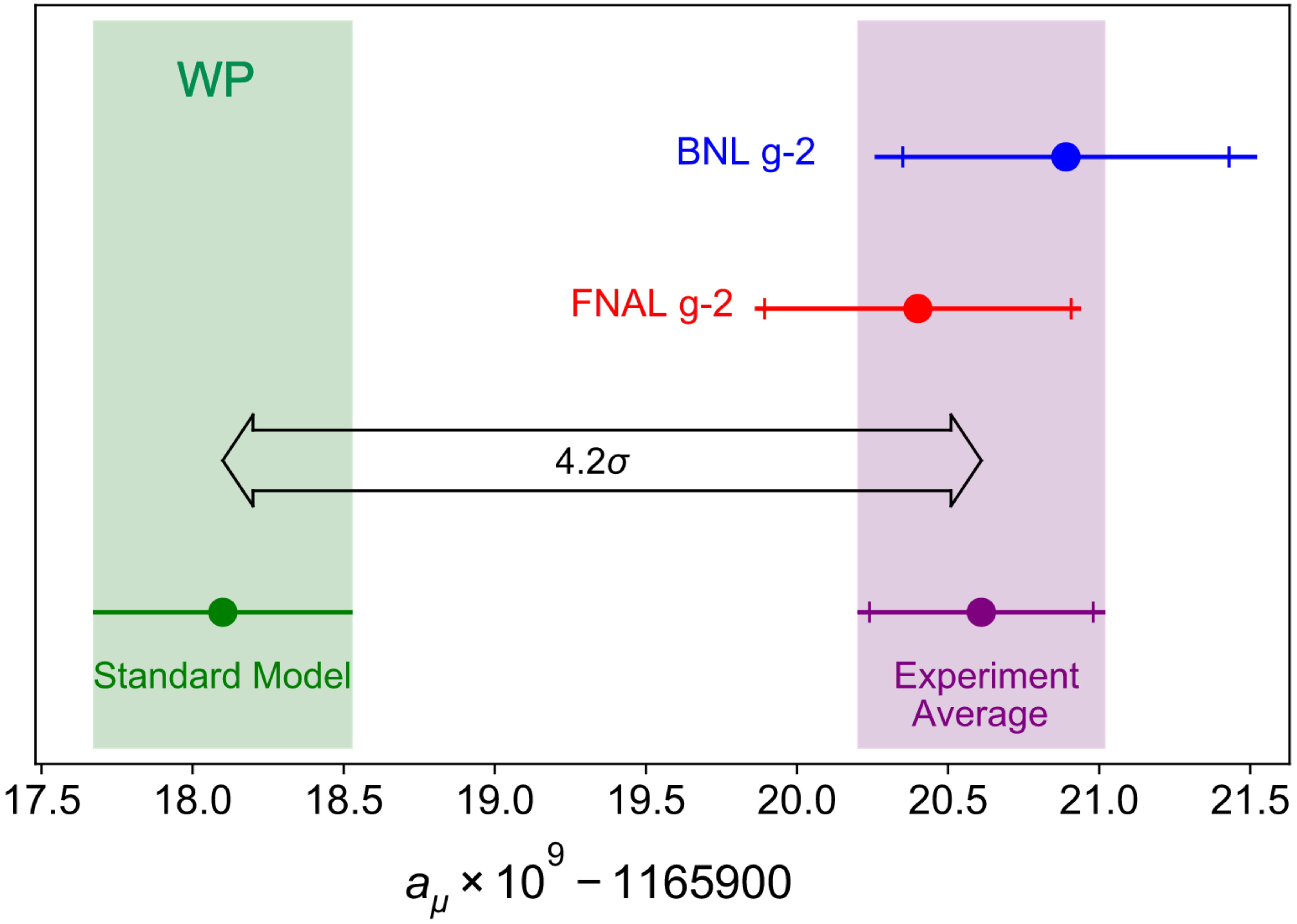
Expt: BNL E821 + Fermilab E989

- Current err ~ 60×10^{-11}
- Expected err ~ 15×10^{-11}



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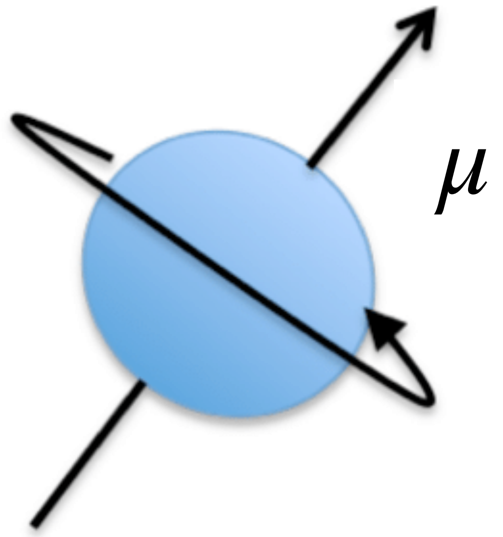

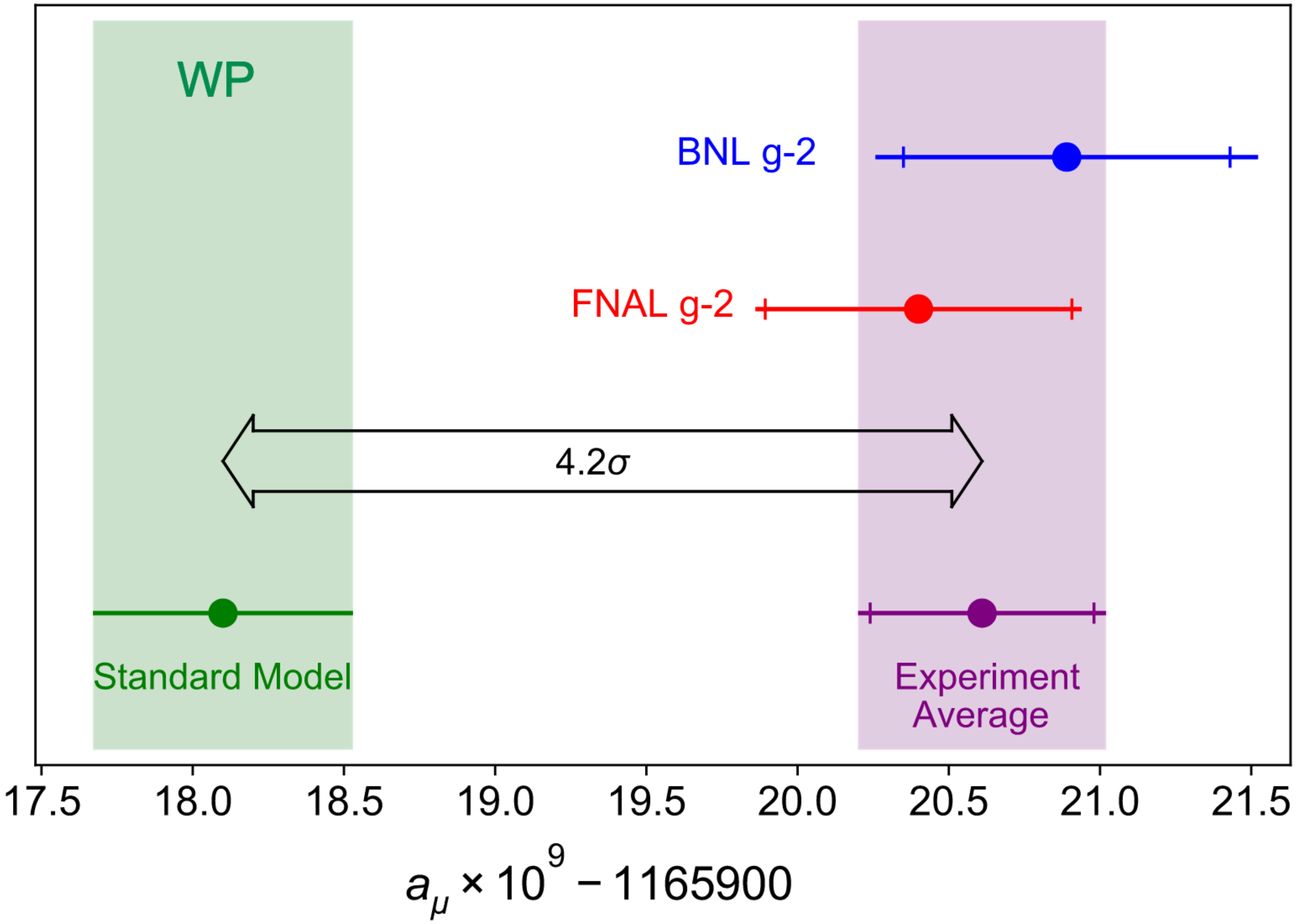

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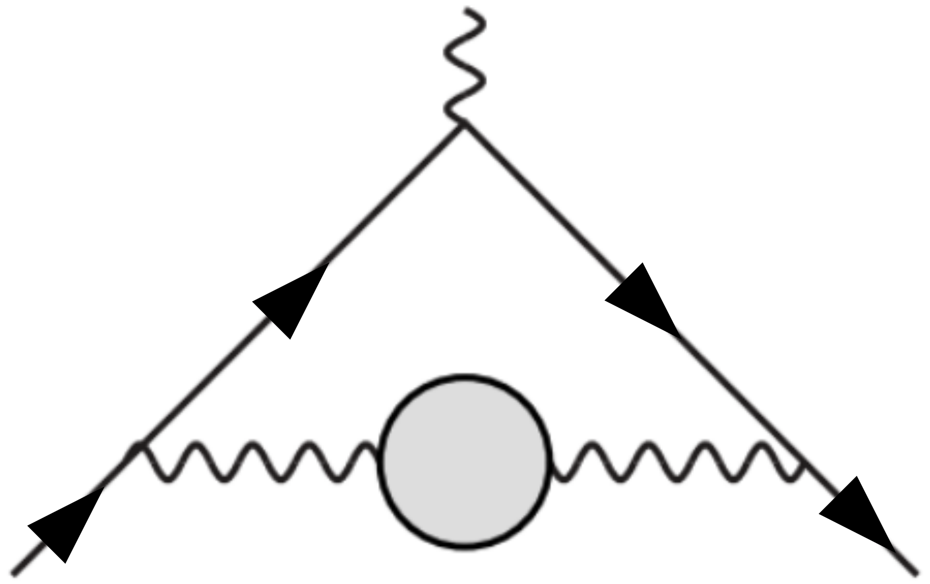
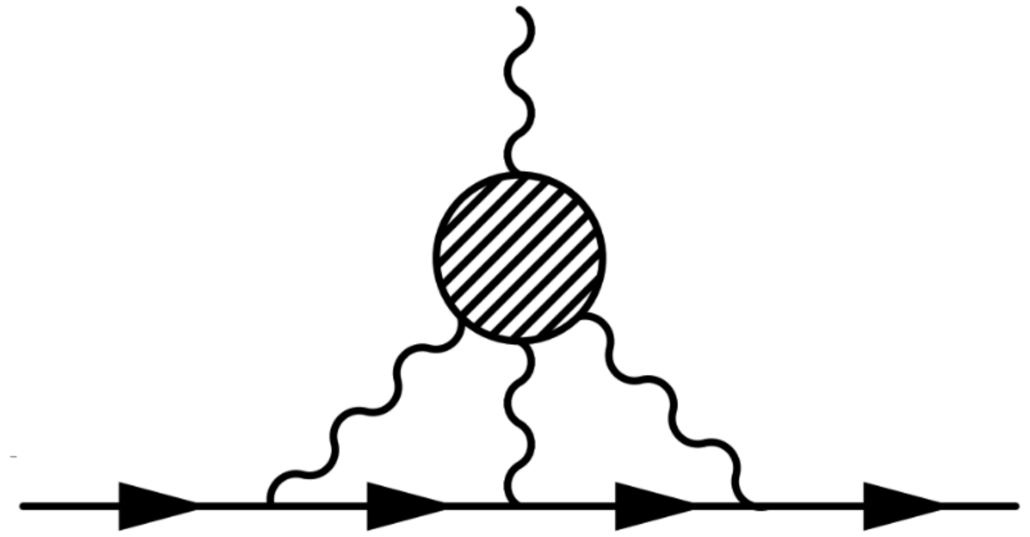
- HVP err ~ 40×10^{-11}
  HLbL err ~ 20×10^{-11}





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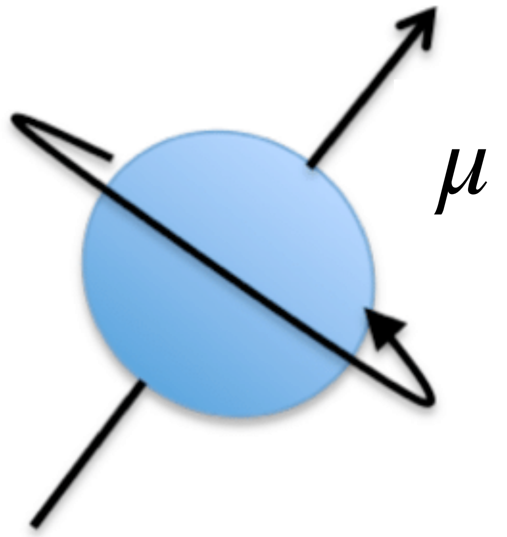

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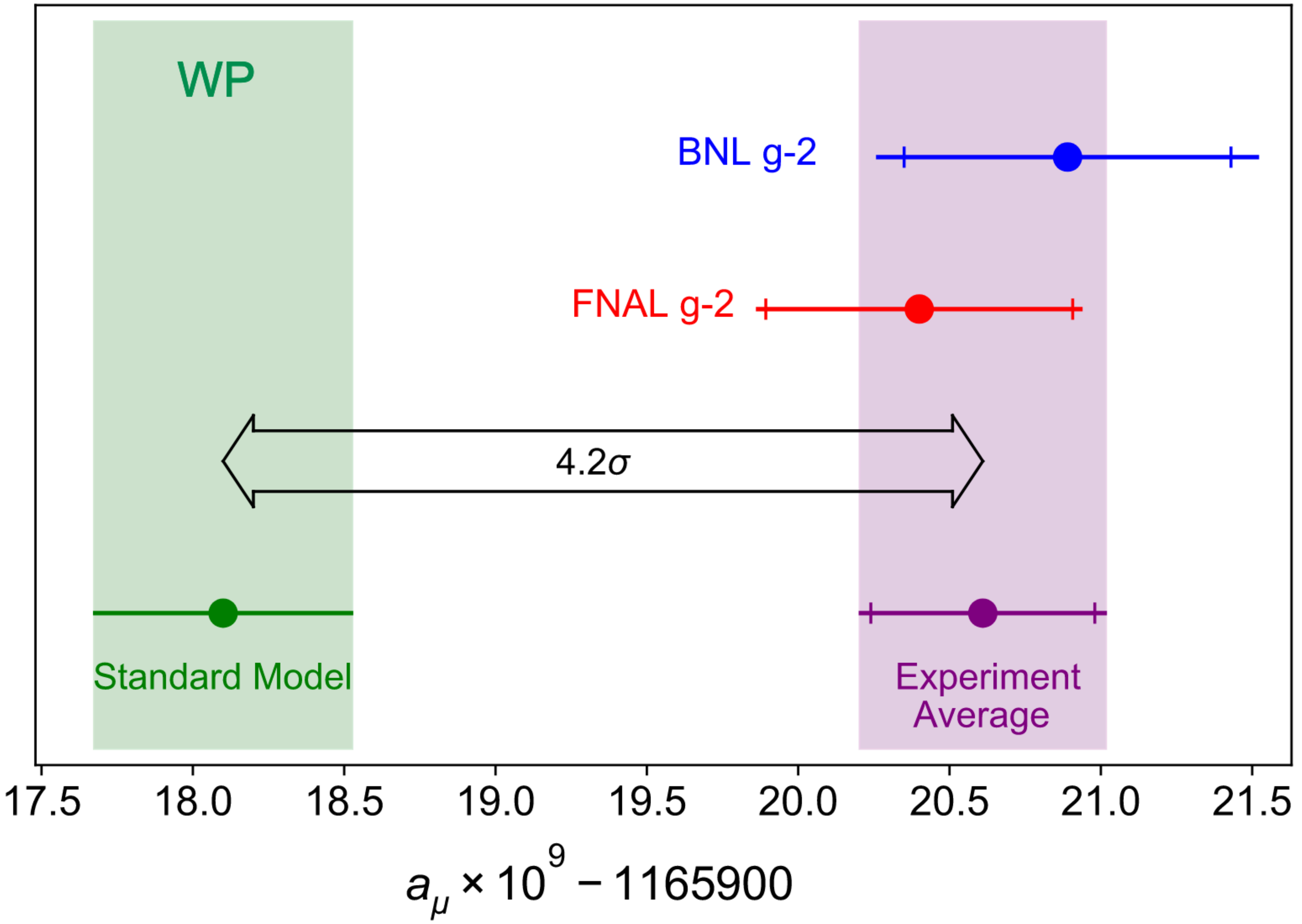
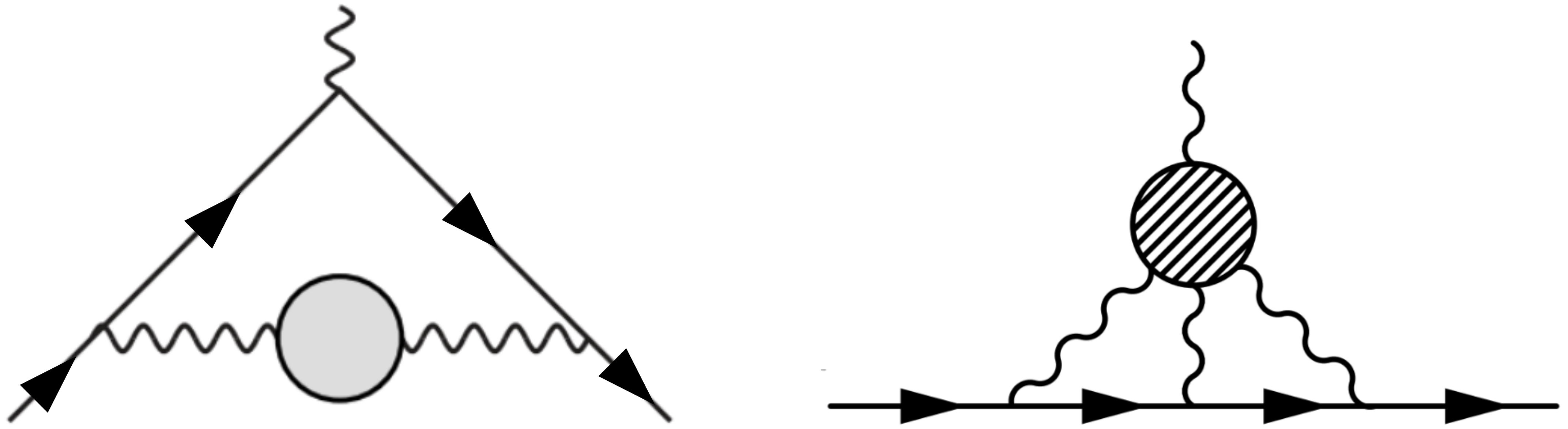
At the order of E989 projected err

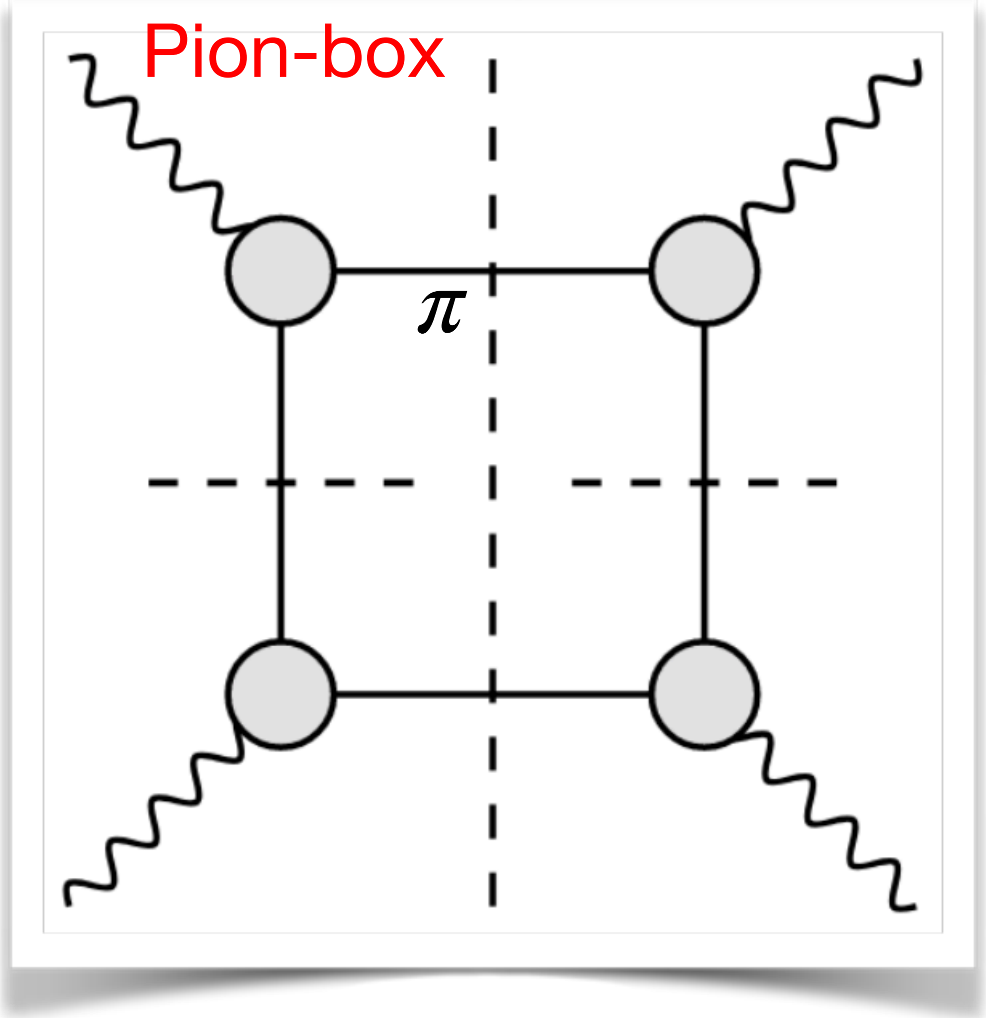




[g-2 WP, Phys. Rep. 887 (2020)]

[Colangelo+ JHEP 1409, 091 (2014)]
[Colangelo+ PLB 738, 6 (2014)]
[Colangelo+ JHEP 1509, 074 (2015)]

HLbL decomposition



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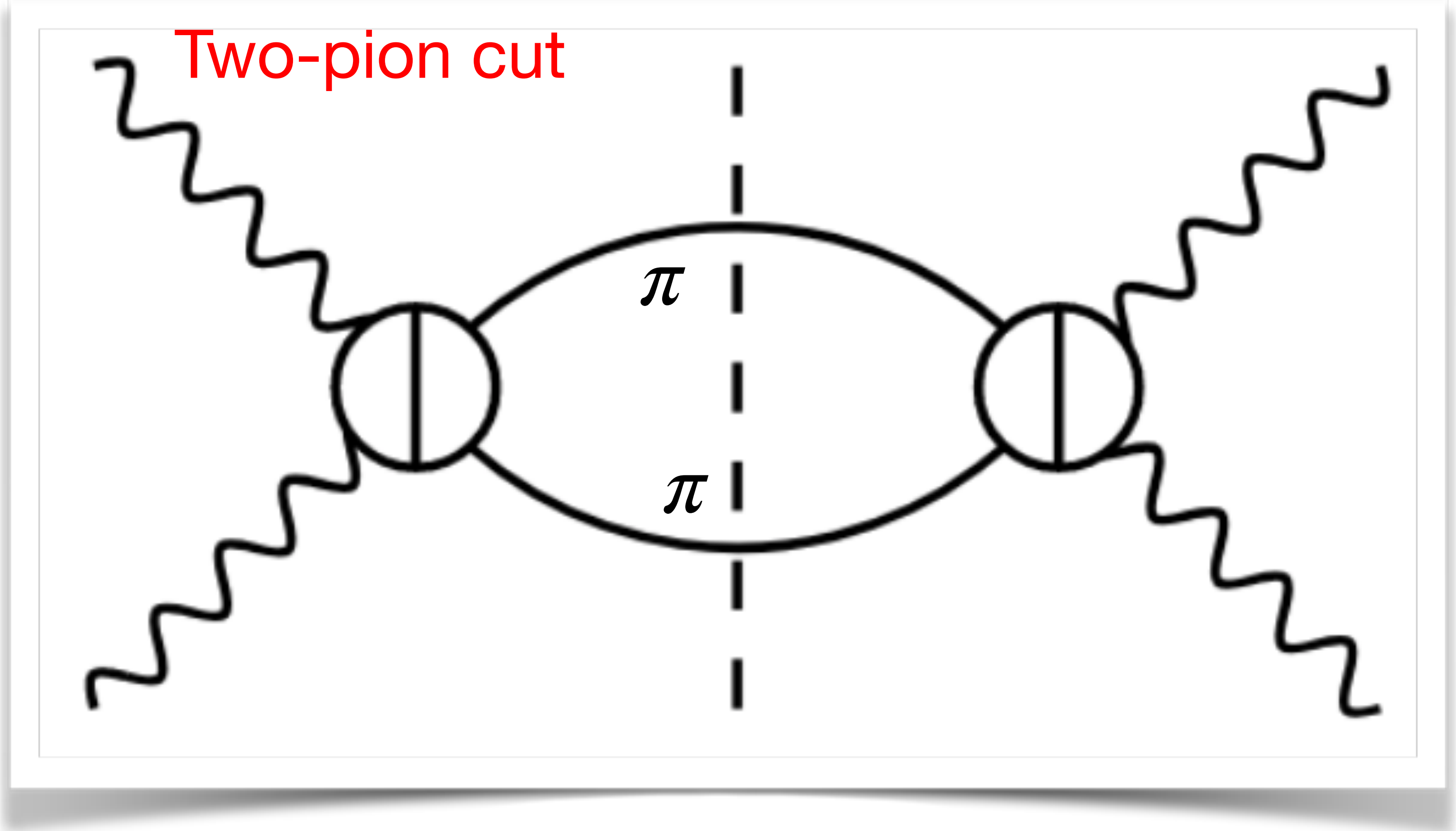

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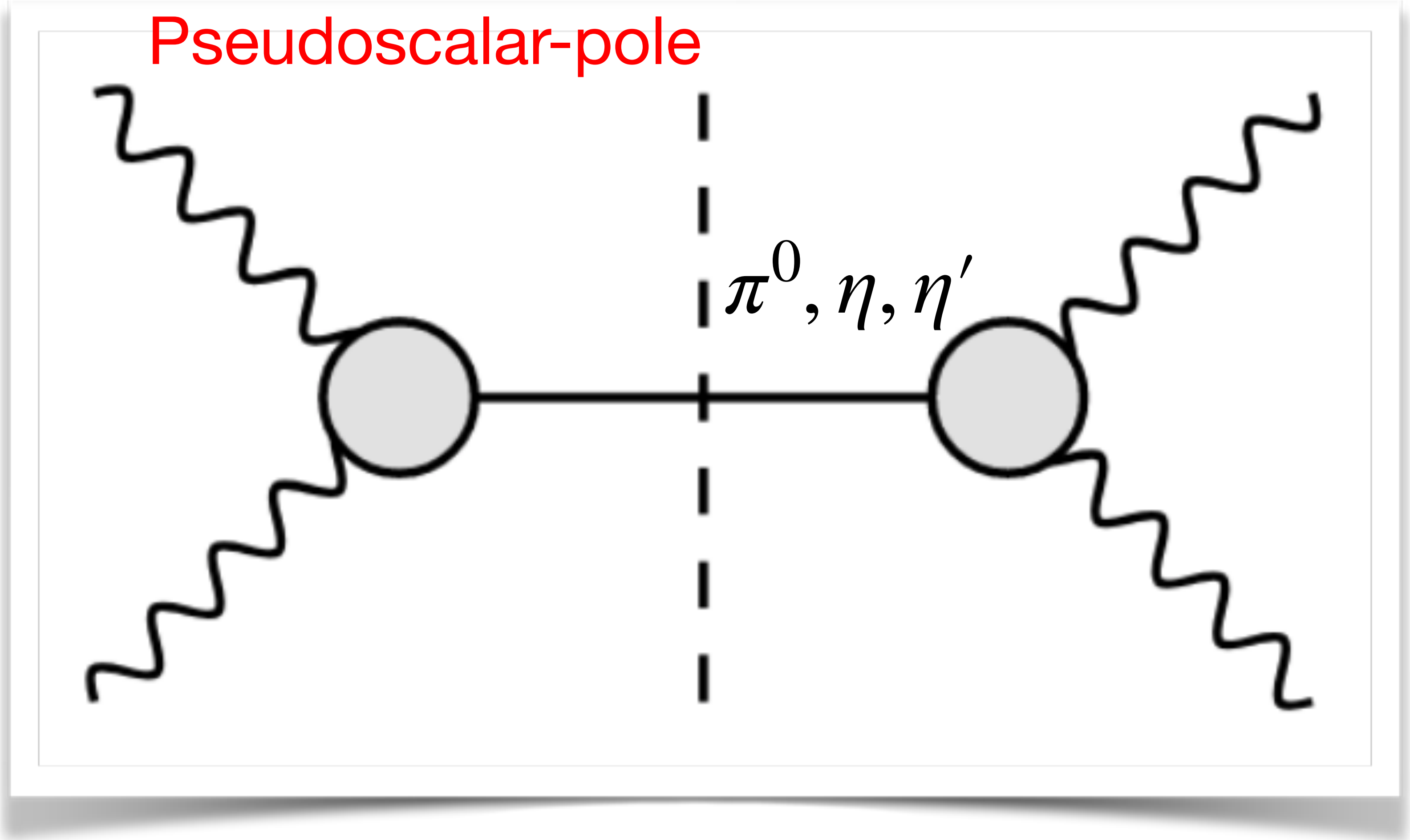
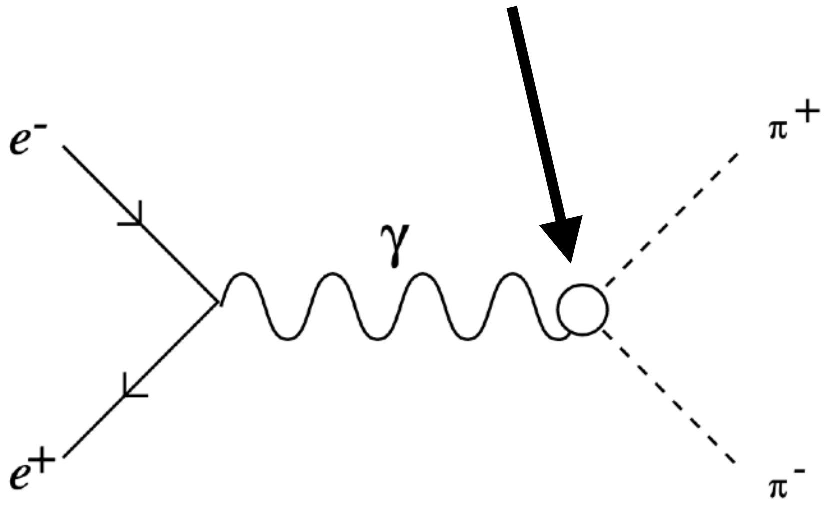


+ ...

