Emergent strongly coupled ultraviolet fixed point with 8 fundamental flavors

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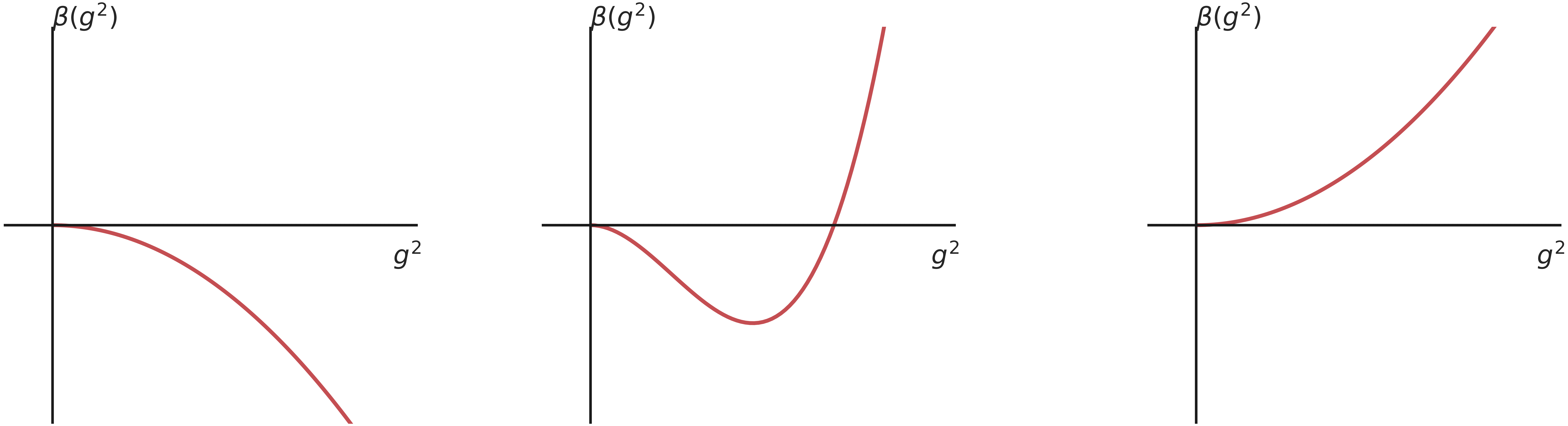
Based on a recent publication: PRD 106 (2022) 014513

Phases of gauge-fermion systems

SU(3) gauge + N_f (fundamental) fermions

small N_f — Confining | $N^* < N_f < N^{IF}$ — Conformal | $N_f > N^{IF}$ — Infrared free



Perturbatively: the IR fixed point emerges at $g_0^2 = \infty$ at $N_f = N^*$, moves to $g_0^2 = 0$ as $N_f \to N^{IF}$

Nonperturbatively: the IR fixed point could emerge at finite g_*^2 if $\beta(g) \sim (\alpha - \alpha_*) - (g - g_*)^2$

Kaplan et al PRD80,125005 (2009)
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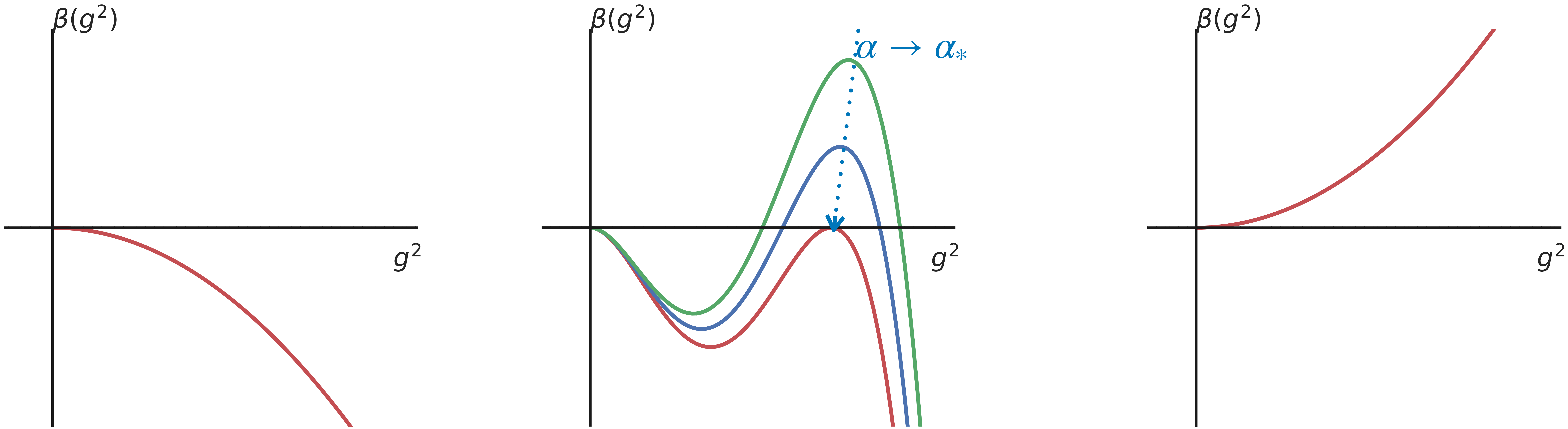

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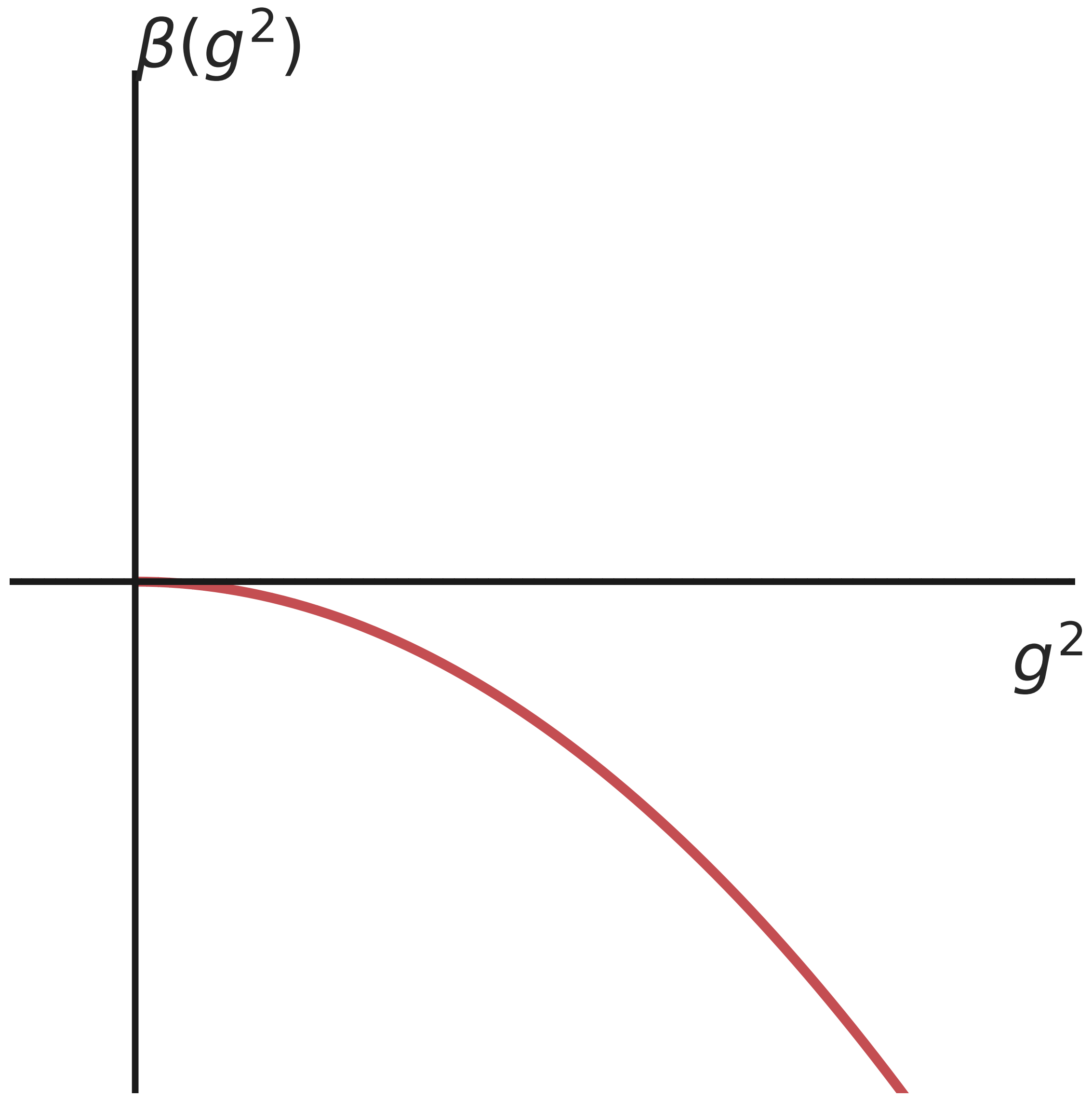

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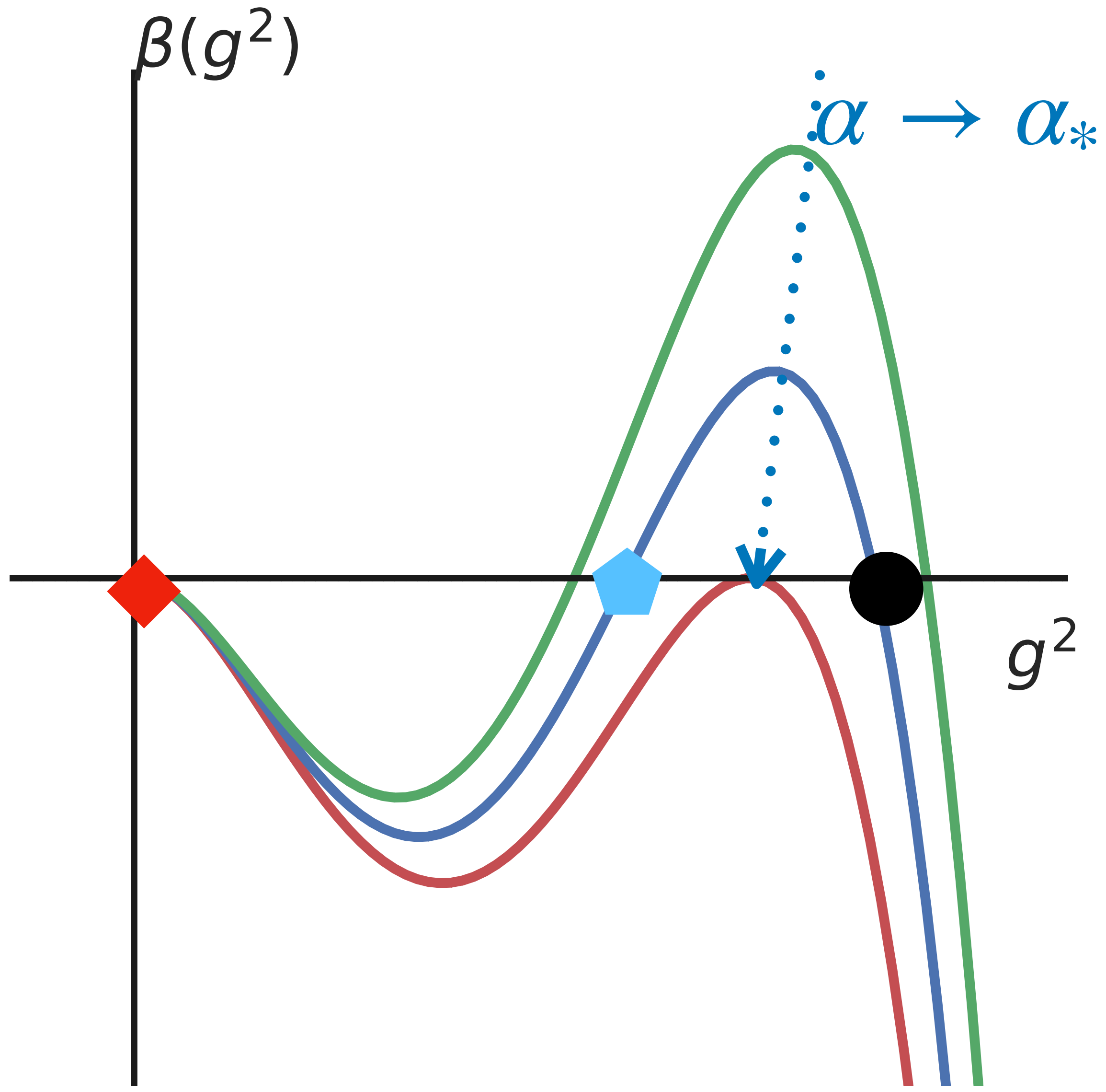

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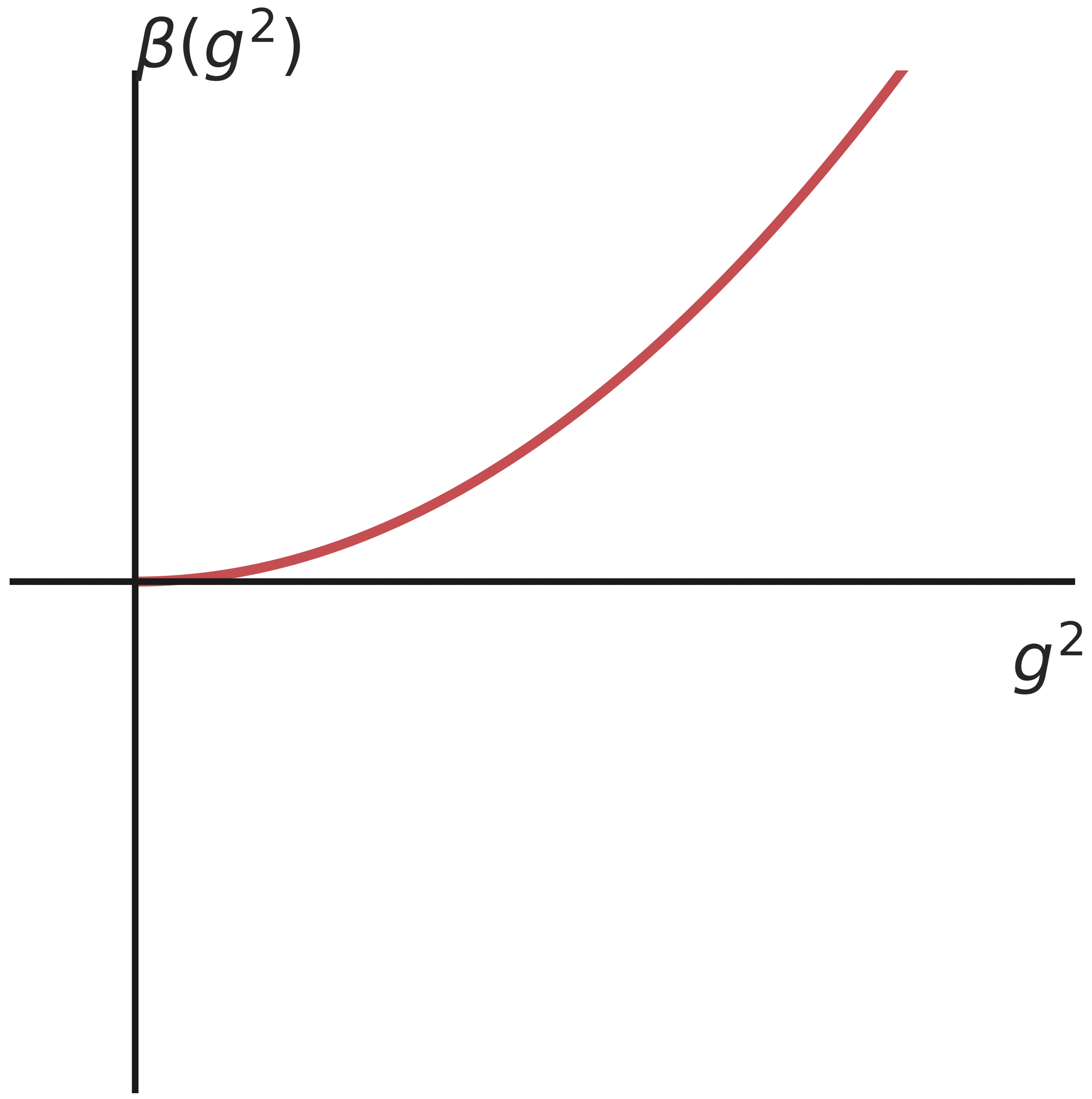
Infrared free



