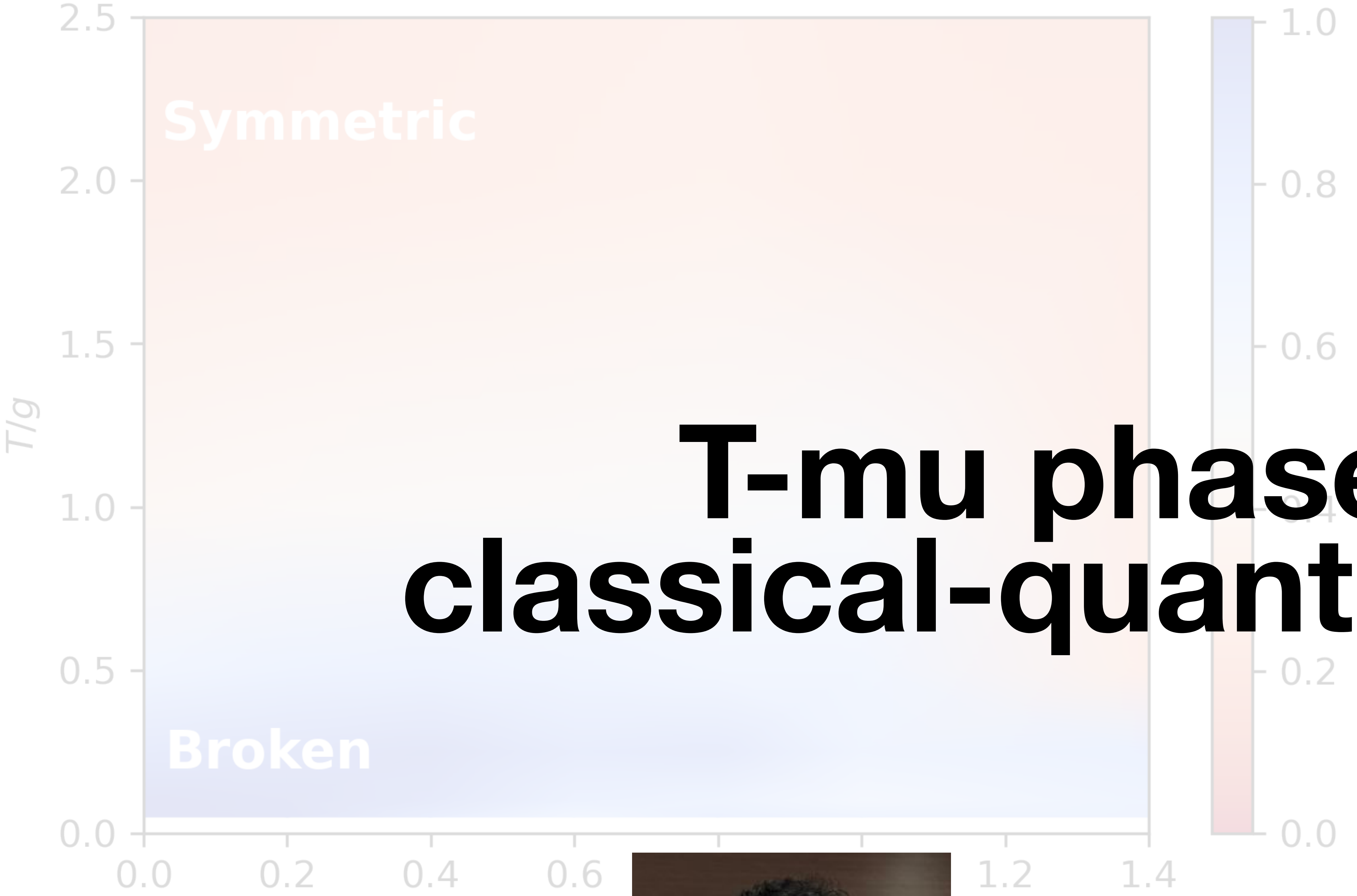

T-mu phase diagram using classical-quantum hybrid algorithm

Akio Tomiya
(Assistant Prof in IPUT Osaka)

Based on arXiv: [2205.08860](https://arxiv.org/abs/2205.08860) →

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NEWS: A new research grant (Japan governmental) initiated!

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- A.Tomiya: Computational physics
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Outline

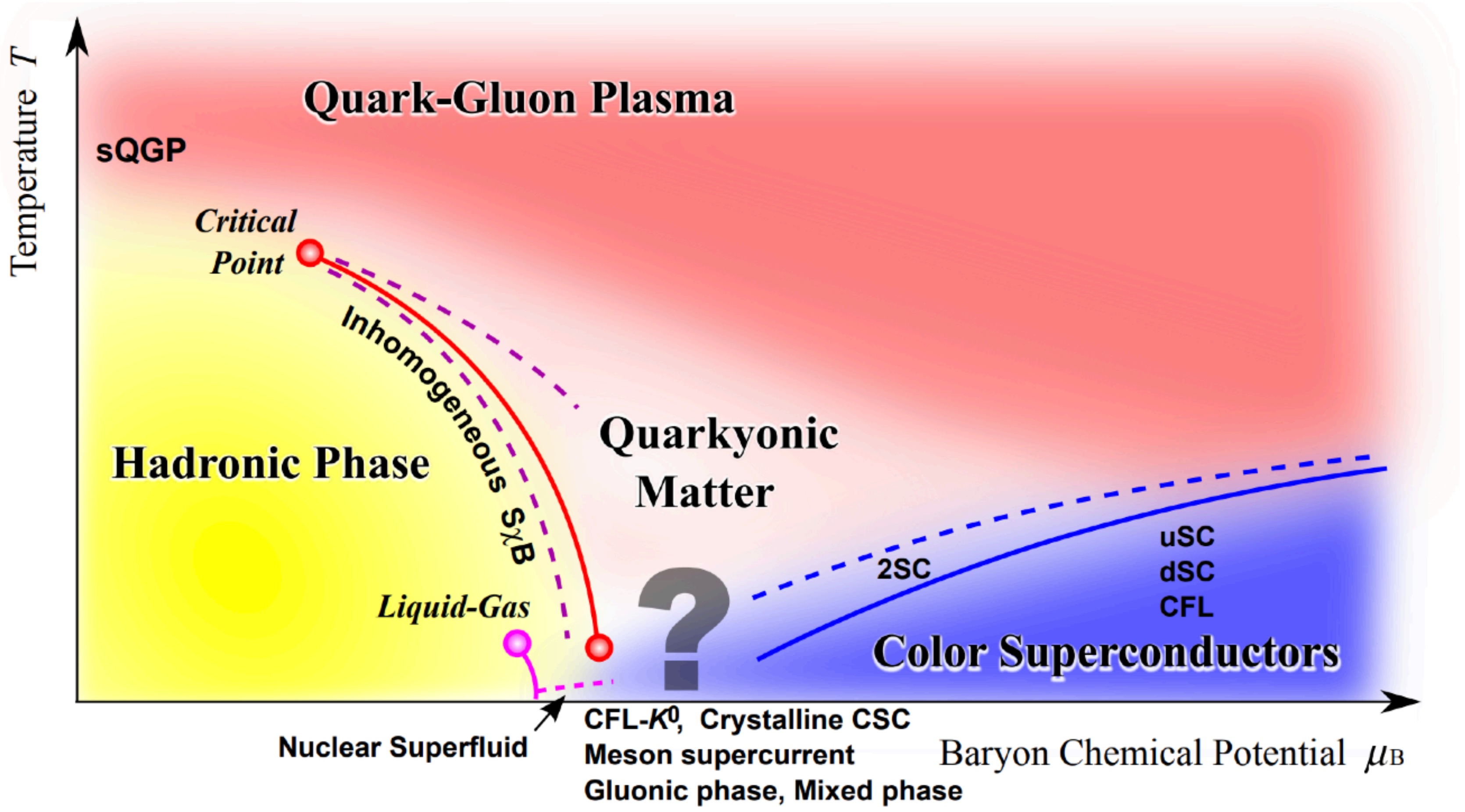
T - mu phase diagram using Quantum/Classical hybrid algorithm