$F_{\gamma^* \to \pi\pi}$ "VFF" well-determined experimentally ✅



Partial waves for $\gamma^*\gamma^* \to \pi\pi$ required

Result is small and well-constrained ✅

$F_{P\to\gamma^*\gamma}$ singly-virtual "TFF" experimentally accessible

$F_{P\to\gamma^*\gamma^*}$ doubly-virtual "TFF" ~**unconstrained**

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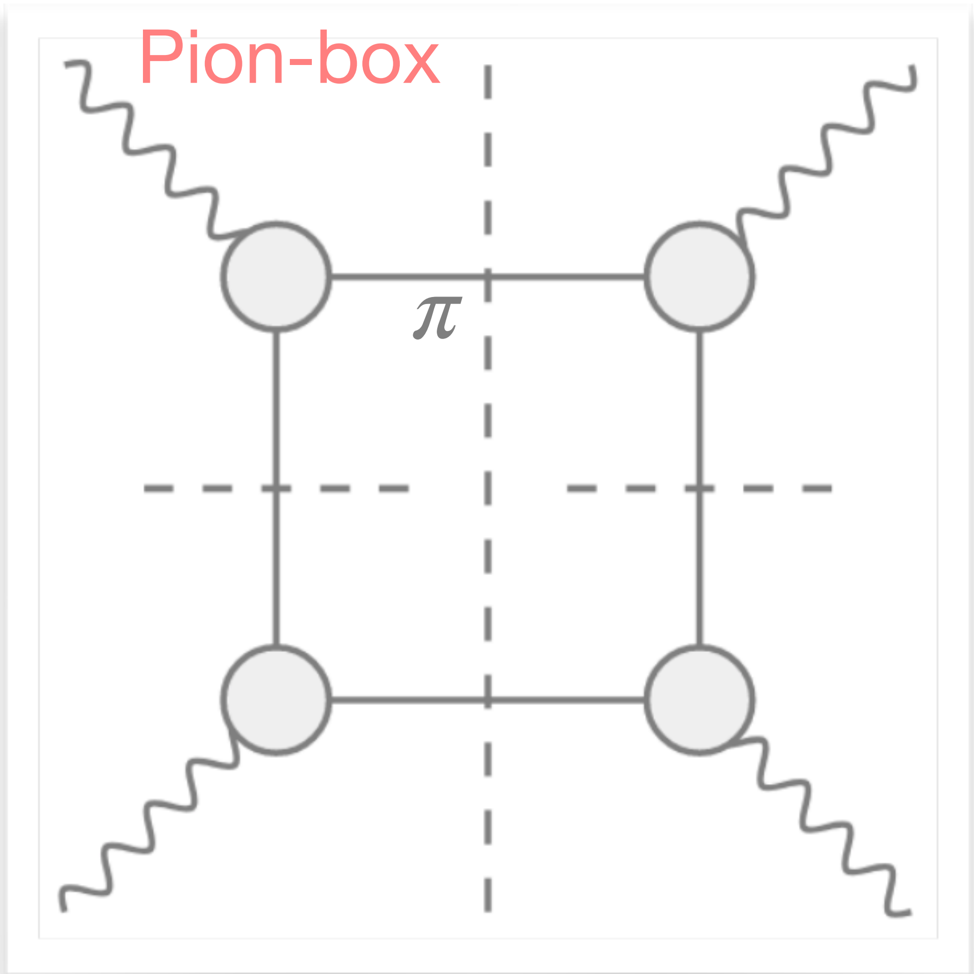

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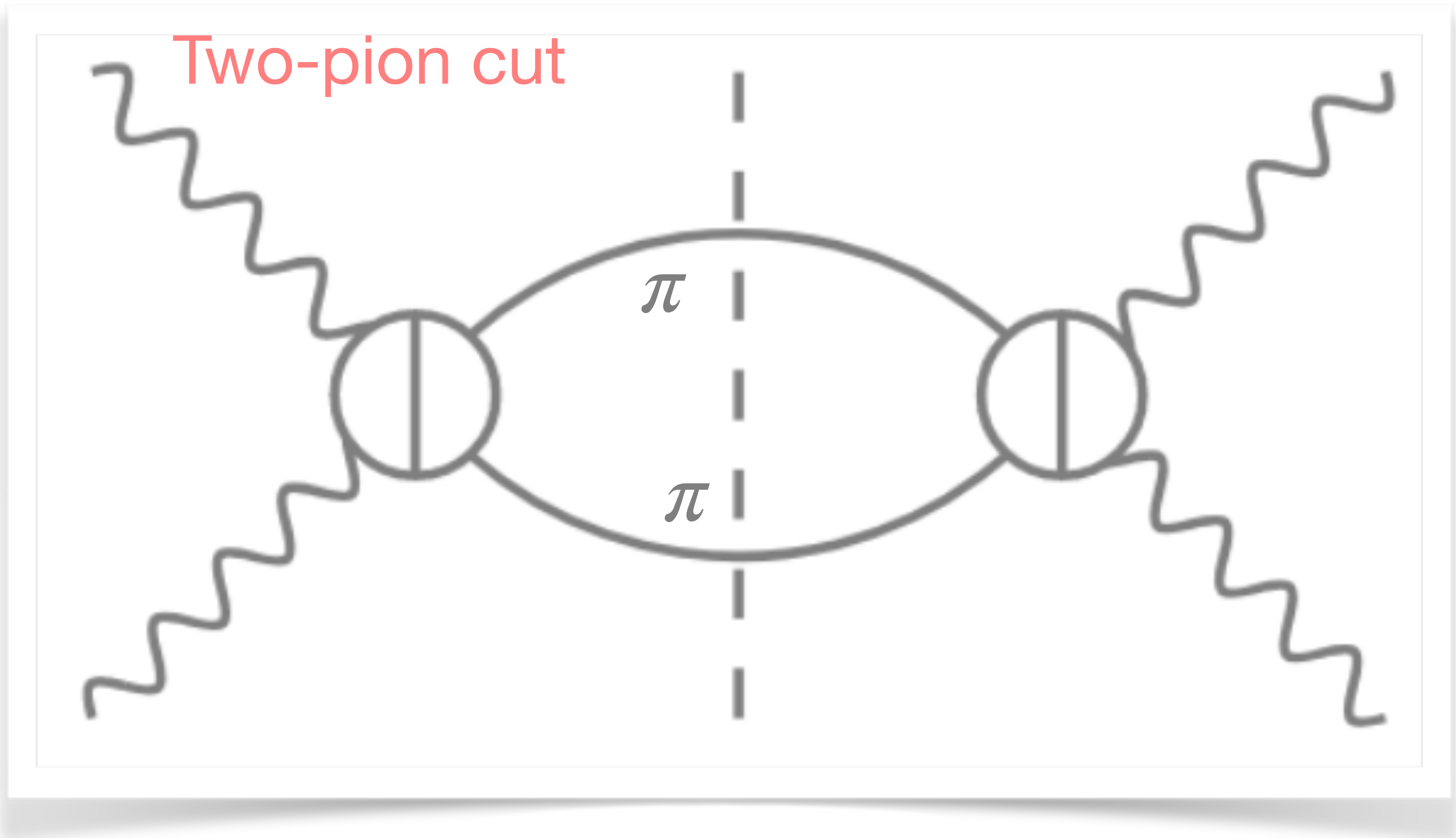

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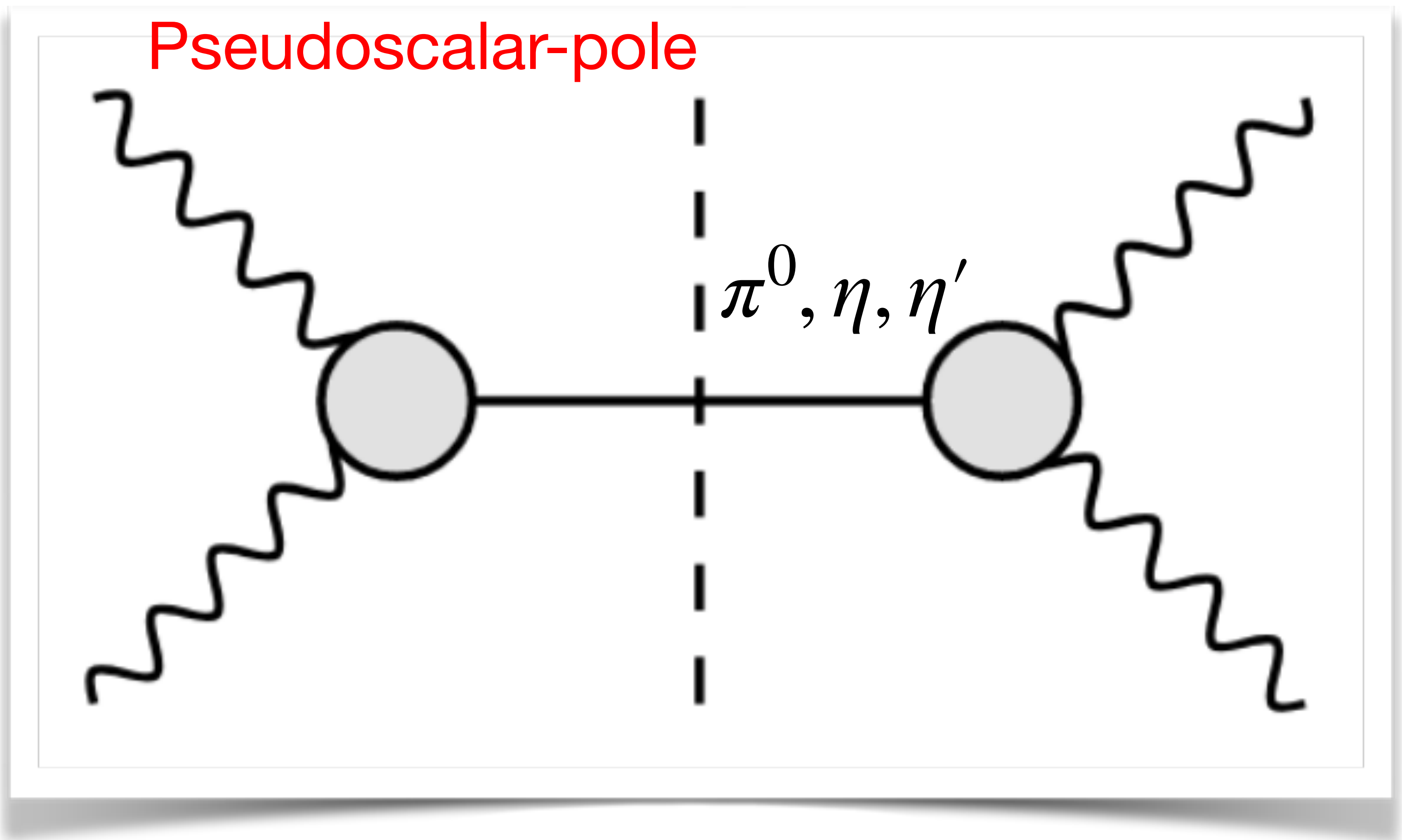

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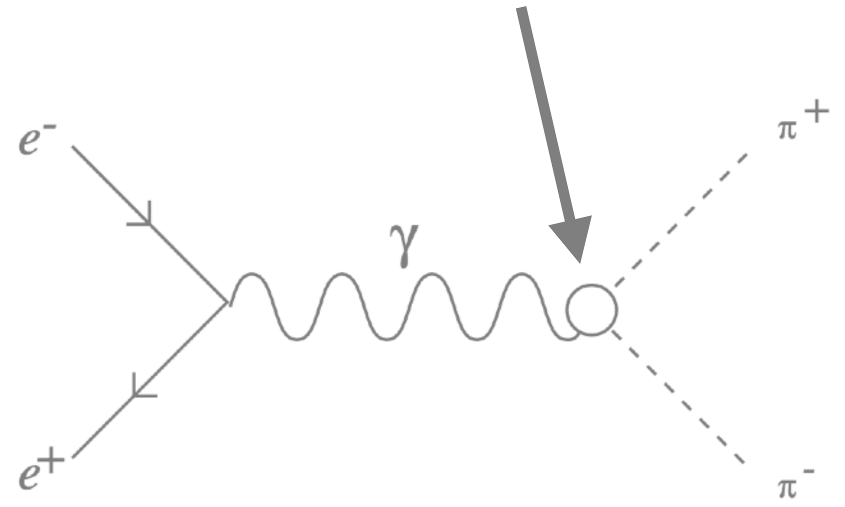

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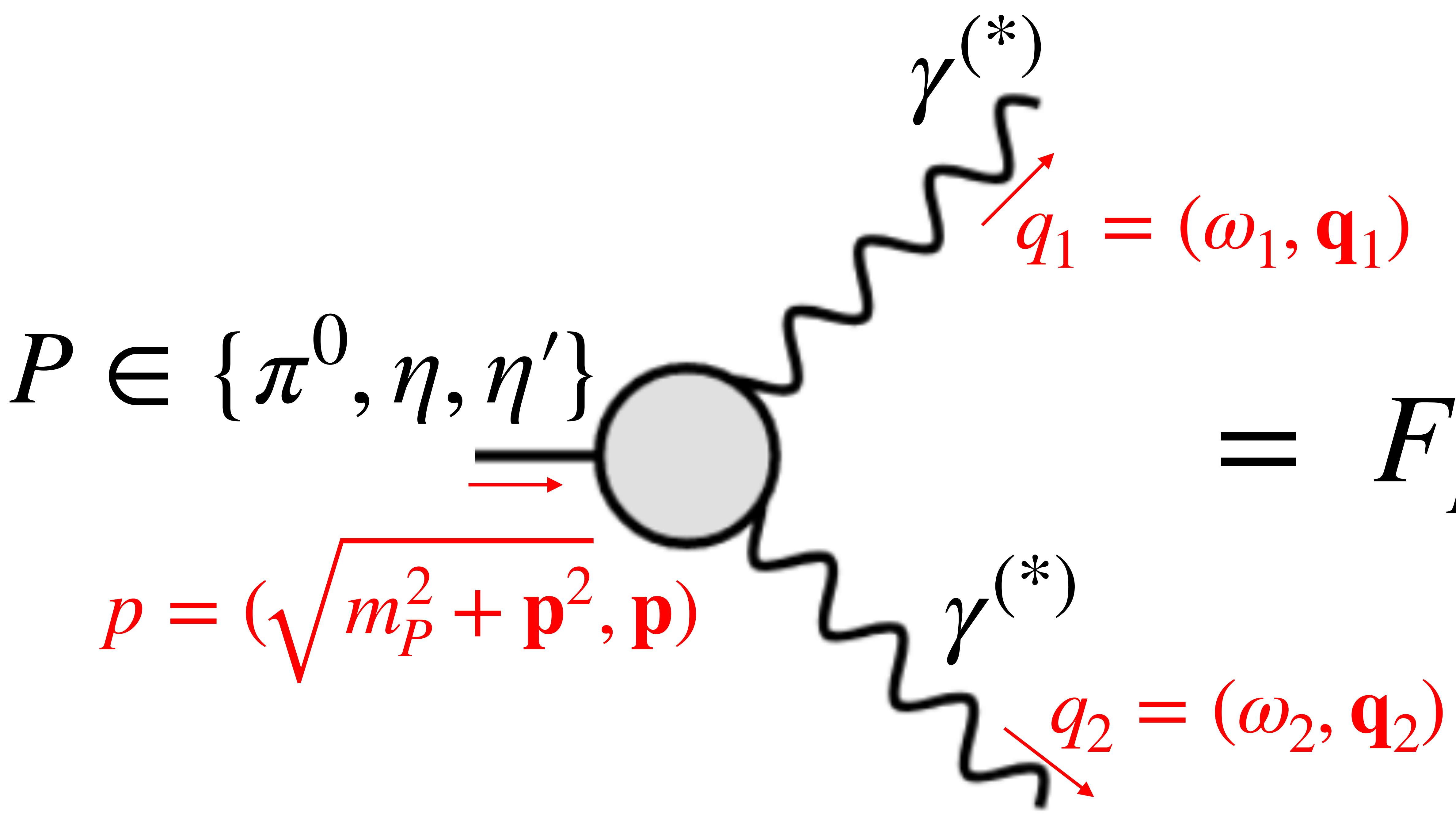
THIS TALK

Pseudoscalar TFFs



$$= \; F_{P\gamma\gamma}(q_1^2, q_2^2)$$

Singly-virtual, spacelike: $F_{P\to\gamma^*\gamma} = F_{P\gamma\gamma}(-Q^2, 0)$

Doubly-virtual, spacelike: $F_{P\to\gamma^*\gamma^*} = F_{P\gamma\gamma}(-Q_1^2, -Q_2^2)$

TFF kinematics

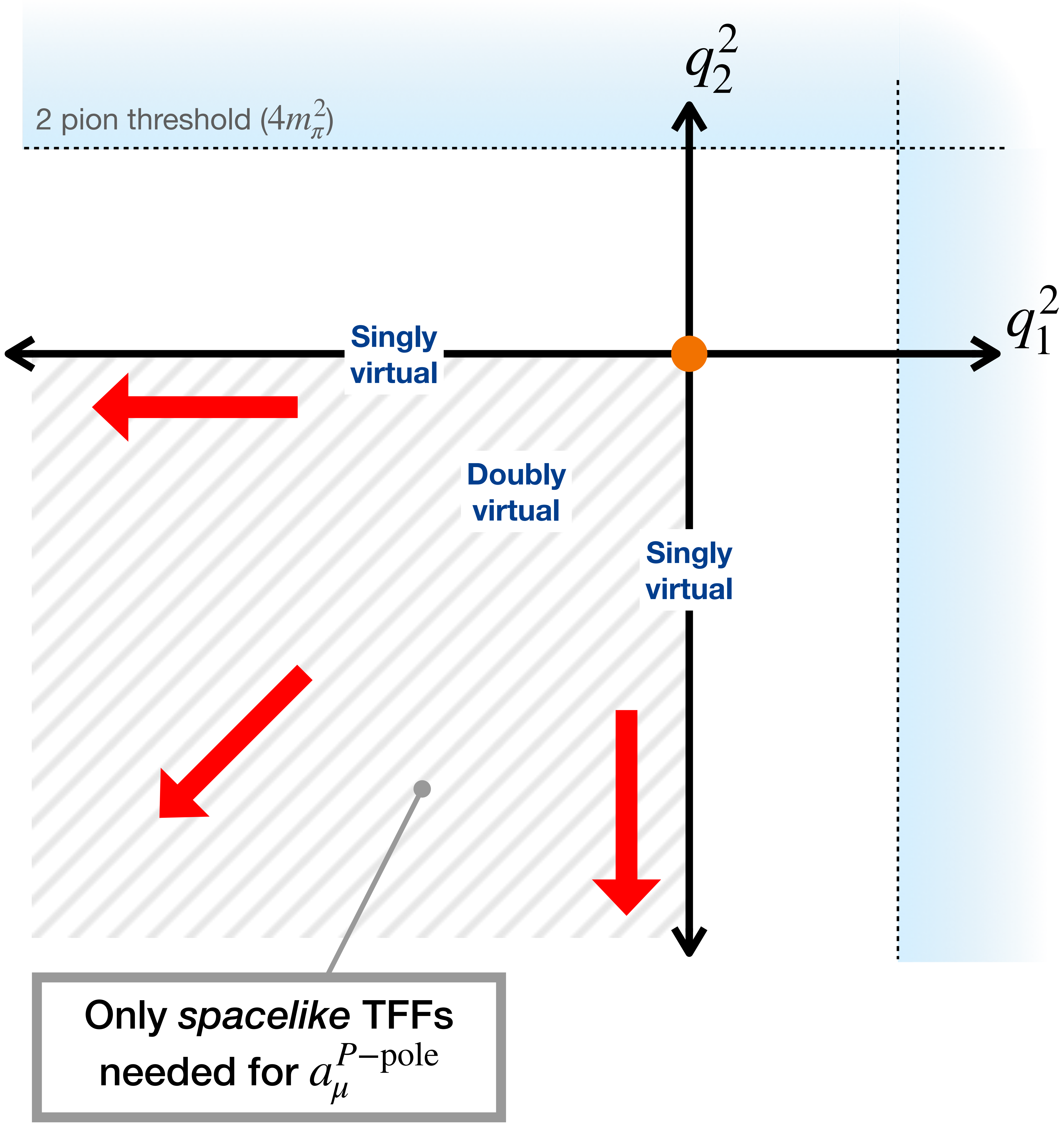
Related to decay width

$$\Gamma(P \to \gamma\gamma) = \left(\frac{\pi\alpha^2 M_P^3}{4}\right) |F_{P\gamma\gamma}(0,0)|^2$$

