D-meson Semileptonic Decays

With Highly Improved Staggered Quarks

William I. Jay - Massachusetts Institute of Technology
(Fermilab Lattice and MILC collaborations)

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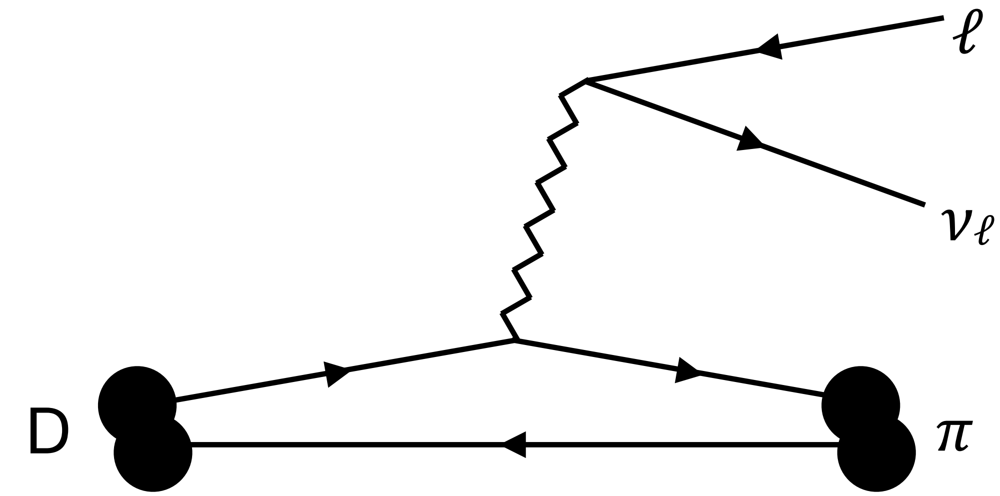
all-HISQ Working Group

- Carleton DeTar
- Aida El Khadra
- Elvira Gamiz
- Steve Gottlieb
- Andreas Kronfeld
- Andrew Lytle
- Jim Simone
- Alex Vaquero

Huge thanks and acknowledgement to my friends and colleagues in the all-HISQ working group. This work would be impossible without their support.

Motivation: Semileptonic decays

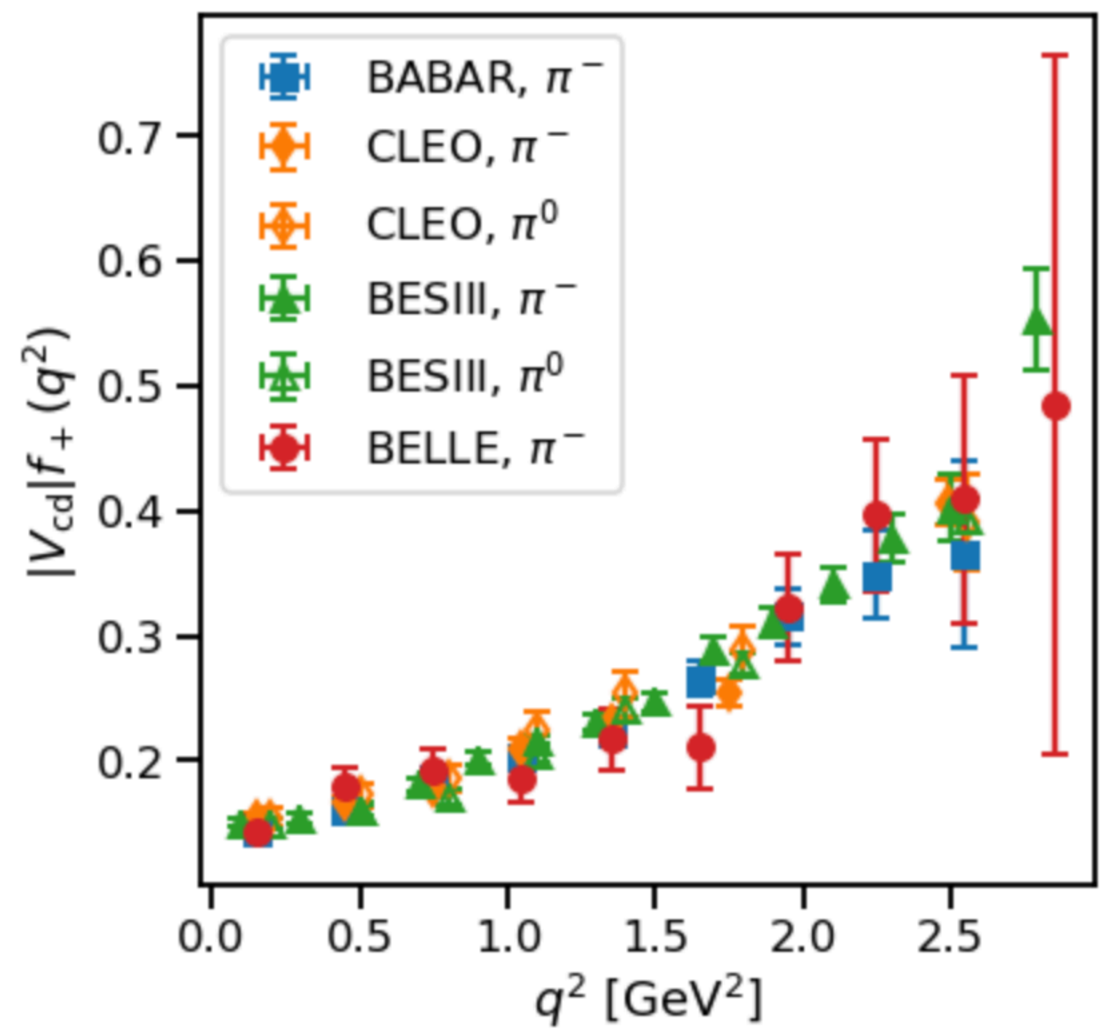
- Consider the decay D→ $\pi\ell\nu$



- Suppose $m_\ell \approx 0$ (excellent approximation for semi-electronic decays)

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cx}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

- For general m_ℓ, the scalar form factor $f_0(q^2)$ also enters ($\propto m_\ell^2$)



Motivation: Semileptonic decays

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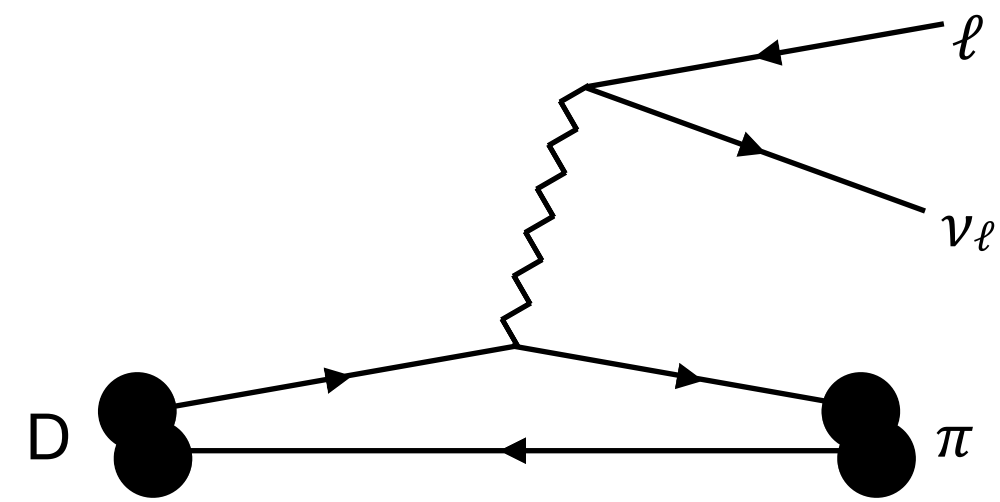


$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cx}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

$$\langle \pi | \mathcal{V}^\mu | D \rangle \equiv f_+(q^2)(p_D^\mu + p_\pi^\mu) + f_-(q^2)(p_D^\mu - p_\pi^\mu)$$

Or can equivalently decompose as:

$$\langle \pi | \mathcal{V}^\mu | D \rangle \equiv \sqrt{2M_D} \left(v^\mu f_{||}(q^2) + p_\perp^\mu f_\perp(q^2) \right)$$

Semileptonic decays: H→Pℓv

Theoretical preliminaries

- Lattice and continuum currents are related via $\mathcal{J} = Z_J J$

- We work in the rest frame of the decay D-meson.