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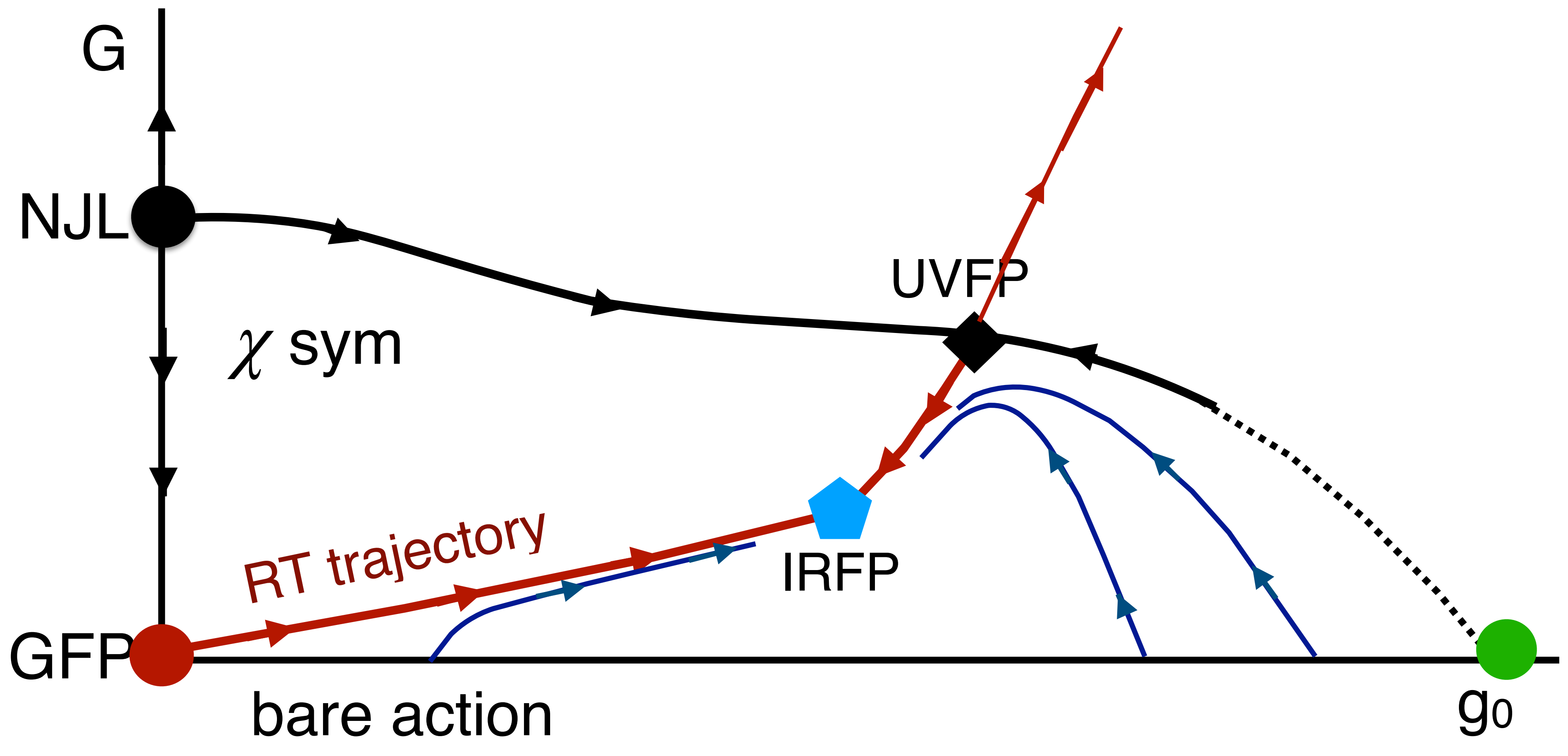
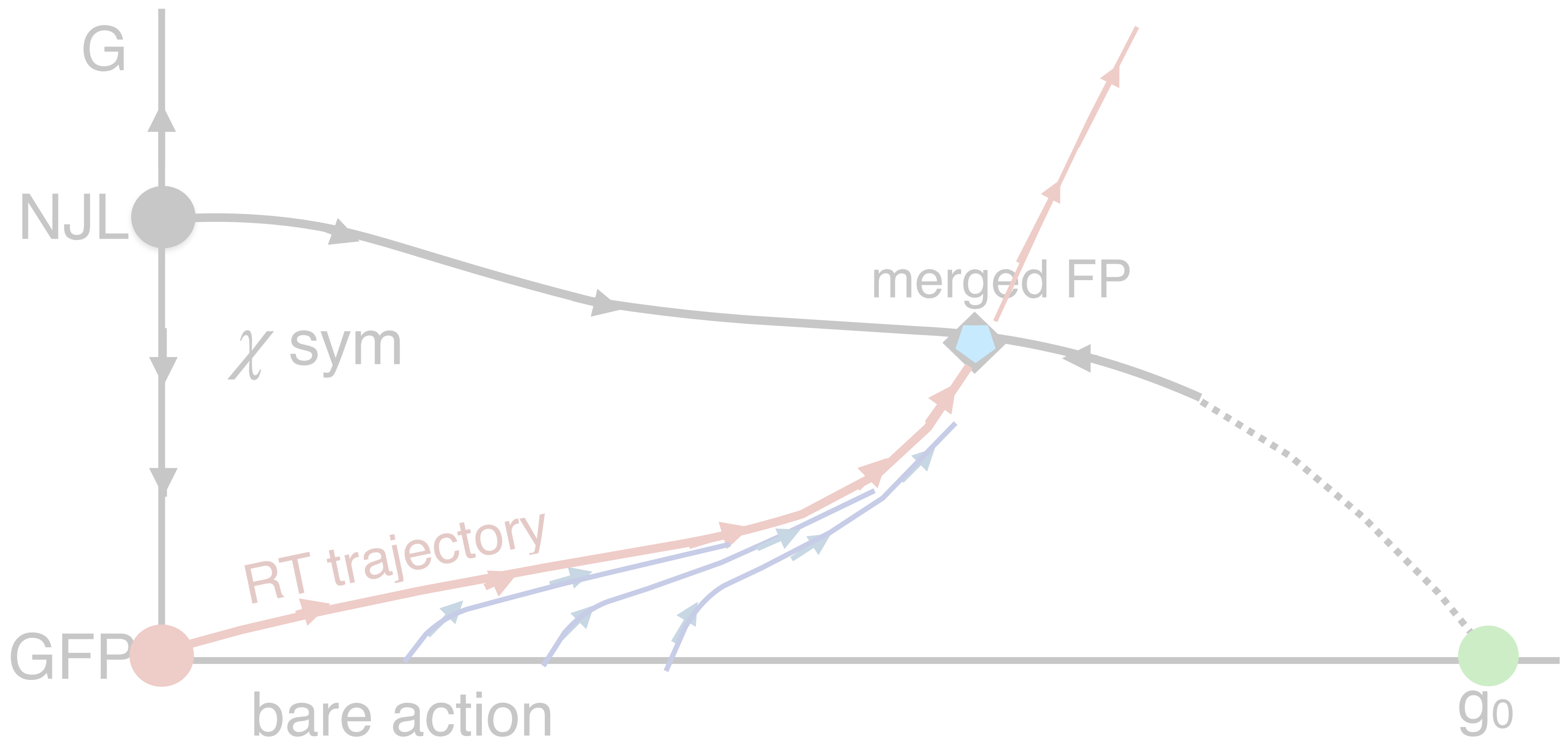
Extended parameter space



- What is the new relevant operator?
- What is the strongly coupled phase?
- What is the continuum theory on the strong coupling side?

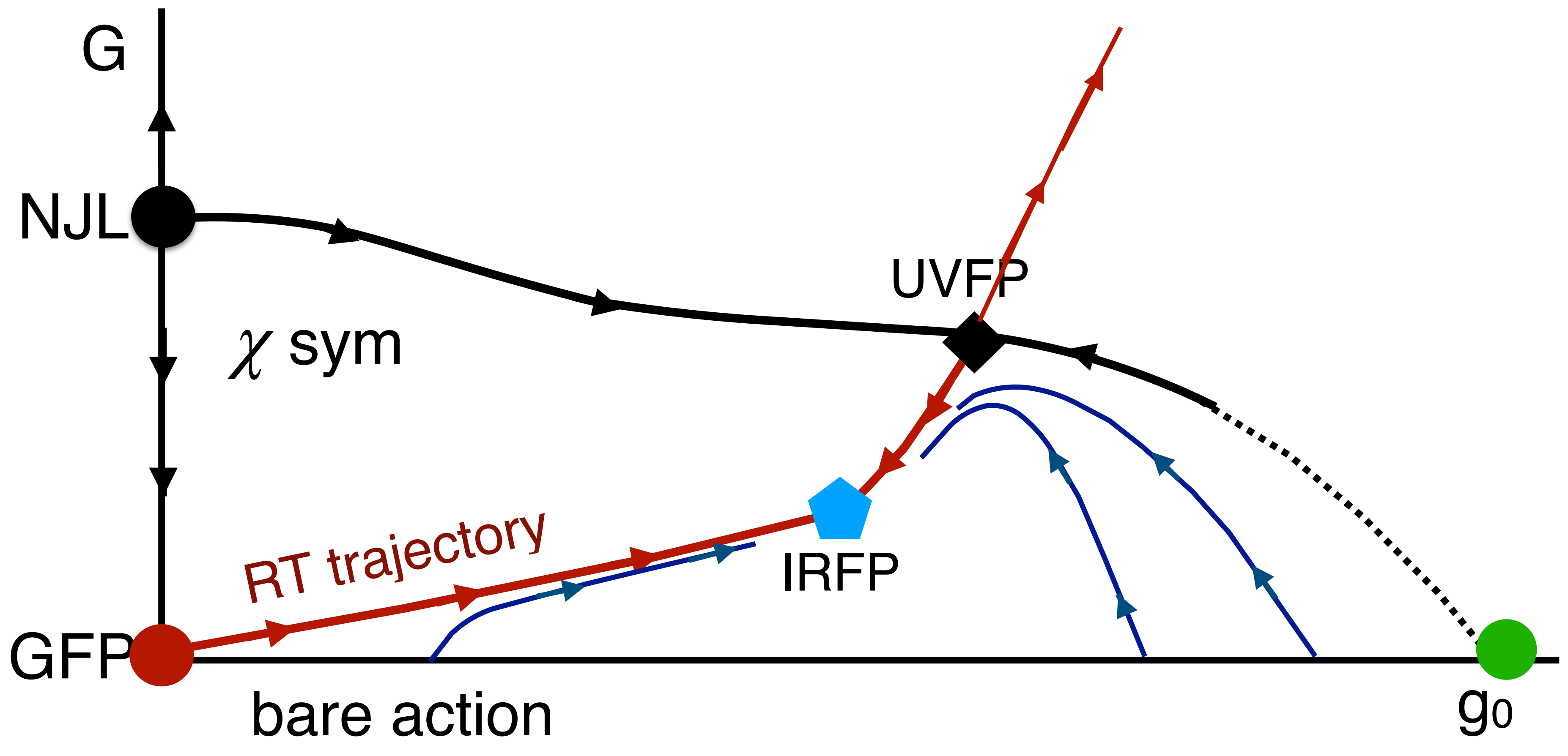


At the moment of FP merger, we have a BKT* "walking scaling" phase transition

- If the phase transition at ● is continuous, it carries the properties of the FP !

* Berezinsky-Kosterlitz-Thouless

Extended parameter space



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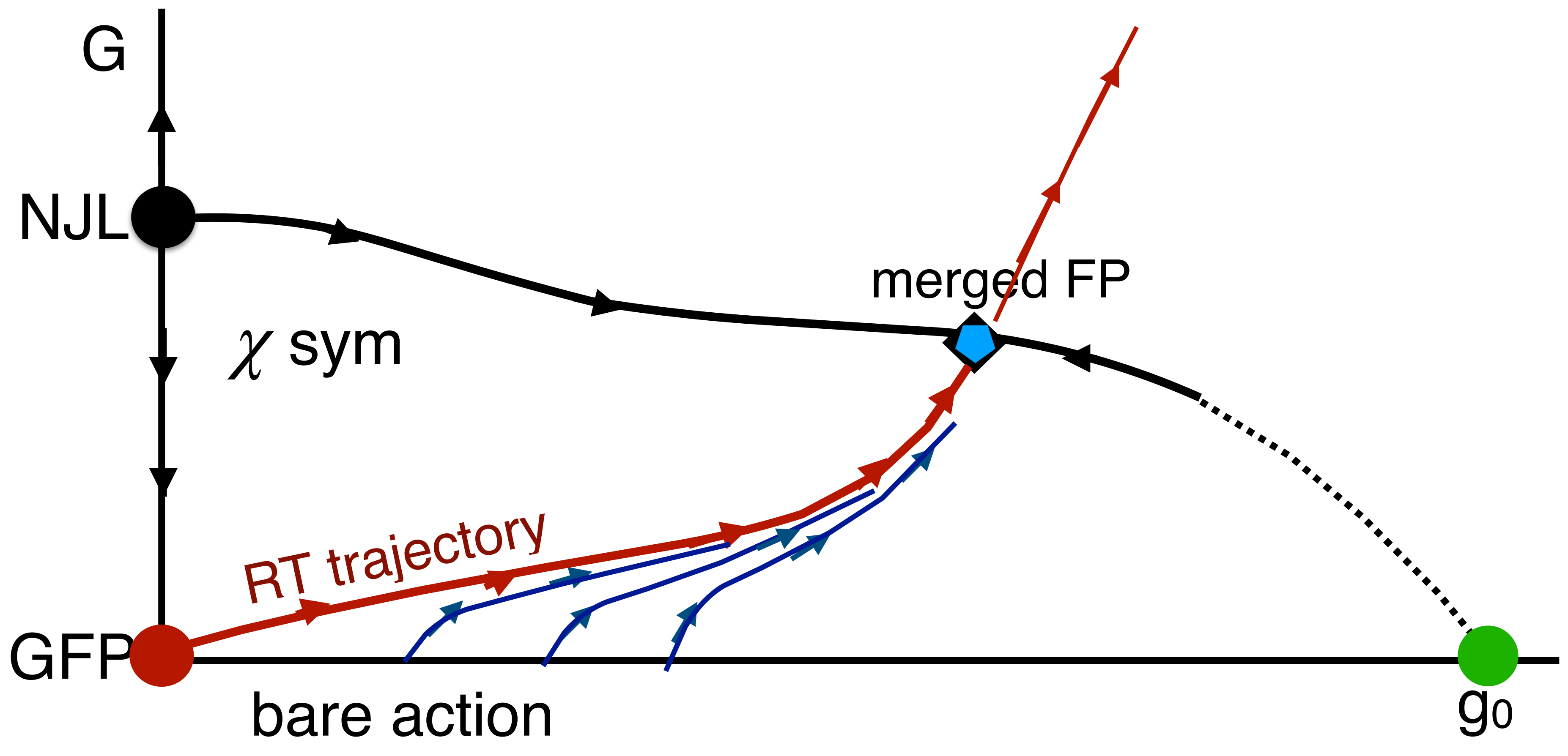