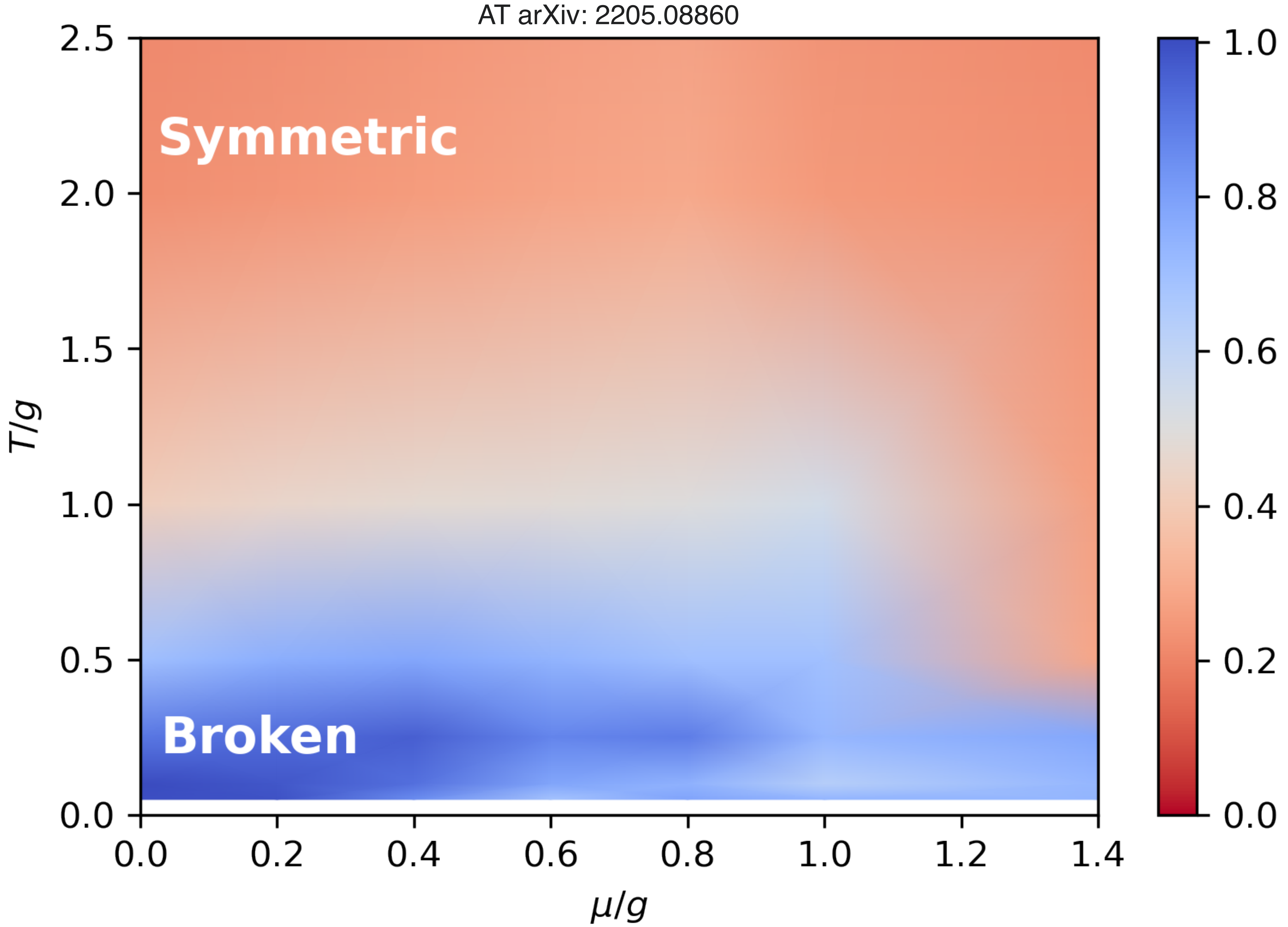
1. Introduction, motivation
2. VQE (a classical-quantum hybrid algorithm)
3. Density matrix, KL-U divergence
4. Beta VQE (VQE for T>0)
5. Results
6. Summary

Introduction

Summary of this talk

Hybrid algorithm = Quantum + “machine learning”





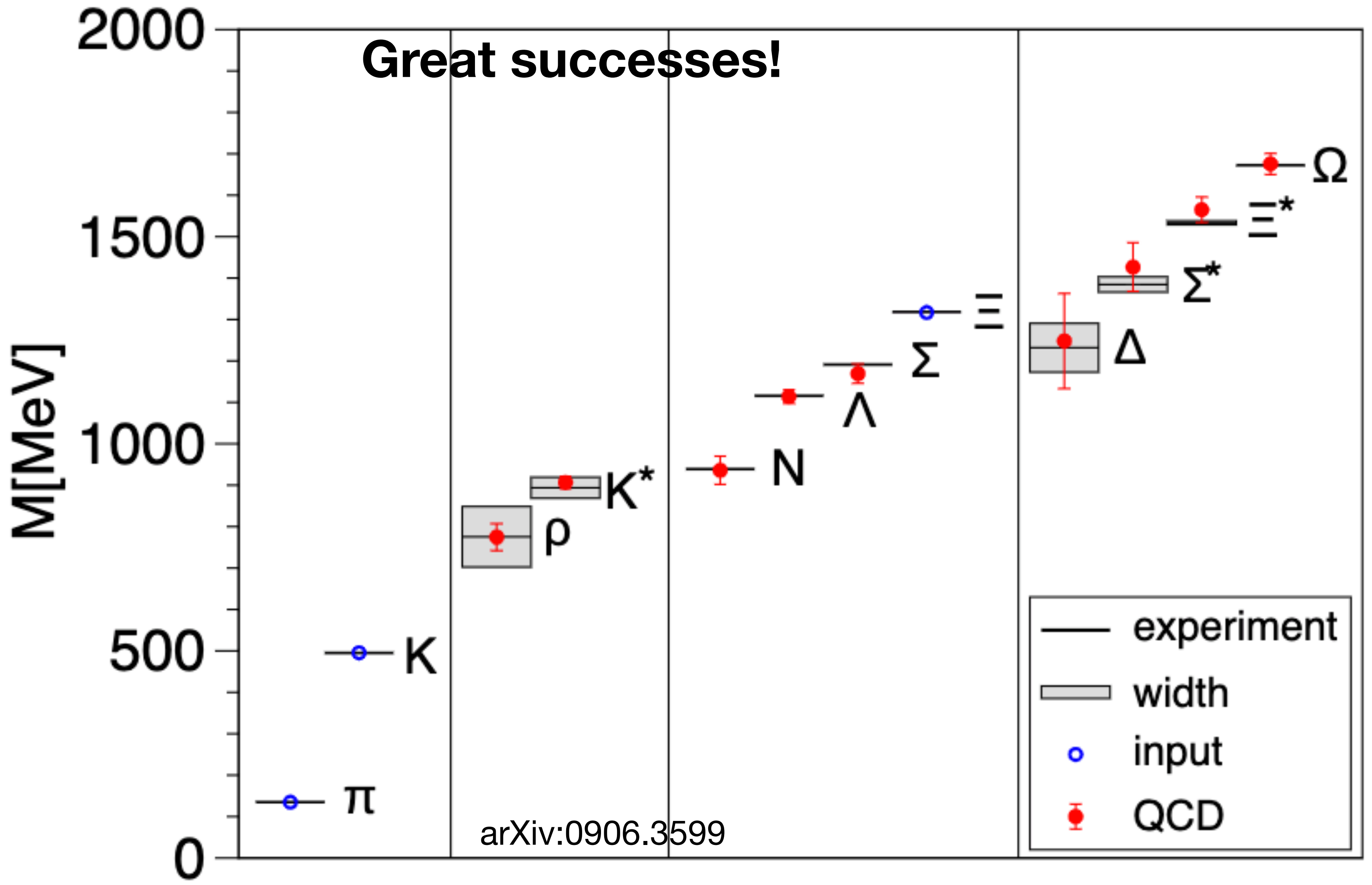
I investigated T-mu phase diagram using a <u>quantum algorithm</u> & <u>neural network</u> (β-VQE, No sign problem) for Schwinger model (toy model of QCD)

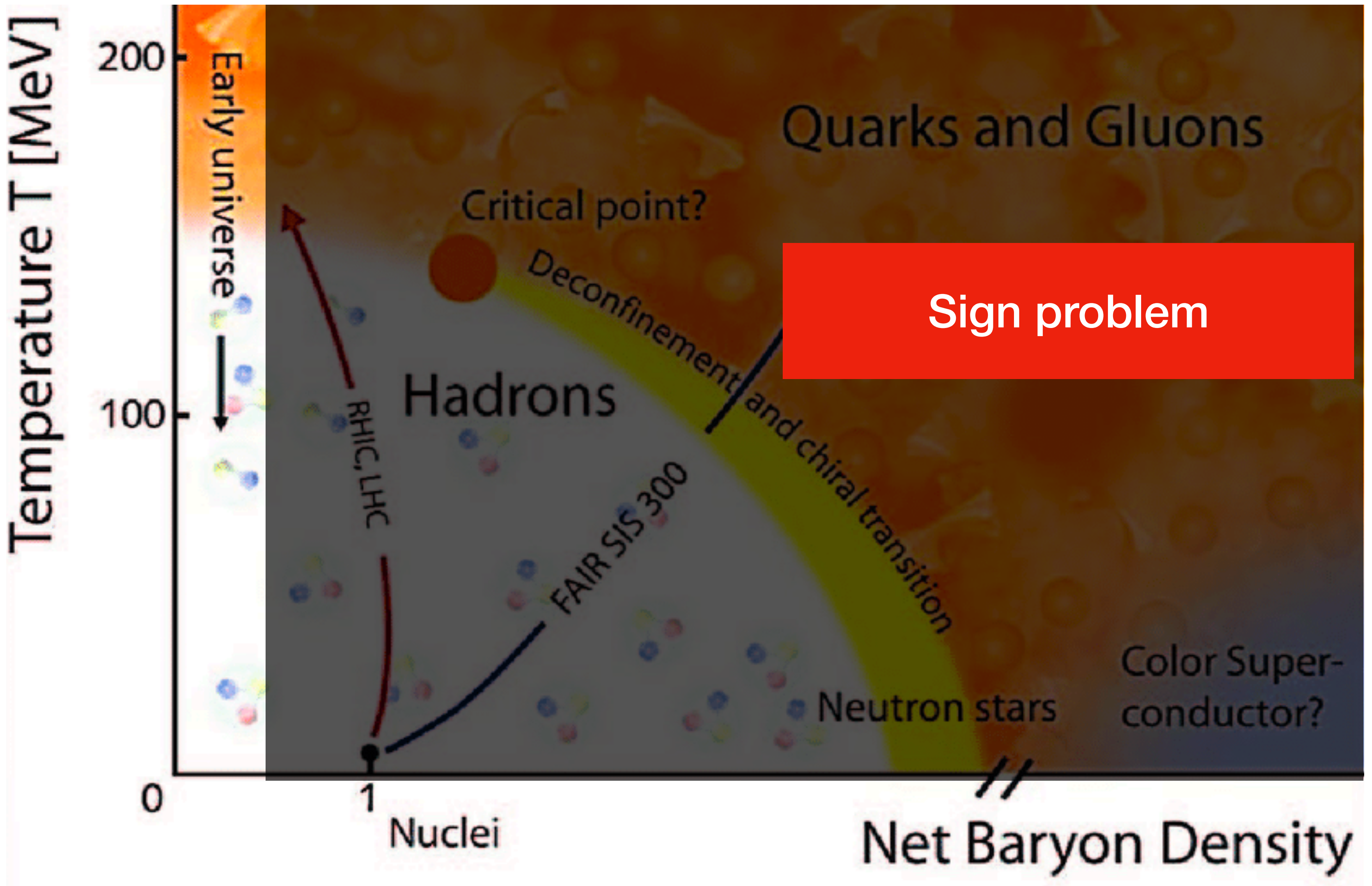
Introduction

MCMC is powerful, if mu = 0

- Monte-Carlo enables us to evaluate expectation values for “statistical system”, like lattice QCD in imaginary time

$$\langle O[U] \rangle = \frac{1}{N_{\rm conf}} \sum_{c}^{N_{\rm conf}} O[U_c] + \mathcal{O}\left(\frac{1}{\sqrt{N_{\rm conf}}}\right) \qquad U_c \leftarrow P(U) = \frac{1}{Z} e^{-S[U]} \in \mathbb{R}_+$$





- If we turn on the baryon number density μ, Monte-Carlo methods do not work because $e^{-S[U]}$ becomes complex. This is no more probability. (sign problem)
- Operator formalism does not have such problem! But it requires huge memory to store quantum states, which cannot be realized even on supercomputer.
- Quantum states should not be realized on classical computer but on quantum computers, as Feynman said.