$$f_{\parallel} = Z_{V^0} \frac{\langle \pi | V^0 | D \rangle}{\sqrt{2M_D}}$$

$$f_{\perp} = Z_{V^i} \frac{\langle \pi | V^i | D \rangle}{\sqrt{2M_D}} \frac{1}{p_\pi^i}$$

$$f_0 = Z_S \frac{m_c - m_\ell}{M_D^2 - M_\pi^2} \langle \pi | S | D \rangle$$

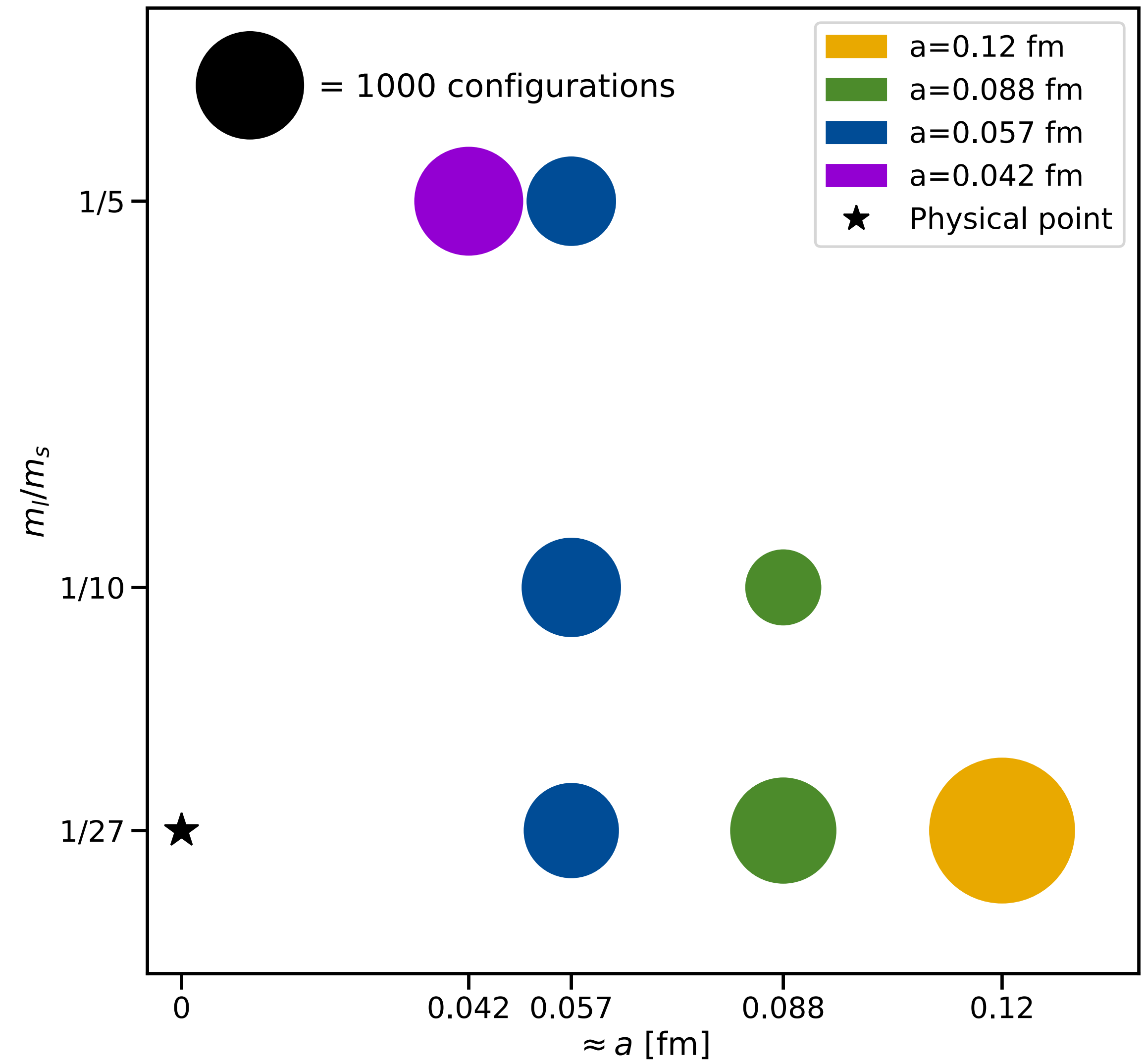
$$f_+ = \frac{1}{\sqrt{2M_D}} \left(f_{\parallel} + (M_D - E_\pi) f_{\perp} \right)$$

All quantities on the RHS are calculable in Euclidean space via lattice gauge theory

Simulation Details

all-HISQ Campaign

- Simulations with Nf=(2+1+1) flavors of dynamical HISQ fermions
- Gauge ensembles generated by the MILC collaboration
- Lattice spacings: 0.12 fm to 0.042 fm
- M_π: 135 MeV to 330 MeV
- Heavy valence masses from 0.9 m_c up to $am_h \approx 1.0$
- Today: $D \to \pi$, $D \to K$, and $D_s \to K$
- See also talk by Andrew Lytle in this session for update on our concurrent calculation of B-decays
- **Our analysis is blinded. "±5% for 3pt functions"**



- Note: on finest HISQ ensembles (0.042, 0.03 fm), $am_b < 1$
- All fermions simulated using the same relativistic light quark action

Simulation Details

Correlation functions

- Simultaneous correlated fit to 2pt + 3pt functions gives transition matrix elements $\langle\pi|J|D\rangle$, i.e. the form factors.

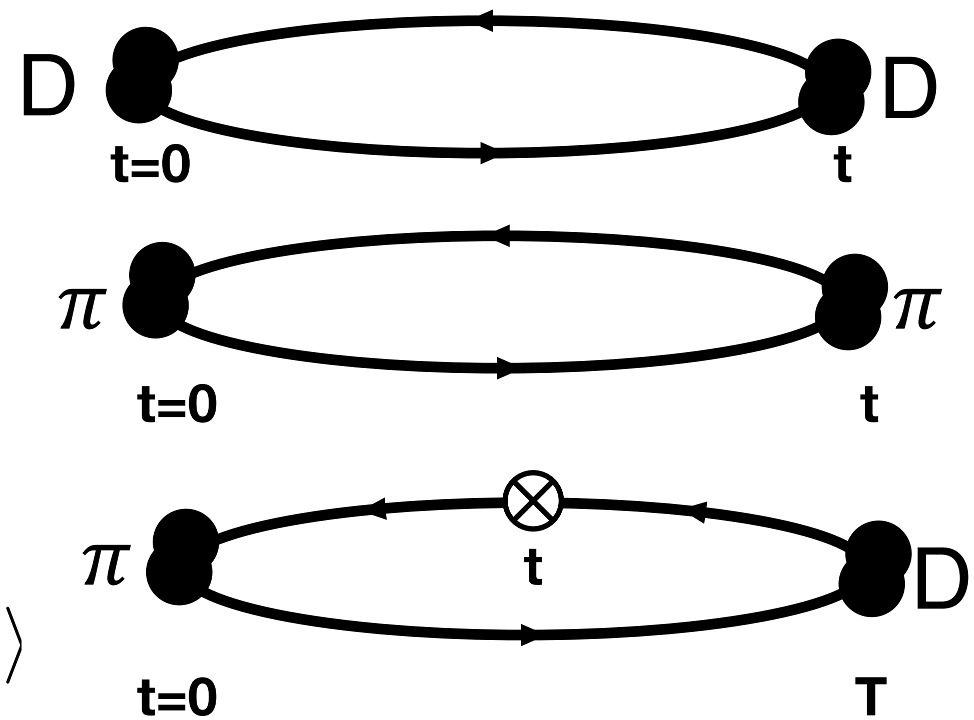
- Methodology: See proceedings from Lattice 2021 [arXiv:2111.05184]

$$C_D(t) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_D(0,\mathbf{0}) \mathcal{O}_D(t,\boldsymbol{x}) \rangle$$

$$C_\pi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}_\pi(0,\mathbf{0}) \mathcal{O}_\pi(t,\boldsymbol{x}) \rangle$$

$$C_3(t,T,\boldsymbol{p}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{p}\cdot\boldsymbol{y}} \langle \mathcal{O}_\pi(0,\mathbf{0}) J(t,\boldsymbol{y}) \mathcal{O}_D(T,\boldsymbol{x}) \rangle$$

$$\longrightarrow \langle 0|\, \mathcal{O}_\pi \,|\pi\rangle \langle \pi|\, J \,|D\rangle \langle D|\, \mathcal{O}_D \,|0\rangle\, e^{-E_\pi t} e^{M_D(T-t)}$$