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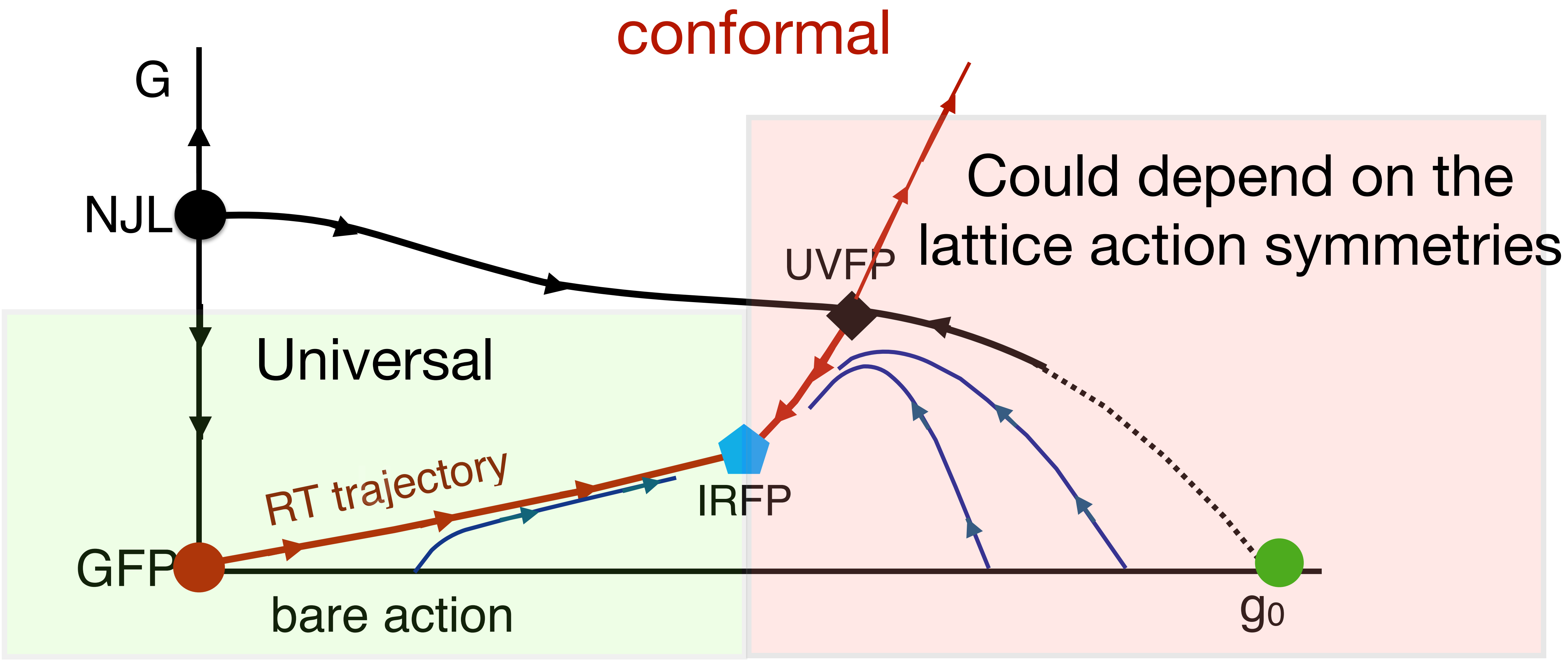
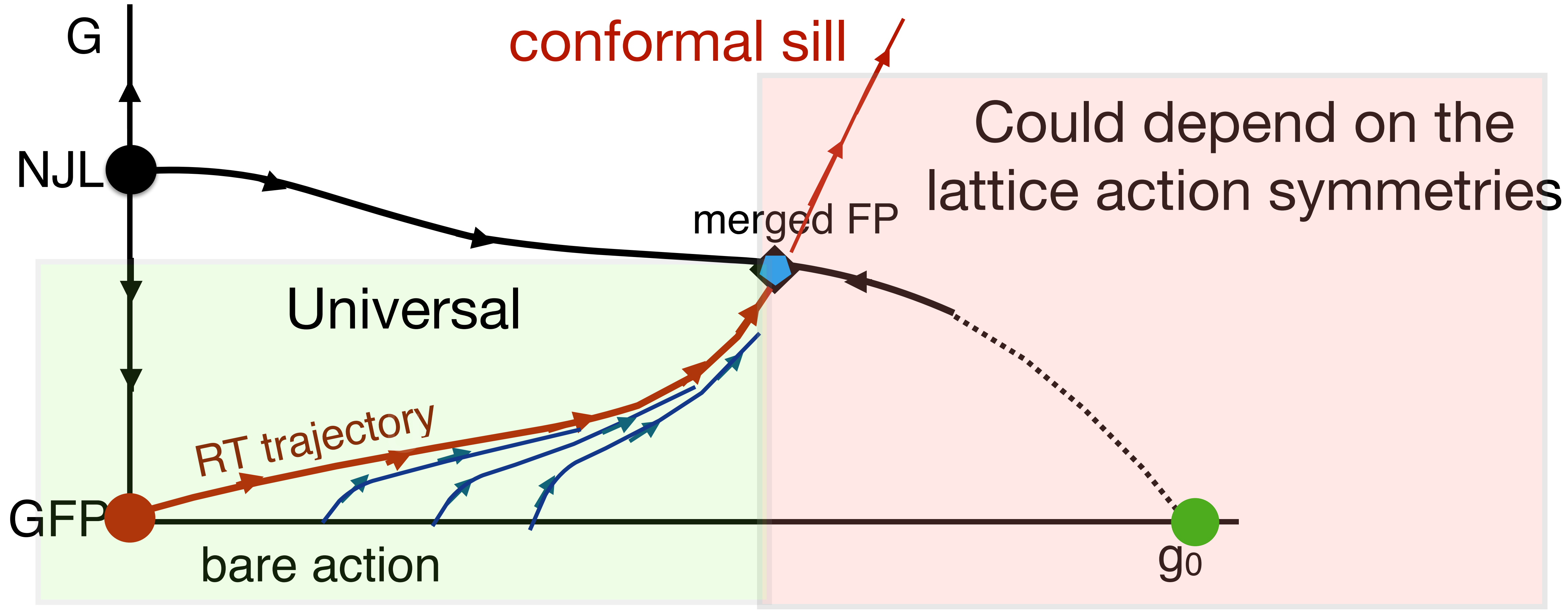
Universality





- IR physics along the RT between Gaussian FP and IRFP is universal

- The emergent UVFP is new; its properties could depend on the symmetries of the lattice action
- PT gives no guidance

The new UVFP could describe an amazingly rich, exciting continuum theory.

Is it realized by nature?

Digress: Taming lattice artifacts with PV

AH, Shamir, Svetitsky, PRD104, 074509 (2021)

- Lattice fermions induce an effective gauge action (hopping expansion)

$$S_{eff} = \frac{N_s}{(2am_f)^4} ReTrV_{\Box} + c\frac{N_s}{(2am_f)^6} ReTrV_{\text{6-link}} \cdots$$

- Bare gauge coupling $\beta = 6/g_0^2$ decreases to compensate, leading to rough gauge configurations, large cutoff effects

➡ Compensate with **heavy Pauli-Villars bosons**

-same interaction as fermions but with bosonic statistics

- $S_{eff} < 0$, β increases

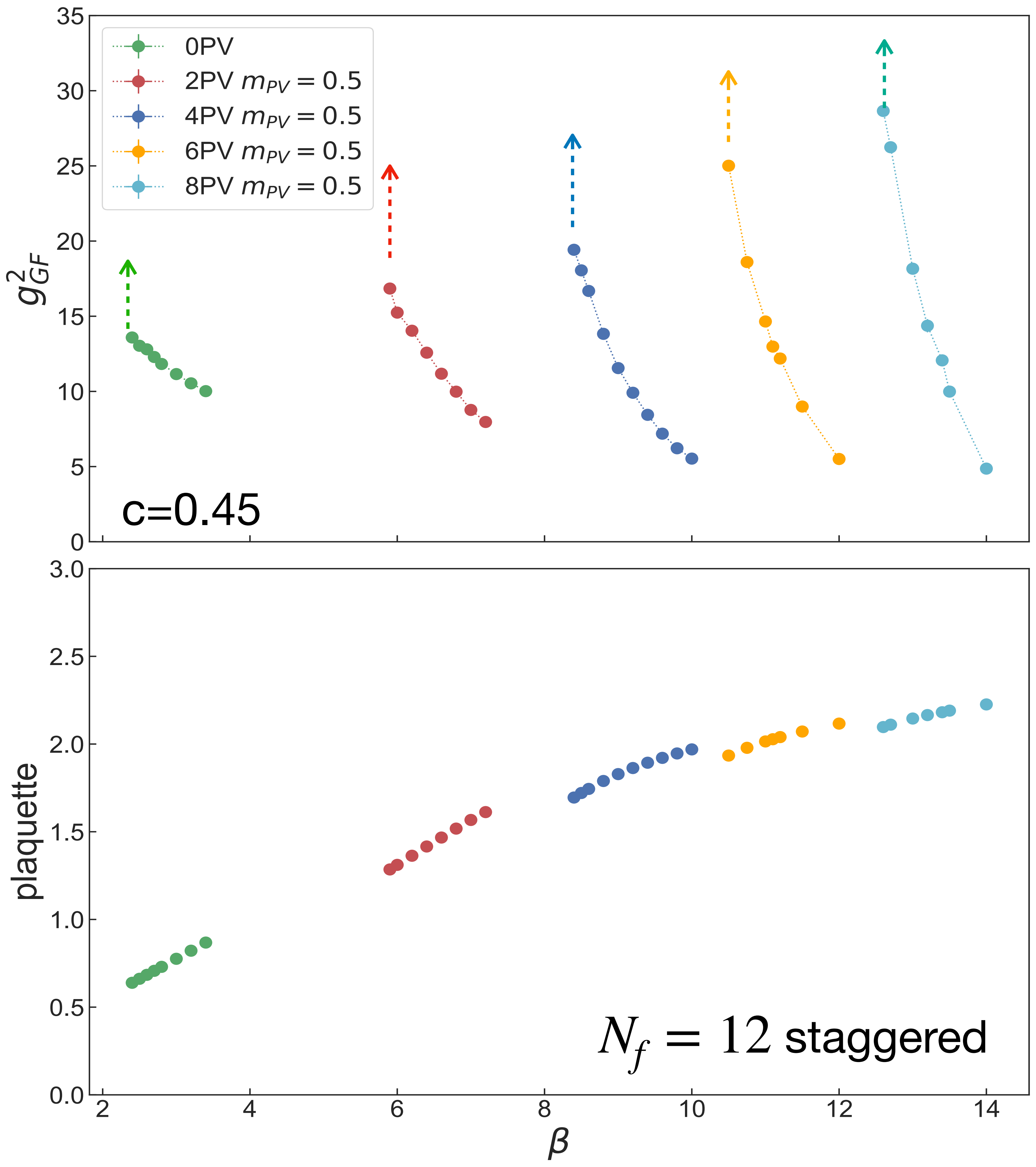
- in the IR the heavy flavors decouple, do not change physics
- equivalently: range of effective gauge action is $\sim exp(-2am_{PV})$

- Add many PV bosons reduce the lattice fluctuations

Digress: Taming lattice artifacts with PV

Test: $N_f = 12$ ($N_s = 3$ staggered action)

- Compare the location and discontinuity of the 1st order bulk transition



Simulations with PV bosons

- have smaller lattice artifacts
- reach stronger renormalized couplings

AH, Shamir, Svetitsky, PRD104, 074509 (2021)

$N_f = 8$ or $N_s = 2$ staggered flavors

Why $N_s = 2$ staggered?

- Staggered fermions are Dirac-Kaehler fermions
  - equivalent to $N_f = 8$ Dirac flavors at the GFP, could be different at $g^2 \neq 0$
- In the chiral limit $N_s = 2$ is anomaly free, allowing symmetric mass generation (SMG)

Catterall et al PRD 104 (2021)
and talk on Wed.

Prior studies (many large-scale, most $am_f > 0$, weak coupling regime)

- Smeared actions show *first order* bulk phase transitions to S4 phase *
- Large scale simulations :
  - Close to the conformal sill
  - both conformal hyperscaling and dilaton χPT interpretation are equally possible

AH,Schaich, Rinaldi, 1506.08791
Kotov et al, 2107.05996

G. Fleming, Poster

* S4 phase : broken single-site shift symmetry with an order parameter

Cheng et al, PRD85, 094509

$N_f = 8$ or $N_s = 2$ staggered flavors

A.H. PRD 106 (2022) 014513

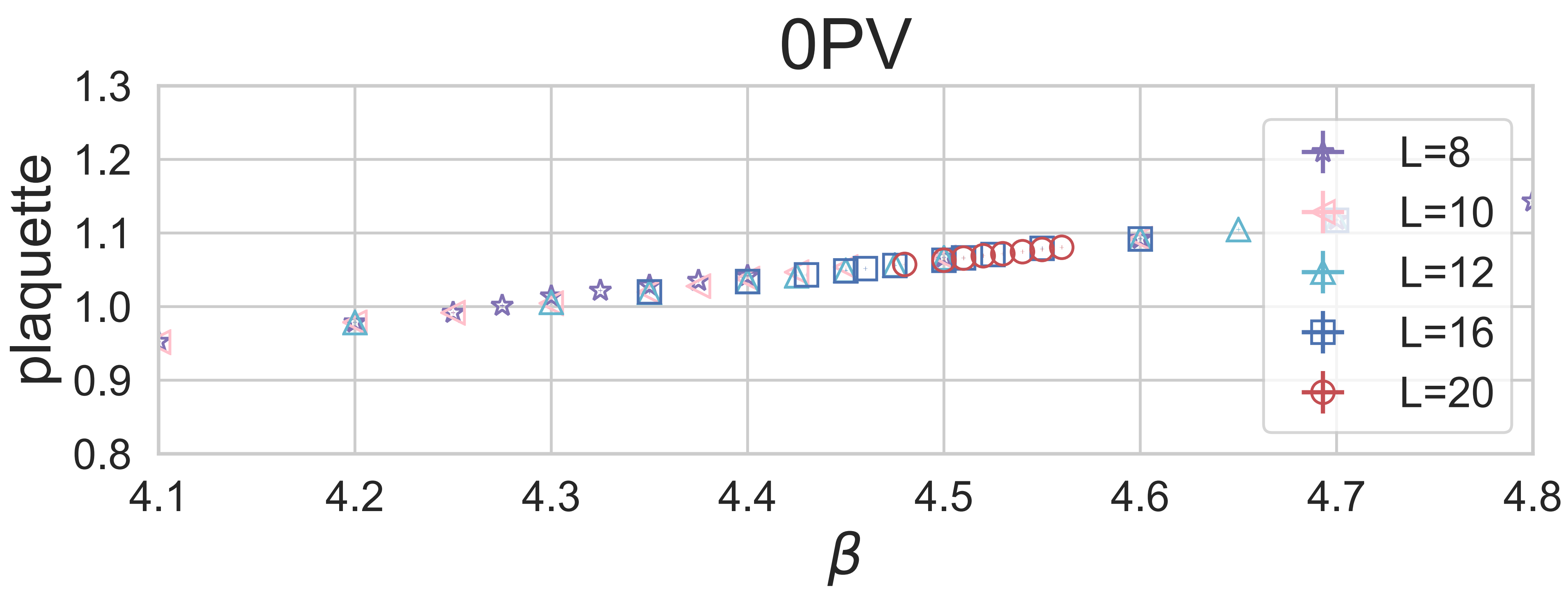
This work: PV improved action, $\mathbf{am_f = 0}$

- Consider two PV actions:
  - $4N_s$ PV bosons with $am_{PV} = 0.5$
  - $8N_s$ PV bosons with $am_{PV} = 0.75$;
- Identify bulk phase transition between weak coupling and S4 phase
- Bulk transition appears continuous

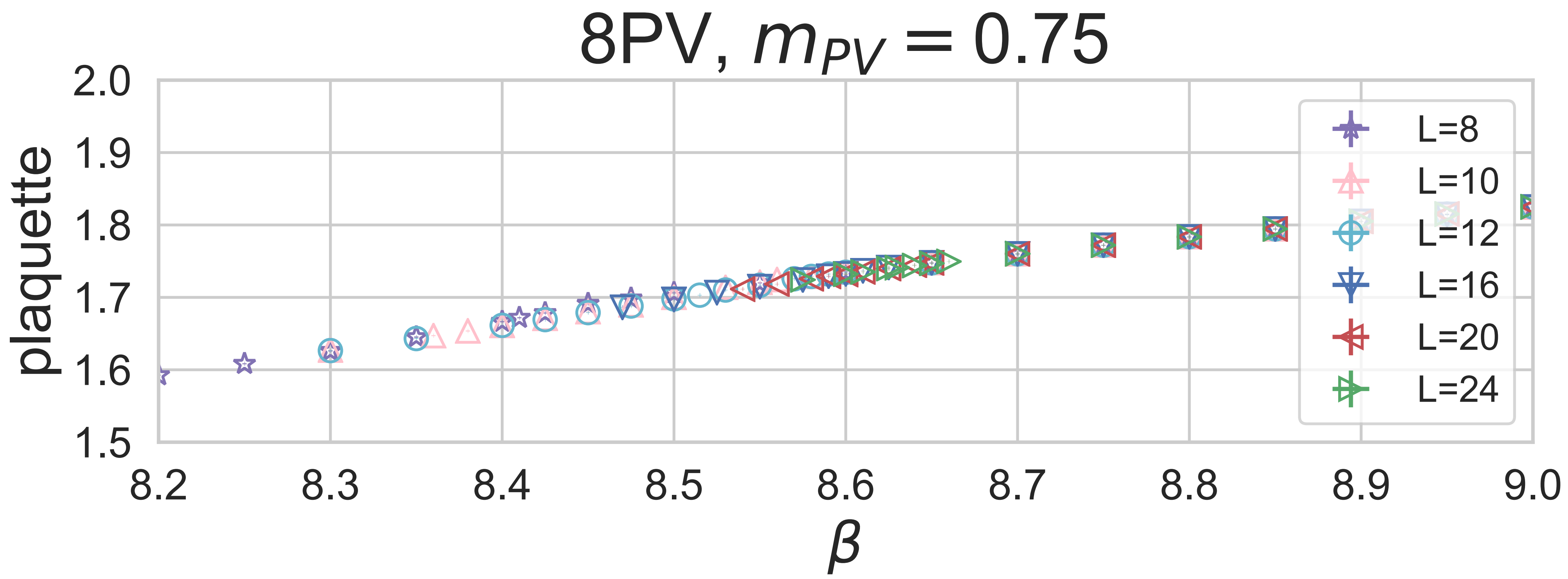
➡ Finite size scaling to test to identify the order