Introduction

Finite T is good for classical machine. Finite mu is good for quantum

Classical machine: Lattice field theory calculations rely on

$$P(U) = \frac{1}{Z} e^{-S[U]} \det(D[U] + m)^2 \in \mathbb{R}_+$$

Since 1980 (M. Creutz)~

- This P cannot be regarded as probability if μ ≠ 0 (sign problem)

Quantum machines can realize (any) unitary evolutions (Solovay Kitaev theorem),

$$U(t) = \mathrm{e}^{-\mathrm{i}\hat{H}t} \quad : \text{unitary}$$

B. Chakraborty, AT+ *Phys.Rev.D* 105 (2022) 9, 094503 and references therein

- No problem for μ≠0 because we can only use unitary gates (operators)
- “Short time evolution” (shallow circuit) is preferred for near-term devices
- (Efficient way of) calculation of non-unitary cases (i.e. Boltzmann weight)?

	Classical Computers	Quantum Computers
Finite Density	Sign Problem	√
Finite Temperature	√	Challenging

Credit: Connor Powers (U. of Maryland)*

We need a method to calculate T>0 and μ≠0 for QCD and for near-term quantum devices!

*https://indico.hiskp.uni-bonn.de/event/40/contributions/484/attachments/358/630/Powers%20Talk%20Final%20Draft%20Updated.pdf

State preparation, VQE

State preparation, VQE

Realization of the exact ground state is difficult

Operator formalism

$\boxed{H : \text{Hamiltonian in QFT}}$ $[A_\mu, E_\nu] = i\hbar\delta_{\mu\nu}\cdots$

(This work)

Real time

Finite temperature/imaginary time

Minkowski in M^{d+1}

Euclid in S^1 x M^d

$$U(t) = e^{-itH}$$

Euclid(t → τ)

$t = -i\tau$

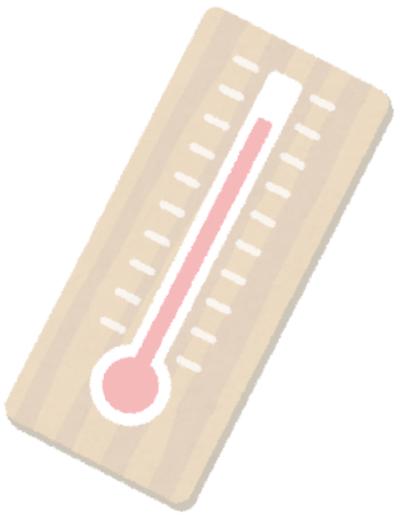
Minkowski(τ → t)

$$U(\tau) = e^{-\tau H}$$

$$\langle OO(\tau) \rangle = \mathrm{Tr}[O(0)O(\tau)\rho]$$

$$\rho = U(\tau)/Z$$

AT+ *Phys.Rev.D* 105 (2022) and a lot!



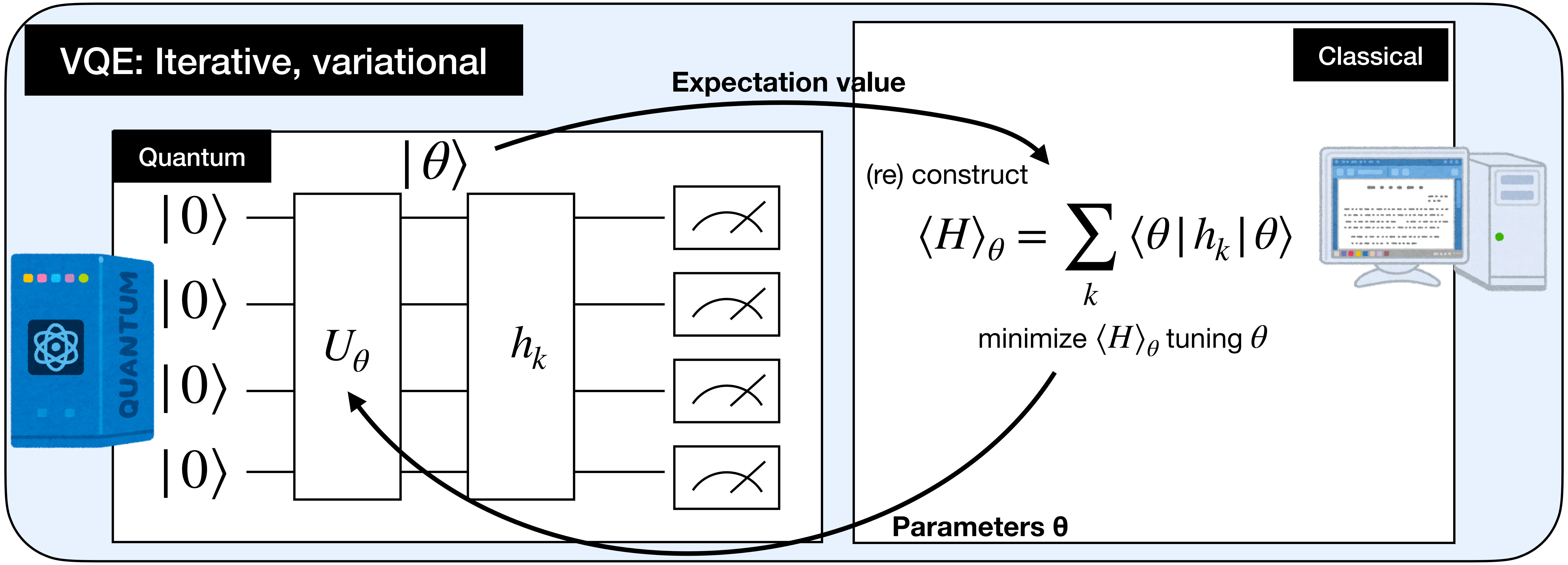
- Typical use of quantum algorithm is real-time simulation (∵ Unitary).
    - Main interest: $\langle \Omega | O | \Omega \rangle$, where $|\Omega\rangle$ is the exact ground state
- Difficulty: State preparation* (as ground state extraction in conventional LQCD)
    -> Variational ansatz (next)
- Our work uses the operator formalism. No problem for μ > 0
- Variational calculations with thermal state? -> Beta VQE

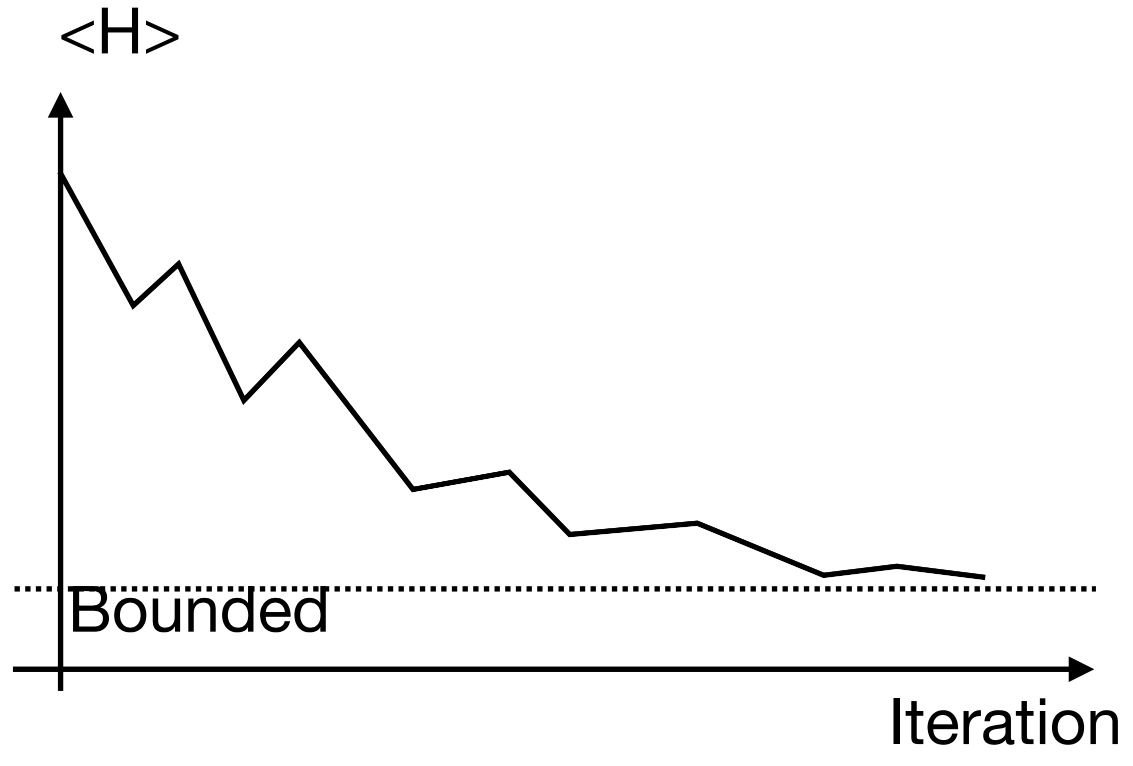
* very good summary in a talk by Alexei Bazavov https://indico.hiskp.uni-bonn.de/event/40/contributions/469/