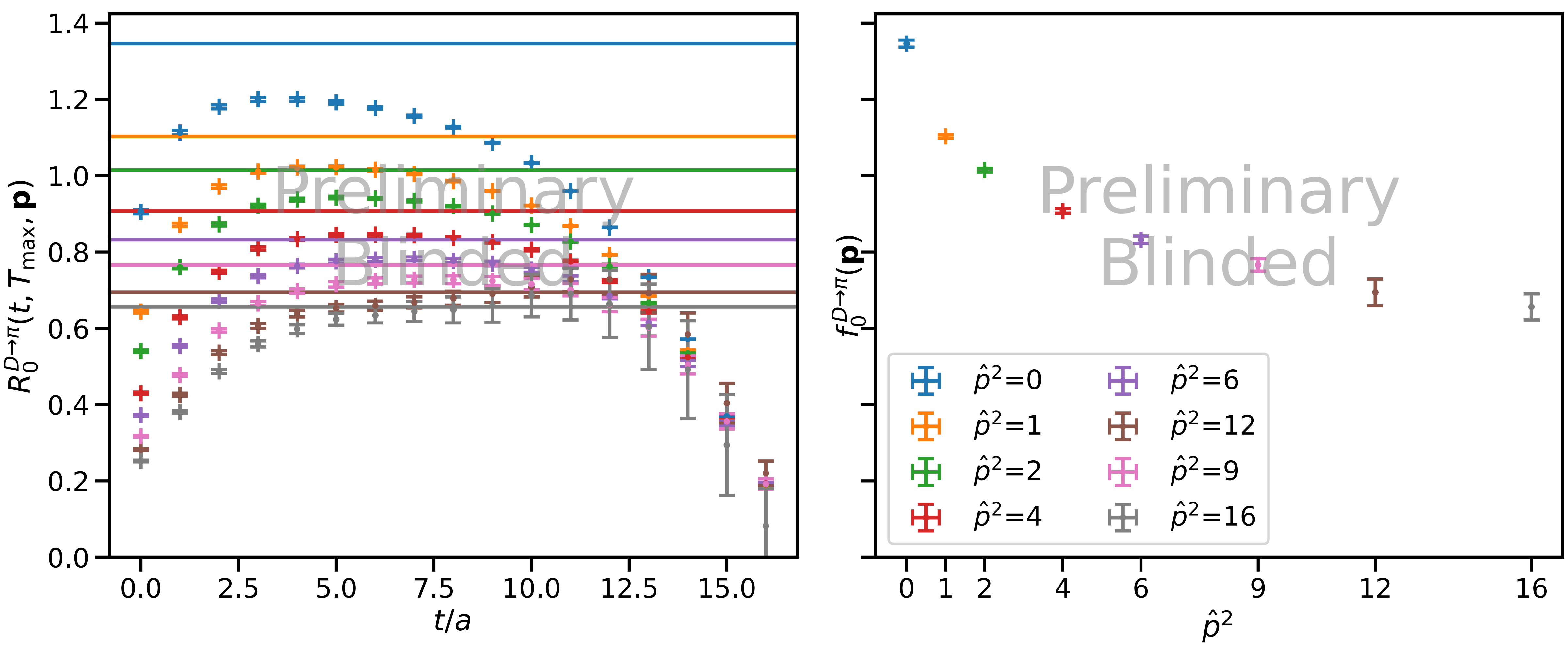


Statistical Analysis

Results: 3pt functions - f_0 for D to π

- A certain ratio is useful to isolate form factors visually:

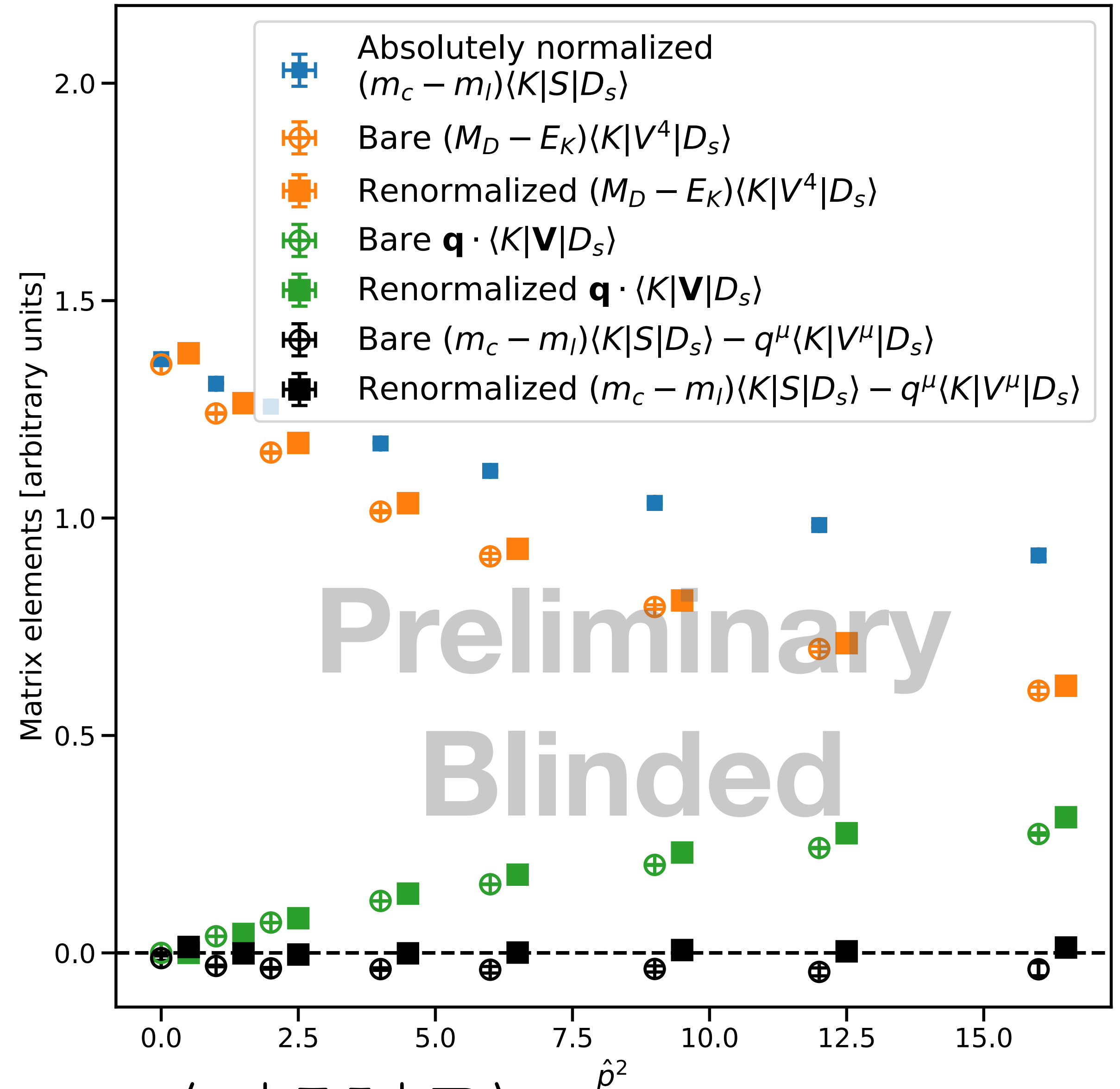
$$R^J(t,T,\boldsymbol{p}) \propto \frac{C_3^J(t,T,\boldsymbol{p})}{\sqrt{C_\pi(t,\boldsymbol{p})C_D(T-t)e^{-E_\pi t}e^{-M_D(T-t)}}} \longrightarrow f_J$$



Renormalization: PCVC

Partial Conservation of the Vector Current

- Recall $\mathcal{J} = Z_J J$
- PVCV: $\partial_\mu \mathcal{V}^\mu = (m_1 - m_2)\mathcal{S}$
- For the HISQ action, the local scalar density is absolutely normalized.
- Imposing PCVC allows us to extract Z_{V0} and Z_{Vi}
- In terms of D→π matrix elements, PCVC reads:



$$Z_{V^0}(M_D - E_\pi)\left\langle \pi \middle| V^0 \middle| D \right\rangle + Z_{V^i}\boldsymbol{q}\cdot\left\langle \pi \middle| \boldsymbol{V} \middle| D \right\rangle = (m_h - m_\ell)\left\langle \pi \middle| S \middle| D \right\rangle$$

Chiral-continuum fits

Hard SU(2) Heavy-meson rooted staggered ChiPT

- With simulations at and above the physical pion mass, the chiral fits are ***interpolations***, not extrapolations

- The shape of the form factors can be modeled using EFT combining:
  - Chiral symmetry $\Sigma = \exp(2i\phi/f)$
  - HQET spin symmetry $H^a = \frac{1+\not{v}}{2}\left[P^{*a}_\mu(v)\gamma^\mu - P^a(v)\gamma_5\right]$
  - Light-quark discretization effects $\frac{1}{16}\sum_\xi M_\xi^2 \log\left(\frac{M_\xi^2}{\Lambda^2}\right)$

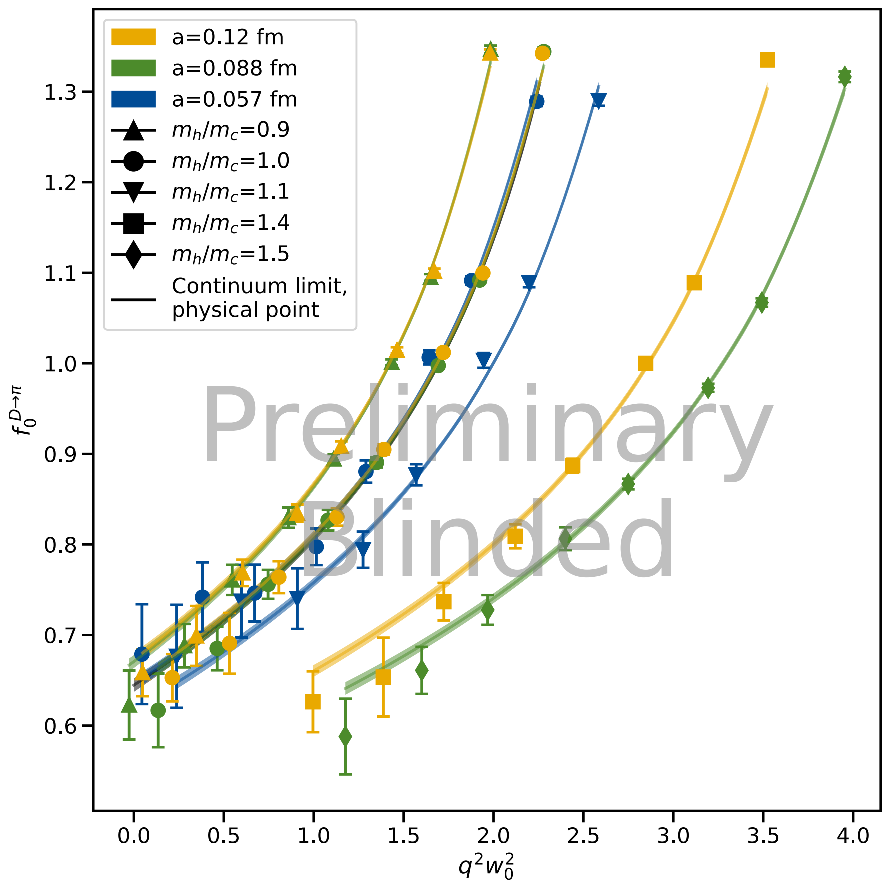
- Schematically:

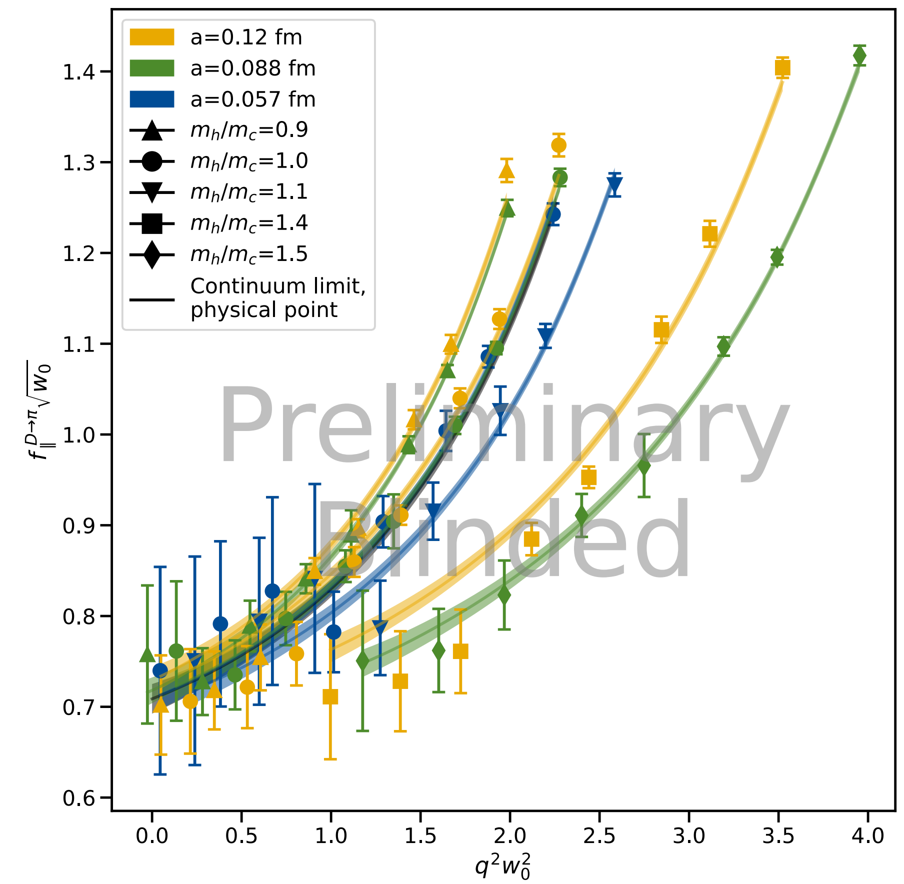
$$f = \frac{\text{const}}{E+\Delta^*}\left(1 + \delta f_{\text{logs}} + \delta f_{\text{artifacts}} + \sum_i c_i \chi_i\right)$$

- Analytic terms "χ_i" are included through NNLO

Chiral-continuum fits: D→π

Example: $f_0(q^2)$ and $f_\parallel(q^2)$

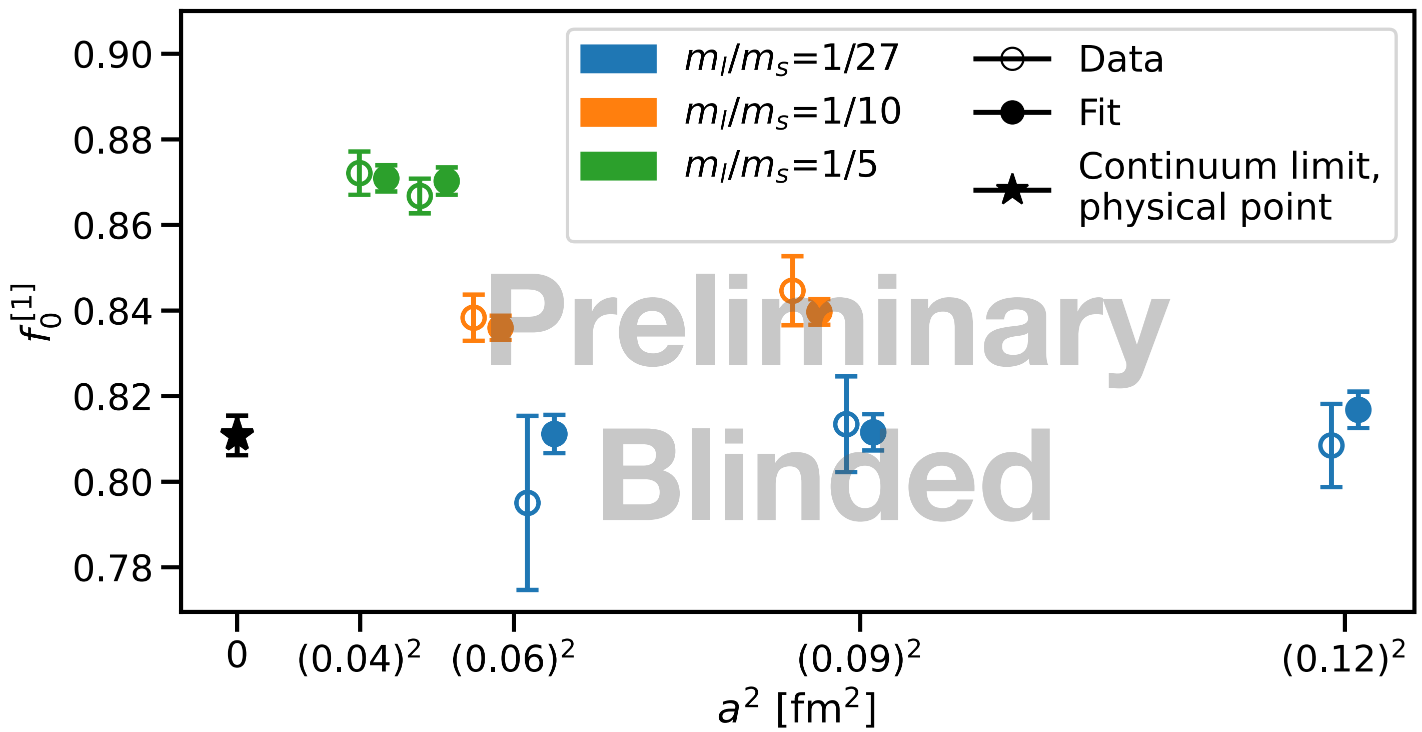




- Displayed: physical-mass ensembles only (but all ensembles included in fit)
- All fits have good quality of fit (e.g., χ^2/DOF ~ 1)
- Curve collapse at $m_h/m_c \approx 1.0$ suggests a mild approach to continuum limit