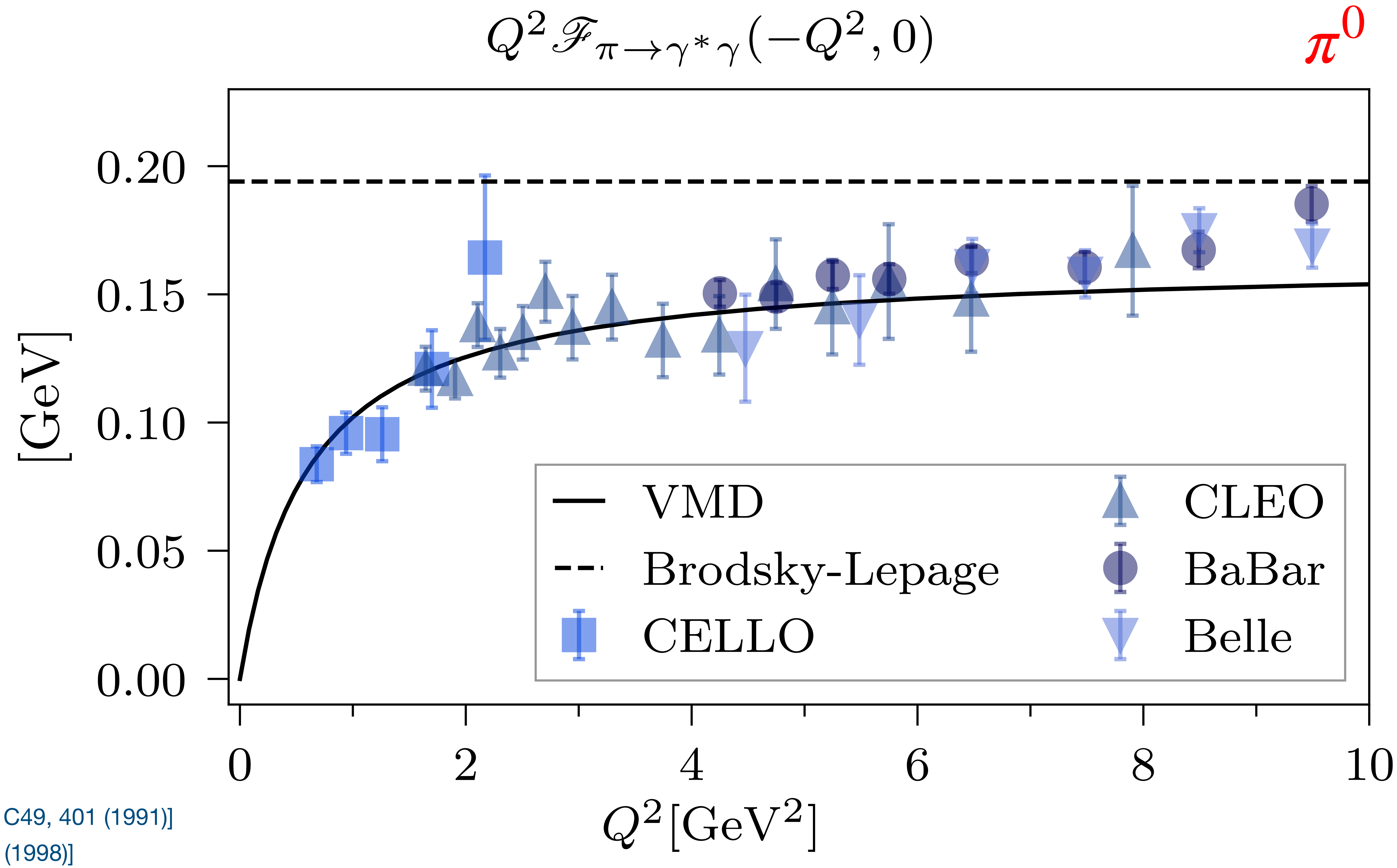
Large-Q^2 limits from pQCD

$$F_{P\gamma\gamma}(-Q^2,0) \to \frac{2F_P}{Q^2} \text{ (Brodsky-Lepage)}$$

$$F_{P\gamma\gamma}(-Q^2,-Q^2) \to \frac{2F_P}{3Q^2} \text{ (OPE)}$$



TFF data: singly virtual



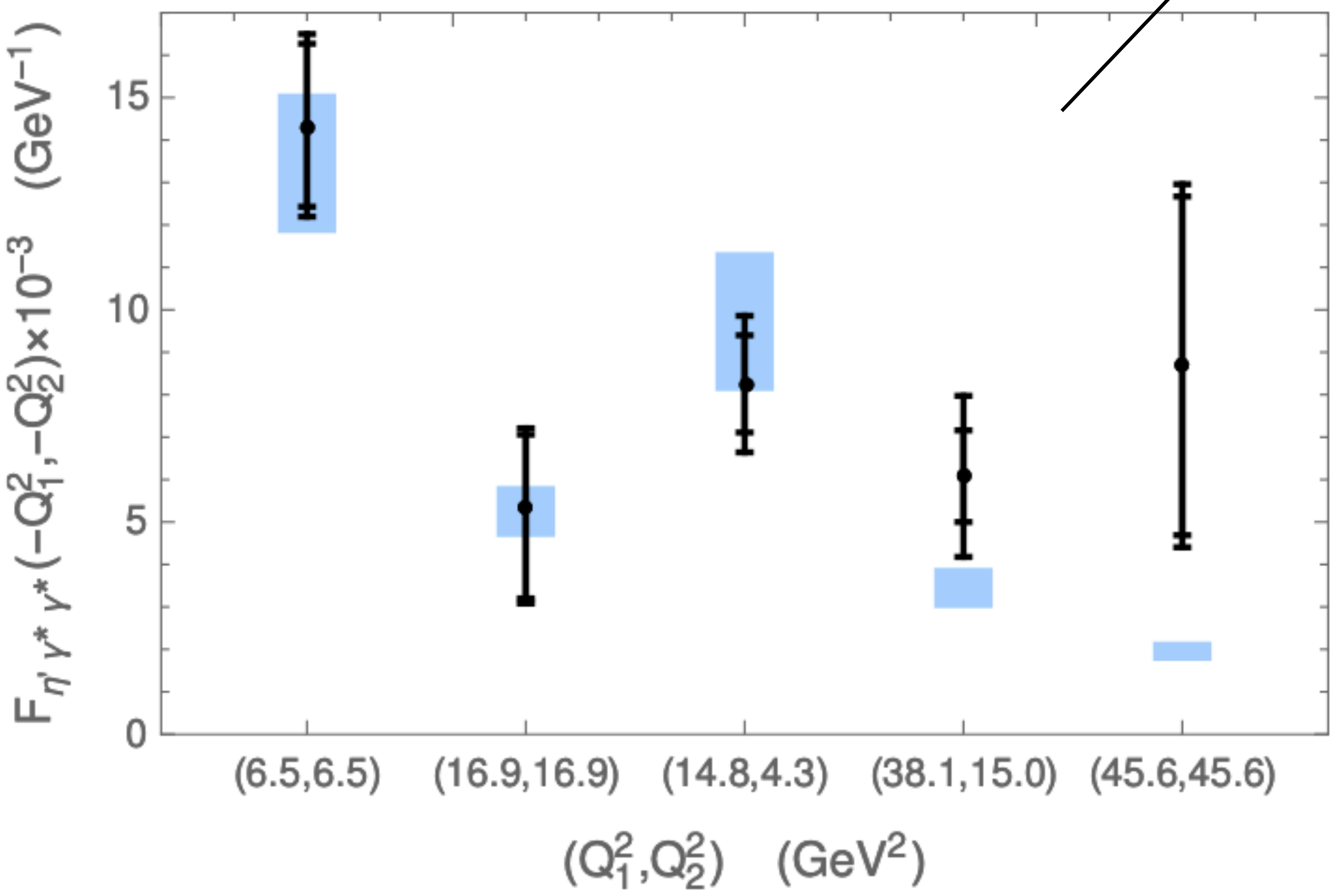
[H. J. Behrend+ (CELLO), Z. Phys. C49, 401 (1991)]
[J. Gronberg+ (CLEO), PRD 57, 33 (1998)]
[B. Aubert+ (BABAR), PRD 80, 052002 (2009)]
[P. del Amo Sanchez+ (BABAR), PRD 84, 052001 (2011)]
[S. Uehara+ (Belle), PRD 86, 092007 (2012)]

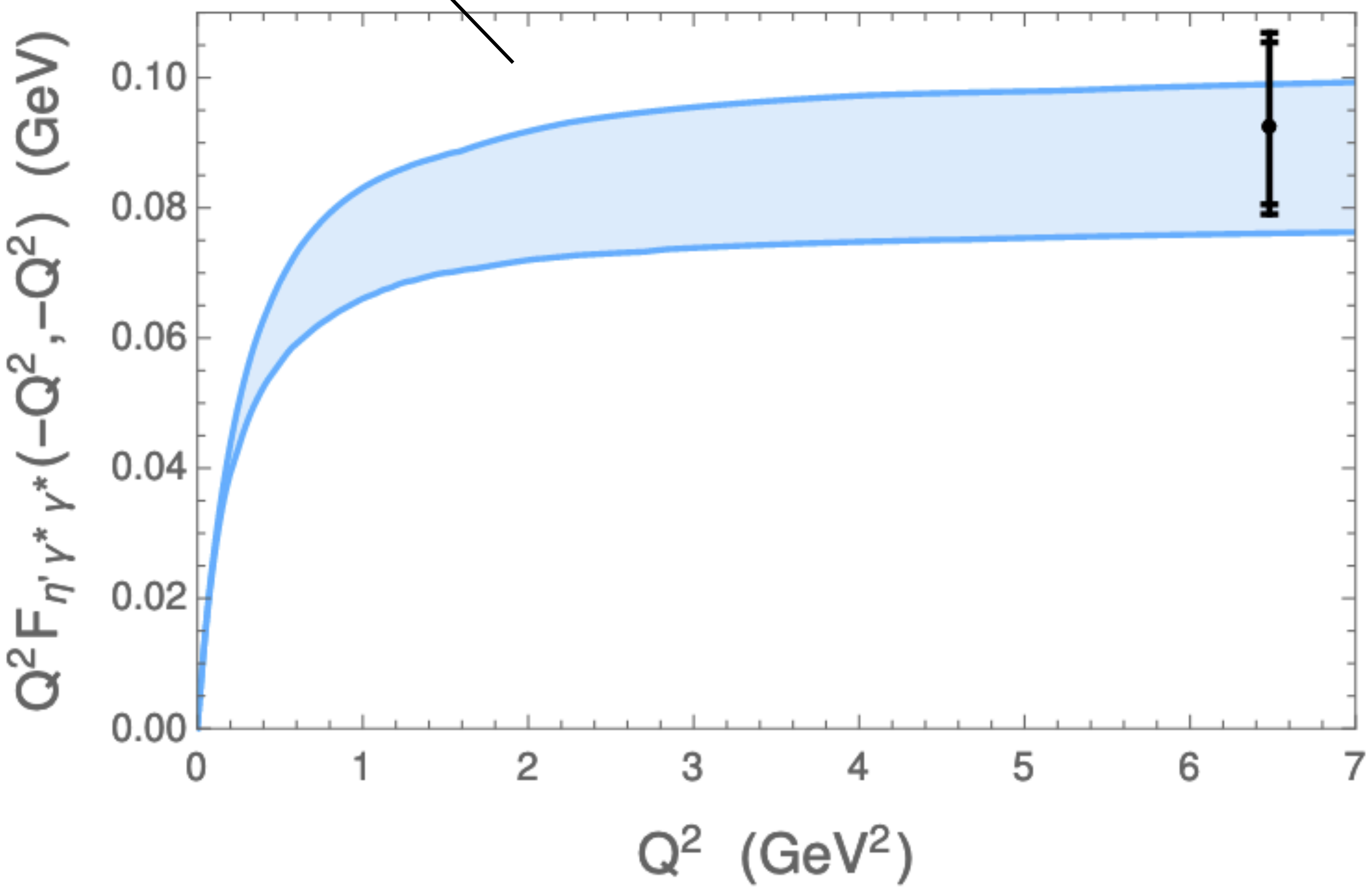
Note: Singly virtual for η, η' are very similar.

TFF data: doubly virtual

Based on ~50 events at BaBar ⚠️

η'

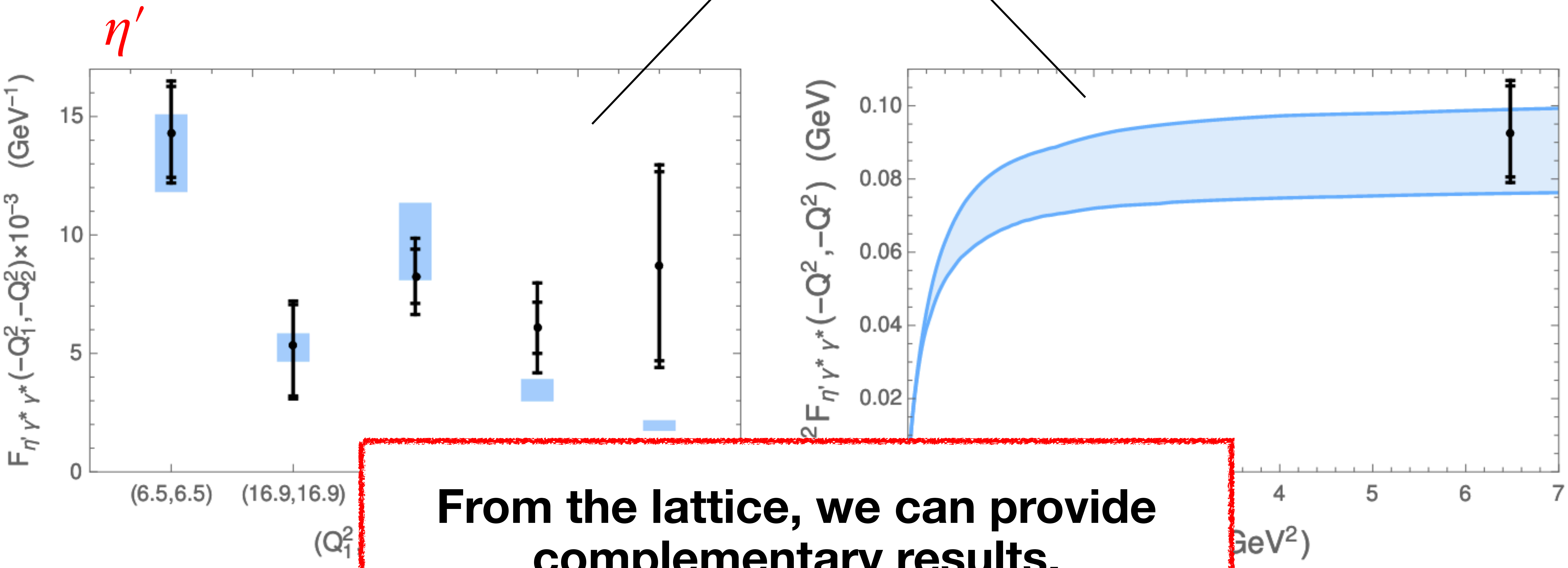




[J. P. Lees+ (BABAR), PRD 98, 112002 (2018)]
Figure from [g-2 WP, Phys. Rep. 887 (2020)]

TFF data: doubly virtual

Based on ~50 events at BaBar ⚠️



From the lattice, we can provide complementary results.

[J. P. Lees+ (BABAR), PRD 98, 112002 (2018)]

Figure from [g-2 WP, Phys. Rep. 887 (2020)]

Lattice details

Action: Nf = 2+1+1 twisted clover, Iwasaki gauge action, *physical point*

Pion analysis:

- Three lattice spacings ✅
- Preliminary continuum extrapolated result 🕐

ensemble	$L^3 \cdot T/a^4$	m_π [MeV]	a [fm]	L [fm]	$m_\pi \cdot L$
cB072.64	$64^3 \cdot 128$	136.8(6)	0.082	5.22	3.6
cC060.80	$80^3 \cdot 160$	134.2(5)	0.069	5.55	3.8
cD054.96	$96^3 \times 192$	140.8	0.057	5.46	3.9

[ETMC PRD 104, 074520 (2021)]

Eta analysis:

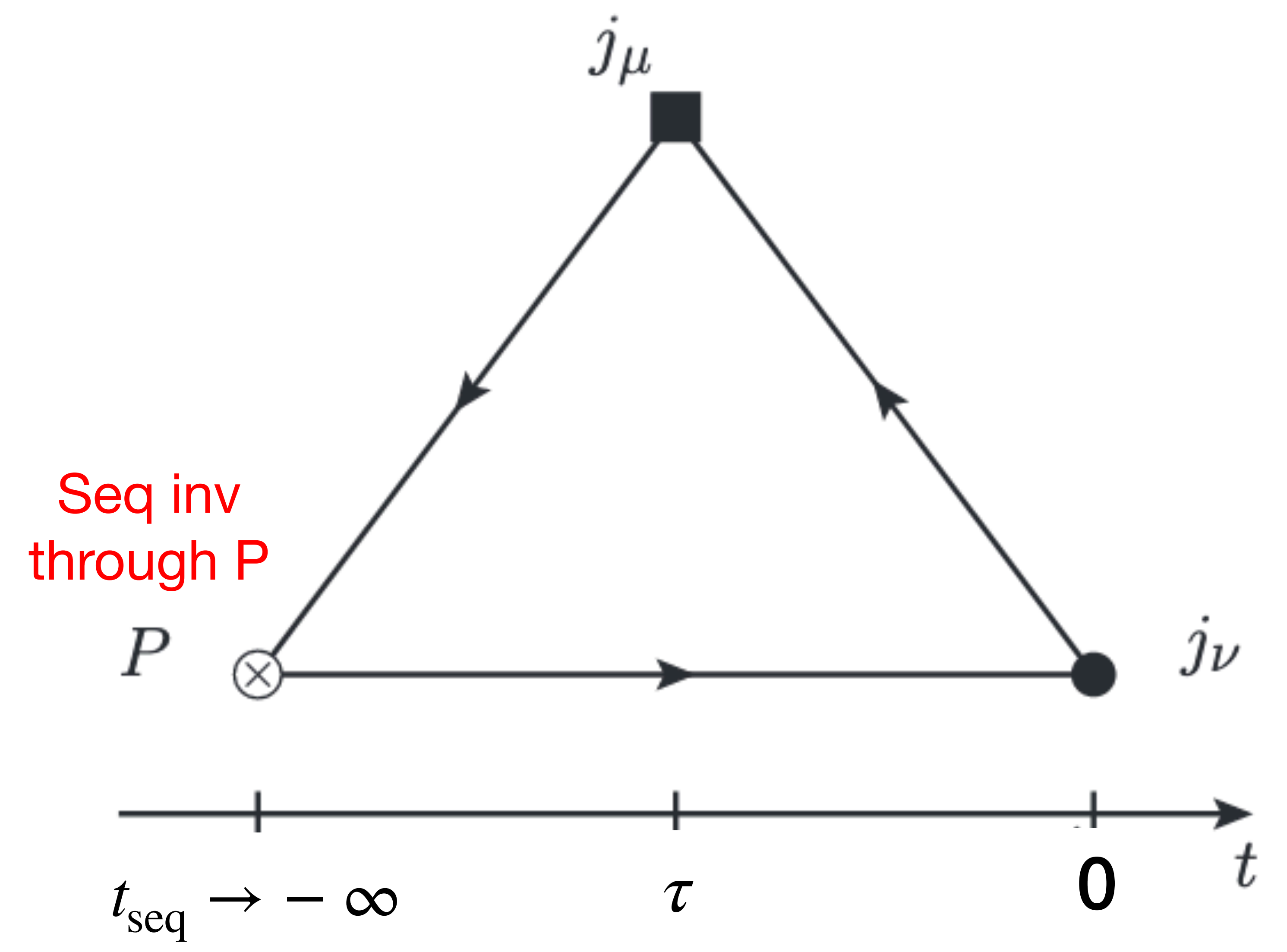
- See talk by Sebastian Burri, today 11:30

Lattice calculation of $F_{P\gamma\gamma}$

1. Euclidean time current-current matrix element

$$\tilde{A}_{\mu\nu}(\tau) = \sum_{\mathbf{x}} e^{-i\mathbf{q}_1\cdot\mathbf{x}} \left\langle 0 \left| j_\mu(\tau;\mathbf{x})\, j_\nu(0;\mathbf{0}) \right| P(\mathbf{p}) \right\rangle$$

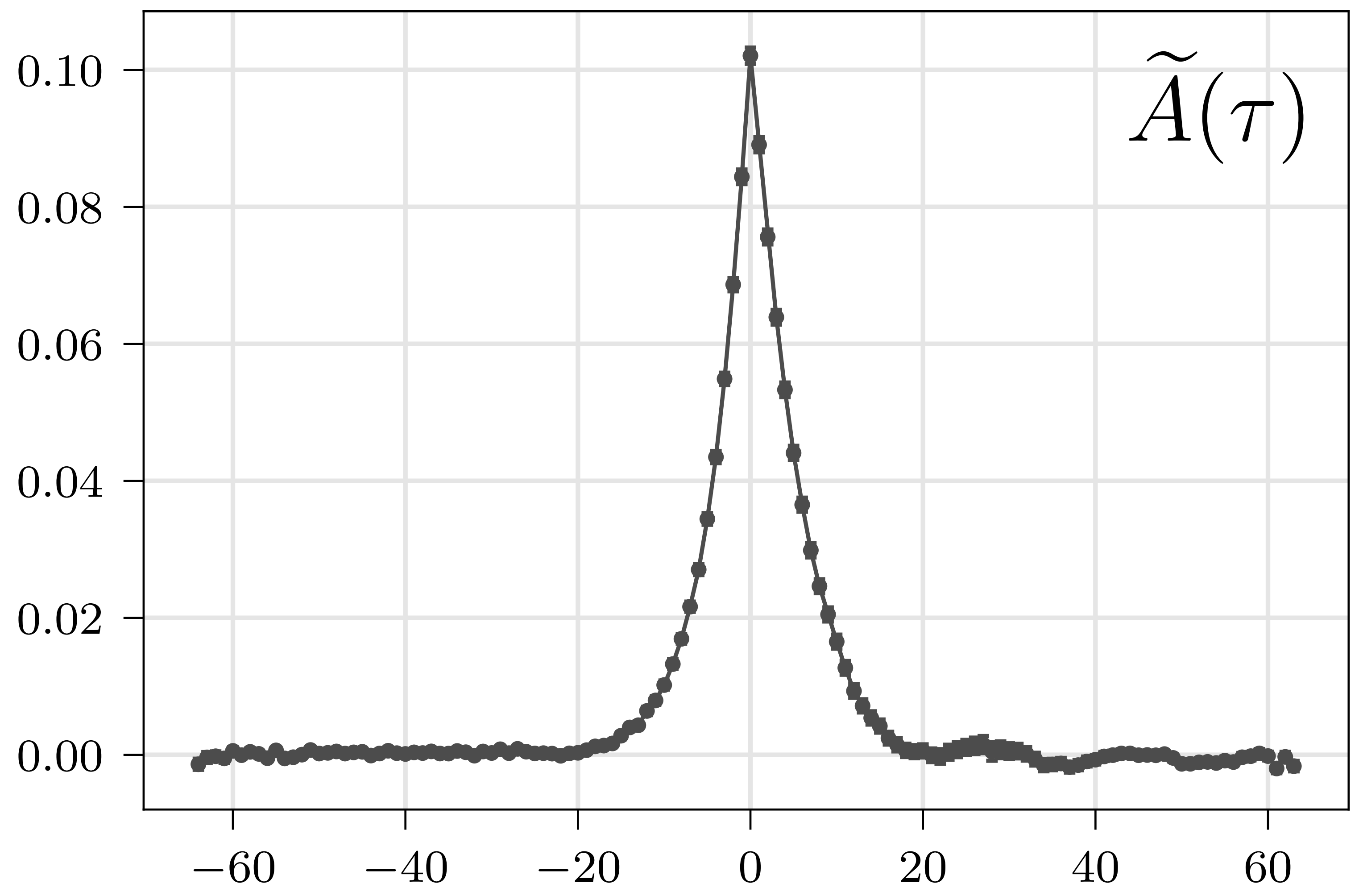
Note: renormalized currents using precisely known Z_A, Z_V



2. Laplace transform

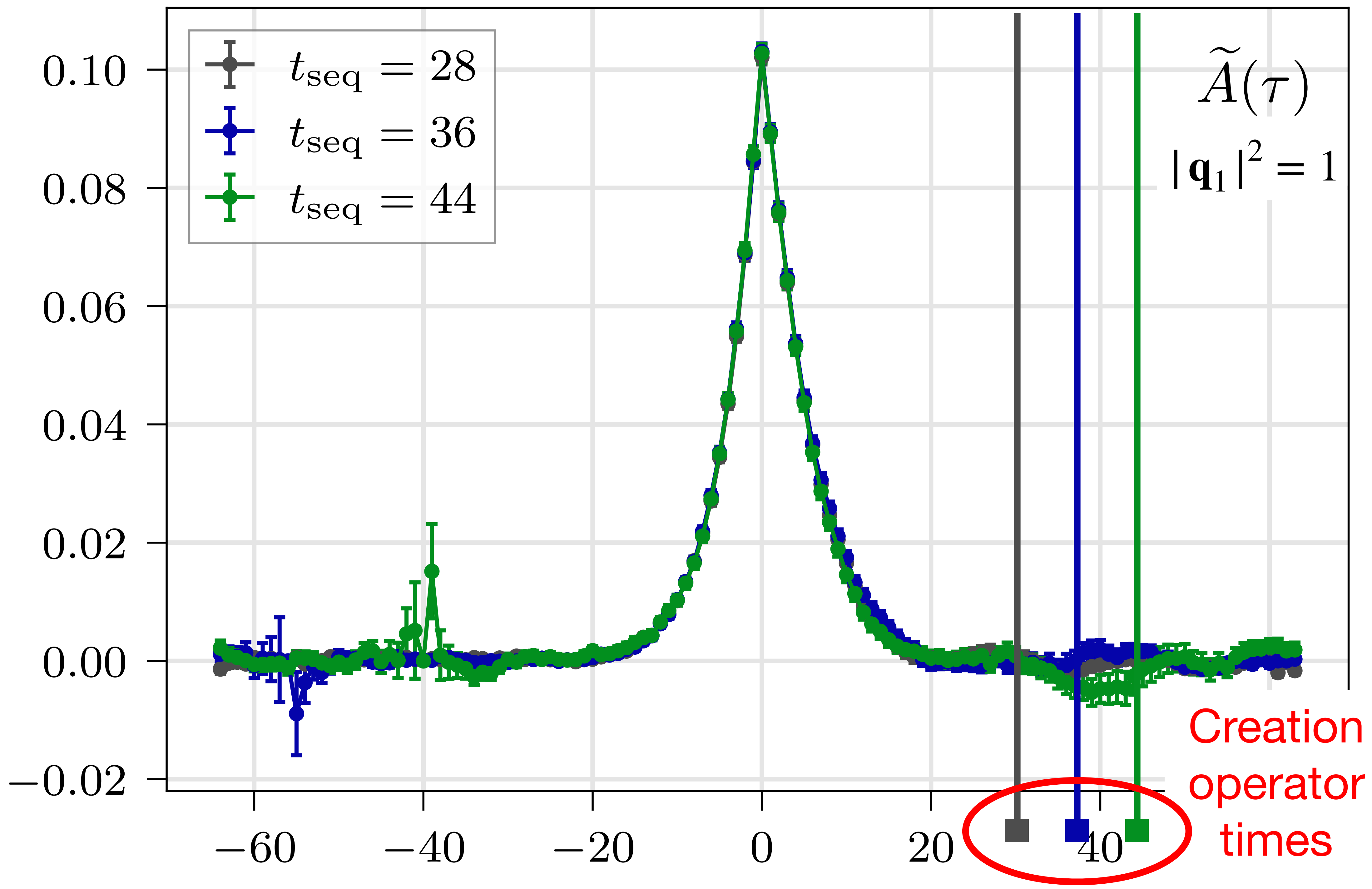
$$\epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} F_{P\gamma\gamma}(q_1^2, q_2^2) = -\, i^{n_0} \int_{-\infty}^{\infty} d\tau\, e^{\omega_1 \tau}\, \tilde{A}_{\mu\nu}(\tau)$$

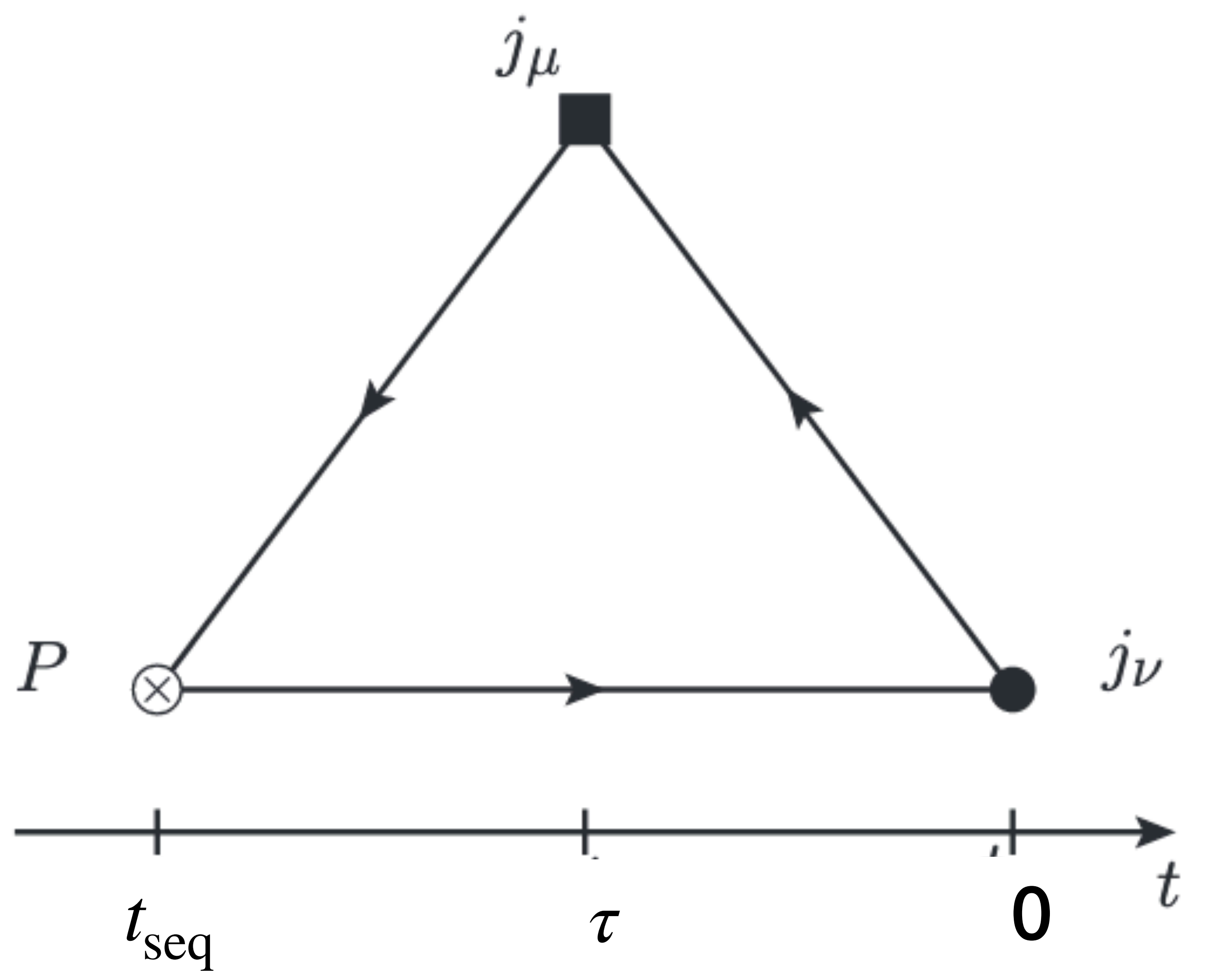
where $\mathbf{q}_2 = \mathbf{p} - \mathbf{q}_1$ by momentum conservation