State preparation, VQE

Mimic the ground state with a parametrized state

- Quantum machine: Exact ground state preparation is hard. In particular, it is difficult on near term devices
- Variational method for a *pure state* with a short circuit (VQE, variational quantum eigen-solver).
    - Quantum/Classical hybrid algorithm, iterative, variational
    - Parametrized **unitary** circuit (~parametrized wave-function $|\theta\rangle$, θ: a set of parameters)
- Basically, it mimics the ground state (pure state)





- Pros: Cheap. NISQ
- Cons: Systematic error (opposite to adiabatic ones)

State preparation, VQE

Thermal … Boltzmann weight is not unitary…

Operator formalism

$\boxed{H : \text{Hamiltonian in QFT}}$ $[A_\mu, E_\nu] = i\hbar\delta_{\mu\nu}\cdots$

Real time

Finite temperature/imaginary time

(This work)

Minkowski in M^{d+1}

Euclid in S^1 x M^d

$$U(t) = e^{-itH} \quad \underset{\text{Minkowski}(\tau \to t)}{\overset{\text{Euclid}(t \to \tau)}{\rightleftarrows}} \quad U(\tau) = e^{-\tau H}$$

$t = -i\tau$

AT+ *Phys.Rev.D* 105 (2022) and a lot!

$$\langle OO(\tau)\rangle = \mathrm{Tr}[O(0)O(\tau)\rho]$$

$$\rho = U(\tau)/Z$$

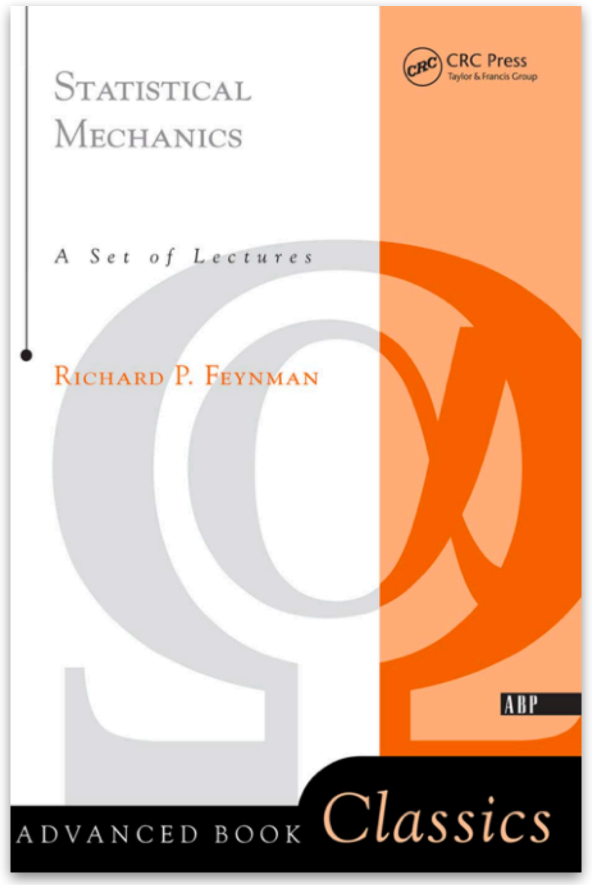
- Typical use of quantum algorithm is real-time simulation (∵ Unitary).
    - Main interest: $\langle \Omega | O | \Omega \rangle$, where $|\Omega\rangle$ is the exact ground state
- Difficulty: State preparation* (as ground state extraction in conventional LQCD)
    -> Variational ansatz
- Our work uses the operator formalism. No problem for μ > 0
- But how can we realized thermal "state"? (Next)

(Alternative approach TPQ: AT Yuki Nagai APLAT, 2020), P. Connor + Lattice 2022); QITE, A. M. Czajka arXiv: 2112.03944)

Density matrix, KL-*U* divergence

Density matrix

can describe statistical mechanics



Pure states: System is purely quantum

$$\rho_{\text{pure}} = |\Psi\rangle\langle\Psi| \qquad \langle O\rangle = \text{Tr}[O\rho_{\text{pure}}] = \langle\Psi|O|\Psi\rangle$$

Mixed states: States are classically mixed (≠ superposition)

$$\rho_{\text{mixed}} = \sum_i w_i |\psi_i\rangle\langle\psi_i| \qquad \langle O\rangle = \text{Tr}[O\rho_{\text{mixed}}] = \sum_i w_i \langle\psi_i|O|\psi_i\rangle$$

$w_i \in \mathbb{R}$ represents probability to find a pure state $|\psi_i\rangle$

Thermal states (Grand-canonical):

$$\rho_{T,\mu} = \frac{1}{Z} e^{-\frac{1}{T}(\hat{H}-\mu\hat{N})} \qquad \langle O\rangle_{T,\mu} = \text{Tr}[O\rho_{T,\mu}]$$

Thermal-quantum average in general

$$\langle O\rangle = \text{Tr}[O\rho]$$

Density matrix

Contain full information of the system, quantum version of probability distributions

*P. Caputa, AT + Physics Letters B 772, 53-57 (and a lot!)

Thermal-quantum average in general

$$\langle O \rangle = \mathrm{Tr}[O\rho]$$

General Properties of density matrix ρ

- It unifies discretion of pure states and mixed states
- Normalized as $\mathrm{Tr}[\rho] = 1$
- This can be used to investigate entanglement/thermalization*
- Density matrix ρ can be regarded as quantum version of probability distribution p(x)
- e.g.) $S = -\int dx\, p(x) \log p(x)$ (Shannon entropy)

  <—> $S = -\mathrm{Tr}[\rho \log \rho]$ (Von-Neumann entropy)
- Distance between two density matrices = quantum relative entropy (next)

KL-*U* divergence

"Distance" between two density matrices

- KL divergence for ρ is Kullback–Leibler *Umegaki* divergence (Pseudo-distance for ρ)
- Classical version: $D(p\,|\,q) = \int dx\, p(x) \log p(x)/q(x)$ (KL divergence)
  - Relative entropy, quantifies difference of two distributions (~distance)
  - Positive definite, Used in machine learning
  - D=0 if and only if p, q are equal
- **Quantum version:** $D(\rho_1\,|\,\rho_2) = \mathrm{Tr}[\rho_1 \log \rho_1 \big/ \rho_2]$ (KL-Umegaki divergence ~ distance)
  - Positive definite
  - D=0 if and only if ρ_1, ρ_2 are equal
- Kullback–Leibler *Umegaki* divergence can be used for variational approaches (as the flow models do)

Ansatz (mdoel) for ρ?

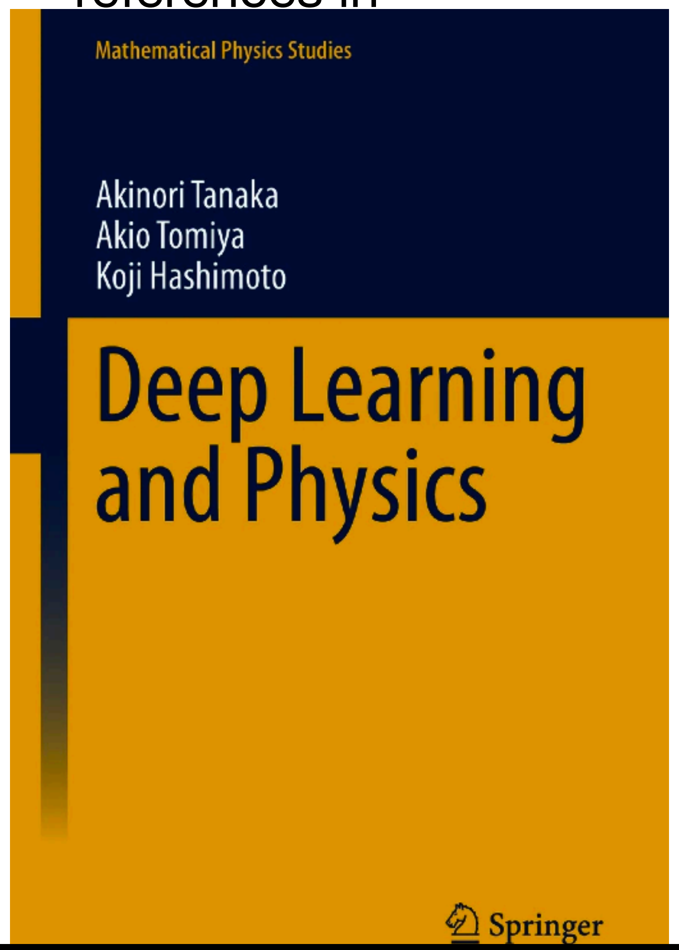
KL-U divergence

Beta VQE = VQE for T>0 = VQE + Neural net

J. -Guo Liu+ 1902.02663
*M. Germain+ 1502.03509

- Variational method for mixed states: Variational method on ρ
  - $\rho_{\Theta}^{\text{ansatz}} = \sum_{\{\vec{x}\}} p_{\phi}[\vec{x}]\, U_{\theta} | \vec{x} \rangle \langle \vec{x} | U_{\theta}\,,\quad \Theta = \theta \cup \phi$ (parameters for state/distribution)
  - $\vec{x} = (x_1, x_2, x_3, \cdots, x_k, \cdots)^{\top}$, and $x_k \in \{0,1\}$: Array of binary numbers
  - $U_{\theta} | \vec{x} \rangle$: parametrized pure states, similar to the conventional VQE, θ = variational parameters in a circuit
  - $p_{\phi}[\vec{x}]$: Classically approximated distribution for a configuration of $\vec{x}$,
    Neural network (Generative model, MADE*) is used.
    ϕ = parameters in the neural network
  - Neural network = Universal approximator of functions**
    Here, it approximate thermal distribution for fermions (generative model)