Chiral-continuum fits: D→π

Approaching the continuum limit: $f_0(q^2)$

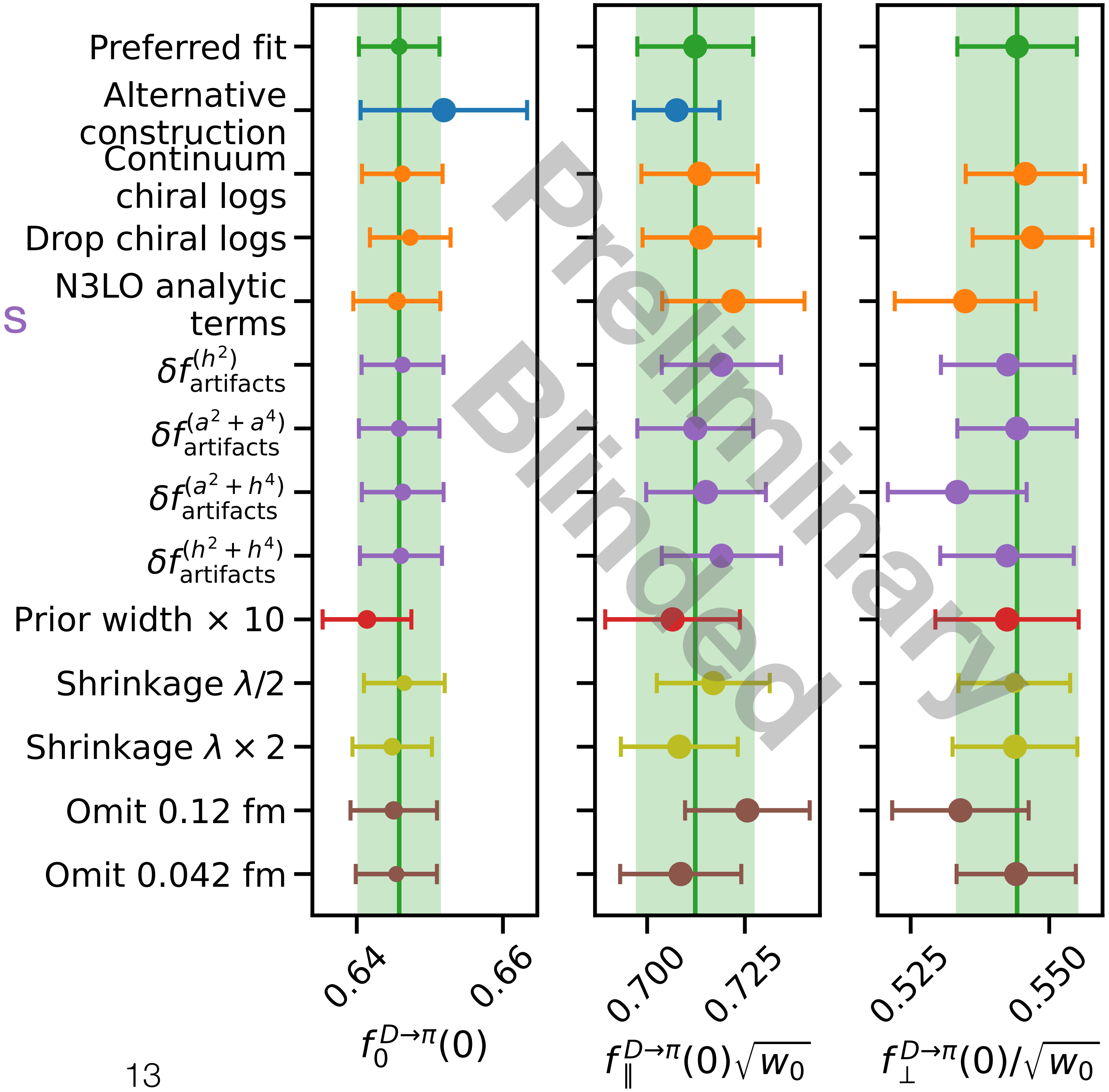


- Compare fit and data for fixed quark masses at $E_\pi \sqrt{w_0} \simeq 0.5$
- Interpolate data to fiducial energy (chosen so that it's interpolation)
- Evaluate fit result (at finite lattice spacing) at fiducial energy

Chiral-continuum fits: D→π

Stability of results

- Preferred analysis
- EFT variations
- Analytic discretization-term variations
- Statistical analysis variations
- Data variations



Results at the physical point

- Re-express the results of the chiral continuum analysis using the model-independent z-expansion

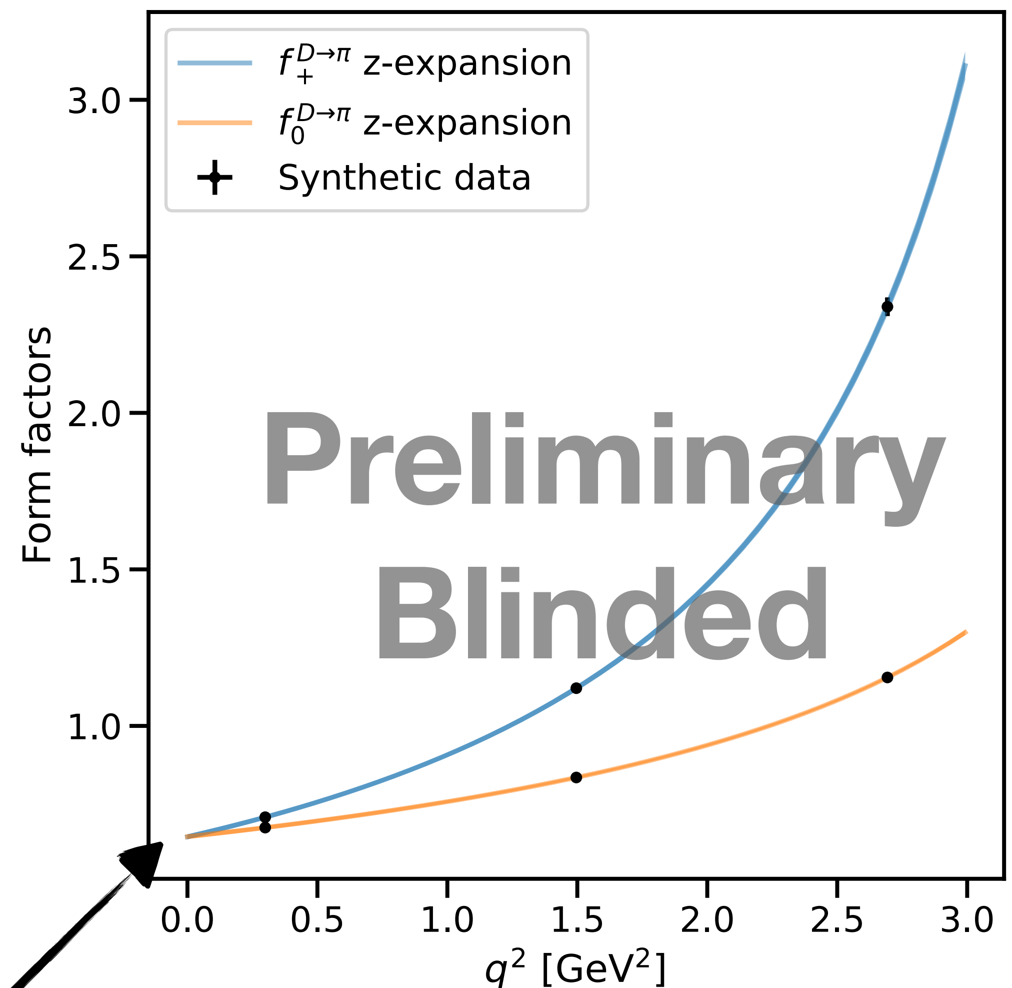
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_\pm = (M_D \pm M_\pi)^2$$

$$f_0(z) = \frac{1}{\left(1 - \frac{q^2(z)}{M_{0+}^2}\right)} \sum_{n=0}^{M-1} b_m z^m,$$

$$f_+(z) = \frac{1}{\left(1 - \frac{q^2(z)}{M_{1-}^2}\right)} \sum_{n=0}^{N-1} a_n \left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right)$$

- Kinematic identity: $f_+(0) = f_0(0)$

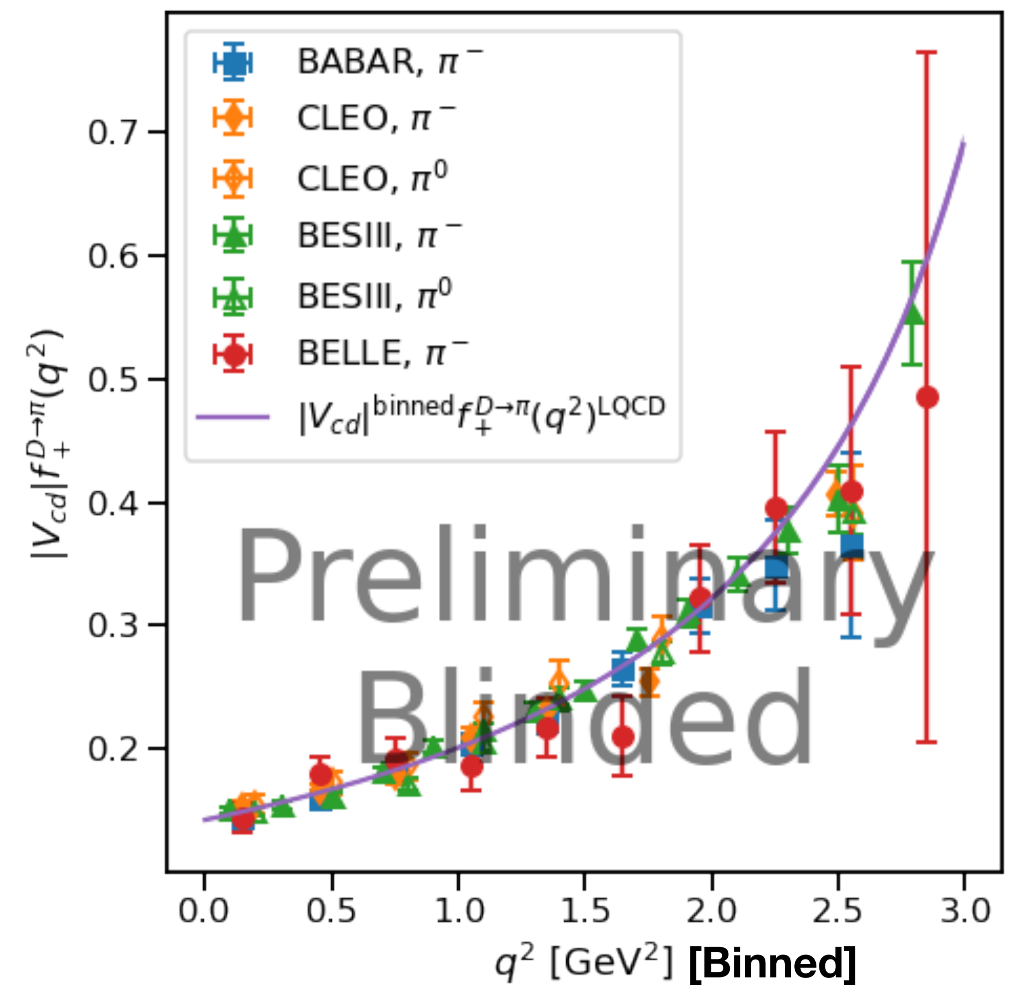
(Imposed in z-expansion fit, but well-satisfied even without the constraint)