Matrix element from 3-pt function

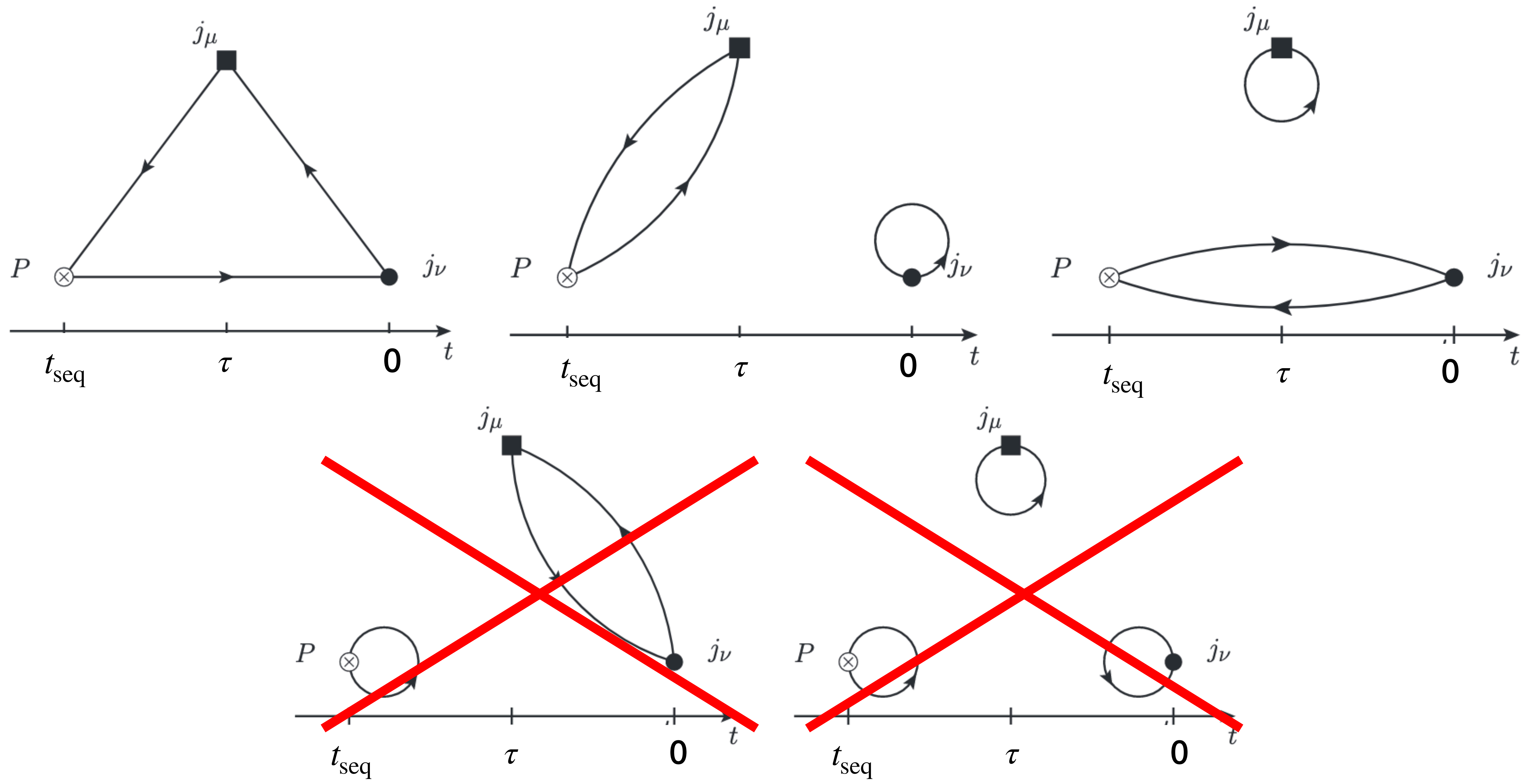
$$\tilde{A}_{\mu\nu}(\tau) = \boxed{\lim_{t_{\text{seq}}\to\infty}} \frac{2E_P}{Z_P} e^{E_P t_{\text{seq}}} \sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{q}_1\cdot\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{y}} \left\langle j_\mu(\tau,\mathbf{x}) j_\nu(0,\mathbf{0}) P^\dagger(-t_{\text{seq}},\mathbf{y}) \right\rangle$$

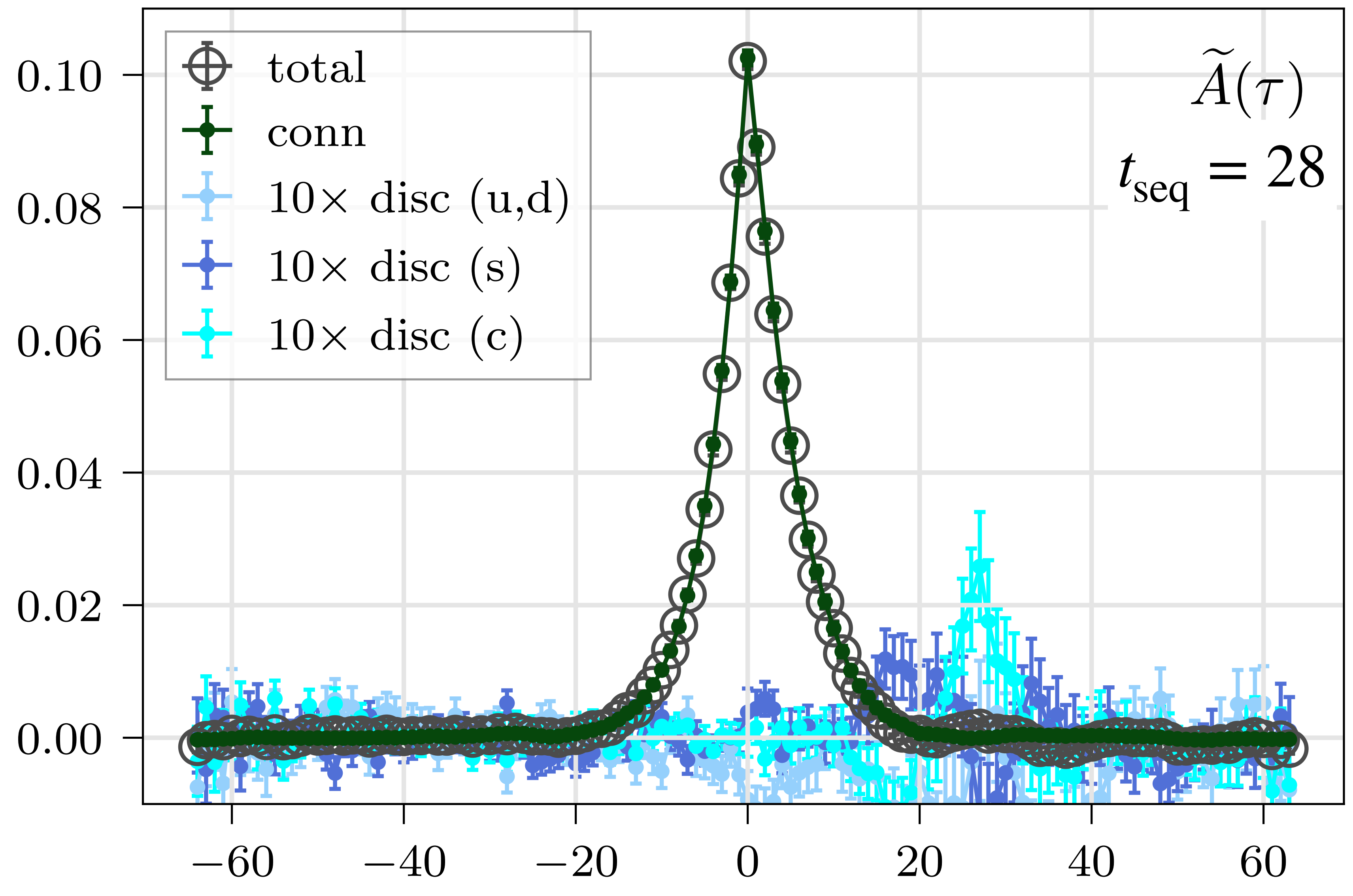




Wick contractions: π^0 isospin rotation

Isospin symmetry allows removing two diagrams:

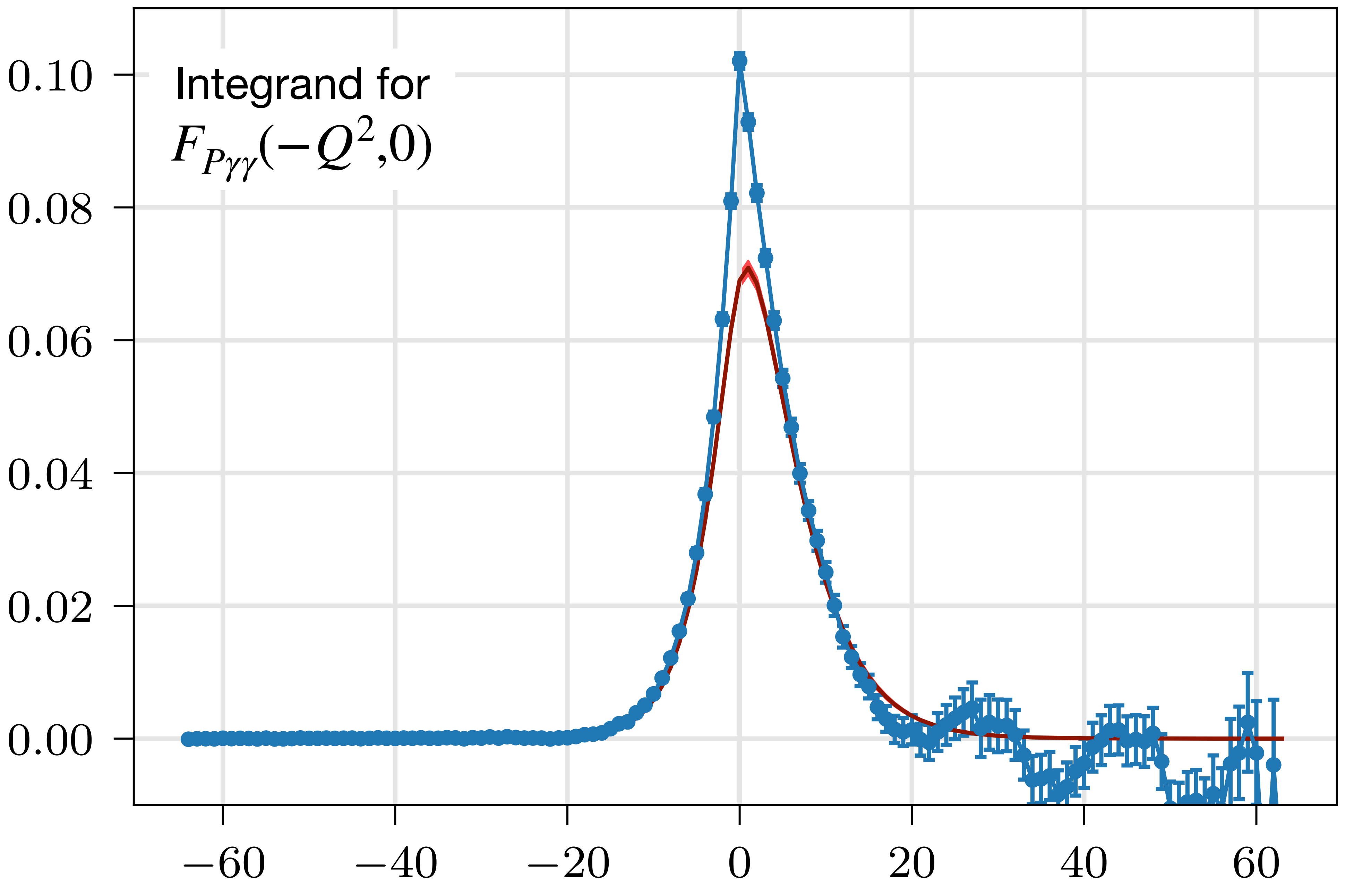


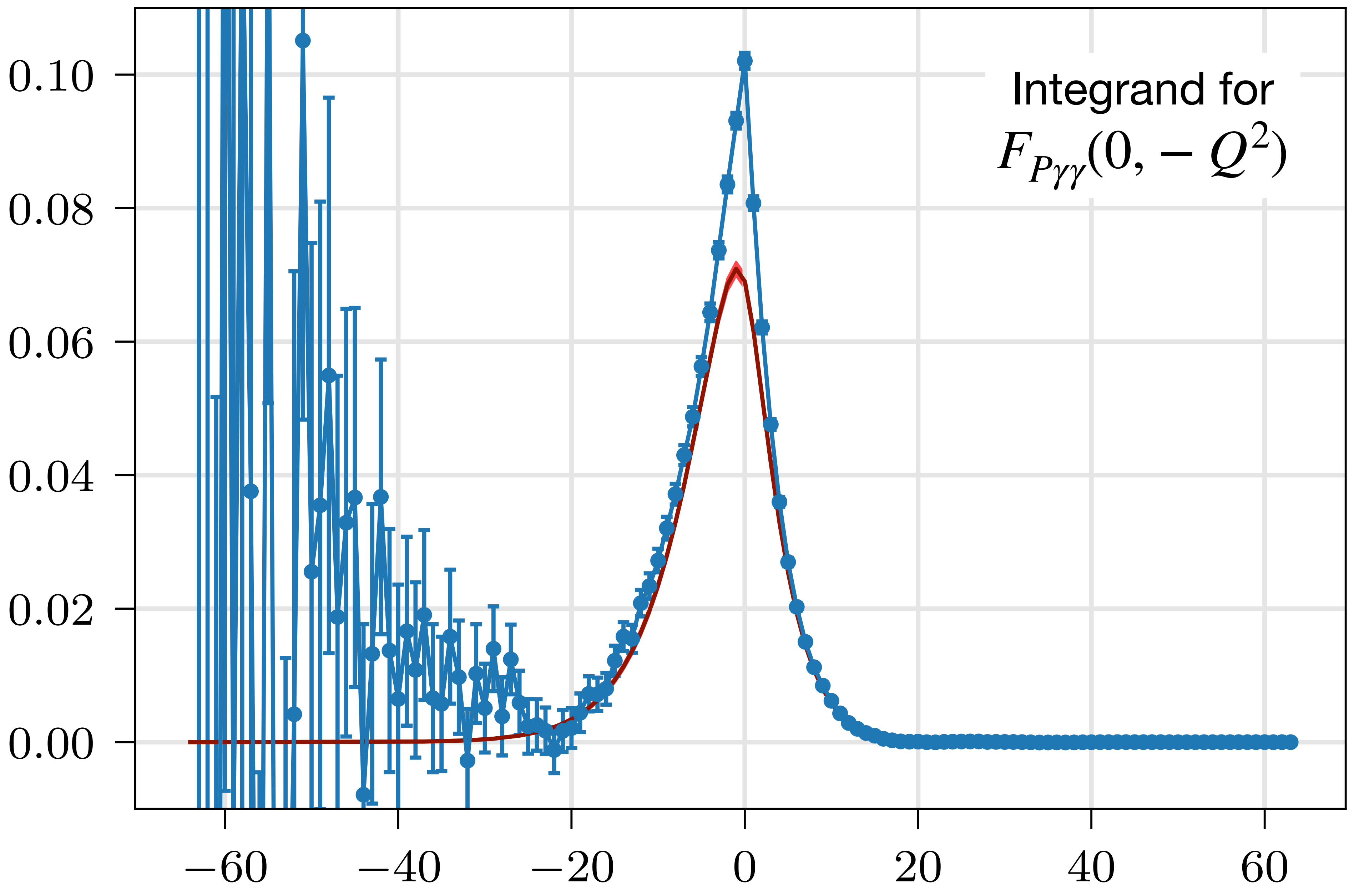


Note: $O(a^2)$ isospin breaking with twisted mass is part of continuum extrapolation

Tail fitting and integration

- Exponentially growing noise in the tails of $\tilde{A}_{\mu\nu}(\tau)$
  - Probed in the Laplace transform $\int_{-\infty}^{\infty} d\tau\, \boxed{e^{\omega_1 \tau}}\, \tilde{A}_{\mu\nu}(\tau)$





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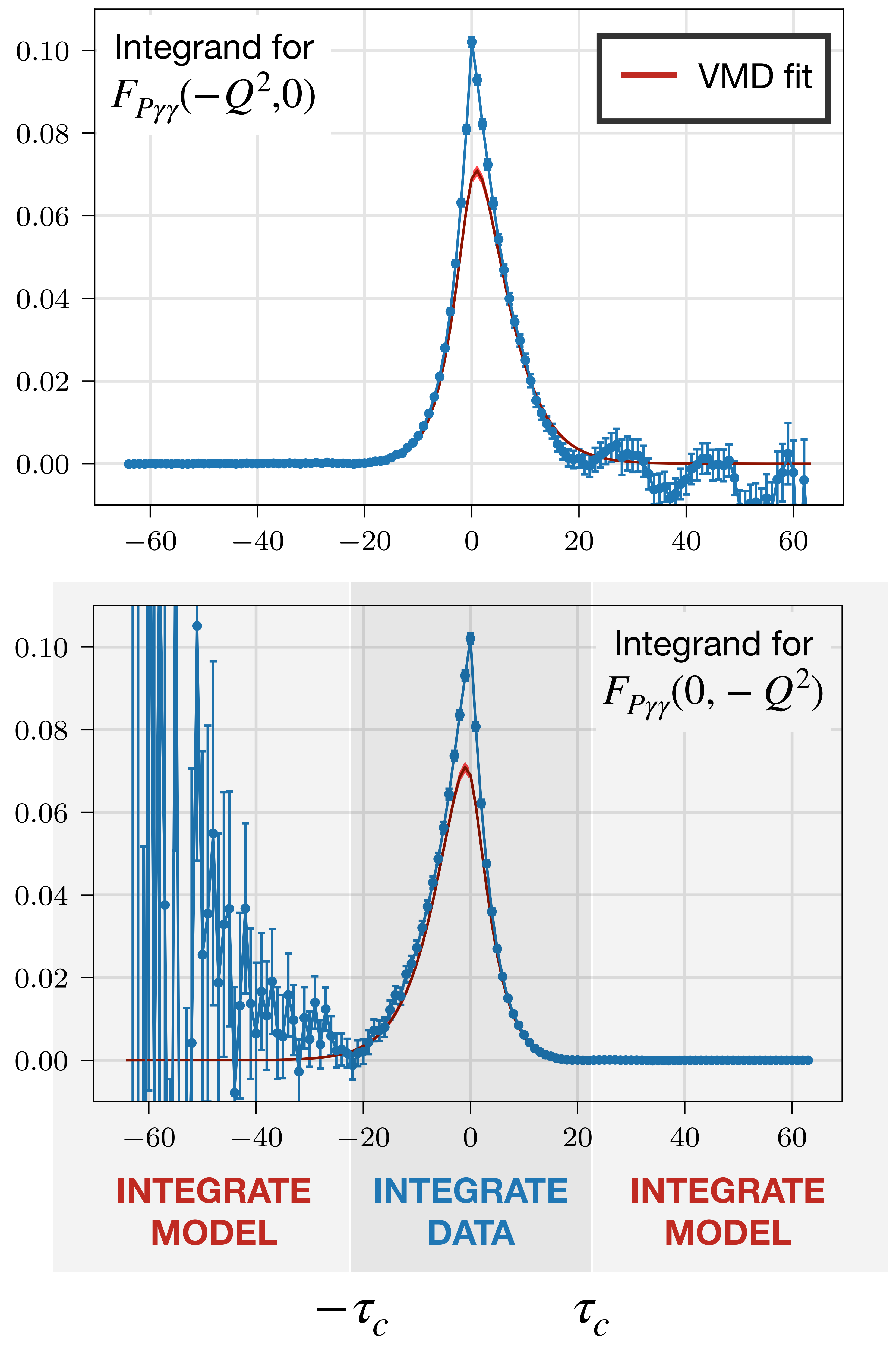
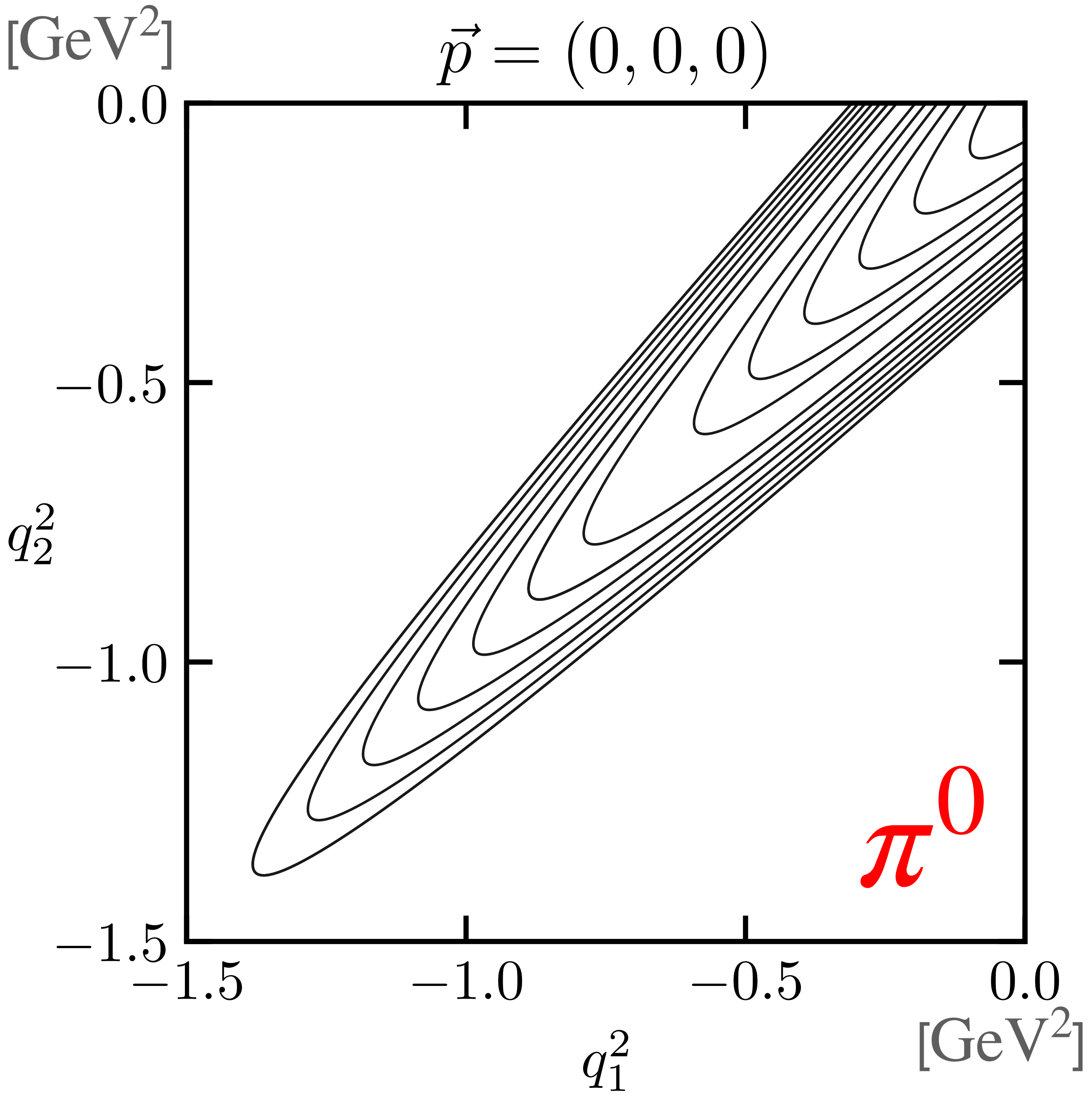
- To handle this…
  - Fit Vector Meson Dominance (VMD) or Lowest Meson Dominance (LMD) models to the tails
  - Integrate **data** in peak ($|\tau| < \tau_c$)
    Integrate **model** in tails ($|\tau| > \tau_c$)



Extrapolating in (q_1^2, q_2^2) plane

Finite volume momenta $\mathbf{q}_1$, $\mathbf{p}$, arbitrary ω_1

- Parabolic "orbits" of (q_1^2, q_2^2) accessible
- Singly virtual coverage ($\mathbf{p} \neq 0$ & larger m_P help!)