** references in



KL-U divergence

Optimized parameters enables us calculate <O> at T and mu

Jin-Guo Liu+ 1902.02663

- We approximate $\rho^{\text{exact}}_{T,\mu} = \frac{1}{Z} \mathrm{e}^{-\frac{1}{T}(\hat{H}-\mu\hat{N})}$ by $\rho^{\text{ansatz}}_{\Theta} = \sum_{\{\vec{x}\}} p_\phi[\vec{x}]\ U_\theta | \vec{x}\rangle\langle\vec{x} | U_\theta$ by tuning/training parameters $\Theta = \theta \cup \phi$ with minimizing $D(\rho^{\text{ansatz}}_{\Theta} | \rho) \geq 0$
  - $\langle O\rangle_{T,\mu} \approx \mathrm{Tr}[\rho_\Theta O]$, if and only if $\rho_\Theta \approx \rho$
- Intuitively:
  - Quantum machine stores a state $U_\theta | \vec{x}\rangle$
  - Classical machine makes thermal distribution $p_\phi[\vec{x}]$ (neural net, generative model)

KL-*U* divergence

Shifted KLU can be used if ln Z is not avilable

J. -Guo Liu+ 1902.02663

*M. Germain+ 1502.03509

- Variational method for mixed states: Variational method on ρ
  - $\rho_{\Theta}^{\text{ansatz}} = \sum_{\{\vec{x}\}} p_{\phi}[\vec{x}]\, U_{\theta} | \vec{x} \rangle \langle \vec{x} | U_{\theta}\,, \quad \Theta = \theta \cup \phi$ (parameters)
  - $\vec{x} = (x_1, x_2, x_3, \cdots, x_k, \cdots)^{\top}$, and $x_k \in \{0,1\}$: (roughly) fermion excitation
  - $U_{\theta} | \vec{x} \rangle$: parametrized pure states, similar to the conventional VQE
  - $p_{\phi}[\vec{x}]$: Classically approximated distribution for a configuration of $\vec{x}$, Neural network (MADE*) is used. ϕ = parameters
- Minimizing $D(\rho_{\Theta}^{\text{ansatz}} | \rho_{T,\mu}^{\text{exact}})$, we get approximated a set of states (= thermal)
- Shifted D (by a constant, minimization is not affected) is used in real applications:
  - $\mathscr{L}(\Theta) \equiv D(\rho_{\Theta}^{\text{ansatz}} | \rho_{T,\mu}^{\text{exact}}) - \underbrace{\ln Z}_{\text{const}} = \mathrm{Tr}[\rho_{\Theta}^{\text{ansatz}} \ln \rho_{\Theta}] + \frac{1}{T} \mathrm{Tr}[\rho_{\Theta}^{\text{ansatz}} (\hat{H} - \mu \hat{N})]$

BUT, in this work, we use exact diagonalization to check the validity of the algorithm and see D

Results

Results

Schwinger model at finite mu and T. Code is written in Julia

AT. 2205.08860

- We apply beta-VQE for Schwinger model = QED in 1+1d.
  - Toy model for QCD in 4d (good for testbed). Common features: Confinement, chiral anomaly, topology …

$$S = \int d^2x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - gA\!\!\!/ - m \right)\psi \right] \Longleftrightarrow H = \int dx \left[-\mathrm{i}\bar{\psi}\gamma^1(\partial_1 + \mathrm{i}gA_1)\psi + m\bar{\psi}\psi + \frac{1}{2}\Pi^2 \right]$$

$$\partial_x E = g\bar{\psi}\gamma^0\psi$$

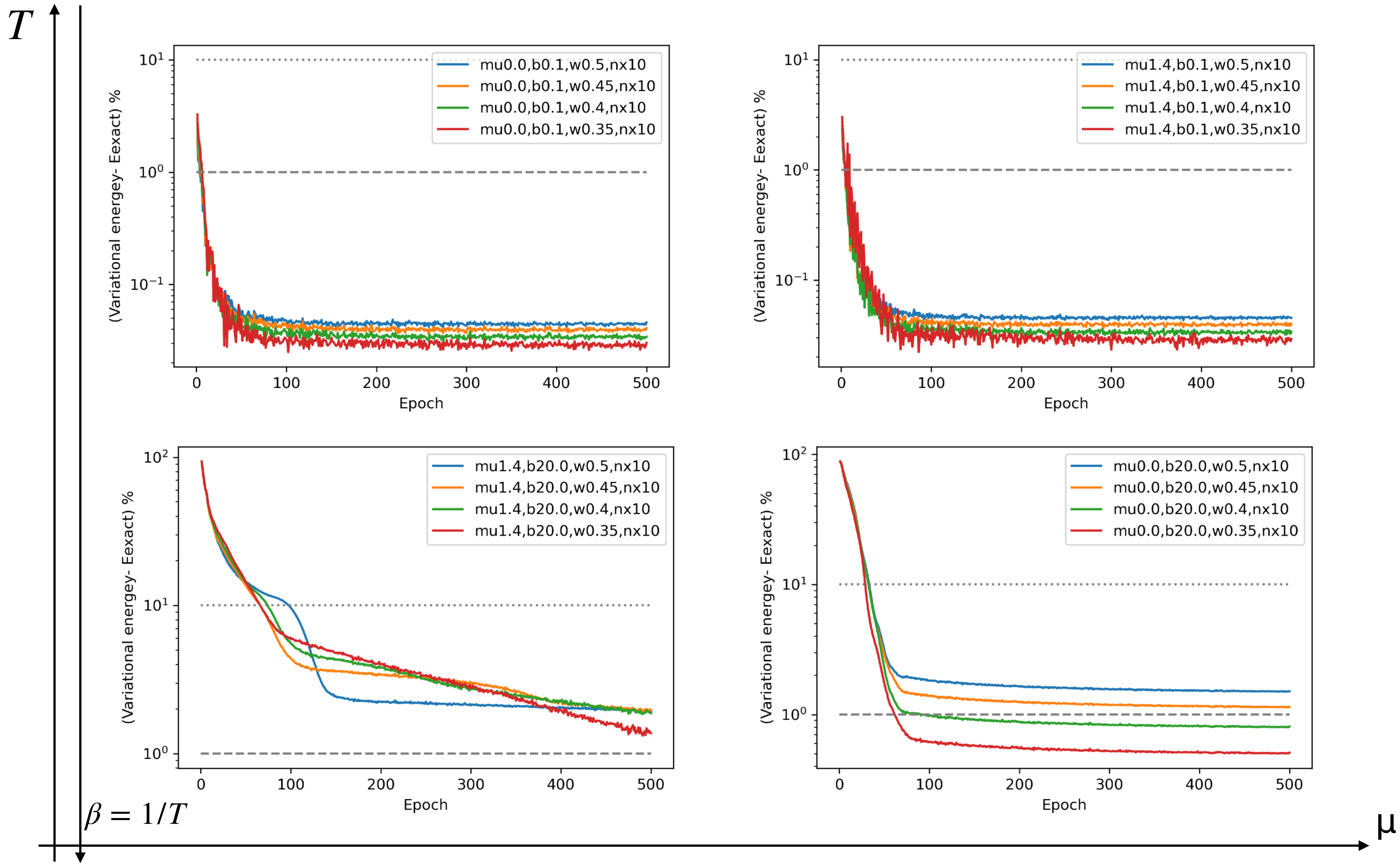
- Staggered discretization, Jordan-Wigner transformation, and open BC are used
- g = 1, Nx = (4, 6), 8, 10, 1/T = [0.5-20.0], mu= [0-1.4], 4 lattice spacings 1/2a = [0.5-0.35]
- We do not take large volume limit but take continuum limit
  - (Practically, Nx>10 cannot be calculated on our numerical resources)
  - (My previous work shows data from Nx>12 are essential to take stable large volume limit though)
- Setup for beta VQE:
  - Unitary part = SU(4) ansatz
  - Classical weight = Masked Auto-Encoder for Distribution Estimation (MADE)
- Training epoch is 500. Sampling = 5000 for classical distribution
- The code is implemented in julia . Calculations on Yukawa21 cluster at Kyoto University

- Observables
  - Variational free energy (exact = lower-bound, and variational one)
  - (Translationally invariant) Chiral condensate
- **Check point: Dependence of variational error on temperature**

Results

Compare the variational energy ~ D. Looks good

AT. 2205.08860



Vertical axis:
Deviation of D
from the exact one(%)