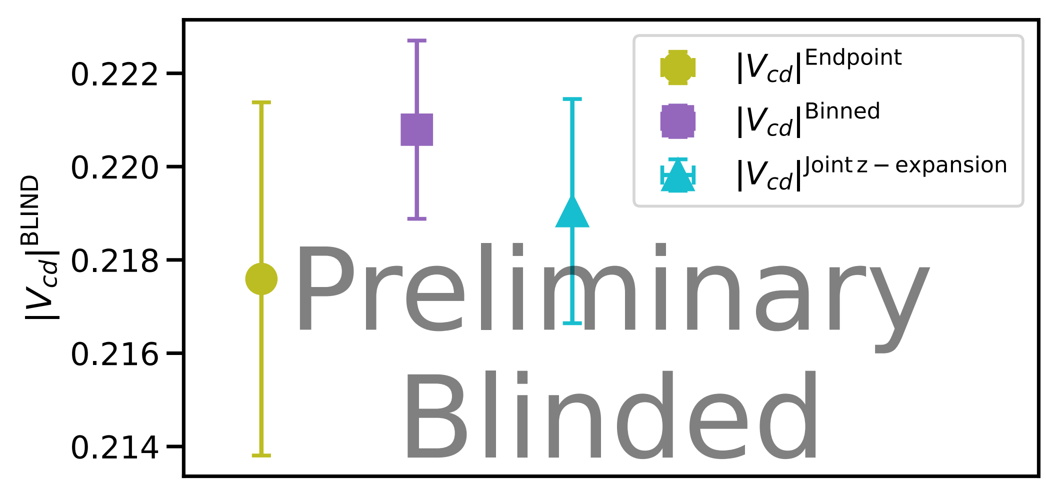


Nota bene: For D-decays, the z-expansion is not an extrapolation, just a convenient change of variables

Extracting $|V_{cd}|$

- Testing 3 methods to obtain |Vcd|
  - Endpoint: $[|V_{cd}|f_+(0)]^{\mathrm{Expt}} / [f_+(0)]^{\mathrm{LQCD}}$
  - Binned: Combine LQCD + experiment in each q^2 bin to construct $[|V_{cd}|]^{\mathrm{binned}}$. Average the results
  - Joint-fit: Fit LQCD + experiment simultaneously to the z-expansion, treating $|V_{cd}|$ as a fit parameter for the relative normalization
- Analysis of statistical and systematic uncertainties still in progress
  - All results are still blinded by an unknown factor ±5%
  - So far: roughly commensurate errors from experiment and LQCD form factor
  - Likely: ≲1% determination of $|V_{cd}|$ (subject to finalization)





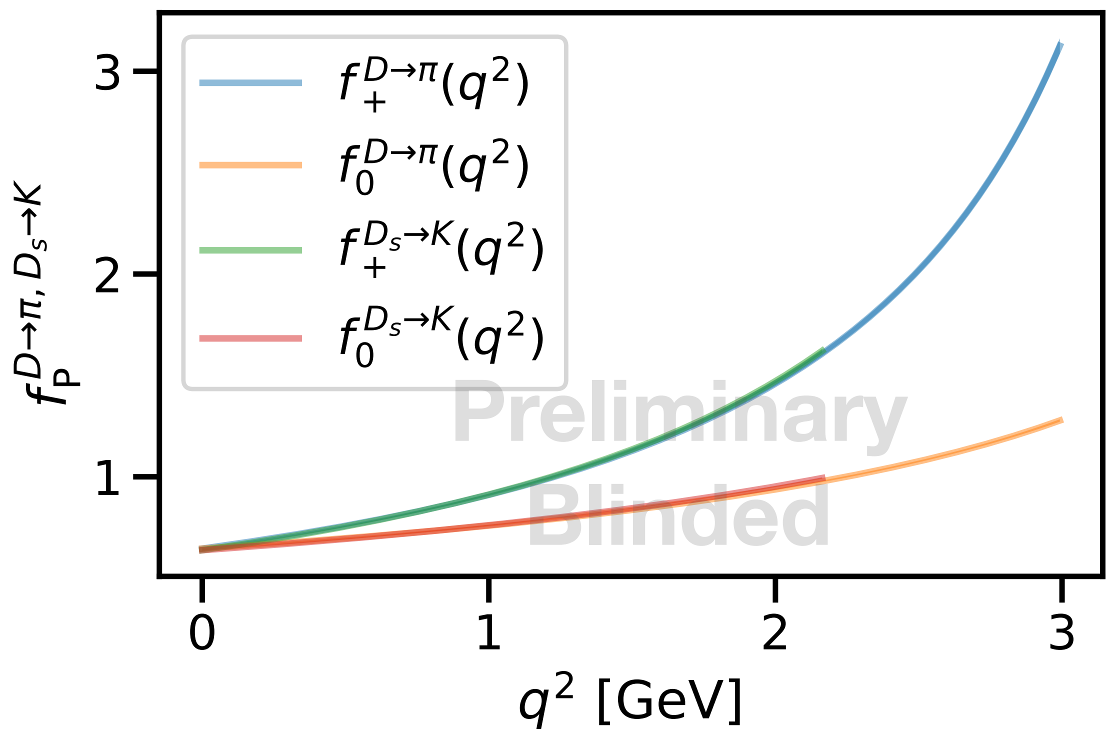
Summary

- We are calculating D-meson semileptonic decays using an all-HISQ setup, and our calculation is an in advanced stage
- Today's talk focused $D \to \pi$ for concreteness, but results are qualitatively similar for $D \to K$ and $D_s \to K$
- Based on the present analysis, we expect
  - Sub-percent results for the scalar and vector form factors
  - Percent or sub-percent determinations of the CKM matrix elements, with roughly commensurate errors from LQCD and experimental measurements

Backup

Spectator dependence

- From the hadronic perspective, D→π and D_s→K only differ by the mass of the spectator quark
- HPQCD has found the spectator dependence to be very mild.
- Our preliminary results seem to confirm this finding, with ≈few percent agreement throughout the full kinematic range of the D_s decay



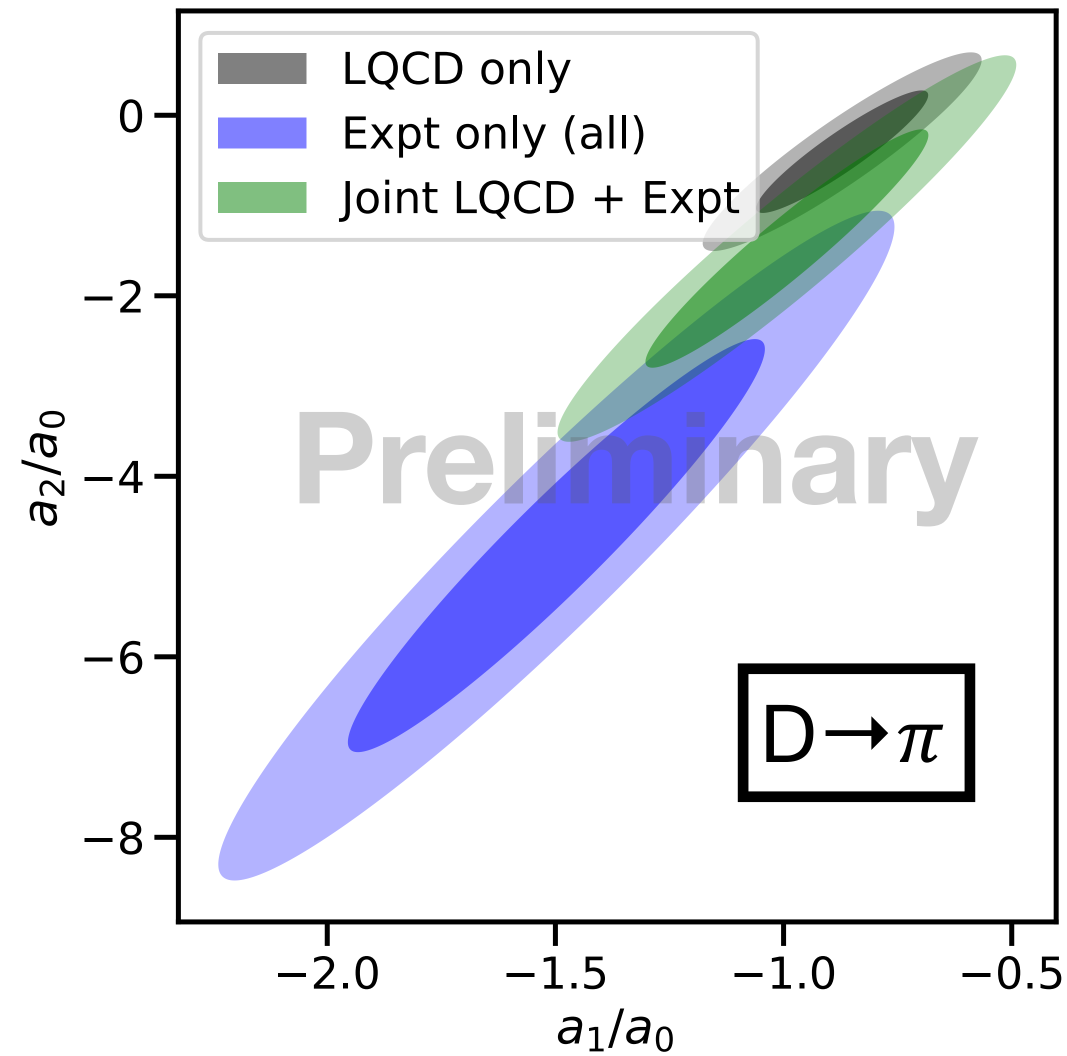
HPQCD [arXiv:1208.6242]
HPQCD [arXiv:1305.1462]