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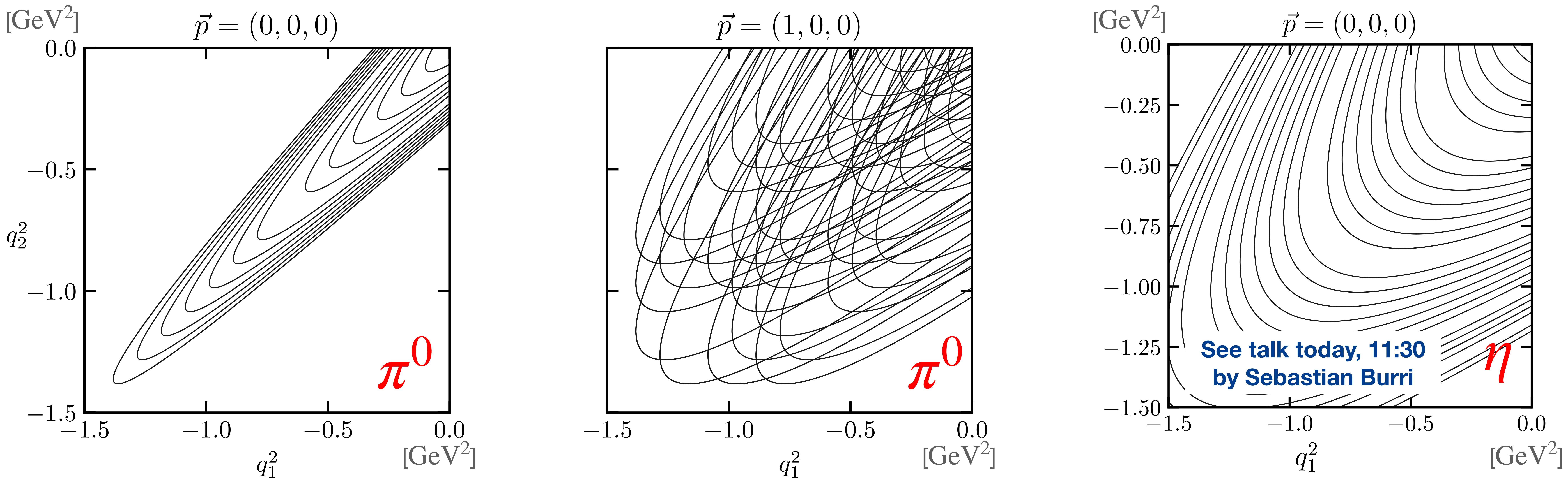

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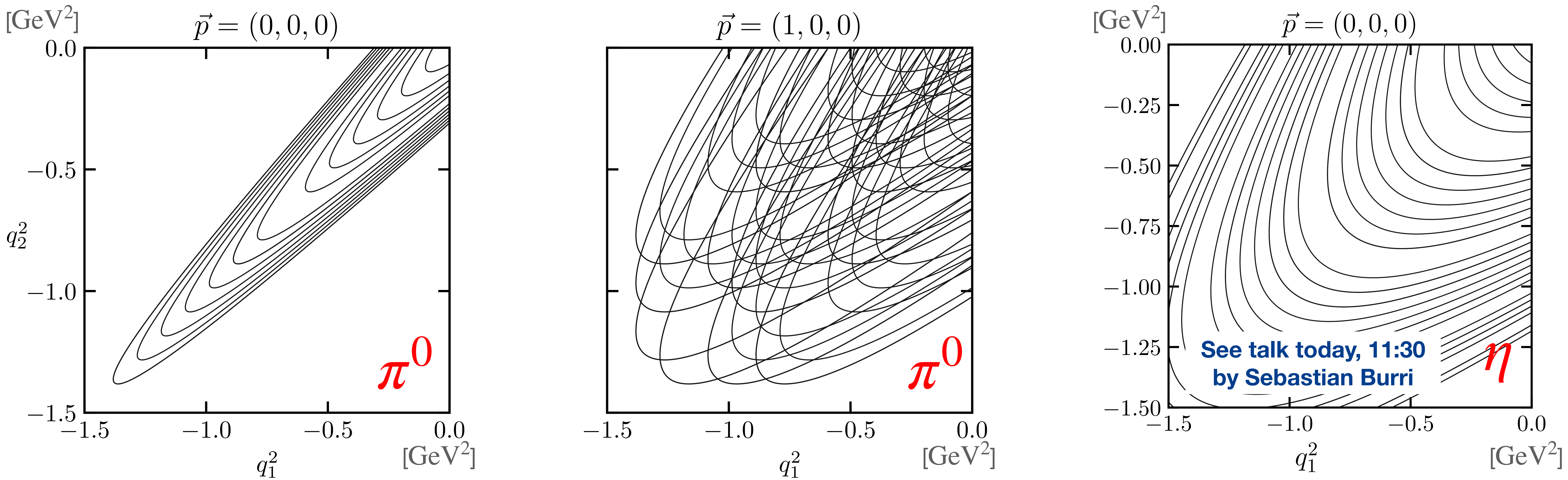
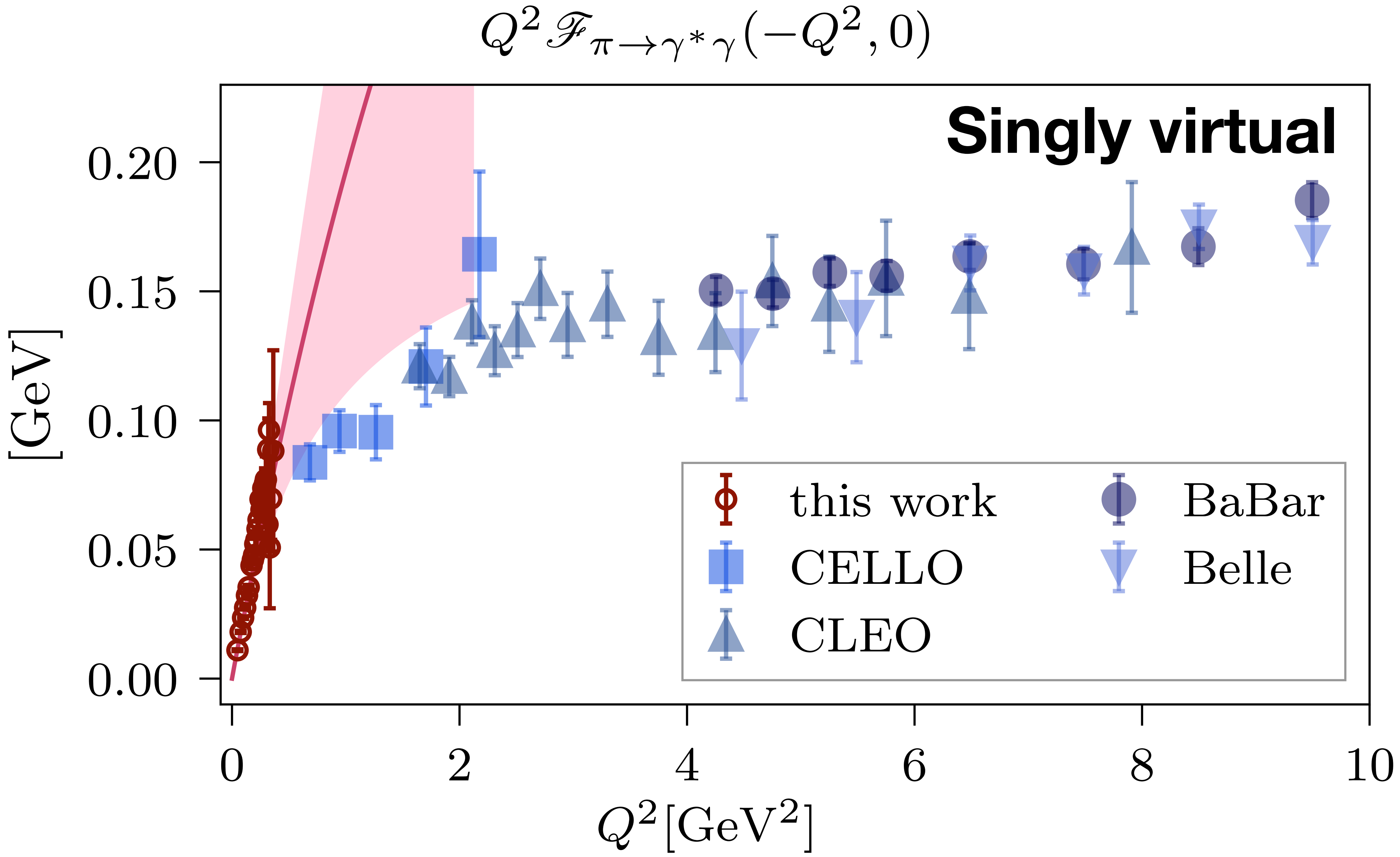
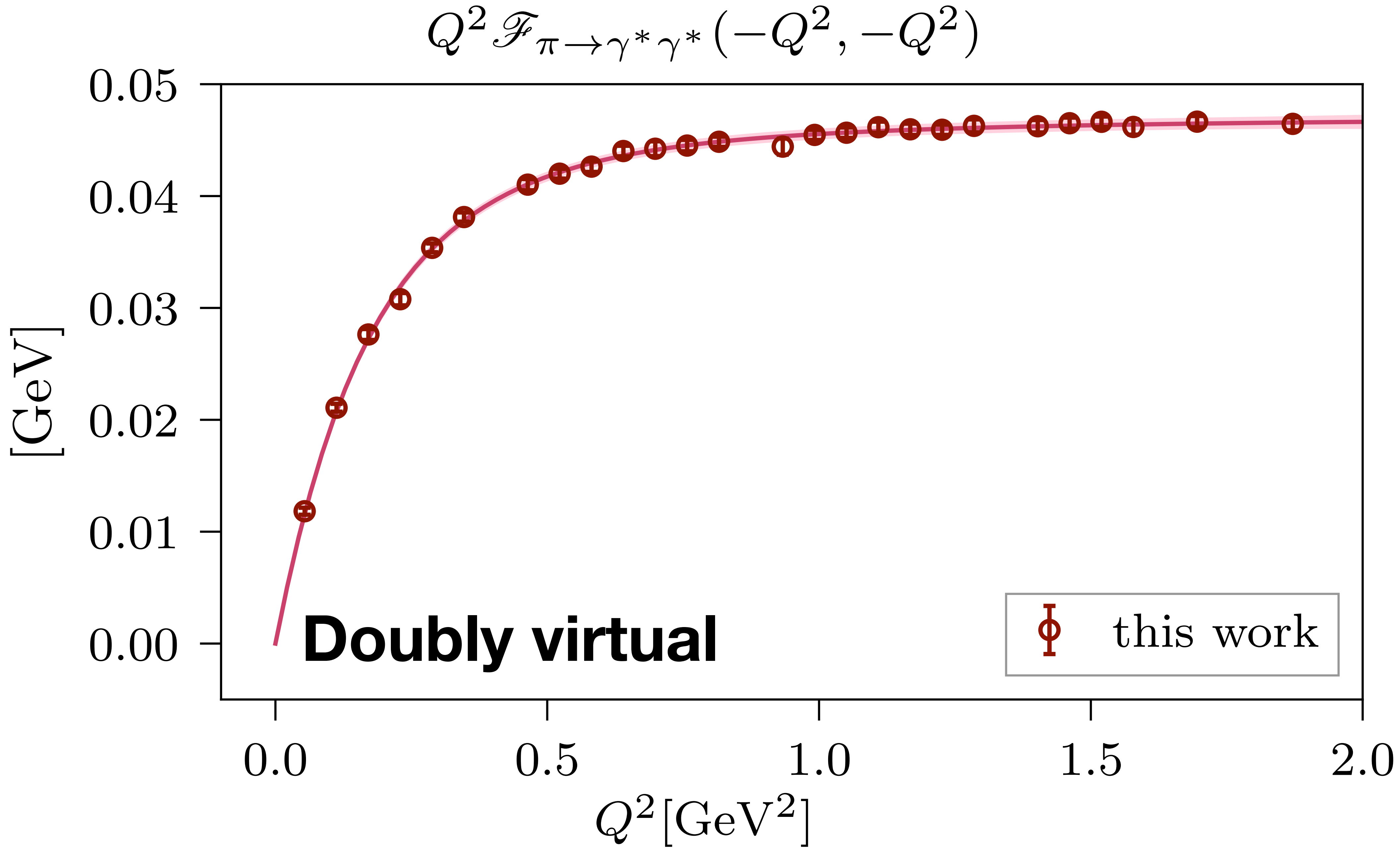
z-expansion for full (q_1^2, q_2^2) dependence



π^0 TFF results





Shown for one choice of ensemble and analysis procedure.

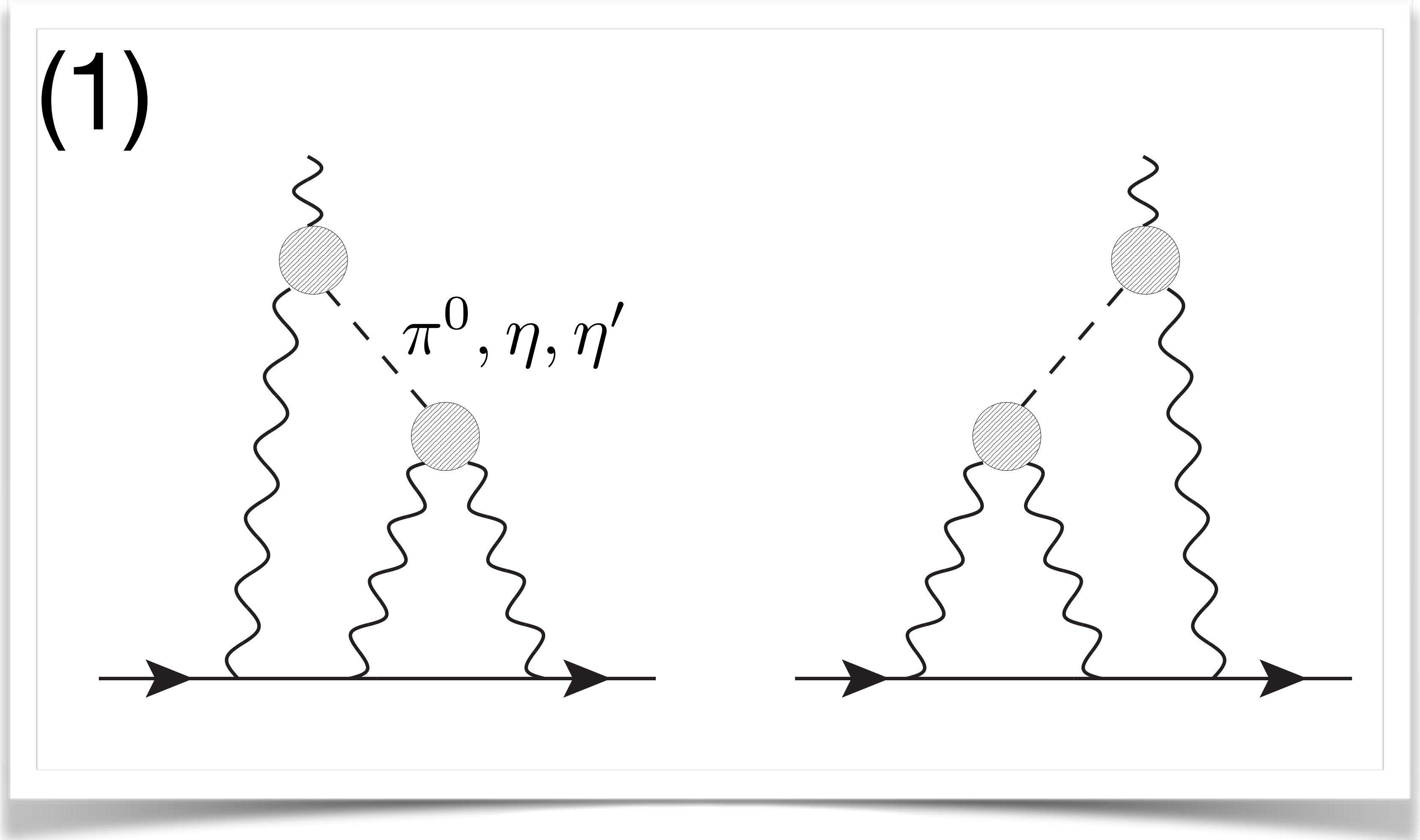
Doubly virtual much more precise than singly virtual due to kinematics.

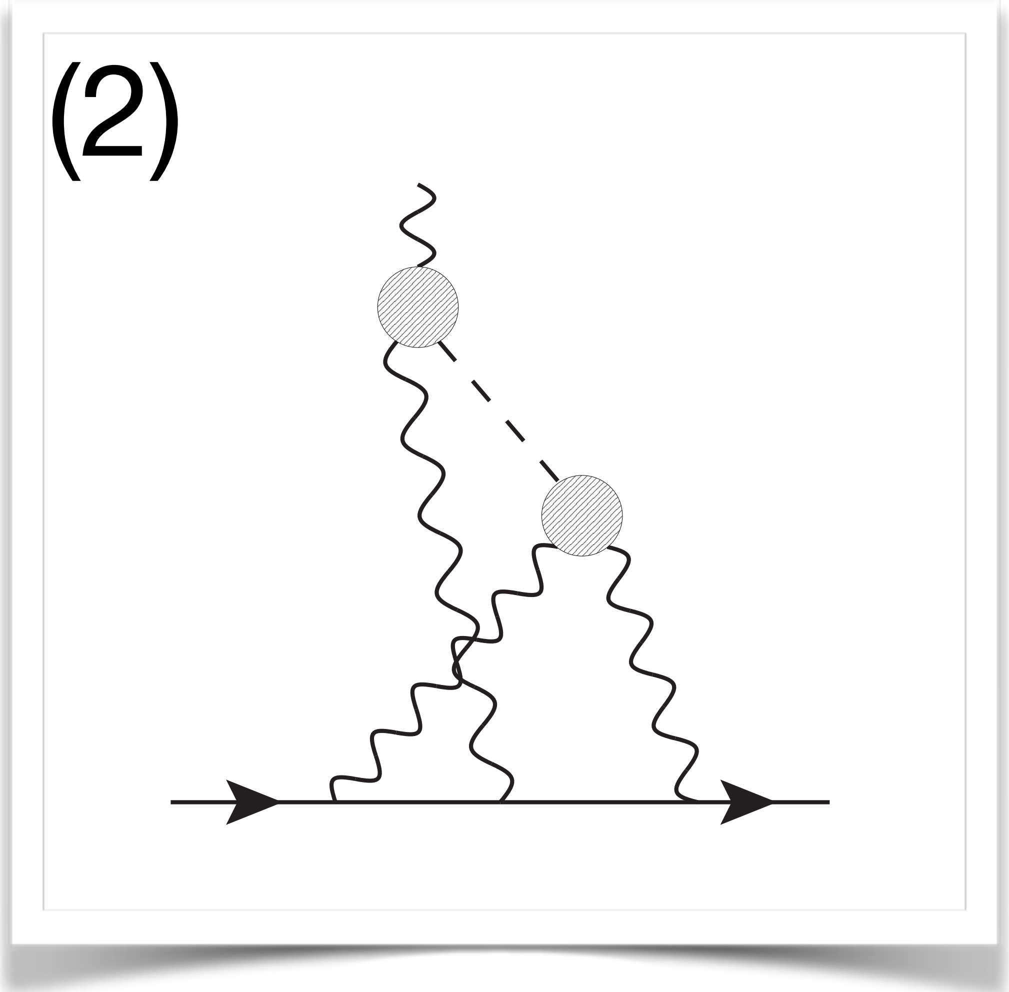
Muon g-2 contribution $a_\mu^{P-\mathrm{pole}}$

$$a_\mu^{P-\mathrm{pole},(1)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\cos\theta\, w_1(Q_1, Q_2, \cos\theta)$$
$$\times F_{P\gamma\gamma}(-Q_1^2, -(Q_1+Q_2)^2)\, F_{P\gamma\gamma}(-Q_2^2, 0)$$

$$a_\mu^{P-\mathrm{pole},(2)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\cos\theta\, w_2(Q_1, Q_2, \cos\theta)$$
$$\times F_{P\gamma\gamma}(-Q_1^2, -Q_2^2)\, F_{P\gamma\gamma}(-(Q_1+Q_2)^2, 0)$$

$$a_\mu^{P-\mathrm{pole}} = \left(\frac{\alpha}{\pi}\right)^3 \left[a_\mu^{P-\mathrm{pole},(1)} + a_\mu^{P-\mathrm{pole},(2)}\right]$$





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Known kinematic weight functions

$$a_\mu^{P-\mathrm{pole},(1)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\cos\theta \; w_1(Q_1, Q_2, \cos\theta) \times F_{P\gamma\gamma}(-Q_1^2, -(Q_1+Q_2)^2)\, F_{P\gamma\gamma}(-Q_2^2, 0)$$

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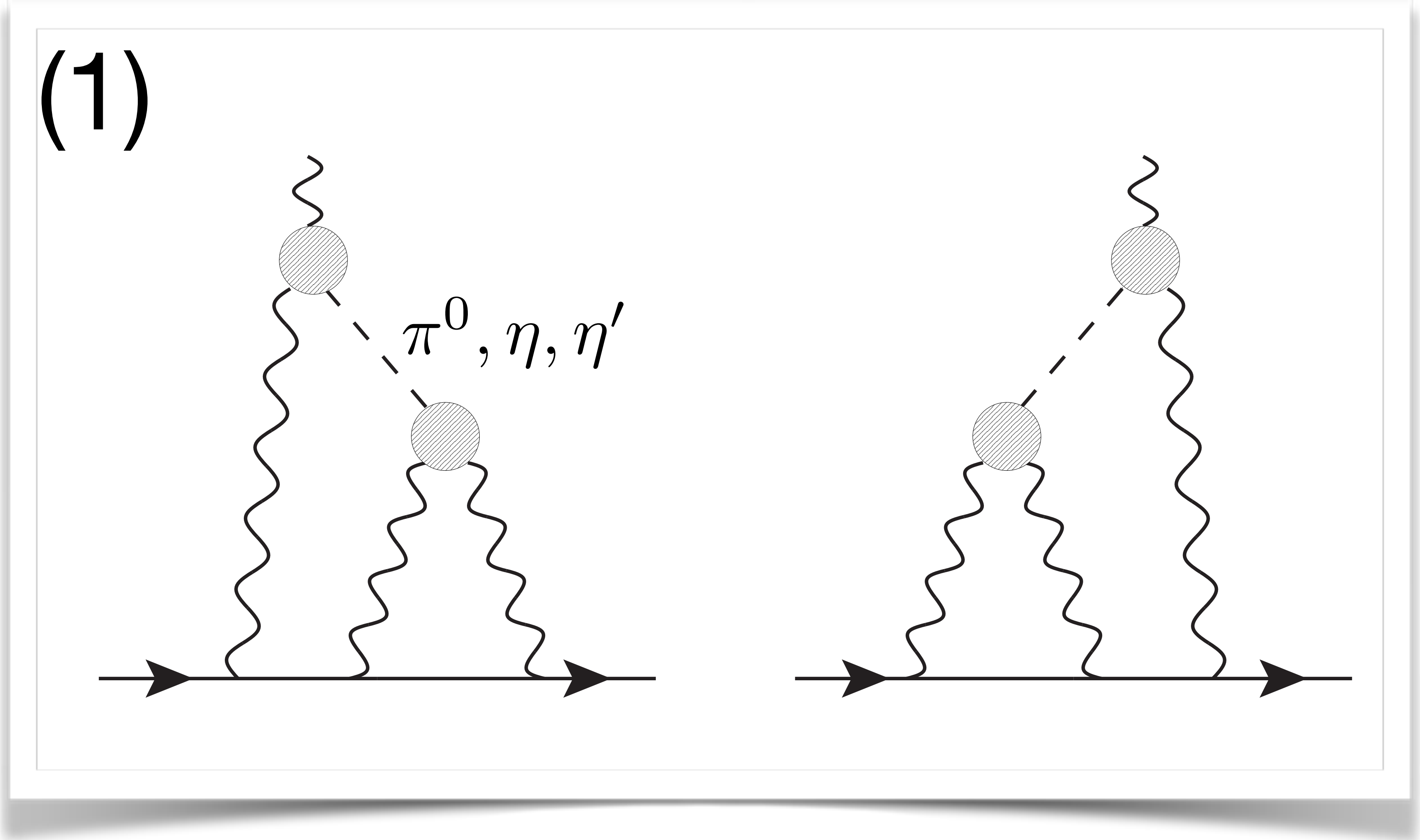