Horizontal axis:
Training epoch

1. Mild dependence on μ
2. Hard for T -> 0 (large deviation) as expected

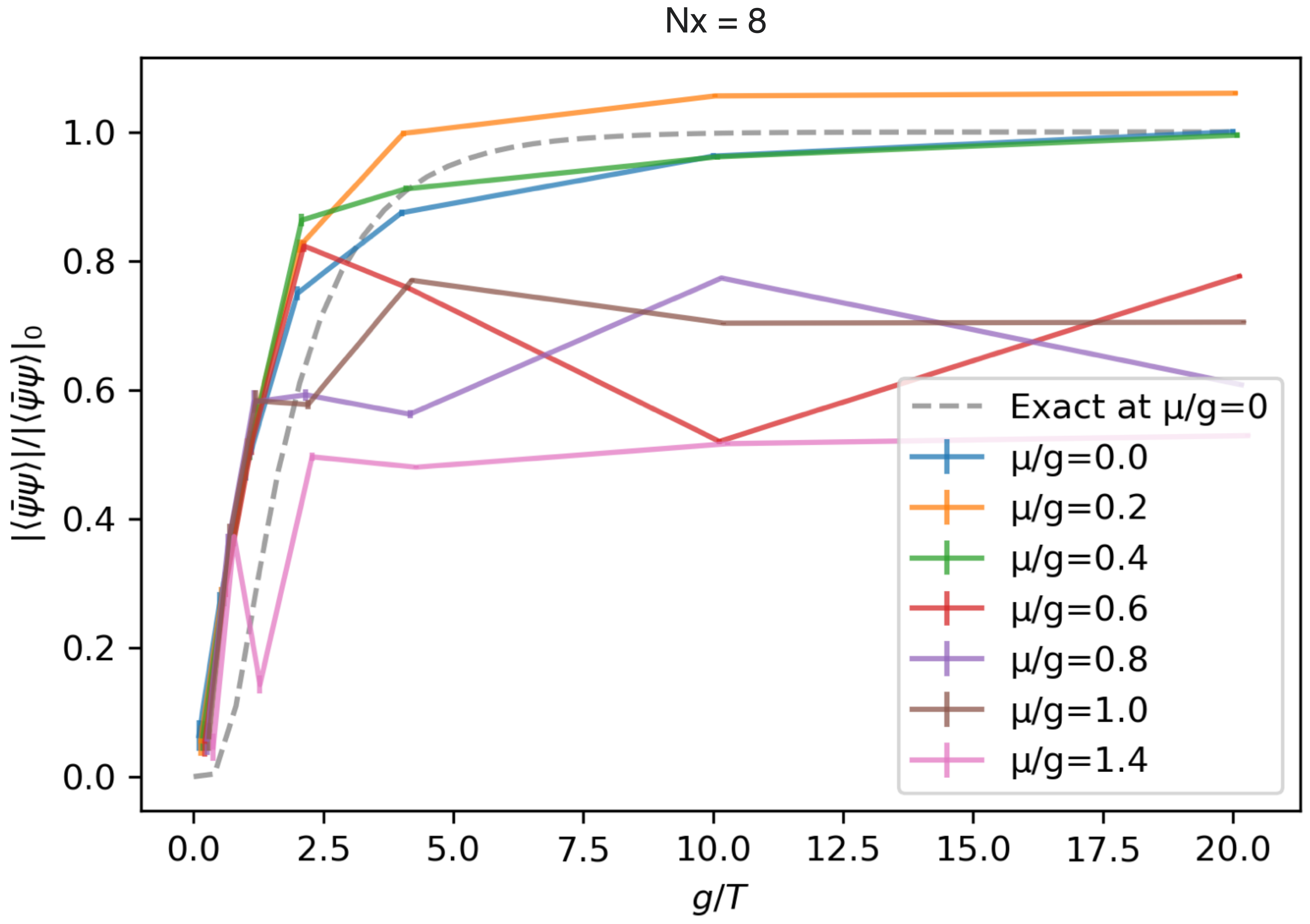
Results

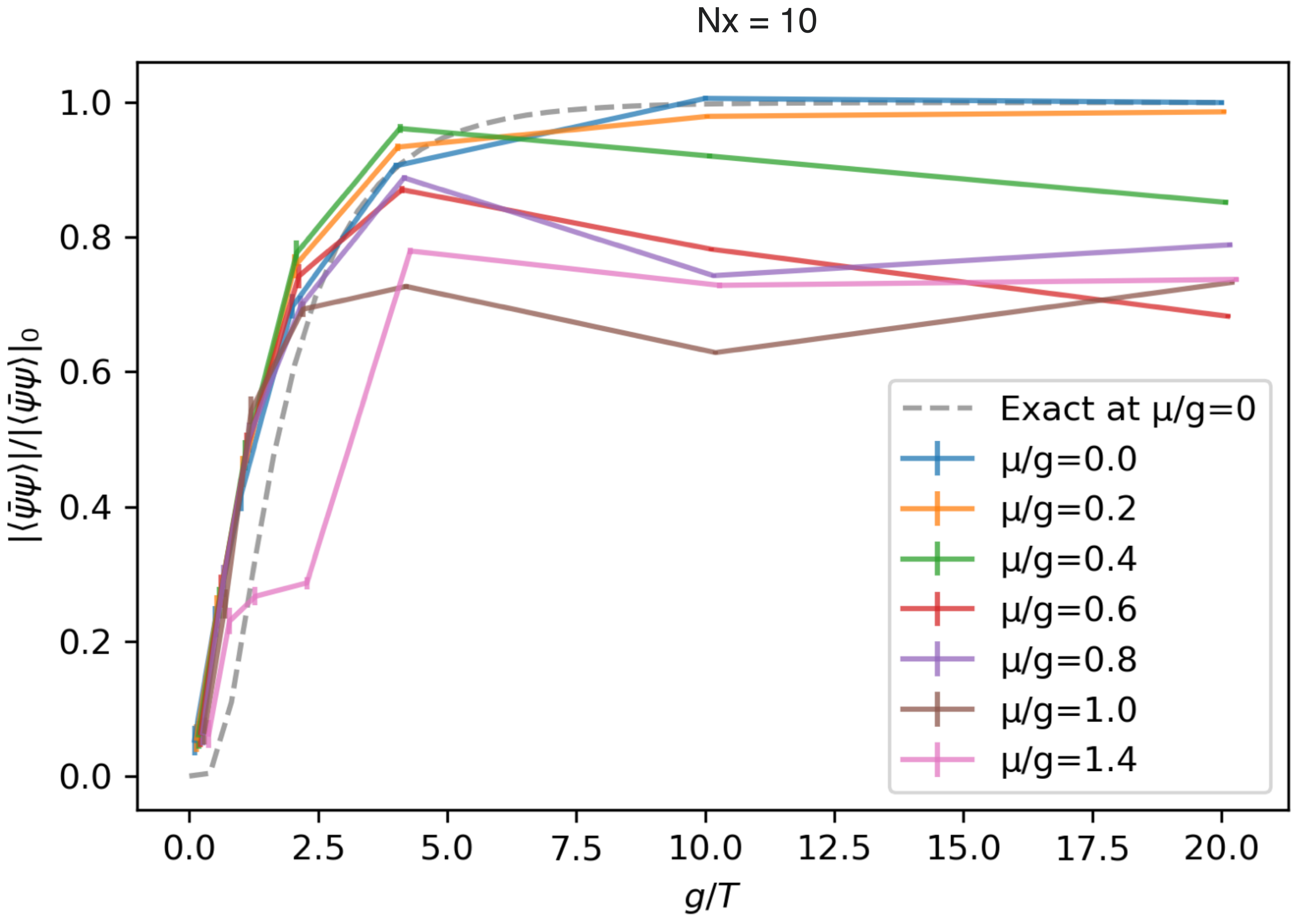
Chiral condensate as a function of T and mu

Akio Tomiya

AT arXiv: 2205.08860

Qualitatively consistent at μ = 0 (*), and results seems reasonable





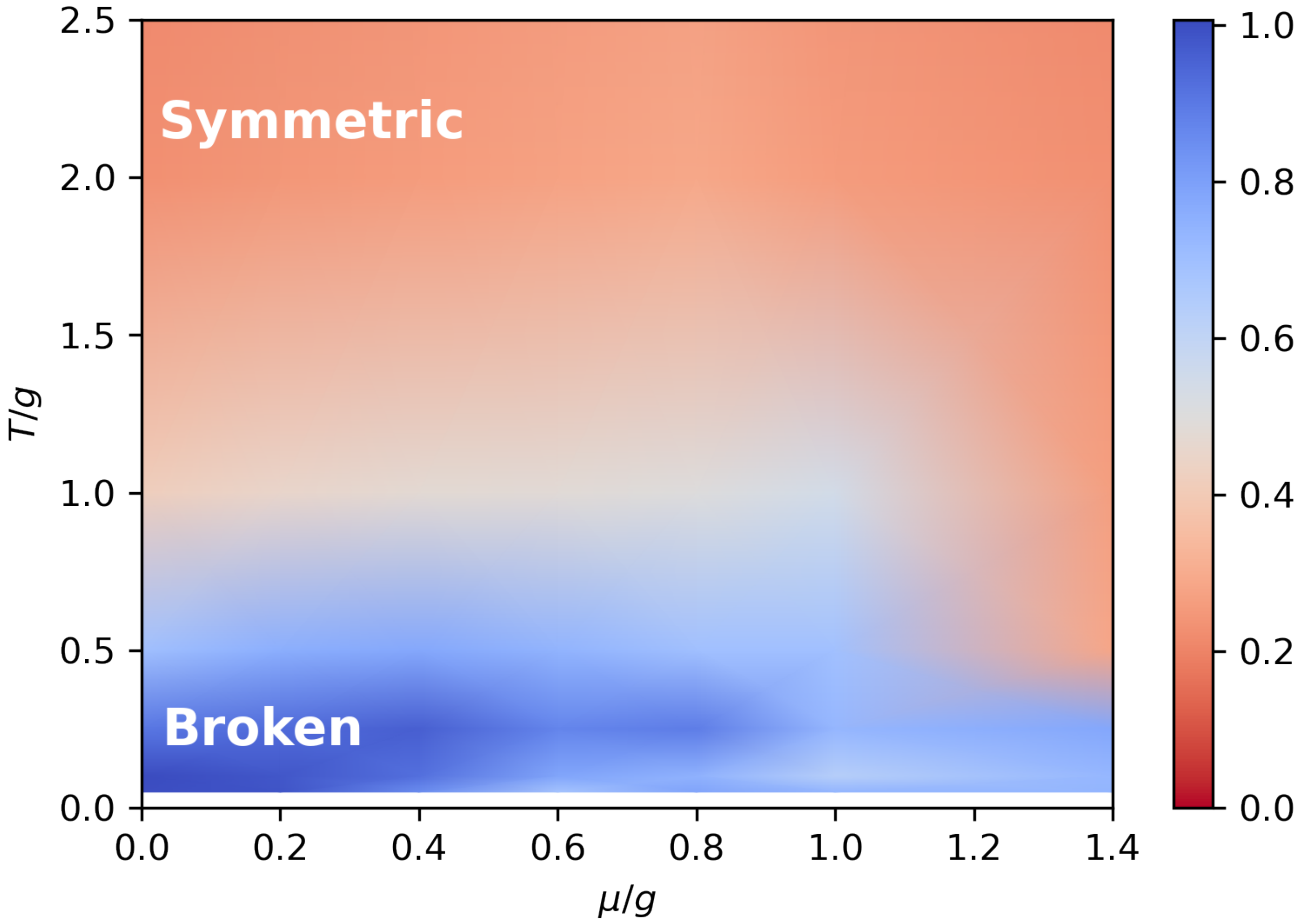
*(I did not include additive mass shift (Ross Dempsey+ arXiv: 2206.05308).
I thank to Takis Angelides (DESY) and Etsuko Itou (RIKEN) for letting me know this important reference!)