Form factor shapes

- The z-expansion results offer a normalization-independent comparison of shapes:

$$f_+(z) = \frac{1}{\left(1 - \frac{q^2(z)}{M_{1^-}^2}\right)} \sum_{n=0}^{N-1} a_n \left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right)$$

- Construct ratios a_1/a_0 and a_2/a_0, for which the normalization cancels
- All fits to z-expansion have good quality
- Joint fit lies between LQCD and experiment



Differential Decay Rate

P-to-P decays

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} \left(\eta_{\mathrm{EW}} \left|V_{cd}\right|\right)^2 \left(1 - \frac{m_\ell^2}{q^2}\right)^2 (1 + \delta_{\mathrm{EM}}) \times$$

$$\left(\left|\boldsymbol{p}_\pi\right|^3 \left(1 + \frac{1}{2}\frac{m_\ell^2}{q^2}\right) \left|f_+(q^2)\right|^2 + \left|\boldsymbol{p}_\pi\right| M_D^2 \left(1 - \frac{M_D^2}{M_\pi^2}\right)^2 \frac{3}{8}\frac{m_\ell^2}{q^2} \left|f_0(q^2)\right|^2 \right)$$

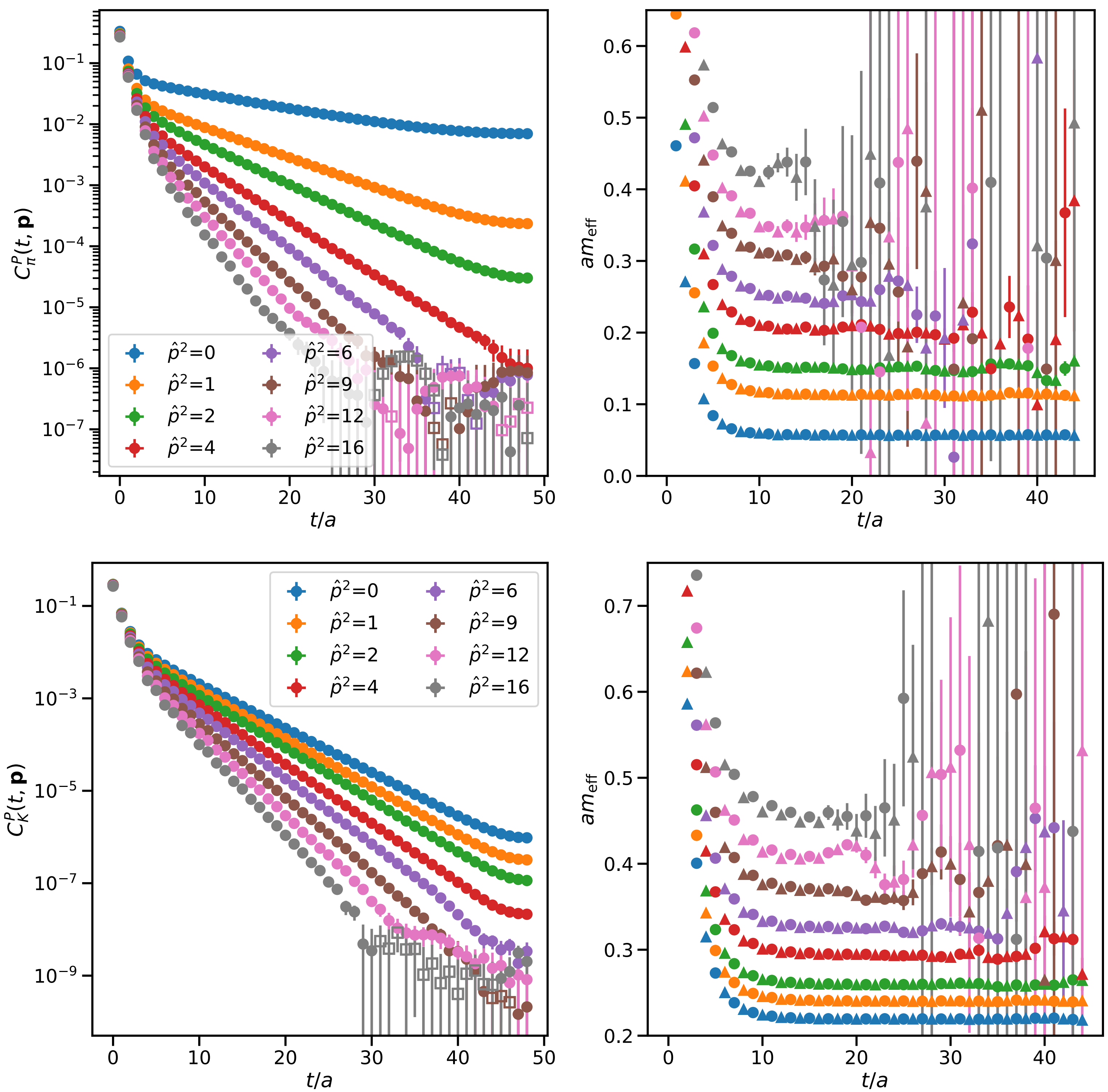
Ensemble details

$\approx a$ [fm]	$N_s^3 \times N_t$	m_ℓ	m_h/m_c	w_0/a	$N_{\text{src}} \times N_{\text{configs}}$	$T = t_{\text{snk}} - t_{\text{src}}$	$\approx M_{\pi,P}$ [MeV]
0.12	$48^3 \times 64$	physical	$0.9, 1.0, 1.4$	1.4168(10)	32×1352	{12, 13, 14, 16, 17}	135
0.088	$64^3 \times 96$	physical	$0.9, 1.0, 1.5, 2.0$	1.9470(13)	24×980	{16, 17, 19, 22, 25}	130
0.088	$48^3 \times 96$	$0.1 \times m_s$	$0.9, 1.0, 1.5, 2.0$	1.9299(12)	24×697	{16, 19, 22, 25}	224
0.057	$96^3 \times 192$	physical	$0.9, 1.0, 1.1, 2.2$	3.0119(19)	32×877	{25, 28, 30, 34, 37}	134
0.057	$64^3 \times 144$	$0.1 \times m_s$	$0.9, 1.0, 2.0$	2.9478(31)	36×916	{23, 30, 34, 37}	231
0.057	$48^3 \times 144$	$0.2 \times m_s$	$0.9, 1.0, 2.0$	2.8956(33)	36×823	{23, 30, 34, 37}	325
0.042	$64^3 \times 192$	$0.2 \times m_s$	$0.9, 1.0, 2.0$	3.9222(29)	24×1008	{34, 39, 45, 52}	308

- Approximate lattice spacings are for identification purposes. Intermediate scale setting is done with the Wilson-flow scale w_0/a.
- Light-quark propagators are computing using the truncated solver method, with N_{src} total loose solves and a matching fine solve on each configuration.
- T = source-sink separation. In physical units, values for T ∈ {1.4 - 2.25} fm
- Approximate $M_{\pi,P}$ values are for identification only and refer to the pseudoscalar taste pion.

Effective masses

(Examples from physical-mass 0.12 fm ensembles)



- Top: pion 2pt functions and effective masses
- Bottom: kaon 2pt functions and effective masses

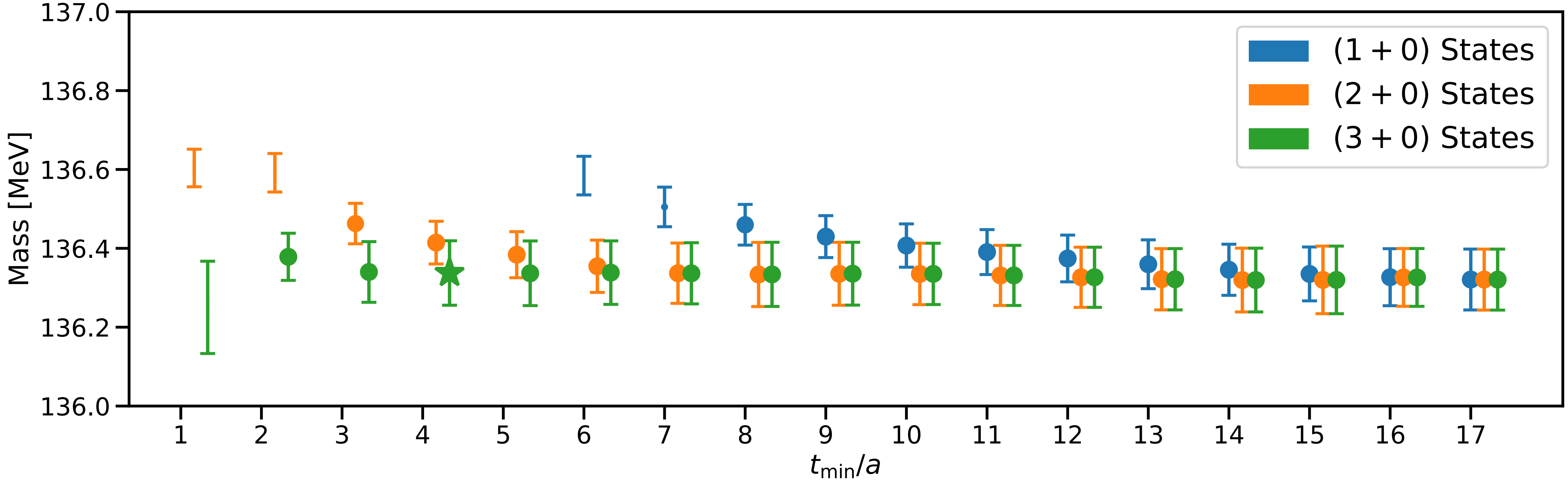
$$am_{\rm eff} \equiv \frac{1}{2}\,\mathrm{arcCosh}\left[(C(t+2)+C(t-2))/C(t)\right]$$