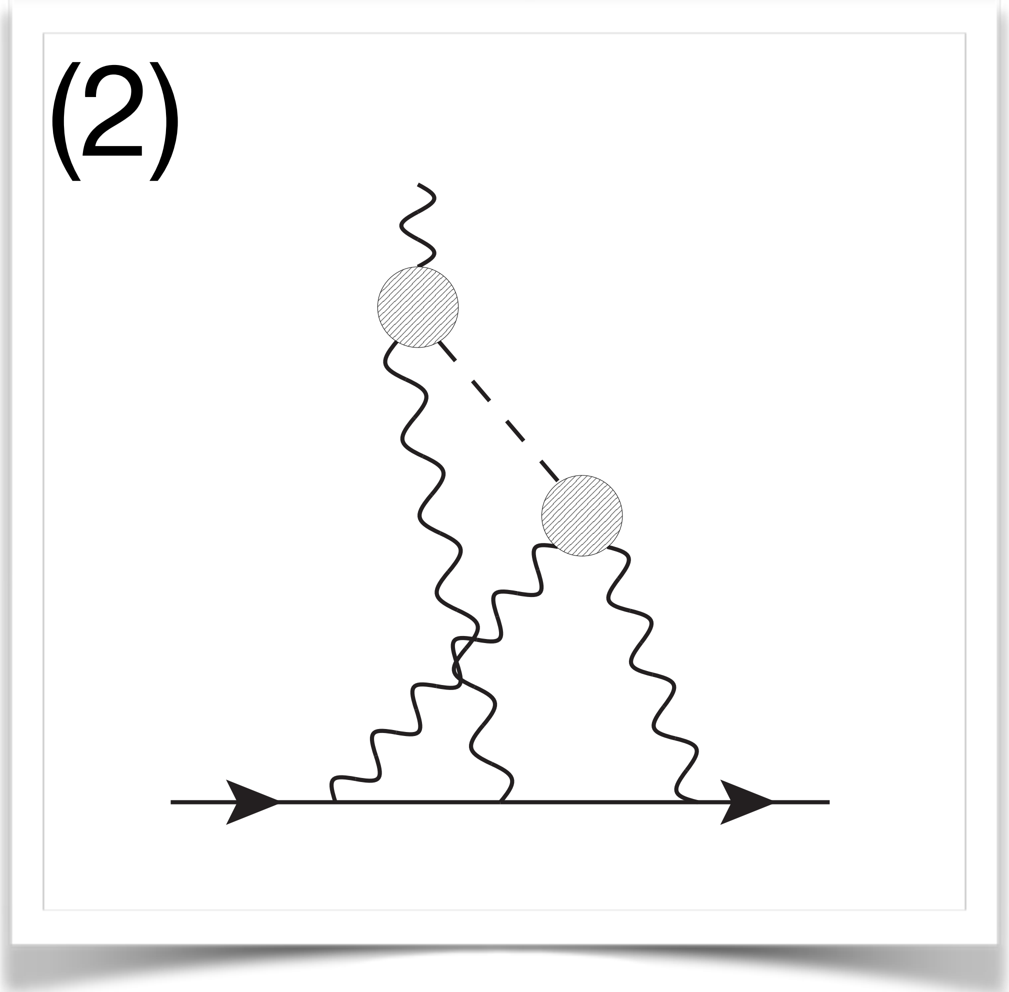

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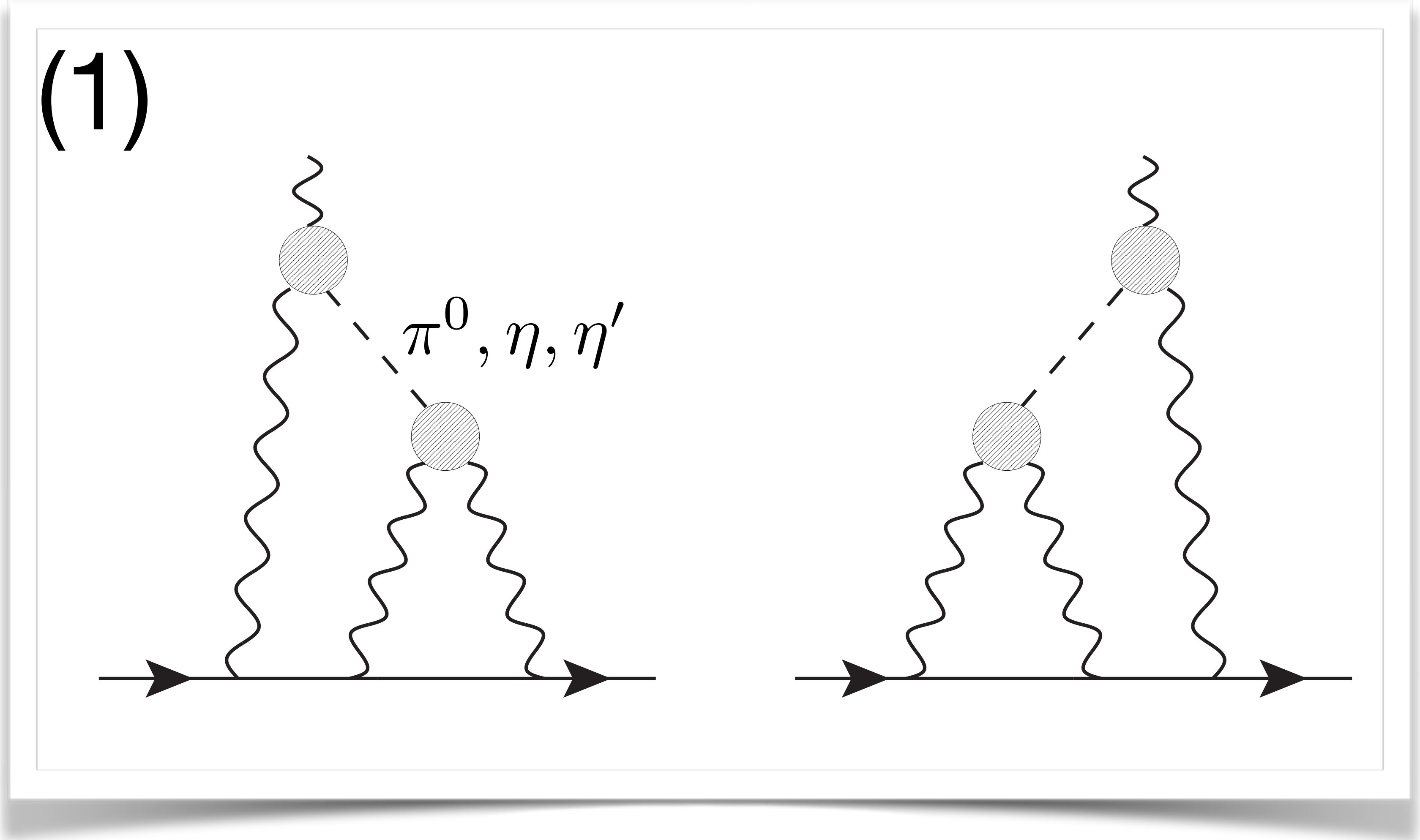
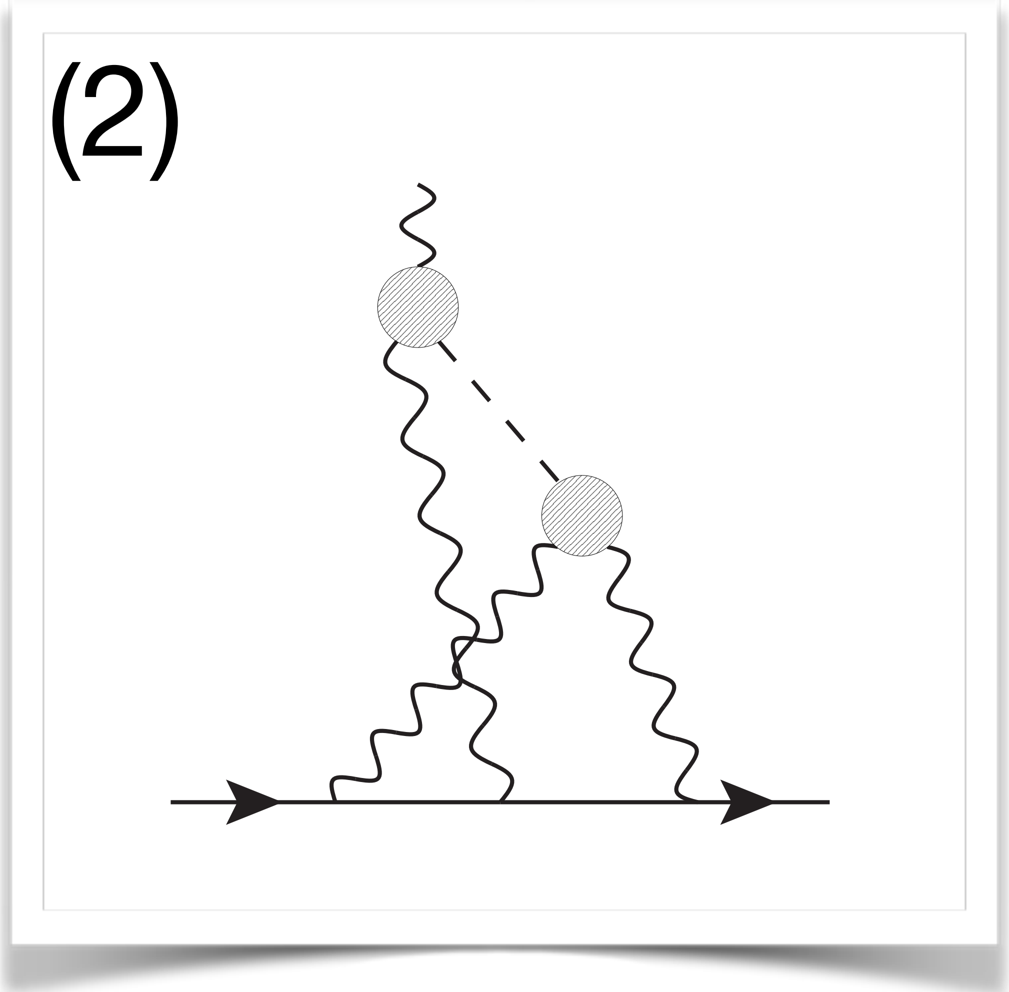
TFFs with DV x SV, spacelike structure

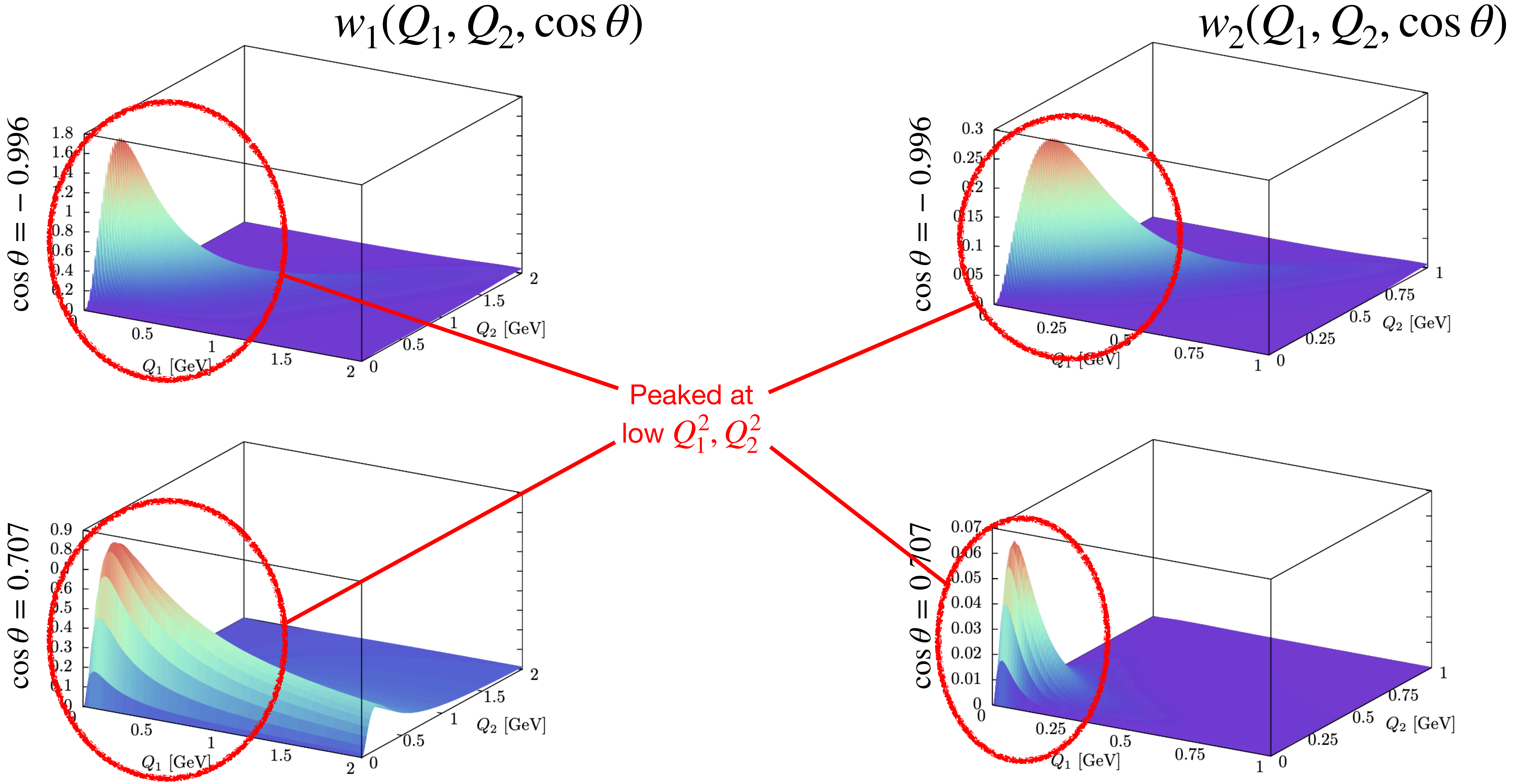
$$a_\mu^{P-\text{pole}} = \left(\frac{\alpha}{\pi}\right)^3 \left[a_\mu^{P-\text{pole},(1)} + a_\mu^{P-\text{pole},(2)}\right]$$





[Nyffeler PRD 94, 053006 (2016)]

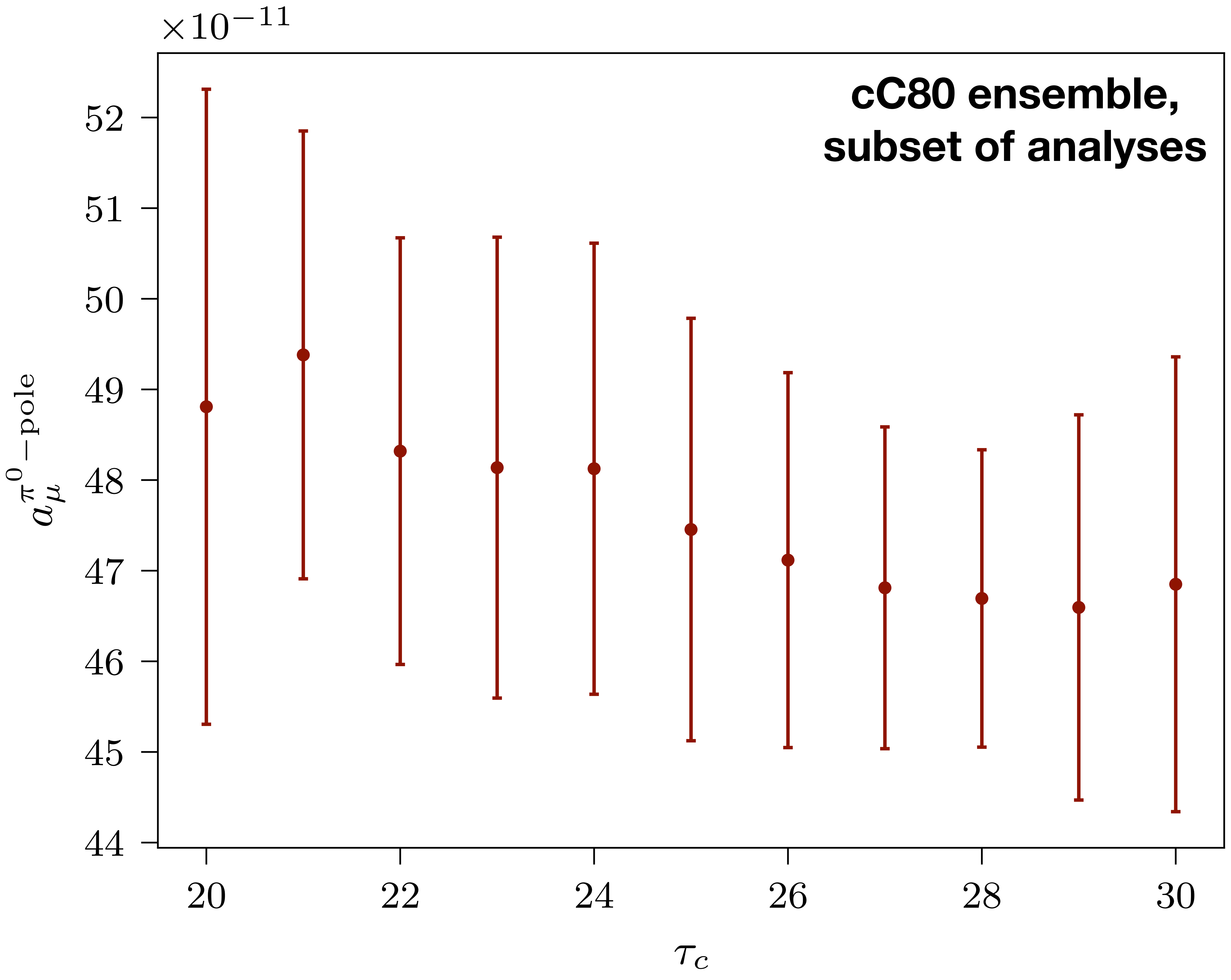
Weight functions for π^0



Muon g-2 contribution from the π^0 pole

1. z-expansion fits to FF data per choice of tail fit window, tail cut τ_c

2. Integrated $a_\mu^{\pi^0-\text{pole}}$ per FF



3. Systematic error accounting:
   - AIC weighted model averaging
     [Borsanyi+ Science 347, 1452 (2015)]
   - Checked variation with analysis choices (z-exp order, z-exp fit ranges, t_{seq})

Muon g-2 contribution from the π^0 pole

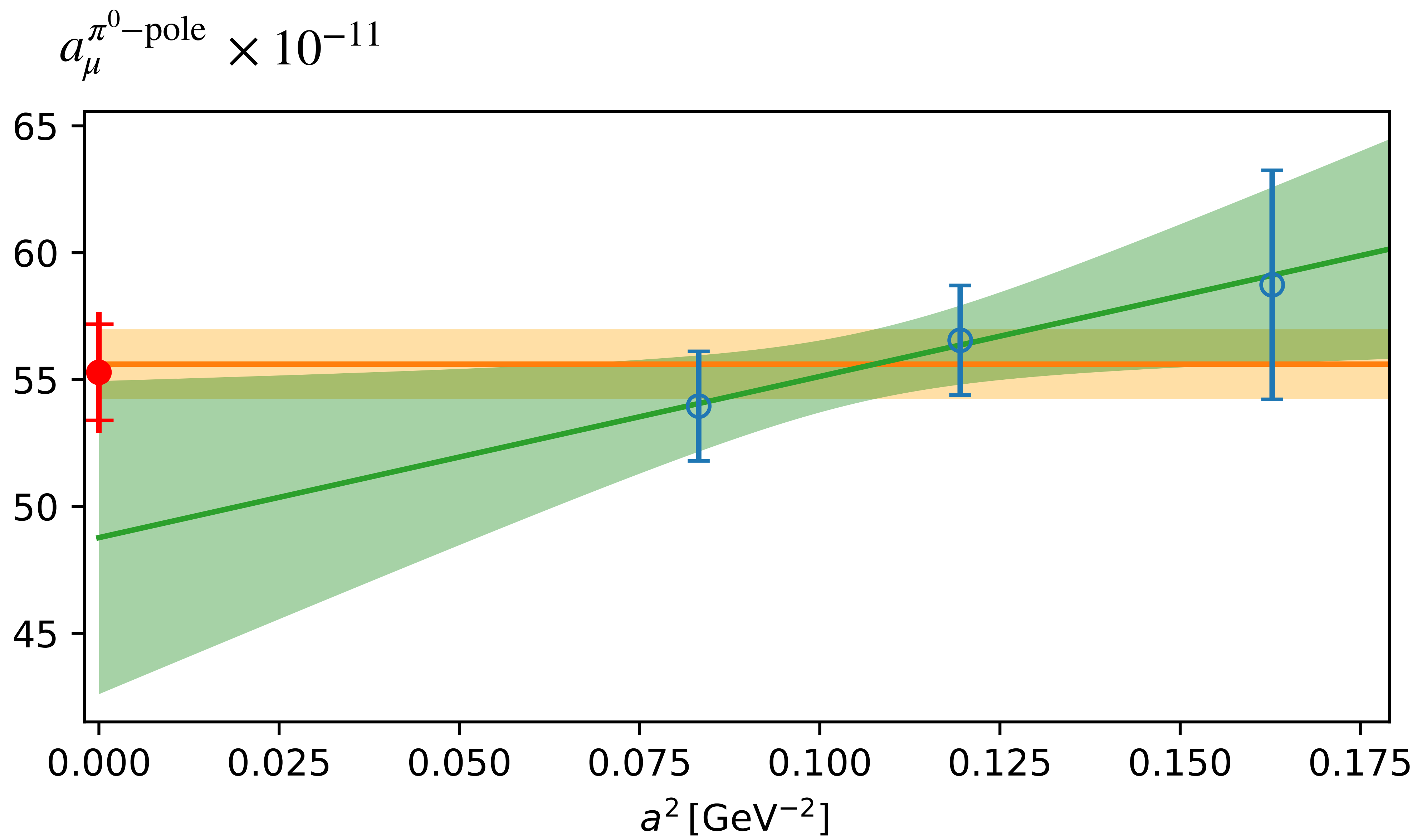
Preliminary continuum limit:

$$a_\mu^{\pi^0-\mathrm{pole}} = 55.3(1.9)_{\mathrm{ctm}}(1.5)_{\mathrm{ctm-syst}} \times 10^{-11}$$

Constant vs linear in a^2 fits.

- Constant: underestimated error
- Linear: overestimated error
- Preliminary result: weighted average + variation between fits as syst. err

[Alexandrou+ (ETMC) 2104.13408]



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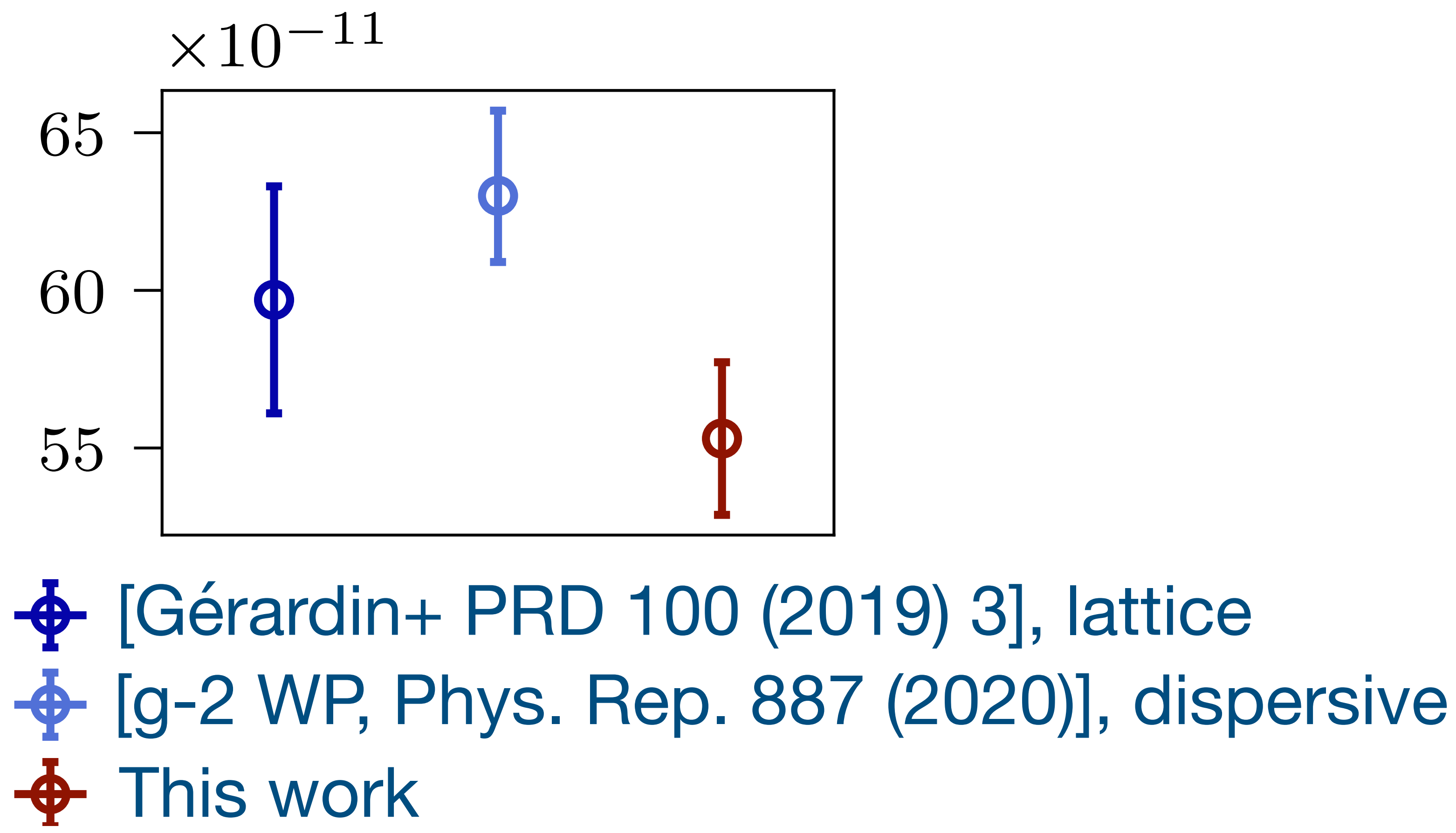

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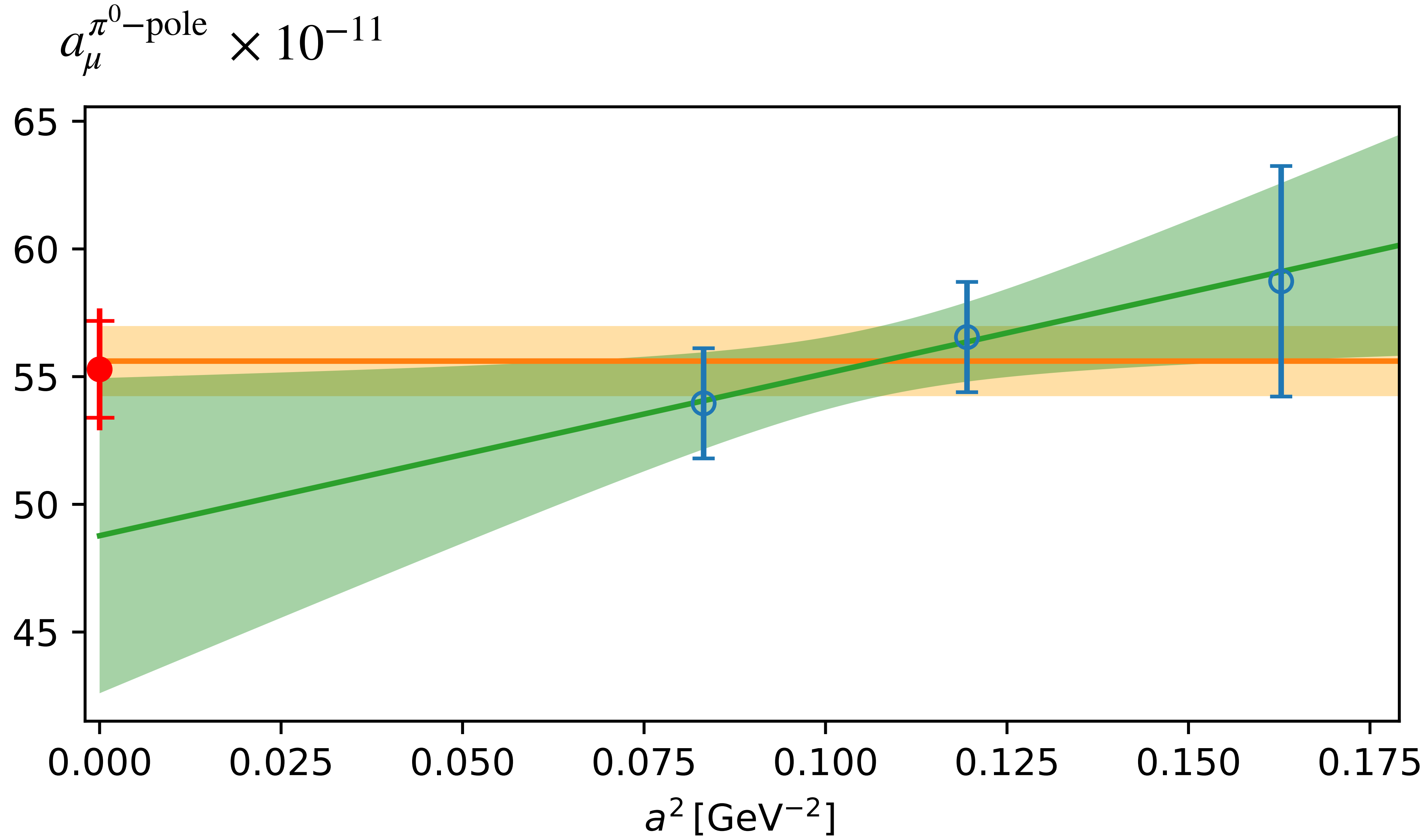
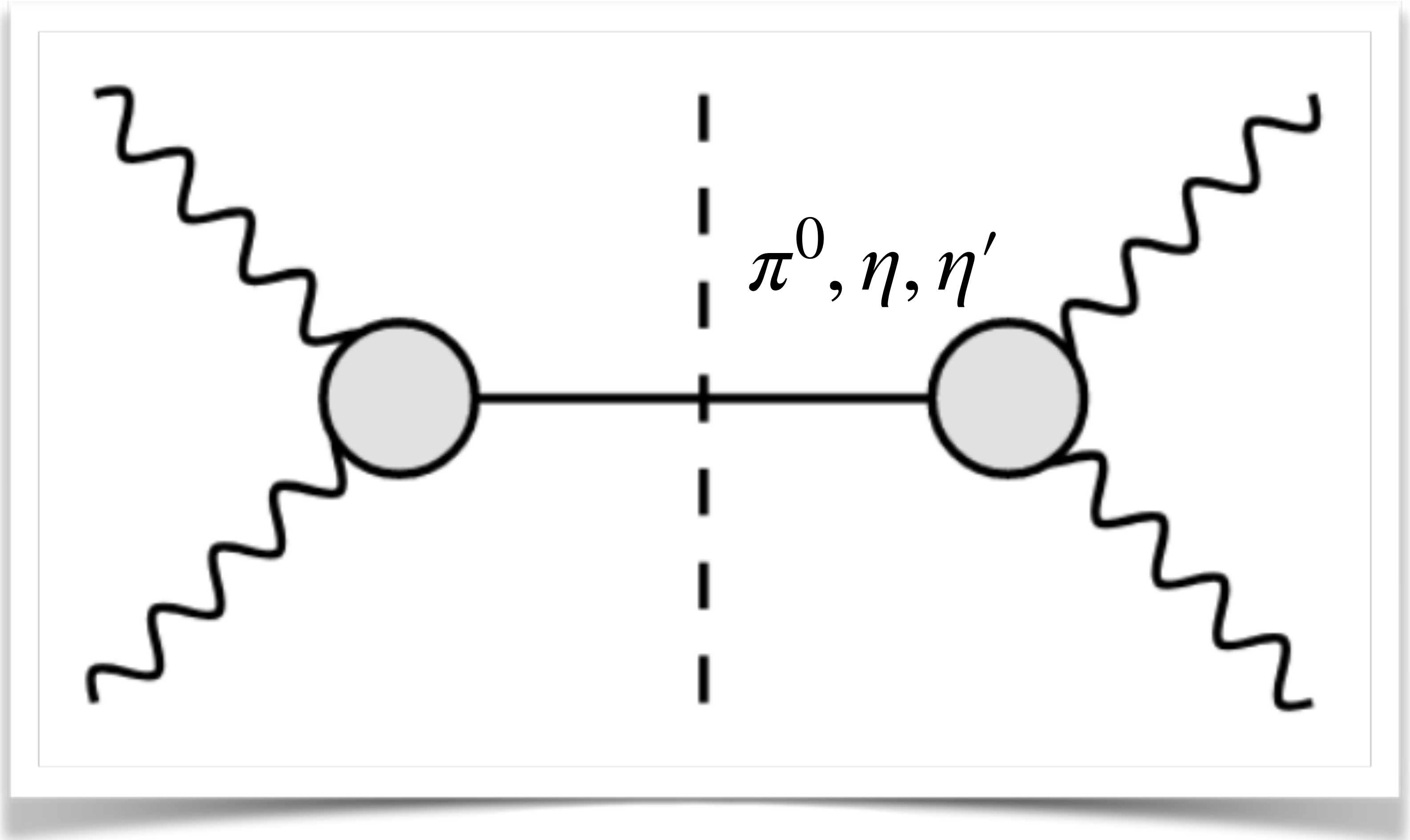