We use Nx = 10 results for the phase diagram

Summary

Classical-quantum hybrid algorithm: T>0 and μ>0

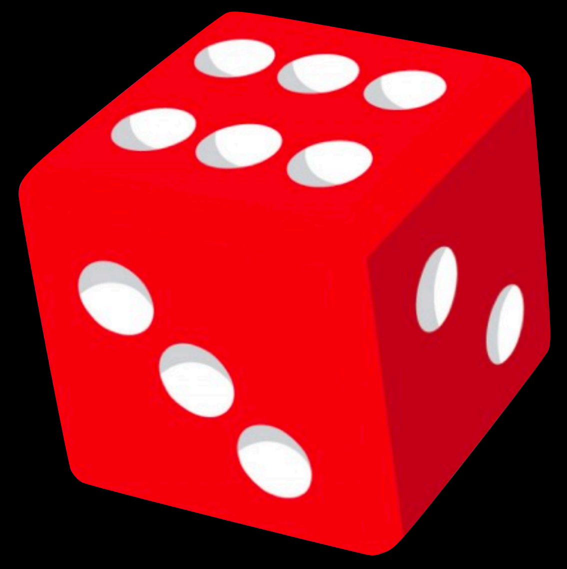
AT. 2205.08860



$$\frac{\langle \overline{\psi}\psi \rangle_{T,\mu}}{\langle \overline{\psi}\psi \rangle_{0.05,0}}$$

- We investigate T-μ phase diagram for Schwinger model. This algorithm works other than this model.
- Continuum extrapolation has been evaluated (w/o mass shift by Ross Dempsey+ arXiv: 2206.05308)
- The variational approach does not show difficulty for our parameter regime
- Future works: Towards to go large volume, optimization of code, GPU version, tensor network. (noise-free) real device!. Finite temperature + finite density + real time??
  Pure state ansatz? (relation to TPQ/QITE?), scaling? SU(N)?

Thanks!



VQE

state

- Quantum machine: Exact ground state preparation is hard. In particular, it is difficult on near term devices
- Variational method for a *pure state* with a short circuit (VQE, variation quantum eigen-solver).
    - Quantum/Classical hybrid algorithm, iterative
    - Parametrized unitary circuit (~parametrized state $|\theta\rangle$, θ: a set of parameters)
        - $|\theta\rangle = \hat{U}(\theta)\big(|0\rangle_1|0\rangle_2|0\rangle_3\cdots\big)$, and $\hat{U}(\theta)$ is a short circuit (entanglement + rotations)
    - If $\langle\theta|H|\theta\rangle = 0$, $|\theta\rangle \approx |\Omega\rangle$, where $|\Omega\rangle$ is the exact ground state
- Systematic error since $|\theta\rangle \neq |\Omega\rangle$ (cannot be exact in practice)

- How about thermal states? Thermal evolution is not unitary
- (TPQ/QITE on a classical emulator is an option but short circuit rep is not known)

Beta VQE

O(0.1)% for T>>0. T ~0 is difficult

μ/g	g/T	N_x	~1/a w/g	Approx $\mathcal{L} - \ln Z$	Exact $-\ln Z$	Diff (%)
0.0	0.1	4	0.5	-27.779	-27.781	0.00804
0.0	0.1	4	0.35	-27.807	-27.808	0.005
0.0	0.1	10	0.5	-70.686	-70.718	0.0459
0.0	0.1	10	0.35	-71.744	-71.765	0.0302
0.0	0.5	4	0.5	-5.792	-5.802	0.185
0.0	0.5	4	0.35	-5.885	-5.891	0.105
0.0	0.5	10	0.5	-17.133	-17.25	0.68
0.0	0.5	10	0.35	-18.849	-18.934	0.448
0.0	10.0	4	0.5	-1.748	-1.75	0.161
0.0	10.0	4	0.35	-1.829	-1.829	0.0184
0.0	10.0	10	0.5	-8.218	-8.341	1.48
0.0	10.0	10	0.35	-9.98	-10.03	0.496
0.0	20.0	4	0.5	-1.492	-1.739	14.2
0.0	20.0	4	0.35	-1.653	-1.806	8.46
0.0	20.0	10	0.5	-8.202	-8.328	1.51
0.0	20.0	10	0.35	-9.955	-10.006	0.509

			~1/a	Approx	Exact	
1.4	0.1	4	0.5	-28.021	-28.023	0.00697
1.4	0.1	4	0.35	-27.989	-27.991	0.00755
1.4	0.1	10	0.5	-70.842	-70.874	0.0453
1.4	0.1	10	0.35	-71.742	-71.763	0.0291
1.4	0.5	4	0.5	-6.784	-6.789	0.0609
1.4	0.5	4	0.35	-6.644	-6.647	0.0327
1.4	0.5	10	0.5	-17.989	-18.104	0.636
1.4	0.5	10	0.35	-19.445	-19.534	0.456
1.4	10.0	4	0.5	-3.708	-3.71	0.0728
1.4	10.0	4	0.35	-3.63	-3.669	1.07
1.4	10.0	10	0.5	-10.067	-10.243	1.71
1.4	10.0	10	0.35	-11.763	-11.862	0.837
1.4	20.0	4	0.5	-3.673	-3.681	0.218
1.4	20.0	4	0.35	-3.621	-3.669	1.31
1.4	20.0	10	0.5	-10.028	-10.224	1.92
1.4	20.0	10	0.35	-11.699	-11.862	1.37

1. Mild dependence on μ
2. Hard for T -> 0 (large deviation) as expected

state preparation

state

Jin-Guo Liu+ 1902.02663

- We approximate $\rho^{\text{exact}}_{T,\mu} = \frac{1}{Z} e^{-\frac{1}{T}(\hat{H}-\mu\hat{N})}$ by $\rho^{\text{ansatz}}_{\Theta} = \sum_{\{\vec{x}\}} p_\phi[\vec{x}]\ U_\theta|\vec{x}\rangle\langle\vec{x}|U_\theta$
  by tuning/training parameters with minimizing $D(\rho^{\text{ansatz}}_{\Theta}|\rho)$
  - $\langle O\rangle_{T,\mu} \approx \mathrm{Tr}[\rho_\Theta O]$, if and only if $\rho_\Theta \approx \rho$
- Quantum machine can store a state $U_\theta|\vec{x}\rangle$
- Classical machine can sample thermal distribution from $p_\phi[\vec{x}]$ (neural net)

state preparation

state

QCD (Quantum Chromo-dynamics) in 3 + 1 dimension

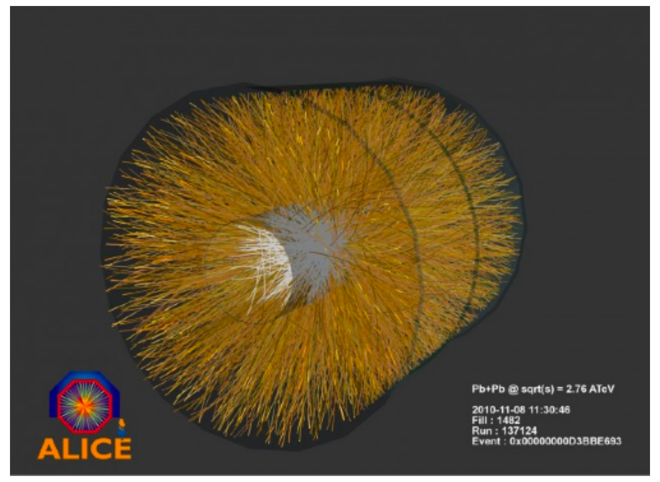
$$S = \int d^4x \left[-\frac{1}{2} \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\partial\!\!\!/ + gA\!\!\!/ - m\right)\psi \right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$A_\mu(x) \in su(3)$, 3x3traceless, harmitian

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

H: Hamiltonian from S



- Generalization of QED, $A_\mu(x)$ is a matrix (Yang-Mills-Uchiyama)
- Action above enables us to calculate followings:
  - Tc of Quark-Hadron, Matrix elements of QCD
  - Forces between nuclei … etc!

Lattice QCD in 4 dimension

F. Wegner 1971
K. Wilson 1974

$U_\mu = e^{aigA_\mu}$

Lattice regulation

$$S[U,\psi,\bar{\psi}] = a^4 \sum_n \left[-\frac{1}{g^2}\mathrm{Re}\ \mathrm{tr}\ U_{\mu\nu} + \bar{\psi}\left(D\!\!\!\!/ + m\right)\psi \right]$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-S}\mathcal{O}(U)$$

$$|\psi(t)\rangle = e^{-H\tau}|\psi(0)\rangle$$

- Lattice QCD has same long-distance physics with continuum QCD
- Euclidean signature, statistic physics

<u>My related talks</u>

U(1)A at fin. temp by Yu Zhang, 28 Jul 2021, 05:45(EDT)
QCD + magnetic field by Xiaodang Wang, 28 Jul 2021, 22:00 (EDT)

Schwinger model

＝2D QED: Solvable at m=0, similar to QCD in 4D.

Schwinger model = QED in 1+1 dimension

$$S = \int d^2x \Big[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\big(\mathrm{i}\partial\!\!\!/ - gA\!\!\!/ - m\big)\psi + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu}\Big]$$

Similarities to QCD in 3+1

- Confinement
- Chiral symmetry breaking (different mechanism), gapped even m=0

$$\langle \overline{\psi}\psi \rangle = -\frac{\mathrm{e}^{\gamma} g}{\pi^{3/2}} = -g0.16\cdots$$

- Topological term can be included as in QCD
- Vacuum decay by external electric field (Schwinger effect)

Hamiltonian of Schwinger model

Akio Tomiya

＝2D QED: Solvable at m=0, similar to QCD in 4D.