- The strong coupling S4

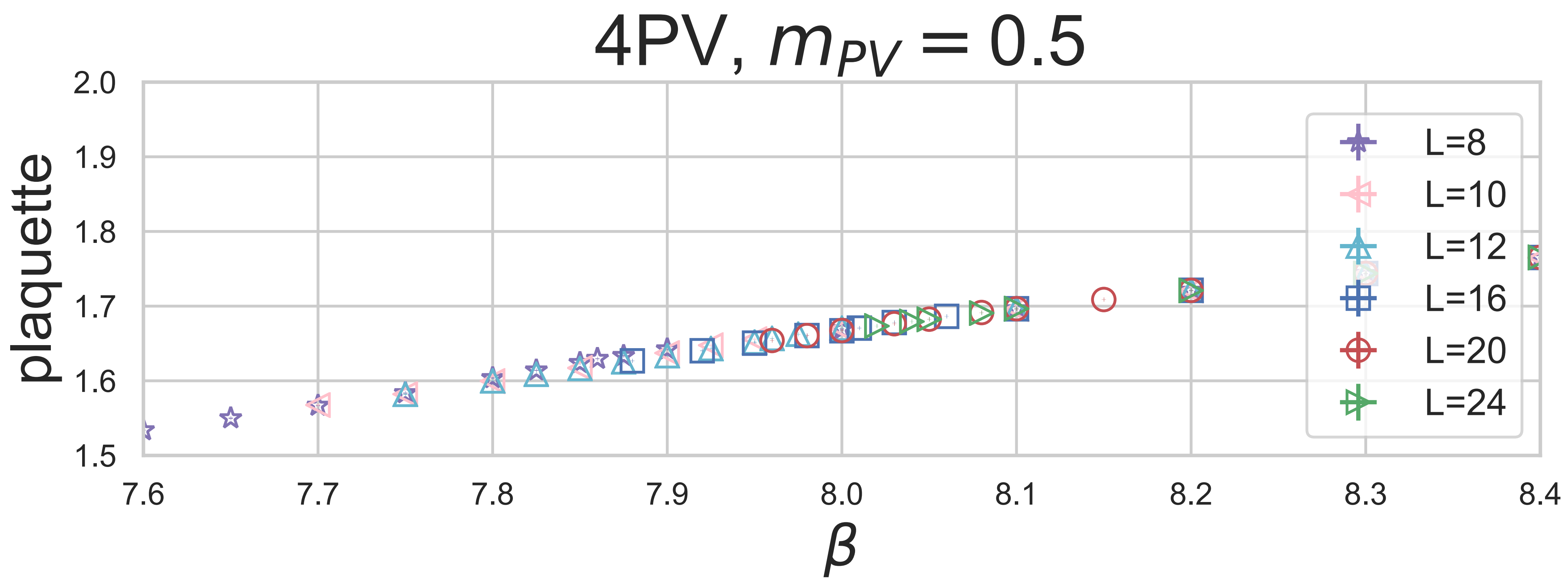
Phase structure - plaquette



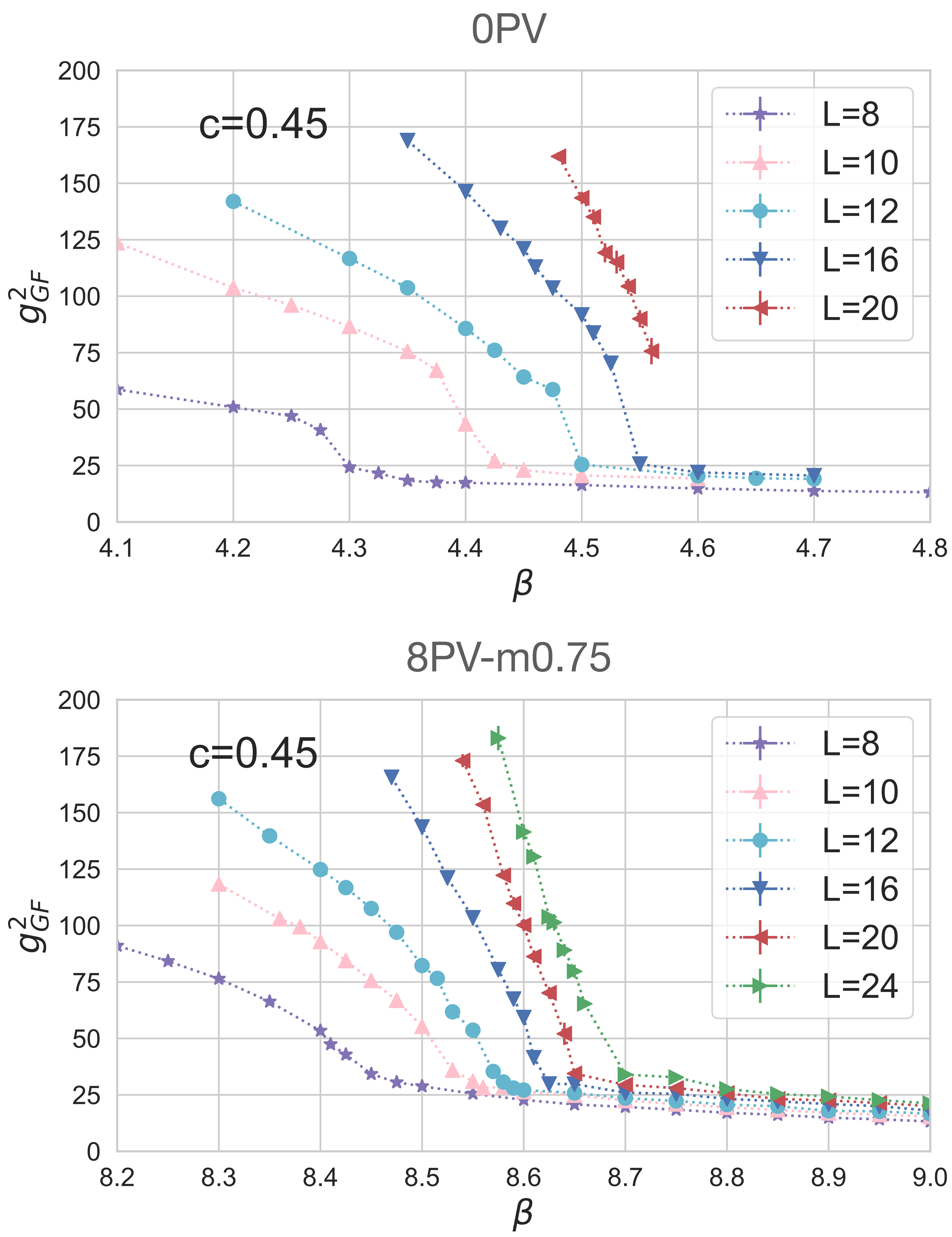
Small discontinuity with 0PV is not resolved



The plaquette value increases from 1.0 to 1.7-1.8 : significant reduction in UV fluctuations



Bulk phase transition - g_{GF}^2



- Gradient flow coupling at $c = \sqrt{8t}/L\,(=0.45)$ shows a qualitative change at the bulk transition
- Significant volume dependence

Finite size scaling

Use the GF renormalized coupling $g_{GF}^2(\beta, L; t) = \mathcal{N} t^2 \langle E(t) \rangle_{\beta,L}$

- has zero anomalous, zero canonical dimension; it measures the flow along the renormalized trajectory
- its use in FSS is a **new application**, in the spirit of MCRG in ~1985

Finite size scaling

- fix $c=\sqrt{8t}/L$ and vary the bare gauge coupling

  - 2nd order scaling: $\xi \propto |\beta/\beta_* - 1|^{-\nu}$

  $$g^2_{GF}(\beta,L;c) = f^{(c)}_{2nd}\left(L\,|\beta/\beta_* - 1|^{\nu}\right)$$

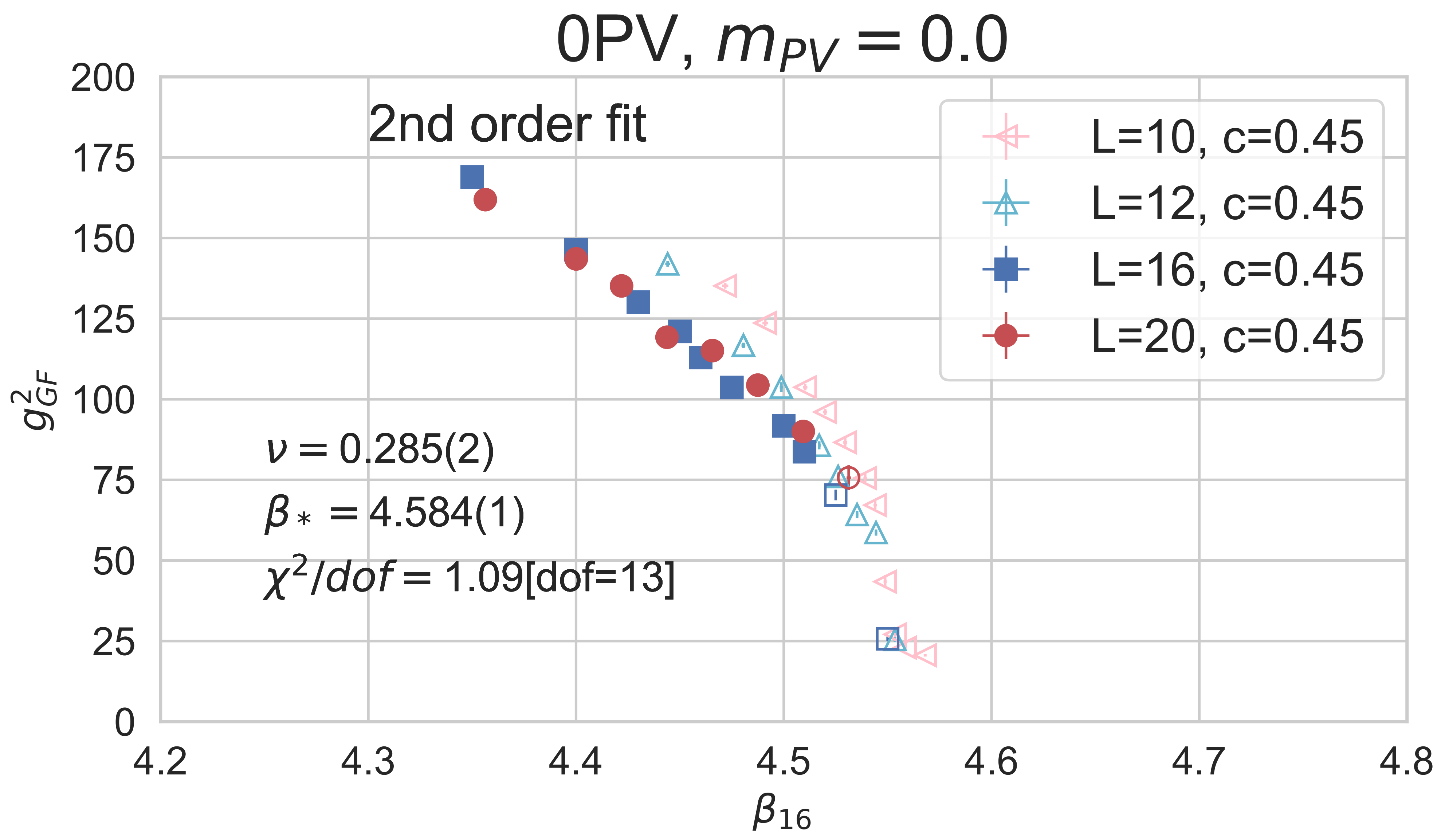
  -1st order scaling: like 2nd order but $\nu = 1/d = 0.25$

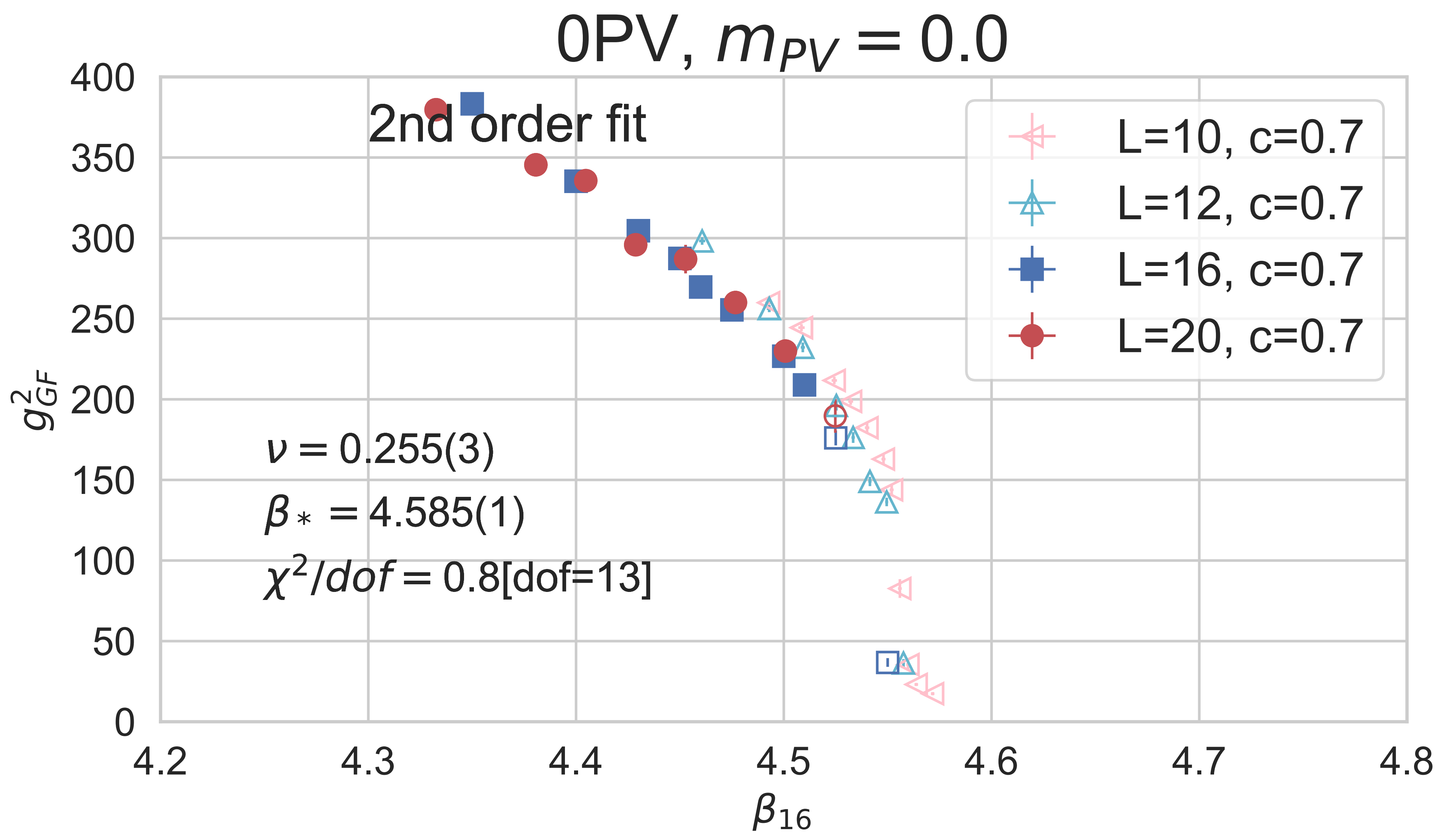
  -BKT or walking scaling: if $\beta(g^2) \sim (g^2 - g^2_*)^{1+\nu} \rightarrow \xi \propto e^{\zeta|\beta/\beta_*-1|^{-\nu}}$ (expect $\nu = 1$)

  $$g^2_{GF}(\beta,L;c) = f^{(c)}_{BKT}\left(L\,e^{-\zeta|\beta/\beta_*-1|^{-\nu}}\right)$$