Summary

1. Unambiguous decomposition of terms of HLbL
   - Pseudoscalar-pole contribution significant uncertainty

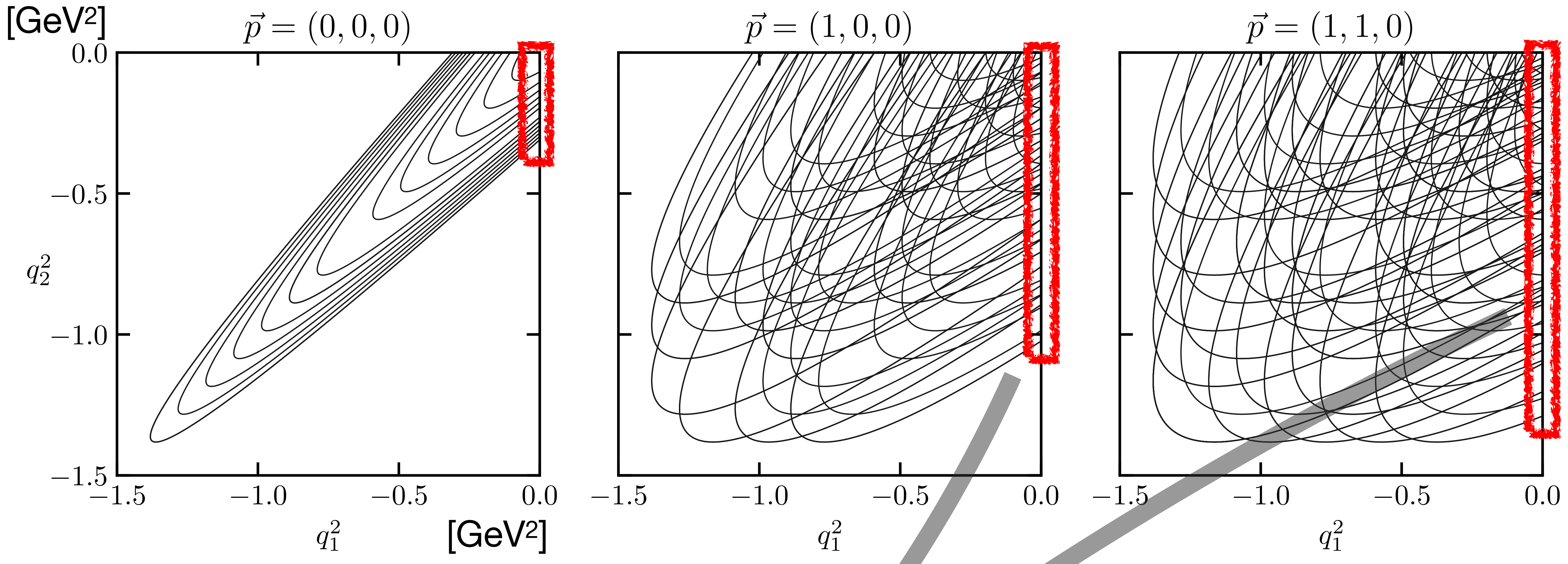
2. Form factors are the key input
   - π^0 dispersively from expt data, η, η' not yet
   - Lattice especially important for **doubly virtual**

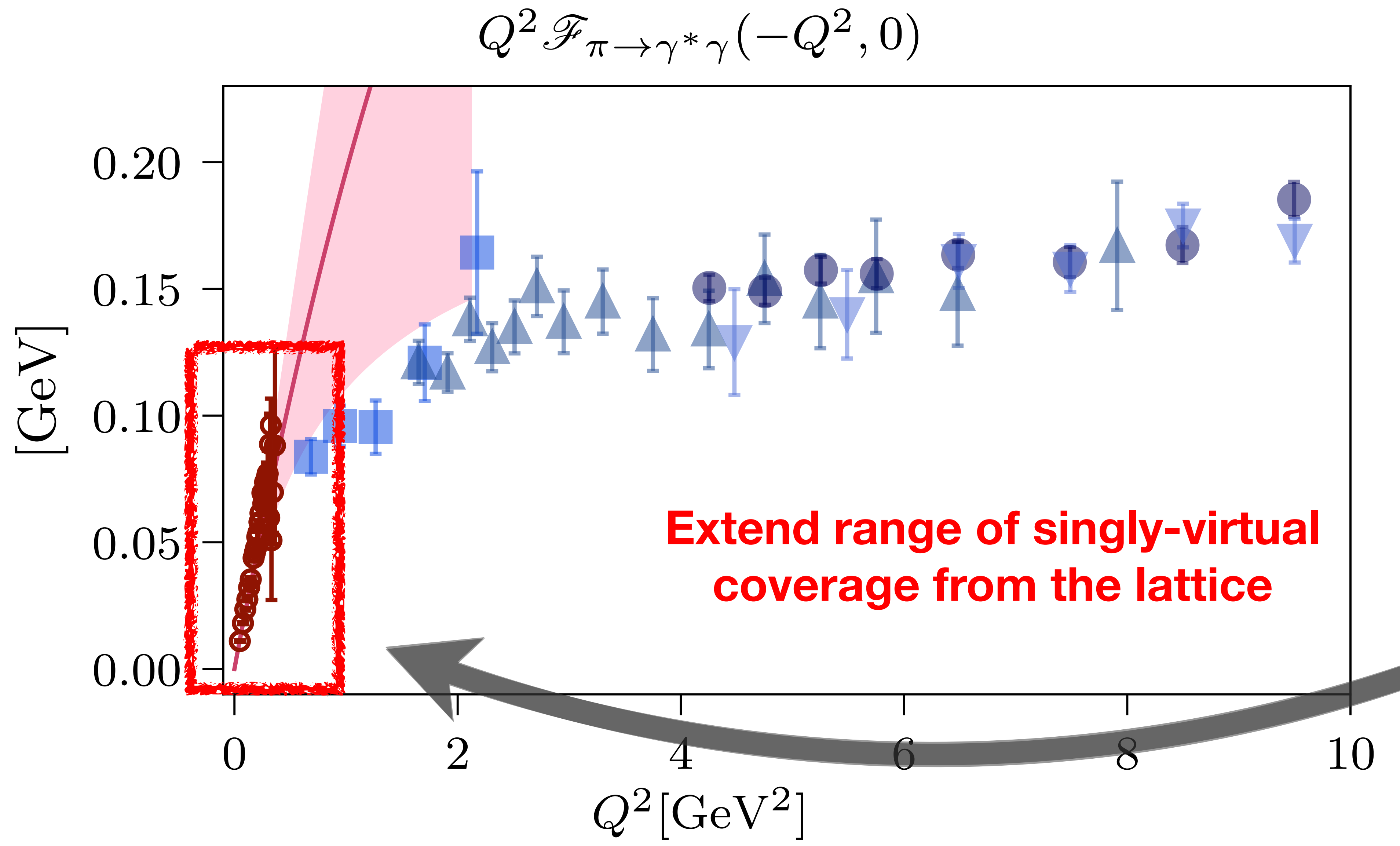


3. Comparable determination of $a_\mu^{\pi^0\text{-pole}}$ at **physical point**

Outlook

1. Boosted frame

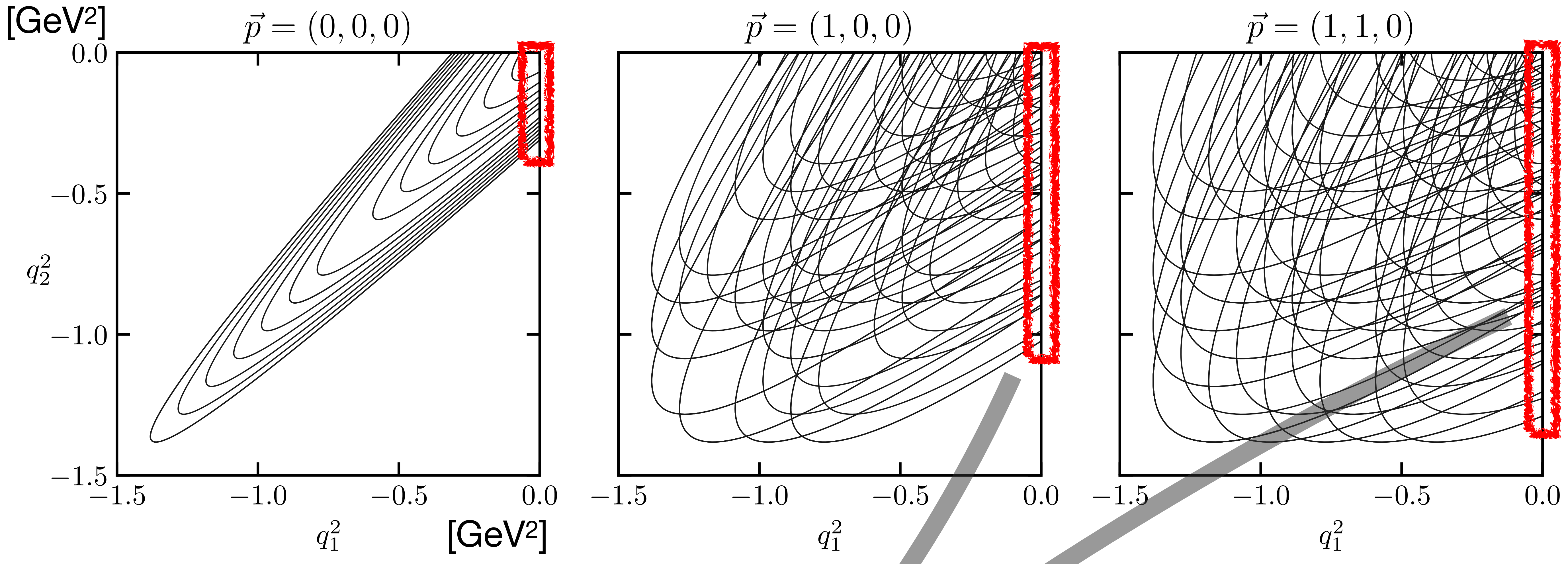


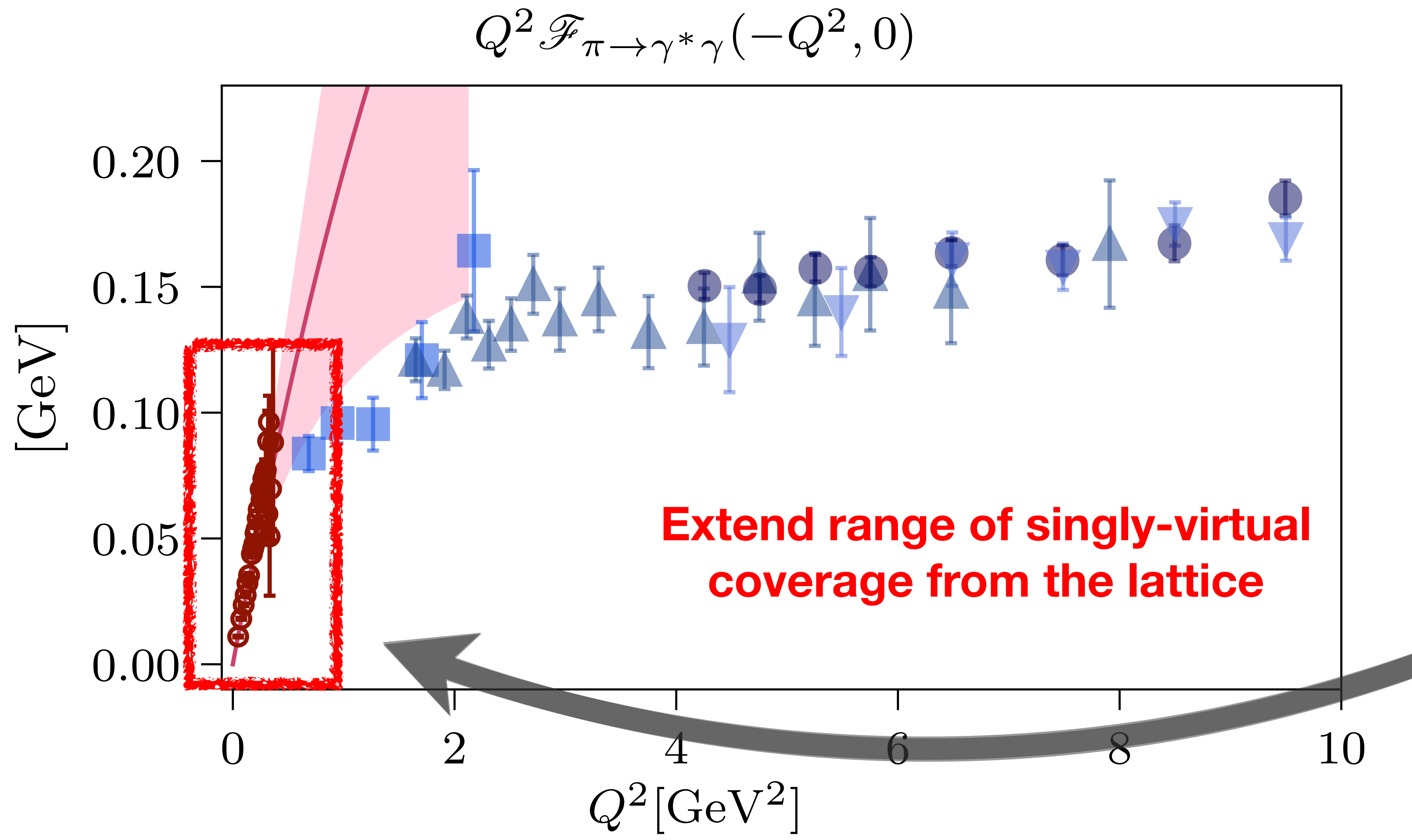


2. Other terms in HLbL decomposition?

Outlook

1. Boosted frame





Thanks for
your attention!

2. Other terms in HLbL decomposition?

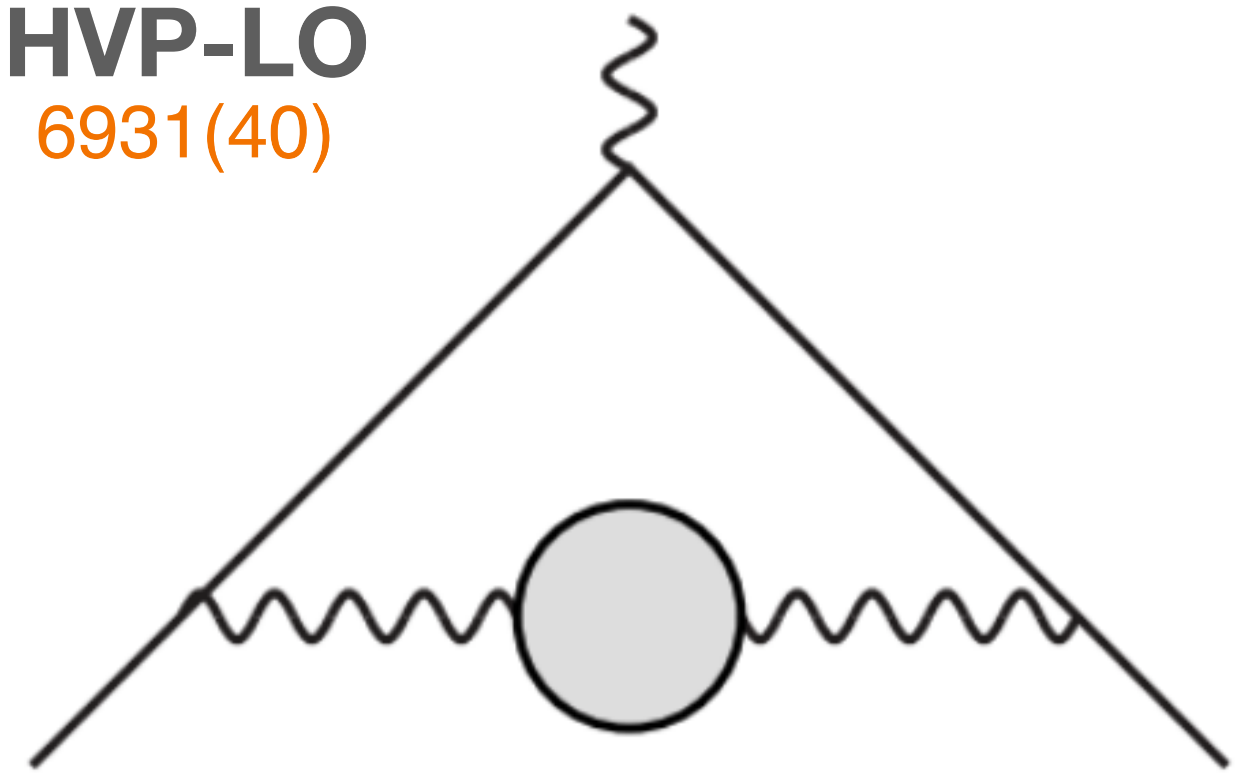
Backup slides

Hadronic contributions to a_μ

All a_μ values $\times 10^{-11}$

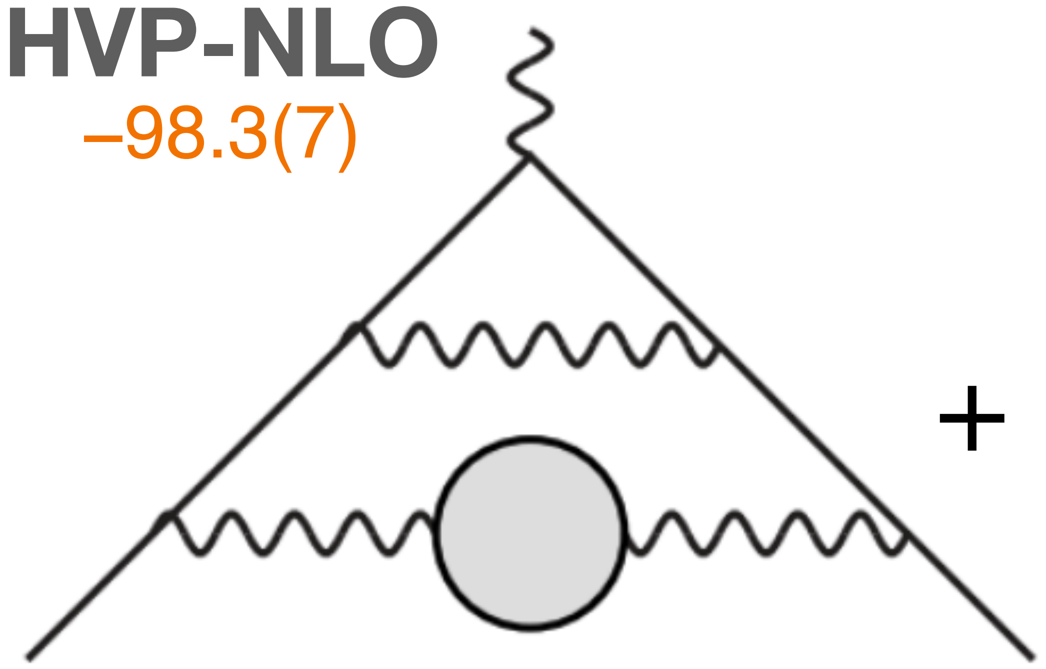
$\mathcal{O}(\alpha^2)$

HVP-LO
6931(40)



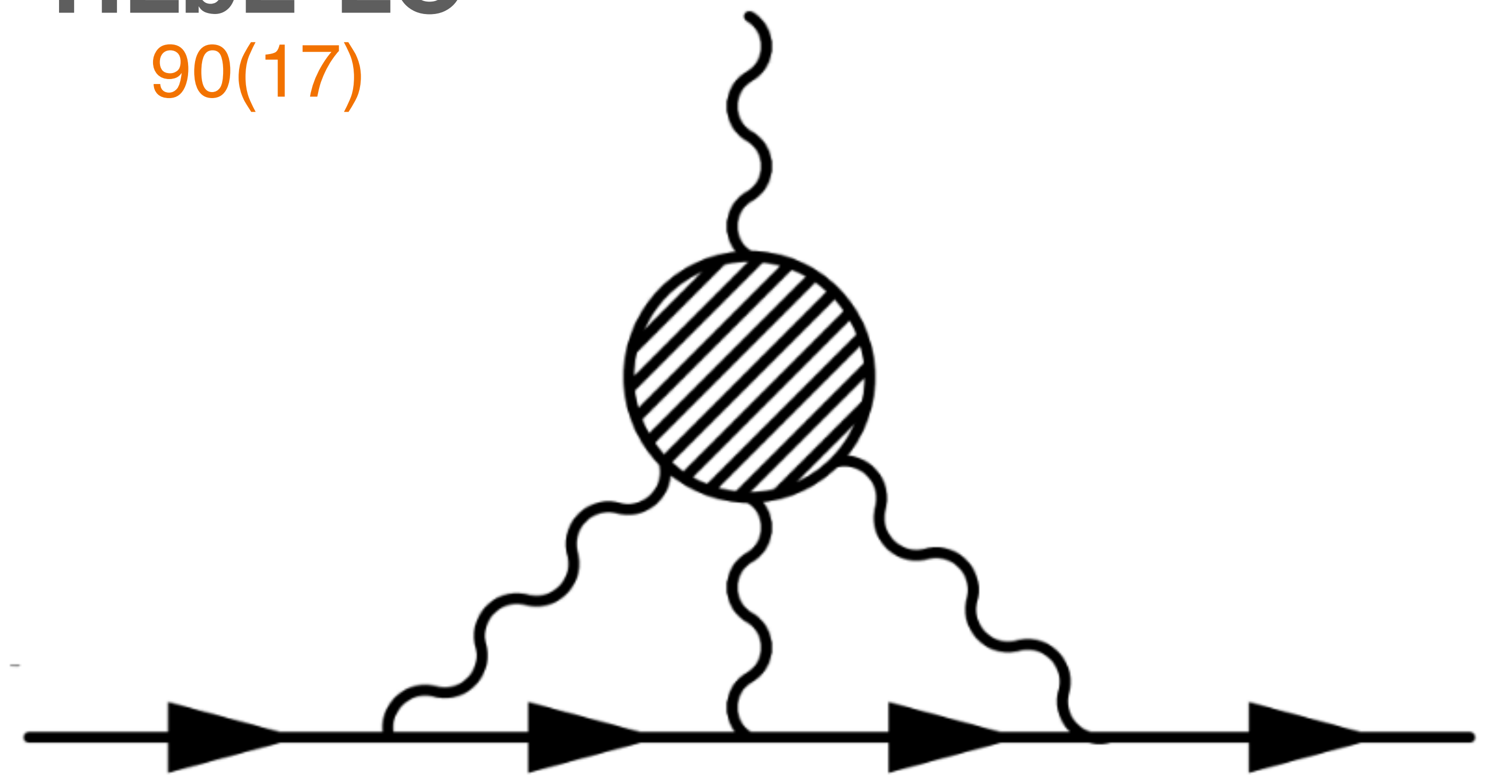
$\mathcal{O}(\alpha^3)$

HVP-NLO
–98.3(7)



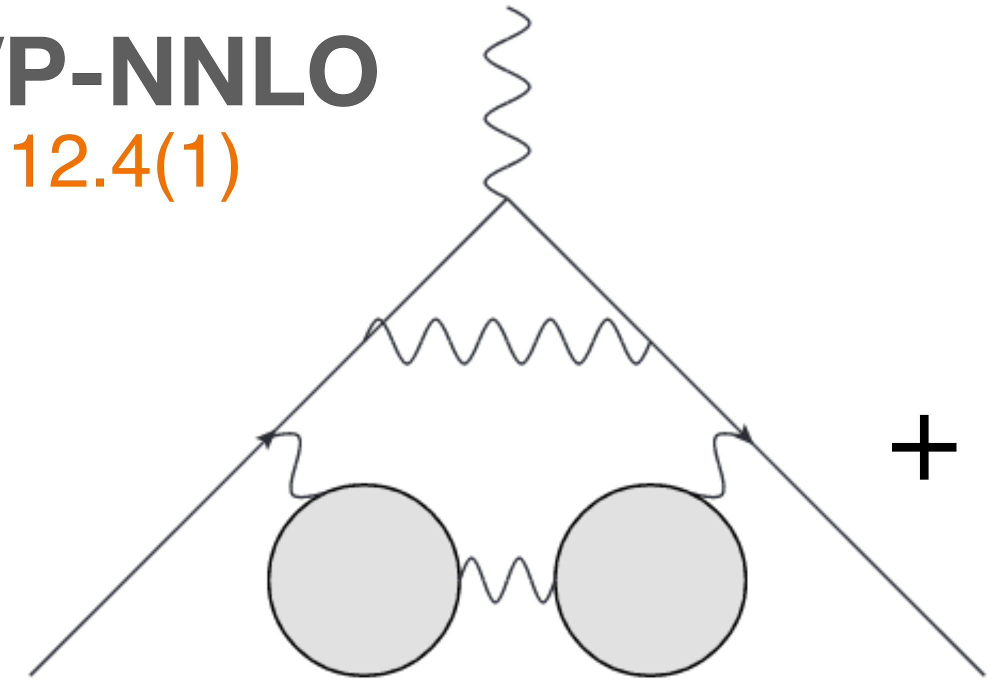
+ …

HLbL-LO
90(17)



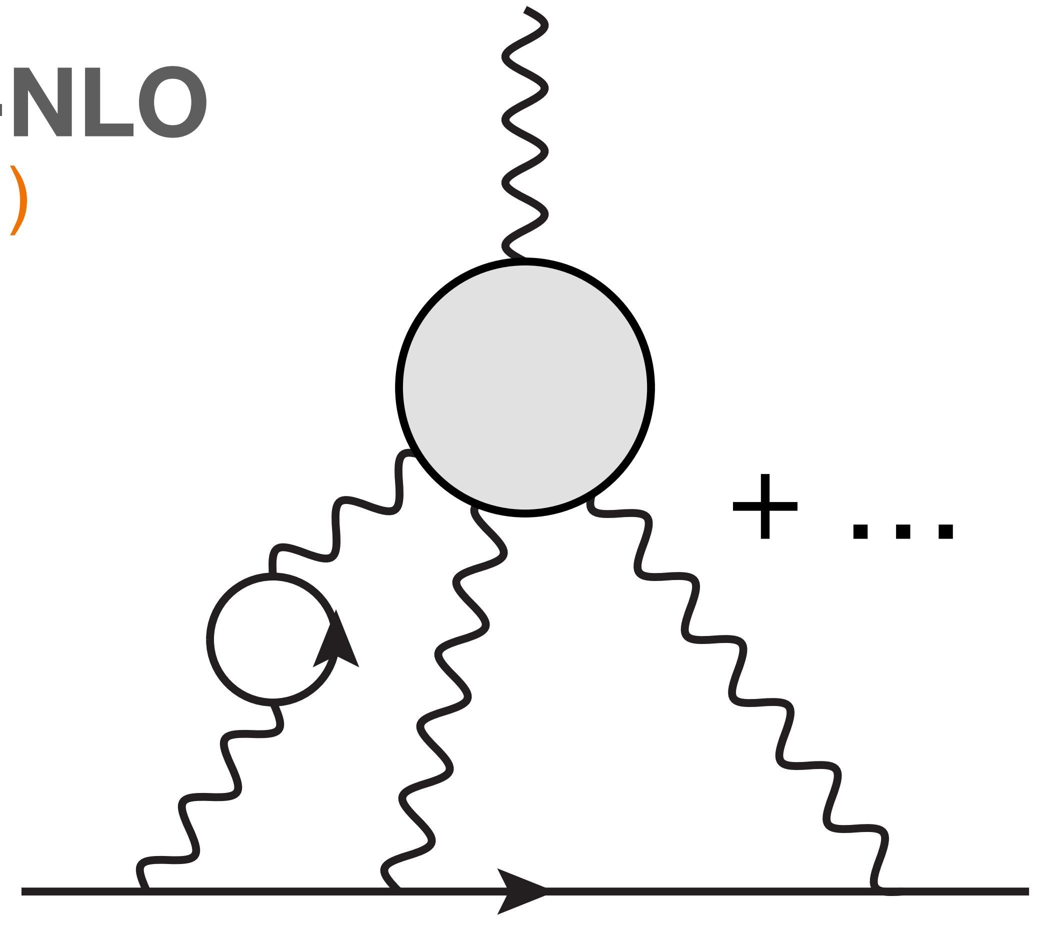
$\mathcal{O}(\alpha^4)$

HVP-NNLO
12.4(1)



+ …

HLbL-NLO
2(1)



+ …

VMD & LMD

$$F_{P\gamma\gamma}^{\mathrm{LMD}}(q_1^2, q_2^2) = \frac{\alpha M_V^4 + \beta(q_1^2 + q_2^2)}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)}$$

and

$$F_{P\gamma\gamma}^{\mathrm{VMD}}(q_1^2, q_2^2) = F_{P\gamma\gamma}^{\mathrm{LMD}}(q_1^2, q_2^2)\,|_{\beta=0}$$

$\tilde{A}^{\mathrm{LMD}}(\tau)$ and $\tilde{A}^{\mathrm{VMD}}(\tau)$ by inverse Laplace transform

According to VMD, $M_V = M_\rho$,
but left as a free fit parameter
in this work

z-expansion

Conformal transformation:

$$z_k = \frac{\sqrt{t_c - Q_k^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q_k^2} + \sqrt{t_c - t_0}}$$

$k \in \{1,2\}$

$t_c = 4m_\pi^2$ given by 2π threshold

t_0 chosen to minimize $|z_k|$ over range of interest

Polynomial fit to cutoff order $N \in \{1,2\}$ used in this work.