Schwinger model = QED in 1+1 dimension

$$S = \int d^2x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left(\mathrm{i}\partial\!\!\!/ - gA\!\!\!/ - m \right) \psi \right]$$

- Strategy

1. Derive Hamiltonian with gauge fixing
2. Rewrite gauge field to fermions using Gauss' law
3. Use Jordan-Wigner transformation → Spin system

Why? next page

Hamiltonian of Schwinger model

Akio Tomiya

Schwinger model in spin language

Schwinger model = QED in 1+1 dimension

$$S = \int d^2x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left(i\partial\!\!\!/ - gA\!\!\!/ - m \right)\psi \right]$$

- Strategy(1gauge fix, 2Gauss' law, 3Jordan-Wigner trf)

Schwinger model on the lattice (staggered fermion, OBC, Spin rep.)

$$H = \frac{1}{4a} \sum_n \left[X_n X_{n+1} + Y_n Y_{n+1} \right] + \frac{m}{2} \sum_n (-1)^n Z_n + \frac{g^2 a}{2} \sum_n \left[\sum_{j=1}^{n} \left(\frac{Z_j + (-1)^j}{2} \right) + \epsilon_0 \right]^2$$

X_j: Pauli matrix of x on site j Y_j: Pauli matrix of y on site j Z_j: Pauli matrix of z on site j

- Spin representation is necessary to use quantum device (Analogous to floating point rep. in classical machine)
- (QCD + QC also requires this strategy)

Hamiltonian of Schwinger model

＝2D QED: Solvable at m=0, similar to QCD in 4D.

(skip)

Schwinger model = QED in 1+1 dimension

$$S = \int d^2x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - gA\!\!\!/ - m \right)\psi \right]$$

$$\Pi(x) = \frac{\partial \mathscr{L}}{\partial \dot{A}^1(x)} = \dot{A}(x) = E(x)$$

$A_0 = 0$

$$\begin{cases} H = \int dx \left[-\mathrm{i}\overline{\psi}\gamma^1(\partial_1 + \mathrm{i}gA_1)\psi + m\overline{\psi}\psi + \frac{1}{2}\Pi^2 \right] \\ \partial_x E = g\bar{\psi}\gamma^0\psi \end{cases}$$

(Gauss' law constraint)

This constrains time evolution to be gauge invariant

Lattice Hamiltonian formalism

Akio Tomiya

Hamiltonian on a discrete space

(skip)

Schwinger model in continuum

$$H = \int dx \left[-\mathrm{i}\overline{\psi}\gamma^1(\partial_1 + \mathrm{i}gA_1)\psi + m\overline{\psi}\psi + \frac{1}{2}\Pi^2 \right]$$

Gauss' law $\partial_x E = g\bar{\psi}\gamma^0\psi$

$$-\frac{1}{g}\Pi(x) \to L_n \qquad \text{upper componentof } \psi \to \chi_{\text{even-site}}$$

$$-agA_1(x) \to \phi_n \qquad \text{lower componentof } \psi \to \chi_{\text{odd-site}}$$

Schwinger model on the lattice (staggered fermion)

$$H = -\frac{\mathrm{i}}{2a}\sum_{n=1}^{N-1}\left[\chi_{n+1}^\dagger \mathrm{e}^{-\mathrm{i}\phi_n}\chi_n - \chi_n^\dagger \mathrm{e}^{\mathrm{i}\phi_n}\chi_{n+1}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n + \frac{g^2a}{2}\sum_{n=1}^{N-1}L_n^2$$

Gauss' law $L_n - L_{n-1} = \chi^\dagger\chi_n - \frac{1}{2}\left(1-(-1)^n\right)$

Lattice Schwinger model = spin system

Akio Tomiya

Gauge trf, open bc, Gauss law -> pure fermionic system

(skip)

Schwinger model on the lattice (staggered fermion)

$$H = -\frac{\mathrm{i}}{2a}\sum_{n=1}^{N-1}\left[\chi_{n+1}^{\dagger}\mathrm{e}^{-\mathrm{i}\phi_n}\chi_n - \chi_n^{\dagger}\mathrm{e}^{\mathrm{i}\phi_n}\chi_{n+1}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^{\dagger}\chi_n + \frac{g^2a}{2}\sum_{n=1}^{N-1}\boxed{L_n^2}$$

Gauss' law $\boxed{L_n - L_{n-1} = \chi^{\dagger}\chi_n - \frac{1}{2}\left(1-(-1)^n\right)}$

$L_0 = \epsilon_0 \in \mathbb{R}$ (**open B.C.**), **and insert "Gauss' law"**

$$\begin{cases} U_n = \prod_{j=1}^{n-1}\mathrm{e}^{-\mathrm{i}\phi_j} \\ \chi_n \to U_n\chi_n \\ \mathrm{e}^{-\mathrm{i}\phi_{n-1}} \to U_{n-1}\mathrm{e}^{-\mathrm{i}\phi_{n-1}}U_n^{\dagger} \end{cases}$$

remnant gauge transformation

Schwinger model on the lattice (staggered fermion, OBC)

$$H = -\frac{\mathrm{i}}{2a}\sum_{n}\left[\chi_{n+1}^{\dagger}\chi_n - \chi_n^{\dagger}\chi_{n+1}\right] + m\sum_{n}(-1)^n\chi_n^{\dagger}\chi_n + \frac{g^2a}{2}\sum_{n}\left[\boxed{\sum_{j}^{n}\left(\chi_j^{\dagger}\chi_j - \frac{1-(-1)^j}{2}\right) + \epsilon_0}\right]^2$$

Lattice Schwinger model

Akio Tomiya

We requires anticommutations to fermions

(skip)

Schwinger model on the lattice (staggered fermion, OBC)

$$H = -\frac{\mathrm{i}}{2a}\sum_n \left[\chi_{n+1}^\dagger \chi_n - \chi_n^\dagger \chi_{n+1}\right] + m\sum_n (-1)^n \chi_n^\dagger \chi_n + \frac{g^2 a}{2}\sum_n \left[\sum_j^n \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right) + \epsilon_0\right]^2$$

System is quantized by assuming the canonical anti-commutation relation

$$\{\chi_j^\dagger, \chi_k\} = \mathrm{i}\delta_{jk} \qquad j, k = \text{site index}$$

On the other hand, Pauli matrices satisfy anti-commutation as well

$$\{\sigma^\mu, \sigma^\nu\} = 2\delta_{\mu\nu}\mathbf{1} \qquad \mu, \nu = 1,2,3$$

Quantum spin-chain case, each site has Pauli matrix, but they are “commute”.

We can absorb difference of statistical property using Jordan Wigner transformation

Jordan-Wigner transformation:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\prod_{j<n}(\mathrm{i}Z_j)$$

X_j: Pauli matrix of x on site j

Y_j: Pauli matrix of y on site j

Z_j: Pauli matrix of z on site j

This guarantees the statistical property

This (re)produces correct Fock space.

We can rewrite the Hamiltonian in terms of spin-chain

Lattice Schwinger model = spin system

Akio Tomiya

Jordan-Wigner transformation: Fermions ~ Spins

(skip)

Schwinger model on the lattice (staggered fermion, OBC)

$$H = -\frac{\mathrm{i}}{2a}\sum_n \left[\chi_{n+1}^\dagger \chi_n - \chi_n^\dagger \chi_{n+1}\right] + m\sum_n (-1)^n \chi_n^\dagger \chi_n + \frac{g^2 a}{2}\sum_n \left[\sum_j^n \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right) + \epsilon_0\right]^2$$

$$\begin{cases} \chi_n = \dfrac{X_n - \mathrm{i}Y_n}{2}\displaystyle\prod_{j<n}(\mathrm{i}Z_j) \\[2ex] \chi_n^\dagger = \dfrac{X_n + \mathrm{i}Y_n}{2}\displaystyle\prod_{j<n}(-\mathrm{i}Z_j) \end{cases}$$

Jordan-Wigner transformation

X_j: Pauli matrix of x on site j

Y_j: Pauli matrix of y on site j

Z_j: Pauli matrix of z on site j

Schwinger model on the lattice (staggered fermion, OBC, Spin rep.)

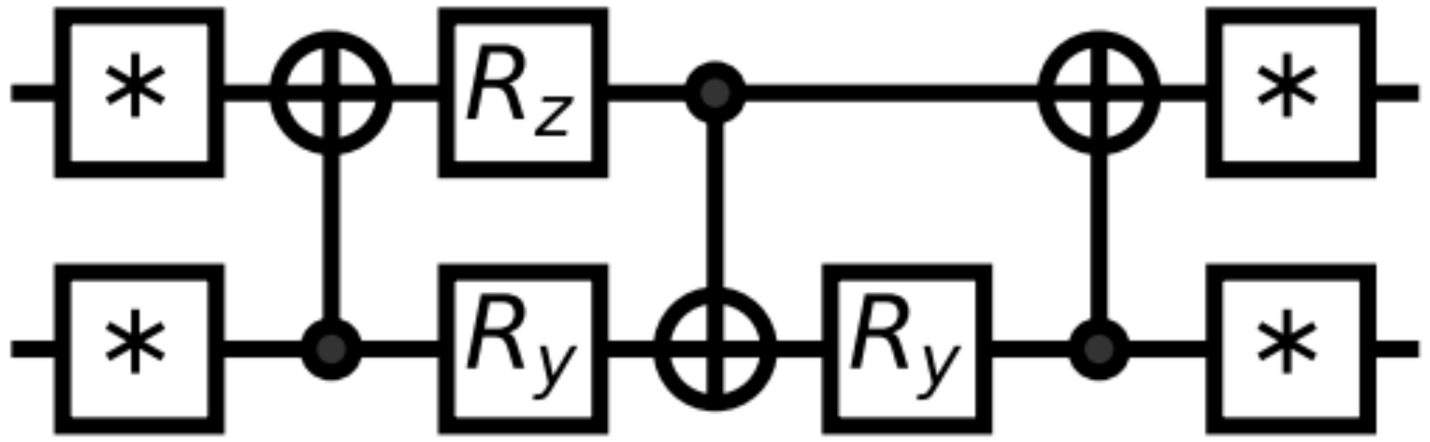
$$H = \frac{1}{4a}\sum_n \left[X_n X_{n+1} + Y_n Y_{n+1}\right] + \frac{m}{2}\sum_n (-1)^n Z_n + \frac{g^2 a}{2}\sum_n \left[\sum_{j=1}^n \left(\frac{Z_j + (-1)^j}{2}\right) + \epsilon_0\right]^2$$

[Y. Hosotani 9707129]

SU(4)

Variational ansatz

J. -Guo Liu+ 1902.02663



The general gate consists of 15 single qubit gates and 3 CNOT gates.
Each two qubit unitary is parametrized by 15 parameters in the rotational gates, which parametrizes the SU(4) group.