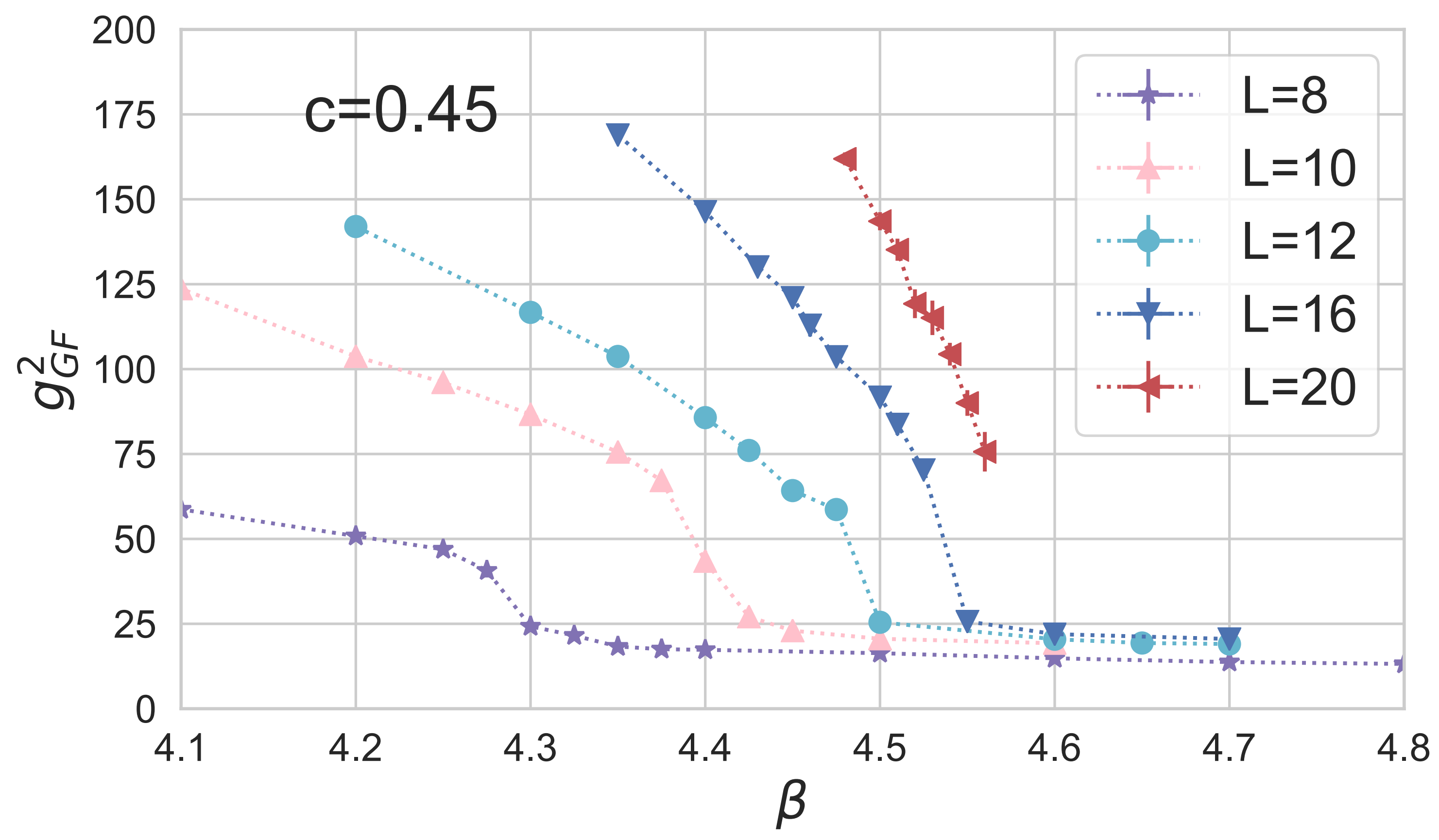
- Find the exponents by standard curve-collapse analysis ;
- Any $c=\sqrt{8t}/L$ can be used, the predicted β_c, ν, ζ must be independent of c
- ν must be independent of the action as well

Curve collapse - **0PV**



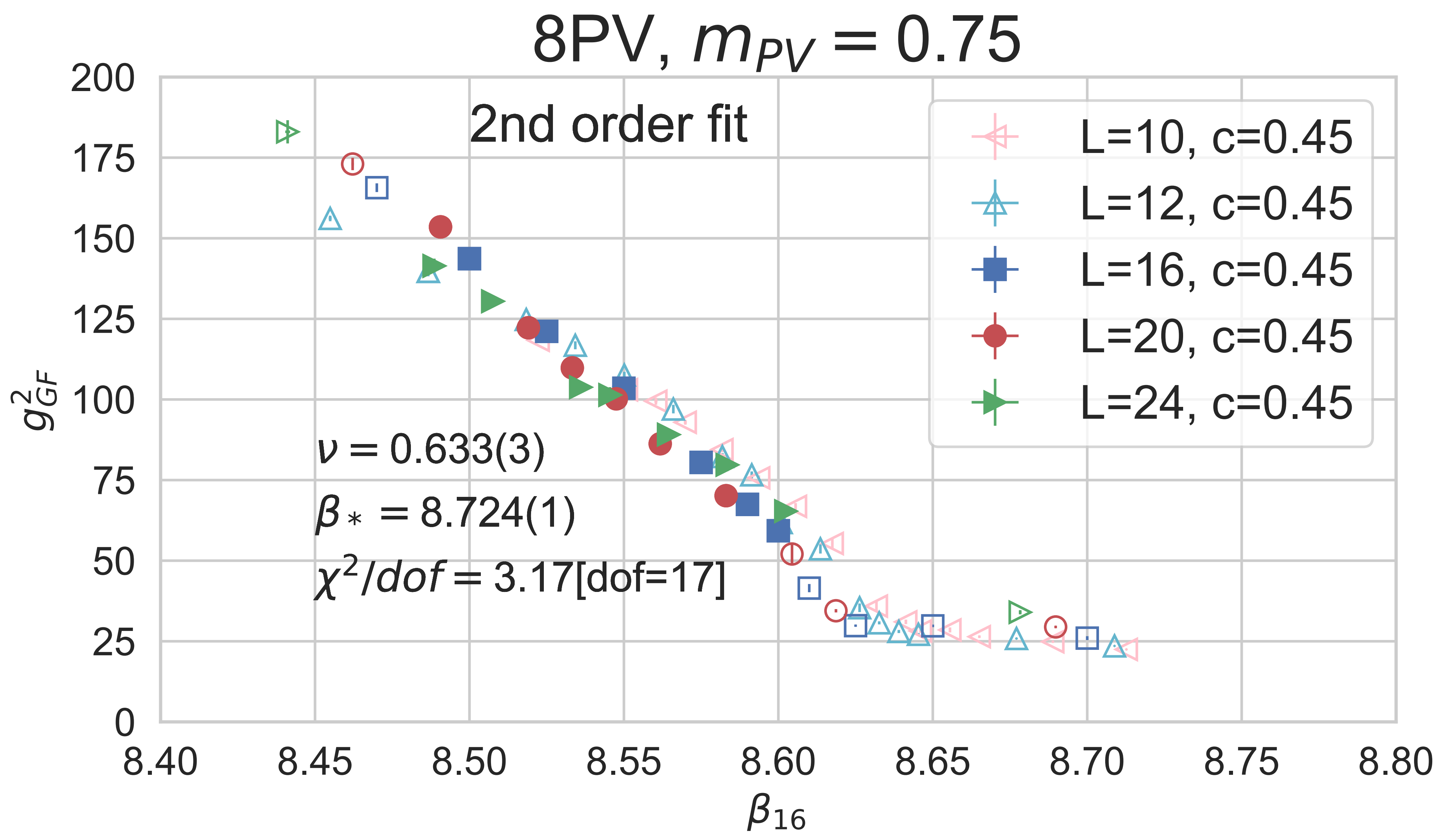


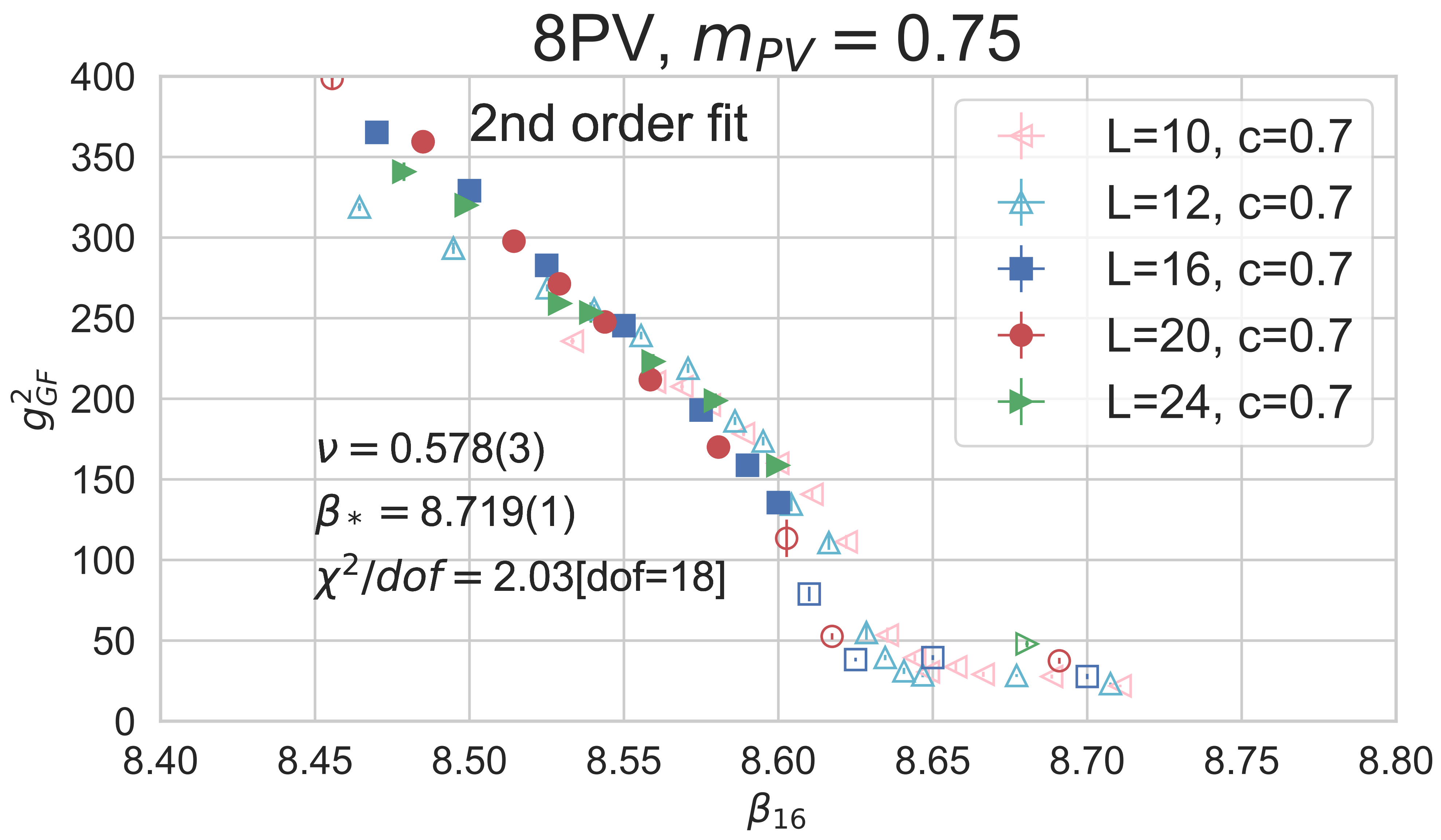
Only $L \geq 16$



- $\beta_{16}(\beta)$ is the solution of the scaling relation
  $L^{1/\nu}(\beta/\beta_* - 1) = L_0(\beta_{16}/\beta_* - 1),\ L_0 = 16$
- Good χ^2/dof ,
  $\nu \approx 0.285 \rightarrow 0.255$
  — consistent with first order transition
- Including $L = 12$ does not change much
- Only filled symbols are included in the FSS fit;