Multiplicative factor to match B-L and OPE asymptotics:

$$P(Q_1^2, Q_2^2) = 1 + \frac{Q_1^2 + Q_2^2}{M_V^2}$$

Relevant momenta and relative contribs

Models for $F_{P\gamma\gamma}$

Λ [GeV]	π^0 [LMD+V]	π^0 [VMD]	η [VMD]	η' [VMD]
0.25	14.4 (22.9%)	14.4 (25.2%)	1.8 (12.1%)	1.0 (7.9%)
0.5	36.8 (58.5%)	36.6 (64.2%)	6.9 (47.5%)	4.5 (36.1%)
0.75	48.5 (77.1%)	47.7 (83.8%)	10.7 (73.4%)	7.8 (62.5%)
1.0	54.1 (86.0%)	52.6 (92.3%)	12.6 (86.6%)	9.9 (79.1%)
1.5	58.8 (93.4%)	55.8 (97.8%)	14.0 (96.1%)	11.7 (93.1%)
2.0	60.5 (96.2%)	56.5 (99.2%)	14.3 (98.6%)	12.2 (97.4%)
5.0	62.5 (99.4%)	56.9 (99.9%)	14.5 (100%)	12.5 (99.9%)
20.0	62.9 (100%)	57.0 (100%)	14.5 (100%)	12.5 (100%)

All contribs saturate around here

[Table 5 of Nyffeler Phys. Rev. D 94, 053006 (2016)]

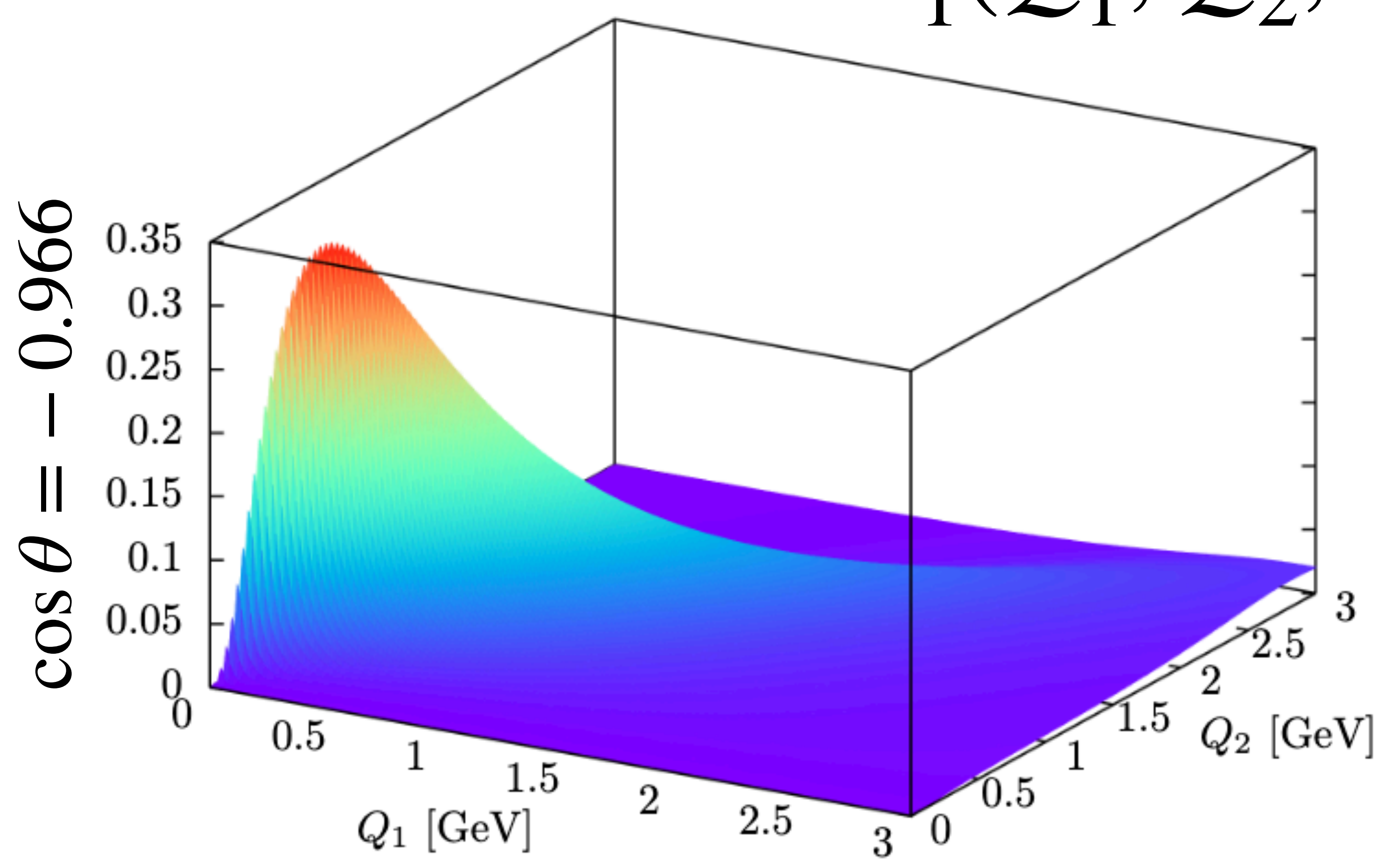
$a_\mu^{P-\mathrm{pole}}$ integrating over

$$0 \le Q_1, Q_2 \le \Lambda$$

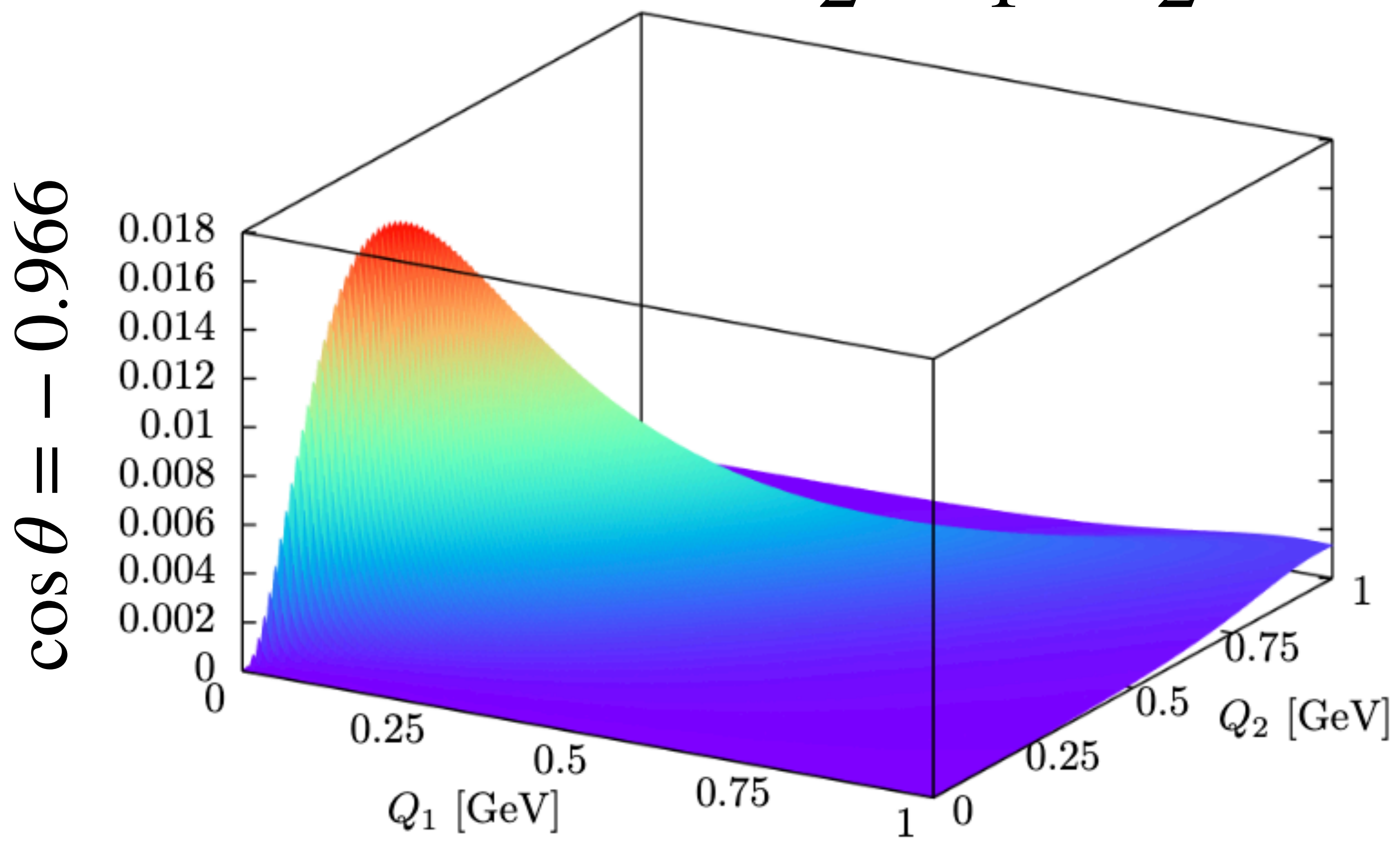
[Nyffeler Phys. Rev. D 94, 053006 (2016)]

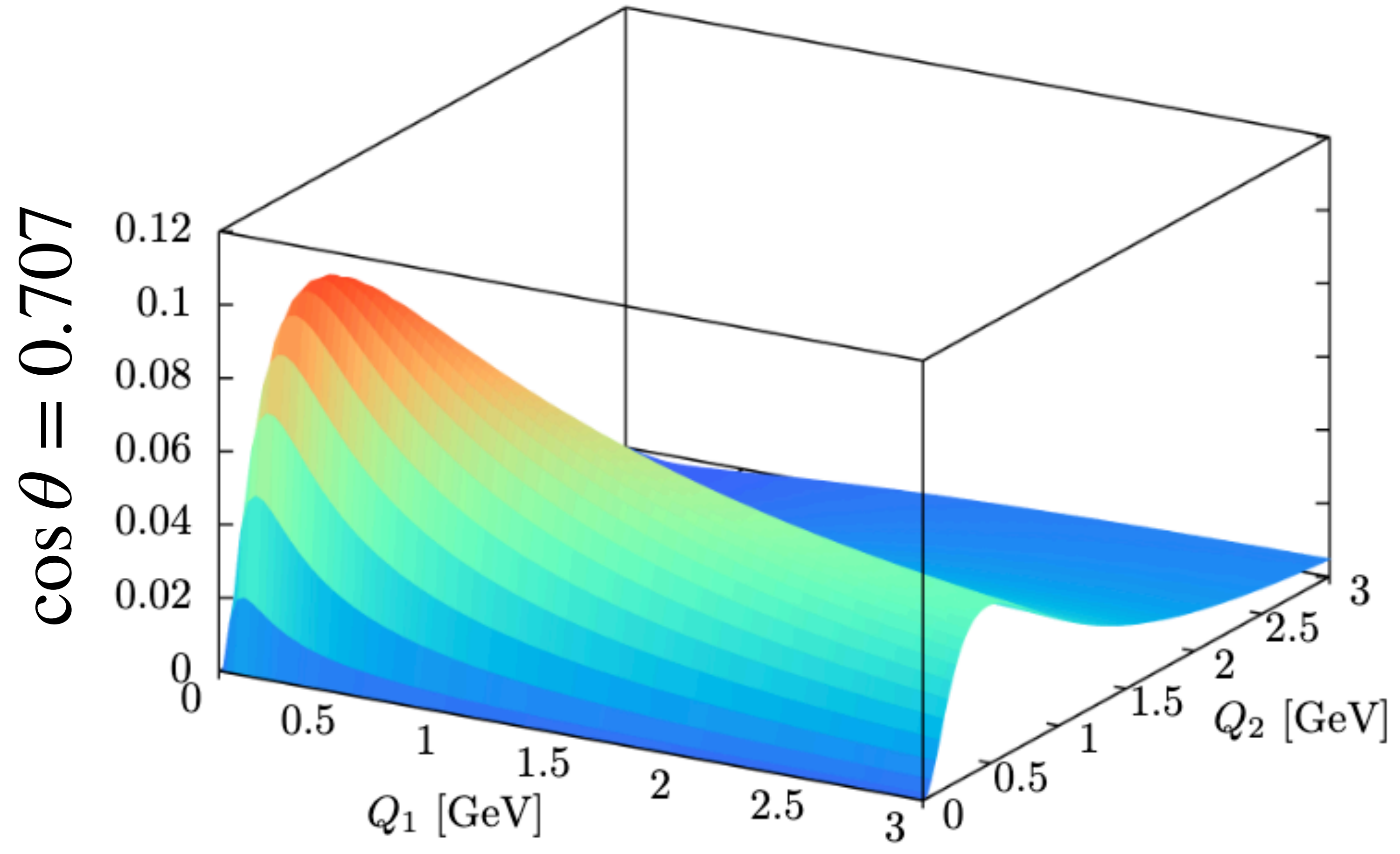
Weight functions: η

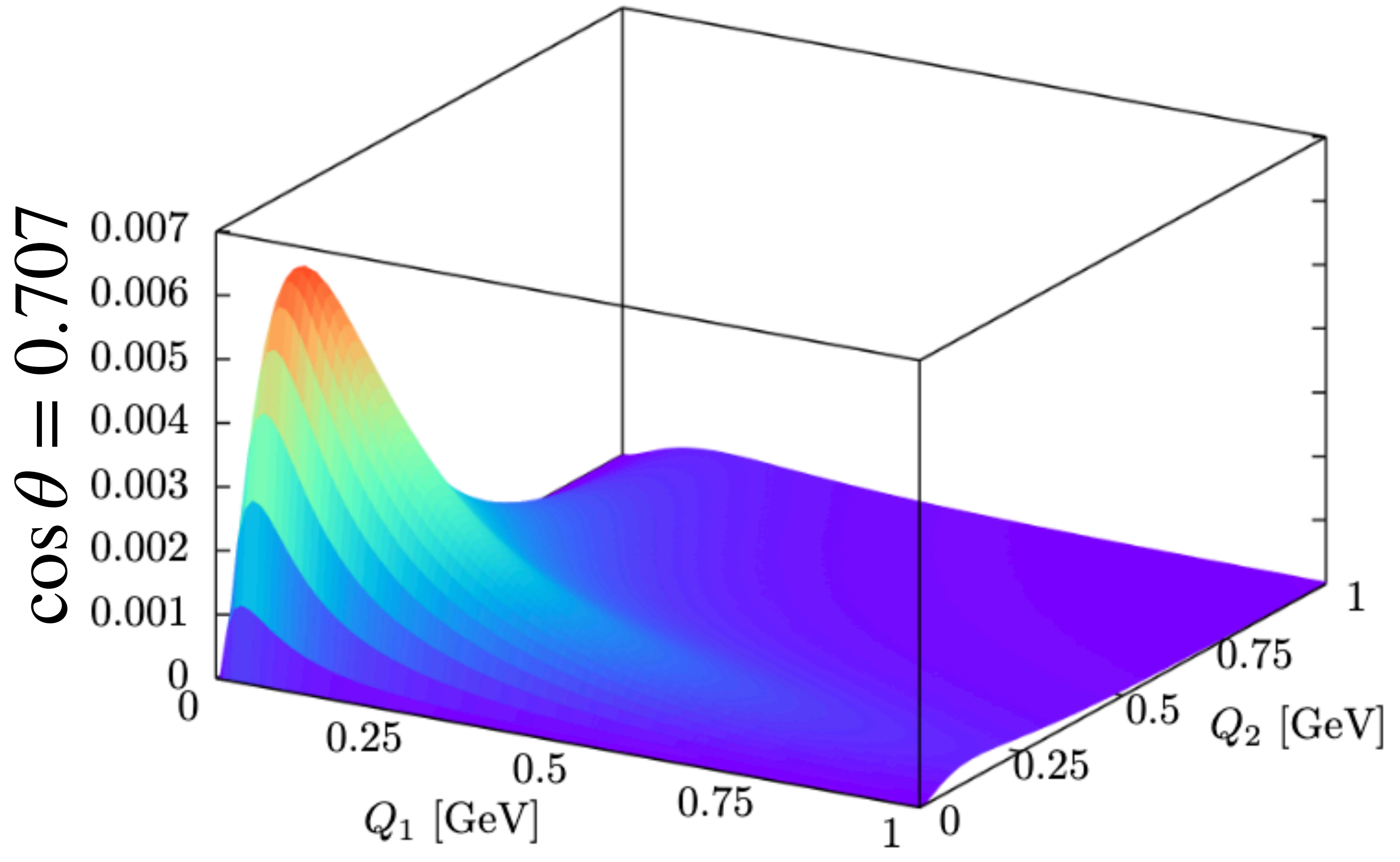
$w_1(Q_1, Q_2, \cos\theta)$



$w_2(Q_1, Q_2, \cos\theta)$

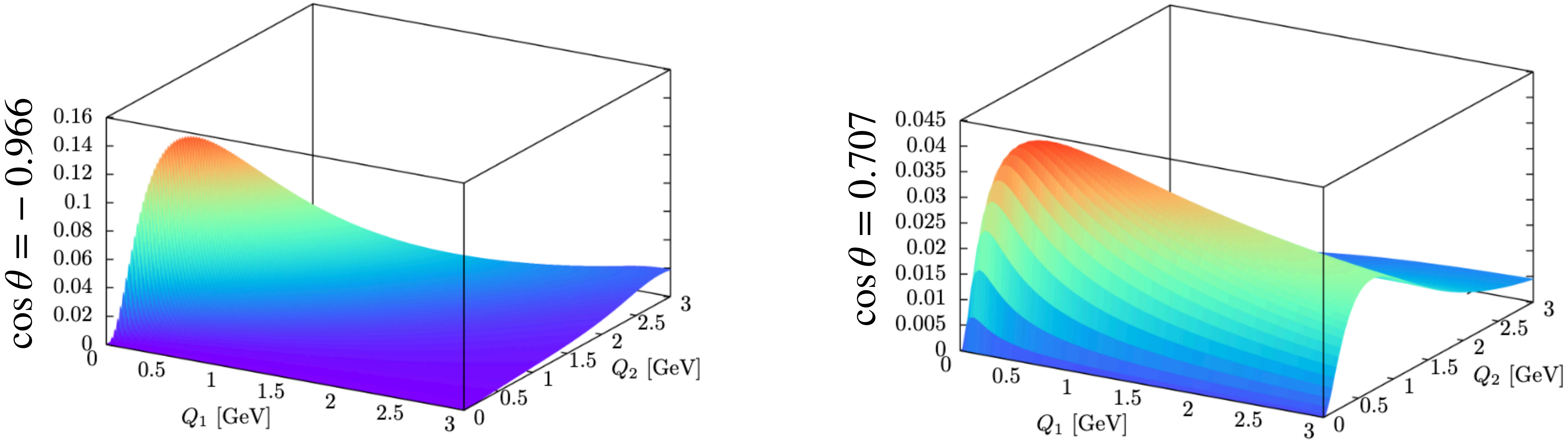




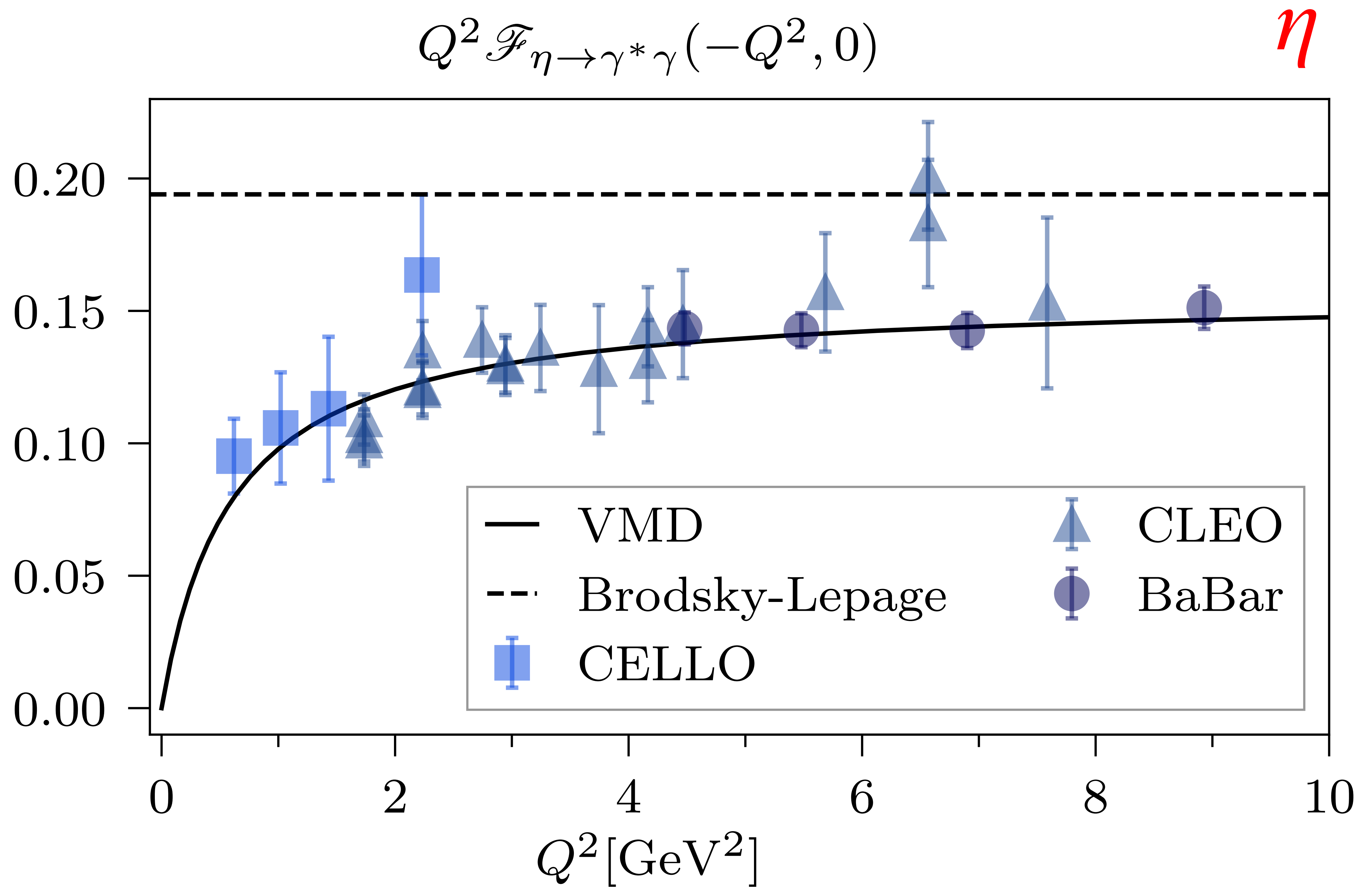


Weight functions: η'

$$w_1(Q_1, Q_2, \cos\theta)$$



TFF data: singly virtual η



Other terms in HLbL decomposition

Contribution	Ref. [6]
π^0, η, η'-poles	93.8(4.0)
π, K-loops/boxes	$-16.4(2)$
S-wave $\pi\pi$ rescattering	$-8(1)$
subtotal	69.4(4.1)
scalars	$-1(3)$
tensors	
axial vectors	6(6)
u, d, s-loops / short-distance	15(10)
c-loop	3(1)
total	92(19)

[Colangelo+ Snowmass White Paper, 2203.15810]

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Addressed by this work

AV form factors accessible on the lattice… future work?

Theoretical ambiguities actively being addressed by data-driven community

[Colangelo+ Snowmass White Paper, 2203.15810]