● = even timeslices

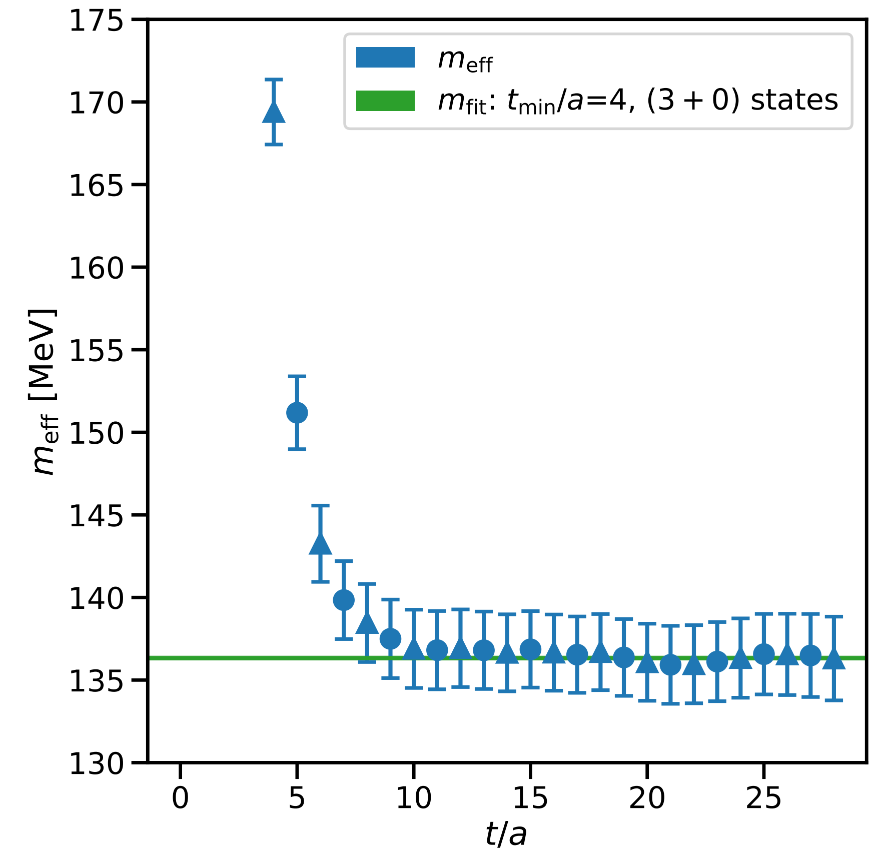
▲ = odd timeslices

Multi-exponential fits

(Examples from physical-mass 0.12 fm ensembles)



- Analysis choices are guided by preliminary fits to 2-point functions. We identify the the minimal number of states required to obtain stable results, at moderate t_{min}, which are consistent with single-exponential fits at large t_{min}.
- Our preferred analysis takes $t_{min} \approx 0.5$ fm.

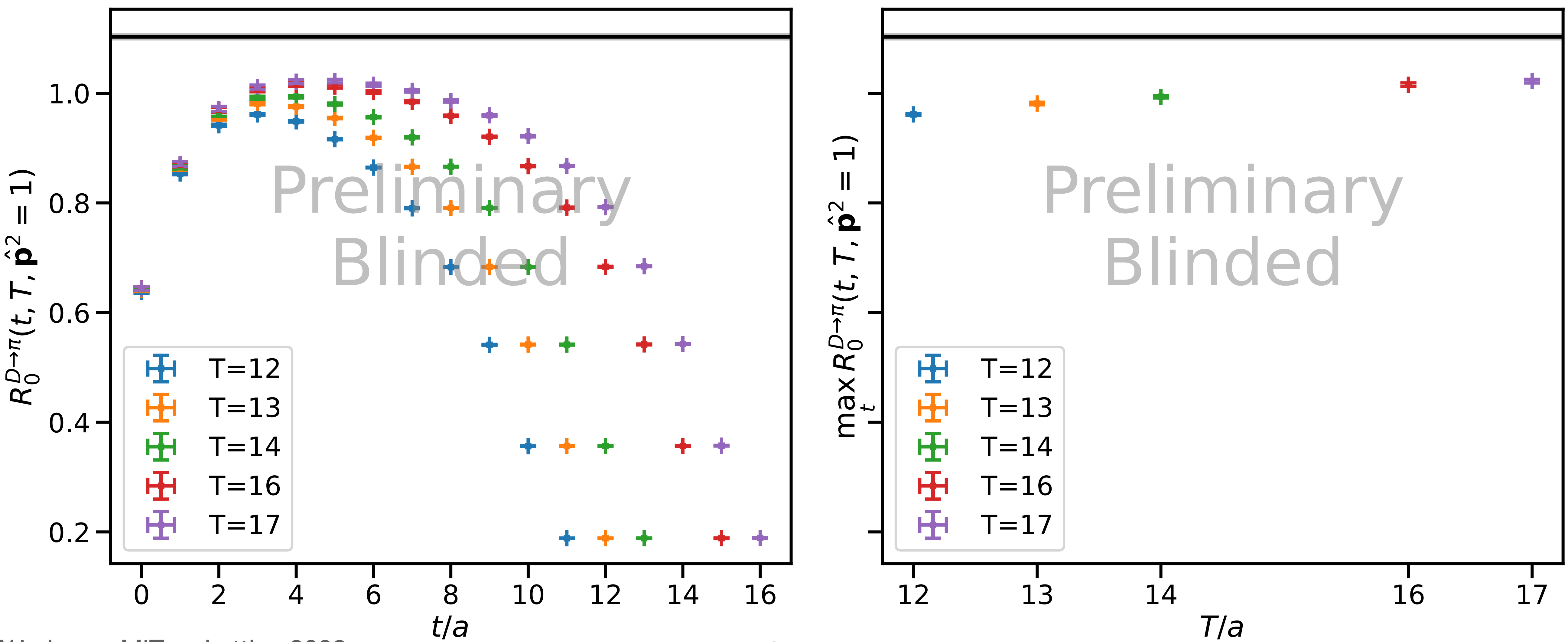


Statistical Analysis

Results: 3pt functions - f_0 for D to π

- A certain ratio is useful to isolate form factors visually

- Can check approach to asymptotic plateau region: $0 \ll t \ll T_{sink}$



Chiral-continuum fit formulae

$$f_P(E) = \frac{c_0}{E + \Delta^*_{yx,\mathrm{P}}} \times \Big[1 + \delta f_{\mathrm{P,logs}} + c_l \chi_l + c_H \chi_H + c_E \chi_E$$
$$+ c_{l^2} (\chi_l)^2 + c_{H^2} (\chi_H)^2 + c_{E^2} (\chi_E)^2$$
$$+ c_{lH} \chi_l \chi_H + c_{lE} \chi_l \chi_E + c_{HE} \chi_H \chi_E$$
$$+ \delta f^{(a^2)}_{\mathrm{artifacts}} \Big],$$

$$\chi_l = \frac{(M^{\mathrm{meas.}}_\pi)^2}{8\pi^2 f^2}$$
$$\chi_E = \frac{\sqrt{2}E}{4\pi f}$$
$$\chi_H = \frac{(M^{\mathrm{meas.}}_{D_{(s)}})^2 - (M^{\mathrm{PDG}}_{D_{(s)}})^2}{8\pi^2 f^2}$$

$$\delta f^{SU(2)}_{\mathrm{P,logs}} = \left(-\frac{1}{16} \sum_\xi \mathcal{I}_1(M_{\pi,\xi}) + \frac{1}{4} \mathcal{I}_1(M_{\pi,I}) + \mathcal{I}_1(M_{\pi,V}) - \mathcal{I}_1(M_{\eta,V}) + [V \to A] \right)$$
$$\times \begin{cases} \frac{1+3g^2}{(4\pi f)^2}, D \to \pi \\ \frac{3g^2}{(4\pi f)^2}, D \to K \\ \frac{1}{(4\pi f)^2}, D_s \to K \end{cases}$$

$$x_{a^2} = \frac{a^2 \bar{\Delta}}{8\pi^2 f^2}$$
$$x_h = \frac{2}{\pi} a m_h.$$

$$M^2_{\pi,\xi} = M^2_{uu,\xi} = M^2_{dd,\xi}$$
$$M^2_{ij,\xi} = \mu(m_i + m_j) + \Delta_\xi$$
$$M^2_{\eta,V(A)} = M^2_{uu,V(A)} + \frac{1}{2}\delta'_{V(A)}$$
$$\bar{\Delta} = \frac{1}{16} \sum_\xi \Delta_\xi.$$