Curve collapse - **8PV**





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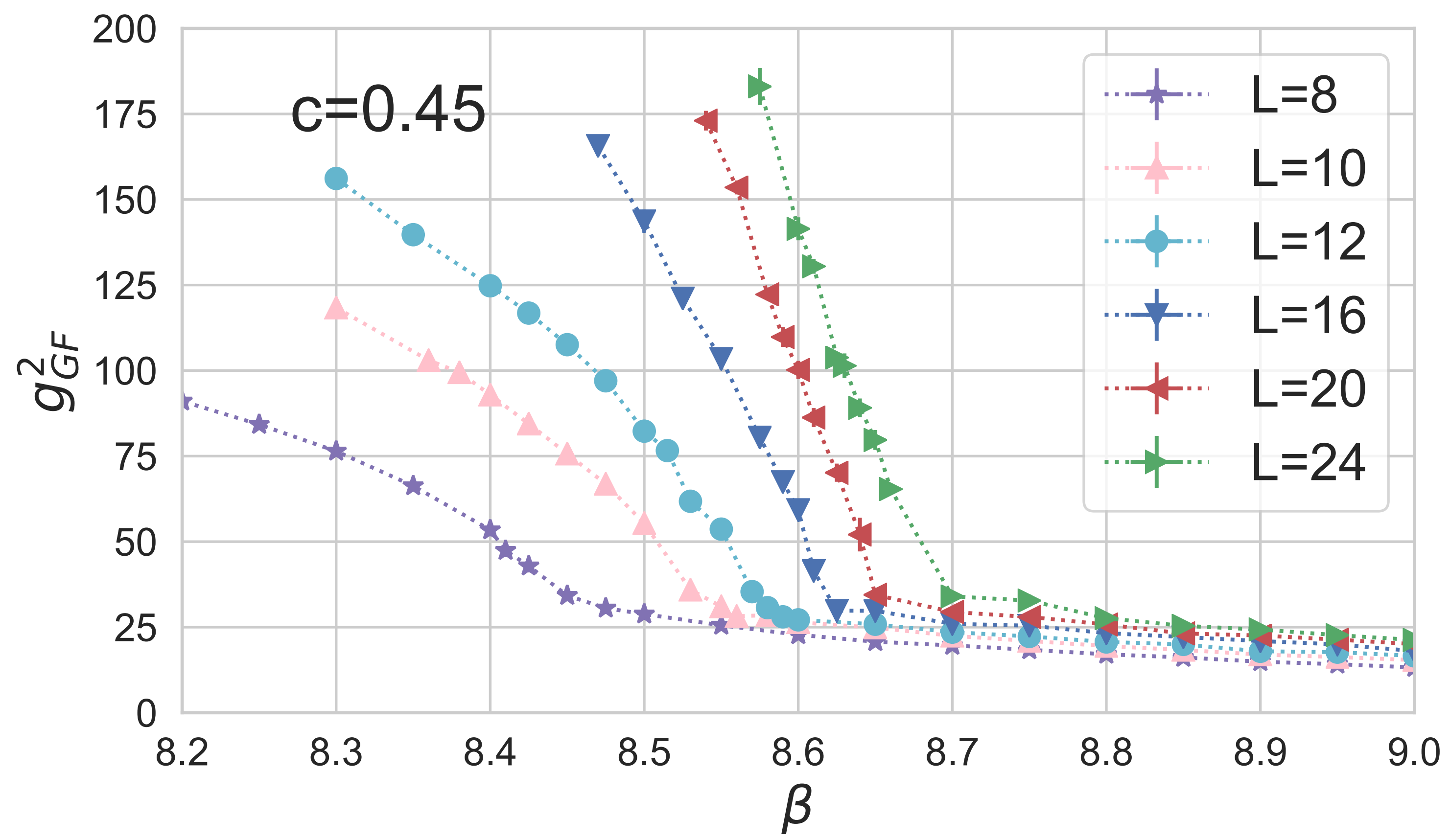
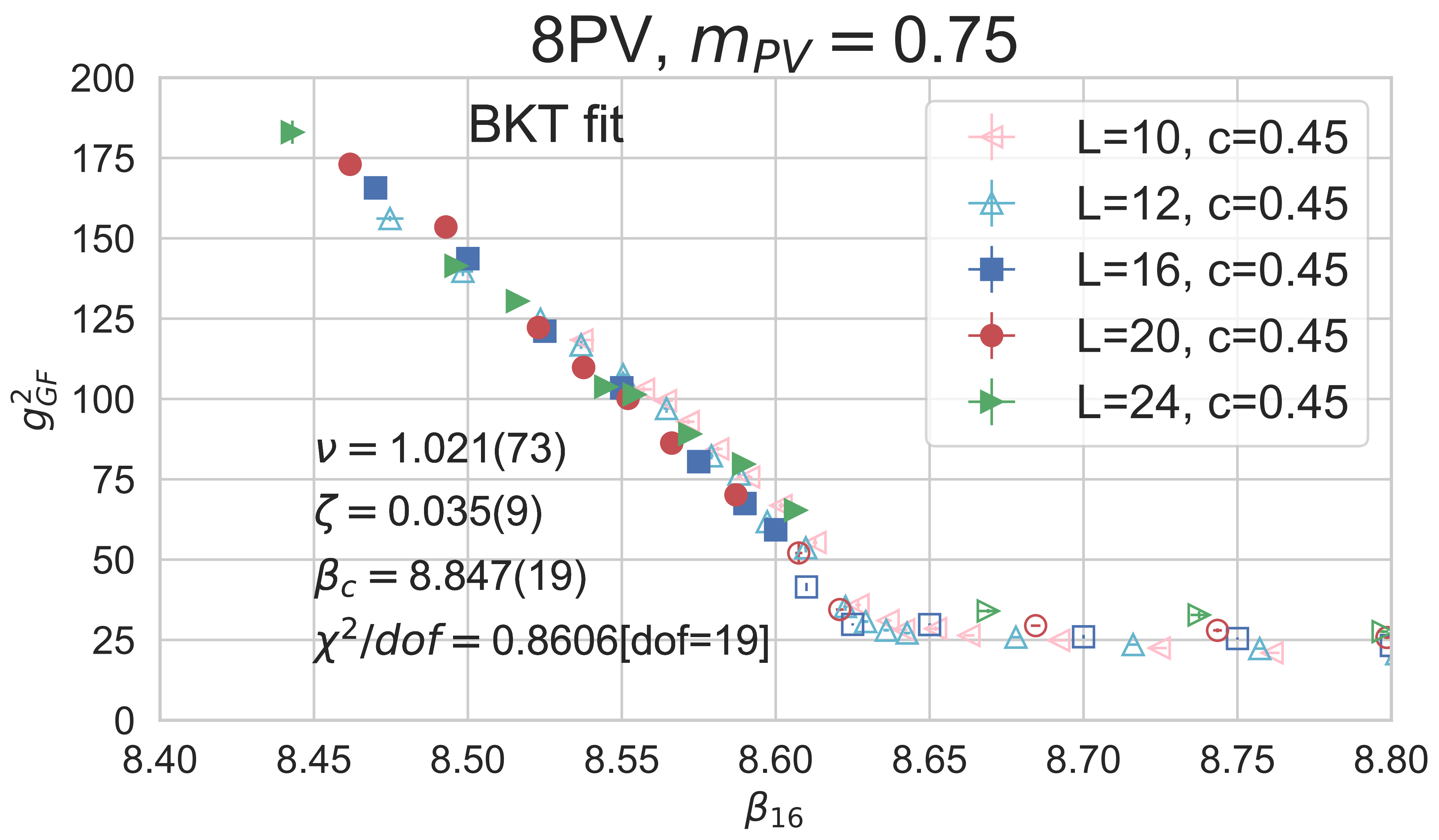
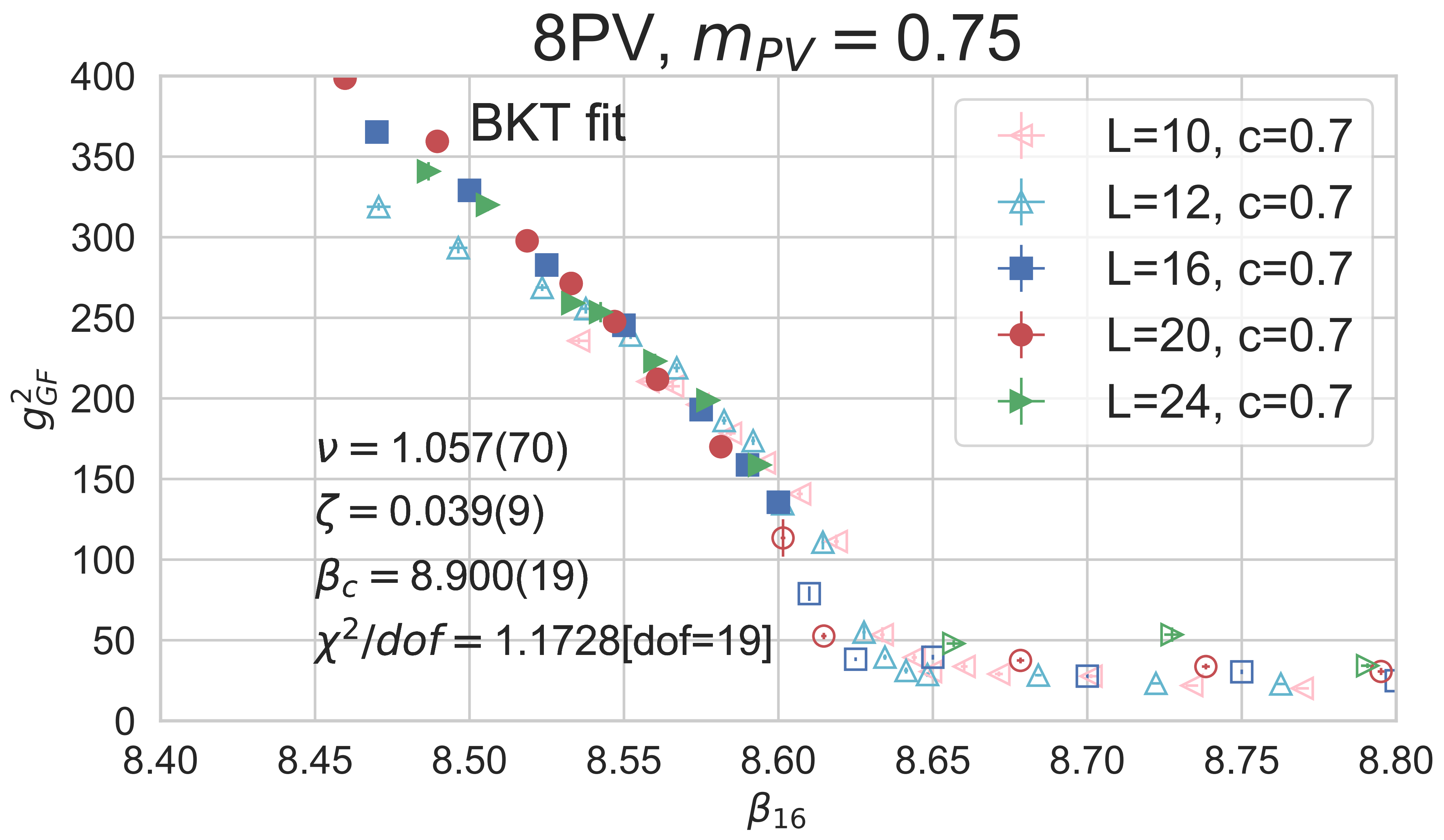


- Acceptable χ^2/dof
  $\nu \approx 0.58 \rightarrow 0.63$
- NOT consistent with first order transition
- Could be 2nd order transition

Curve collapse - **walking scaling, 8PV**





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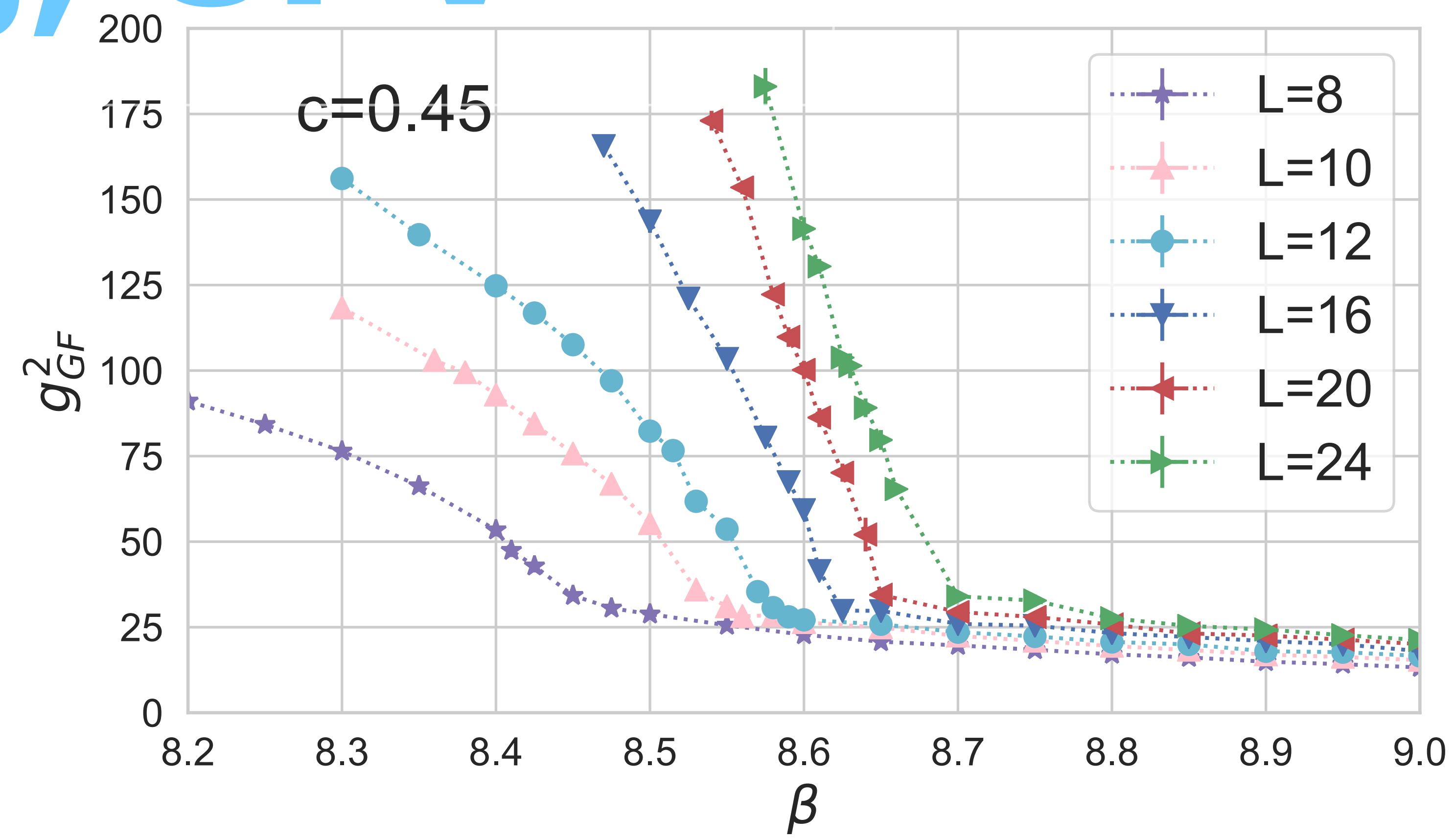
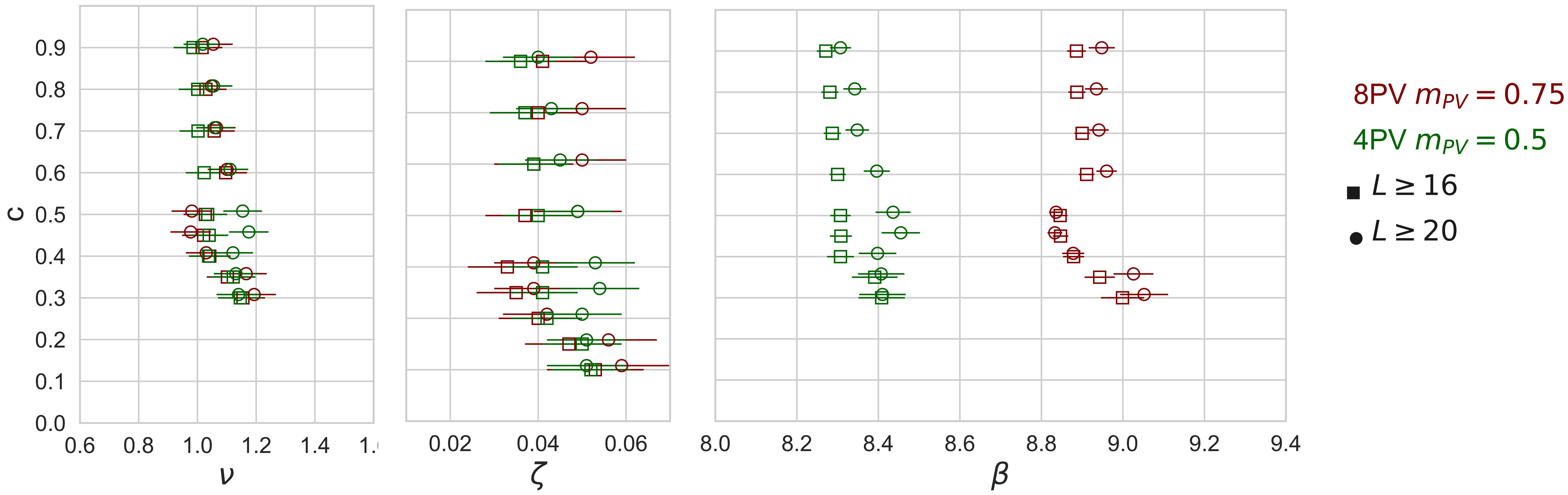


- Good χ^2/dof ,
  $\nu \approx 1.02(7) - 1.06(7)$
- consistent with walking scaling
- dropping $L = 16$ does not change much

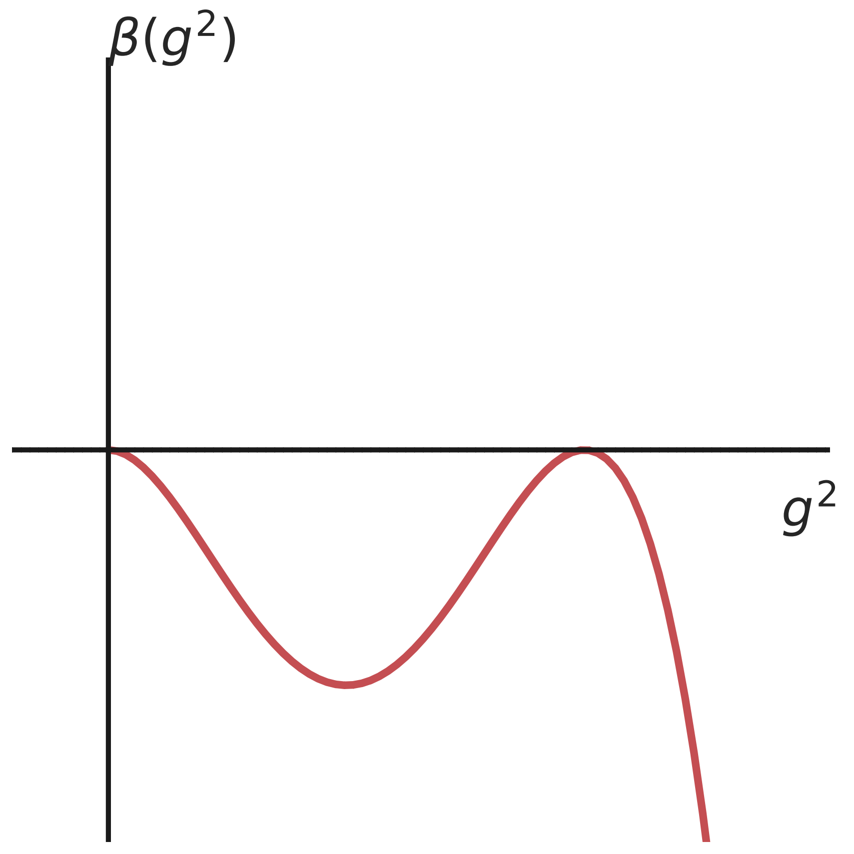
Walking scaling, vary c



Repeat with different c, different volumes, other PV action - all consistent

So far :

No PV action has a 1st order transition with $\nu = 1/4$



Both PV actions show a smooth phase transition that is
- inconsistent with 1st order scaling
- possibly consistent with 2nd order
- prefers BKT or "walking" scaling

Could it be all due to small volumes?
- FSS is reliable at 1st order transition if $L \gtrsim \xi$:
  - not possible in conformal phase
  - but **FSS is done in the S4 strong coupling phase**!

The S4 strong coupling phase

FSS is done in the S4 strong coupling phase

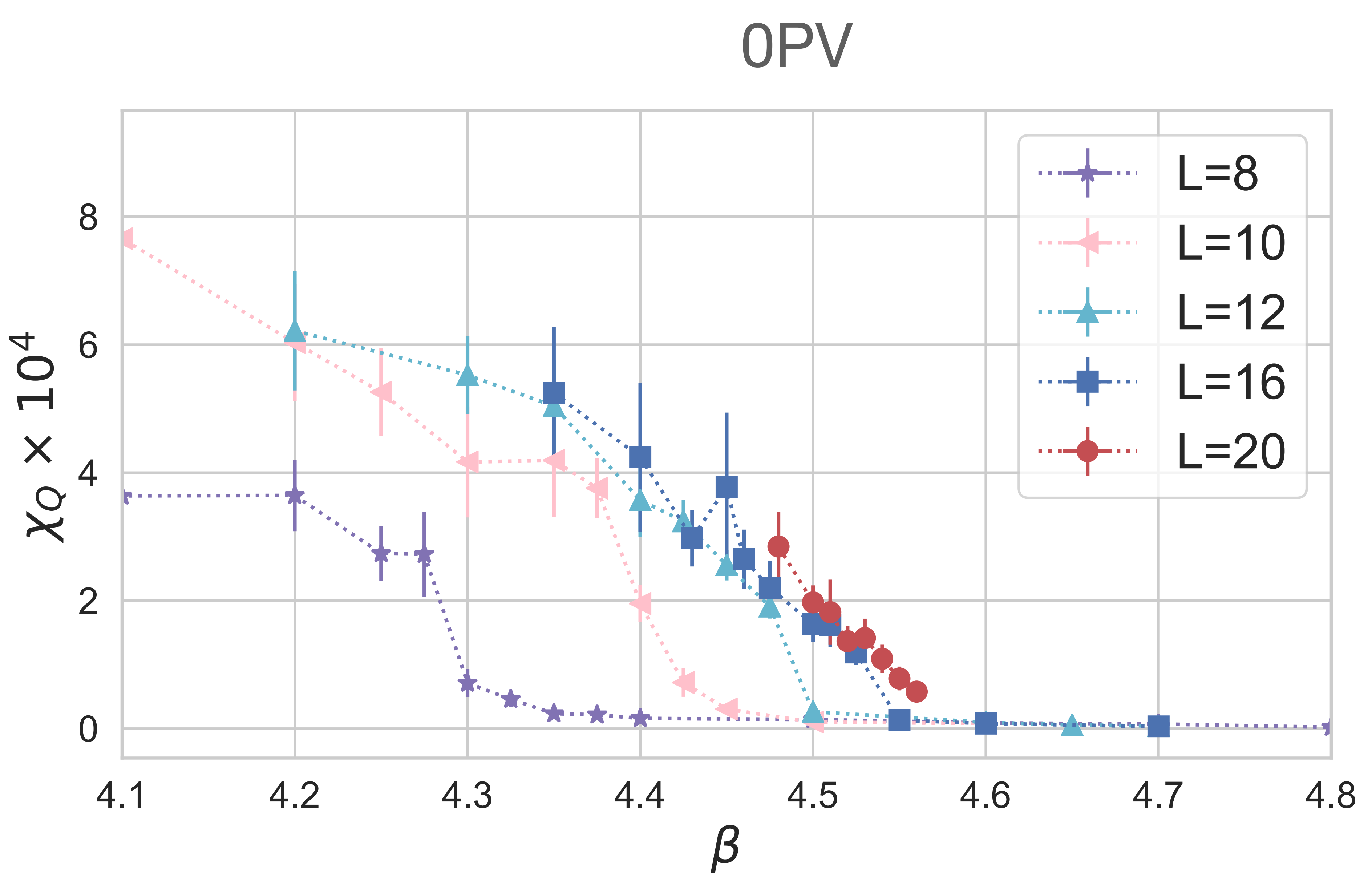
Cheng et al,
Phys.Rev.D 85 (2012) 094509

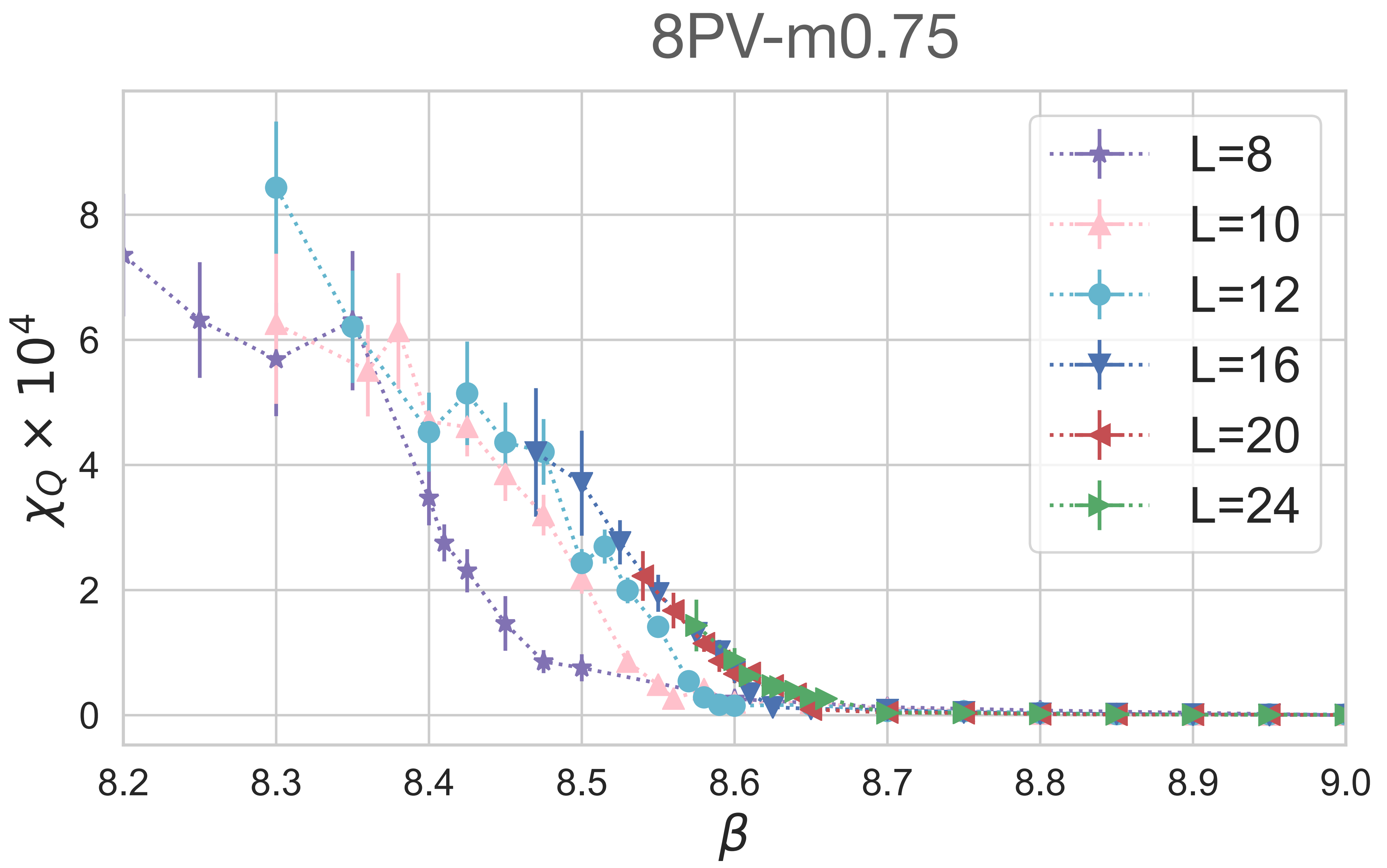
➡ Properties of S4 phase:

- confining
- chirally symmetric
- gapped:
  finite mass even in the chiral limit
- topological (?)

symmetric mass generation
all mesons are massive even
in the chiral limit

S4 phase - topological susceptibility





calculated with GF at large flow time

- Both in conformal and chirally broken systems topology is suppressed **in the chiral limit**

$$\chi_Q = \langle Q^2 \rangle / V = 0$$

- The new strongly coupled phase is full with unpaired instantons
  - how do they avoid the index theorem? (possibly surface modes (?))

S4 phase, meson spectrum

Zero momentum correlators $C(t) = \sum_{\bar{x},\bar{y}} \langle O_S(\bar{x}, t=0) O_S(\bar{y}, t) \rangle$

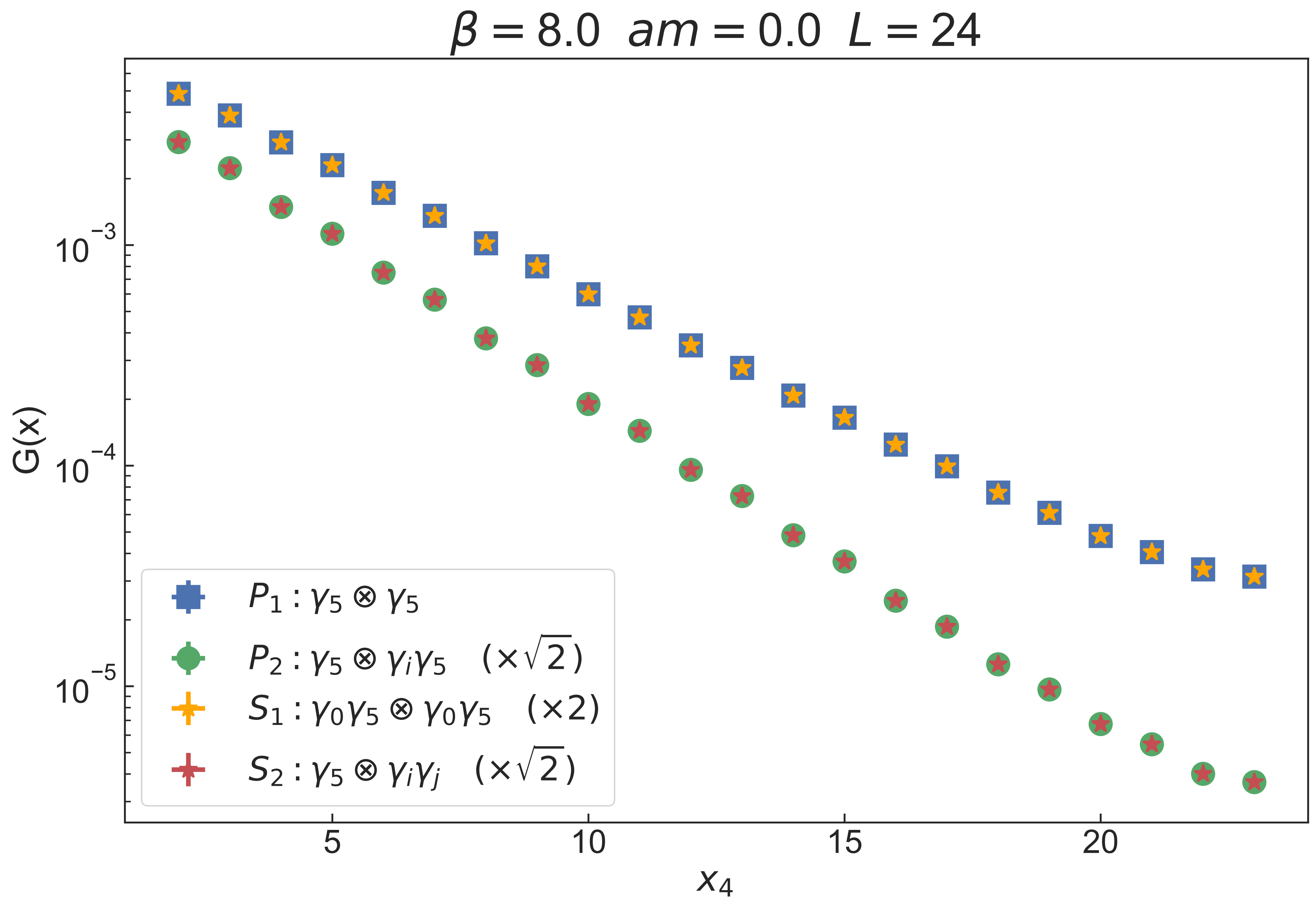
"Pion states" :	spin $\otimes$ taste	in terms of 1-component fields
pseudoscalar :	$P1 = \gamma_5 \otimes \gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x})\, q(\bar{x})\, (-1)^{x_1+x_2+x_3}$
scalar :	$S1 = \gamma_0\gamma_5 \otimes \gamma_0\gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x})\, q(\bar{x})$
pseudoscalar :	$P2 = \gamma_5 \otimes \gamma_i\gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)(-1)^{x_1+x_2+x_3}$
scalar :	$S2 = \gamma_0\gamma_5 \otimes \gamma_0\gamma_i\gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)$

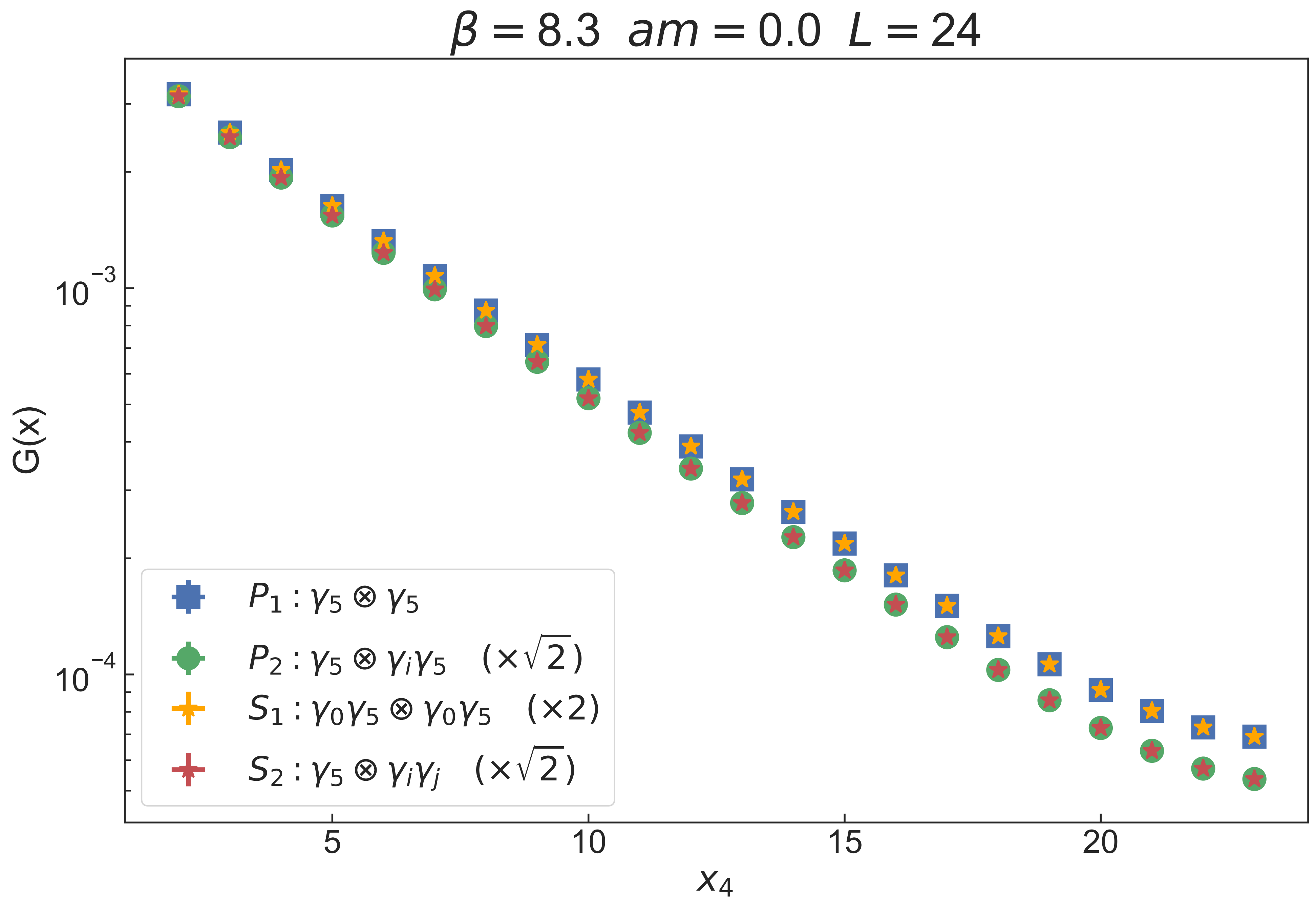
parity partners

parity partners

- all four operators couple to scalar and pseudoscalar, but mostly to one only
- P1 is the lightest state
- Simulations done at $am_f = 0.0 - 0.10$ on $16^3 \times 32$ and $24^3 \times 64$ volumes

S4 phase is chiral symmetric:





S4 phase

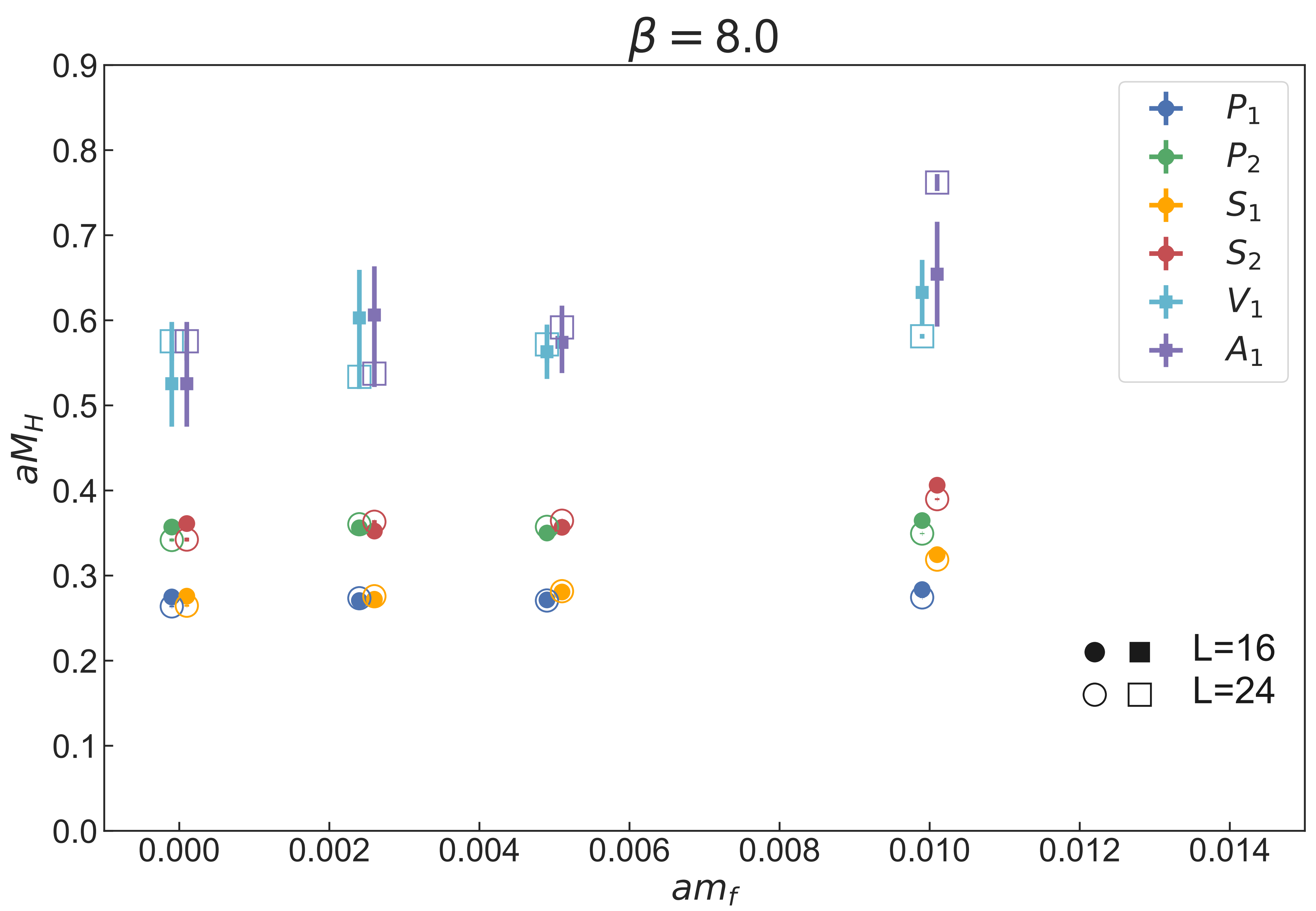
- chirally symmetric (P = S)
- P1-P2, S1-S2 are broken

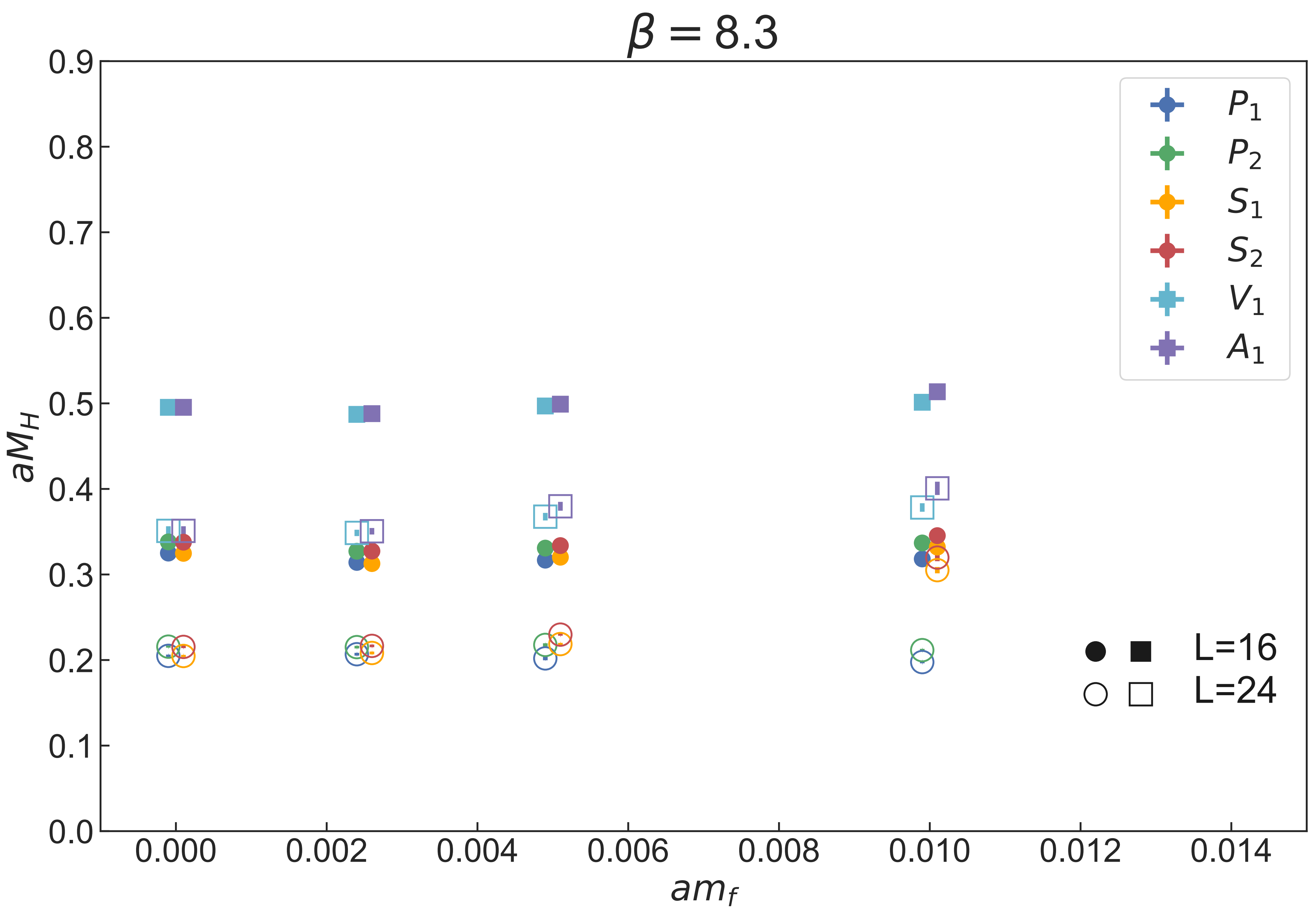
Weak coupling phase

- chirally symmetric (P = S)
- P1,P2, S1,S2 are nearly degenerate
  (taste symmetry / breaking)

S4 phase is gapped

Meson masses, volume dependence





S4 phase :

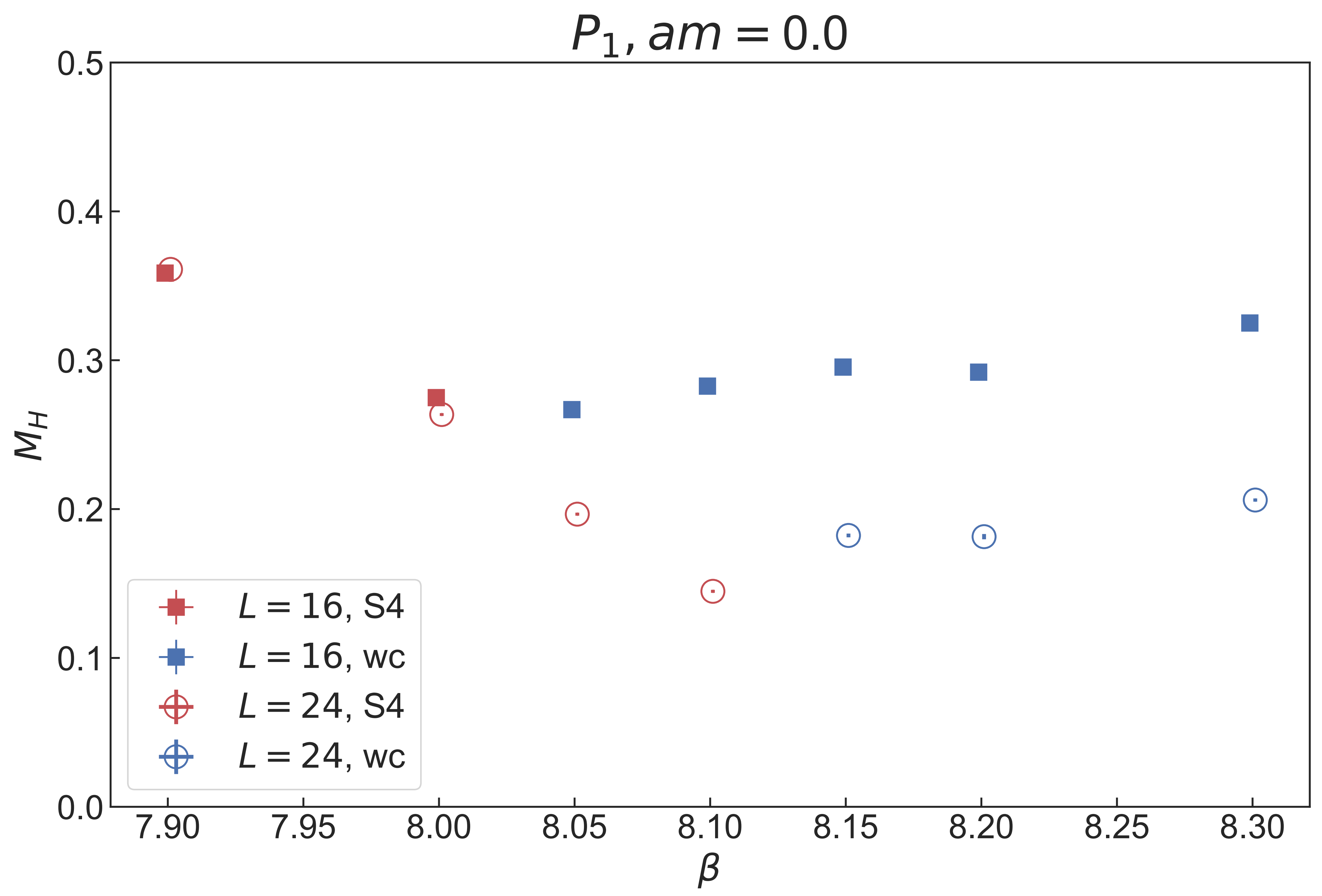
- independent of the fermion mass and volume
- mesons are **massive in the infinite volume chiral limit**

Weak coupling phase :

- $M_H \propto 1/L$ (conformal)
- volume-squeezed for $am_f \lesssim 0.01$

S4 phase is gapped

Meson masses, β dependence



S4 phase :

- mostly independent of fermion mass and volume
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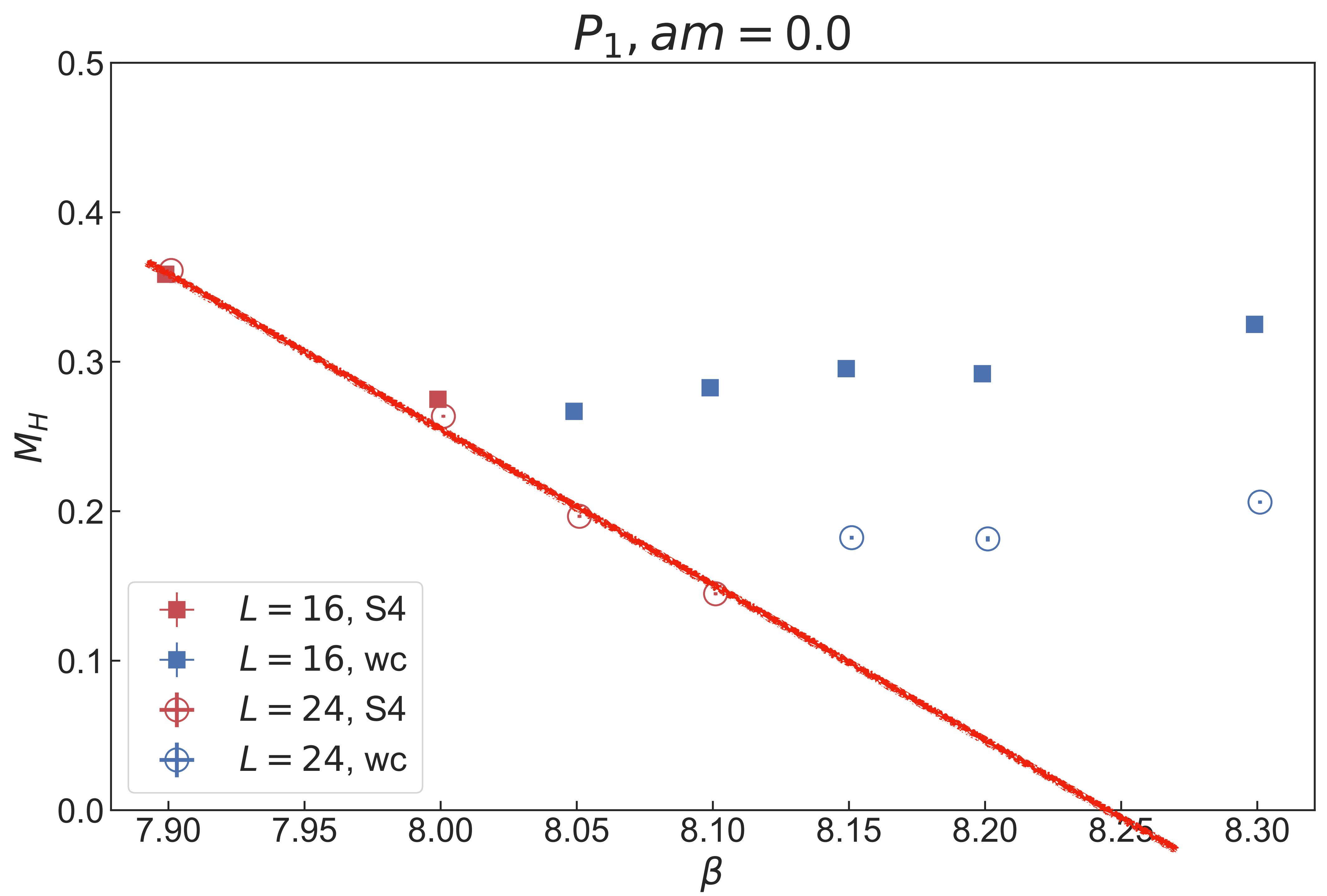

S4 phase :

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Weak coupling phase :

- $M_H \propto 1/L$ (conformal)
- volume-squeezed for $am_f \lesssim 0.01$

Summary: $N_s = 2$ staggered fermions are special

- With PV improved actions show a smooth phase transition
  - Finite size scaling from the strong coupling S4 phase
    - is not consistent with 1st order transition
    - consistent with “walking scaling” transition $(\nu \approx 1)$
- The strong coupling phase (S4):
  - Shows symmetric mass generation:
    - Chirally symmetric and confining
    - Strong topology
  - within the S4 phase $L > 1/M_{PS}$: FSS expected to work
- There is no evidence (yet) if the weak coupling phase is conformal or chirally broken. The phase diagram is only hypothetic
- If $N_f = 8$ is the sill of the conformal window, is anomaly cancellation is likely responsible.

Thank you for your attention!

Special thanks to the organizers for the opportunity to discuss this non-conventional topic!

EXTRA SLIDES

S4 phase

Cheng et al, PRD85, 094509

- Breaks single site translational symmetry
- Confining, all hadrons are heavy in the chiral limit
- Chirally symmetric
- Has a local order parameter that measures staggered symmetry breaking

$$\Delta P_\mu = \langle \mathrm{Re\,Tr}\square_n - \mathrm{Re\,Tr}\square_{n+\mu}\rangle_{n_\mu \mathrm{even}}, \qquad (3)$$

$$\Delta L_\mu = \langle \alpha_\mu(n)\bar{\chi}(n)U_\mu(n)\chi(n+\mu) - \alpha_\mu(n+\mu)\bar{\chi}(n+\mu)U_\mu(n+\mu)\chi(n+2\mu)\rangle_{n_\mu \mathrm{even}}, \qquad (4)$$