The chiral phase transition at nonzero imaginary baryon chemical potential for different numbers of quark flavours

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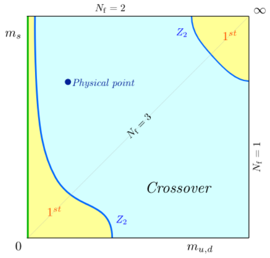
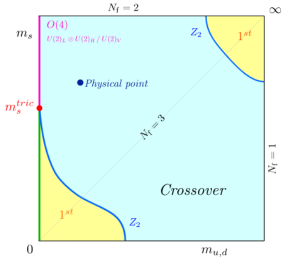
Introduction

- The order of the QCD chiral phase transition has been deeply investigated
- The Columbia Plot summarises the nature of the QCD thermal phase transition for $N_f = 2 + 1$ and zero density



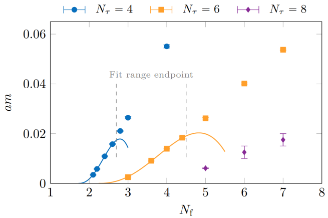


- Two plausible scenarios were suggested [1]

[1] R.D. Pisarski and F. Wilczek, Remarks on the Chiral Phase Transition in Chromodynamics, Phys.Rev.D29(1984)338.

Introduction

- A resolution is proposed through the analytic continuation of N_f from integer to continuous parameter in the (am, N_f) plane [1]
- The projection onto any plane of the bare parameters "space" shows compatibility with the presence of a tricritical point for any $N_f \lesssim 6$



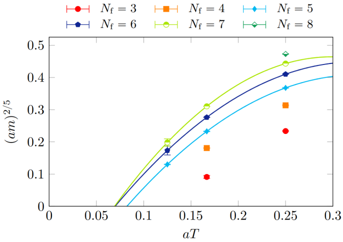


Figure: Figures from [2]

[1] R.D. Pisarski and F. Wilczek, Remarks on the Chiral Phase Transition in Chromodynamics, Phys.Rev.D29(1984)338.

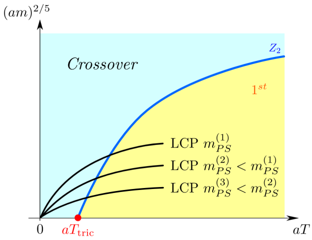
[2] Cuteri, F., Philipsen, O., Sciarra, A. On the order of the QCD chiral phase transition for different numbers of quark flavours. J. High Energ. Phys. 2021, 141 (2021).

Introduction

- Simulations for $N_f \in [2, 6]$ result in a lattice chiral limit compatible with a tricritical point on the N_τ^{-1} axis

- This has implication also on the possible second-order scenario of the Columbia Plot for $N_f \in [2, 3]$

[2] Cuteri, F., Philipsen, O., Sciarra, A. On the order of the QCD chiral phase transition for different numbers of quark flavours. J. High Energ. Phys. 2021, 141 (2021).



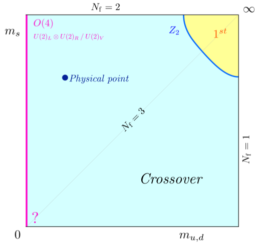


Figure: Figures from [2]

Introduction

- Using a nonzero imaginary baryon chemical potential to extend the Columbia Plot

$$\mathcal{Z}(\mu_i) = \mathcal{Z}(-\mu_i) \qquad\qquad \mathcal{Z}\left(\frac{\mu_i}{T}\right) = \mathcal{Z}\left(\frac{\mu_i}{T} + i\frac{2\pi k}{3}\right), \quad k \in \mathbb{Z} \quad [3]$$

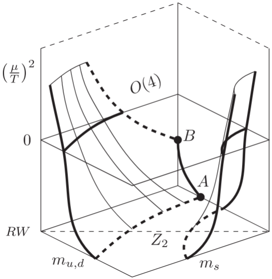


Figure: 3D Columbia Plot [4]

[3] A. Roberge, N. Weiss, Gauge theories with imaginary chemical potential and the phases of QCD. Nuclear Physics B Volume 275,Issue 4, 734-745 (1986)

[4] Claudio Bonati, Philippe de Forcrand, Massimo D'Elia, Owe Philipsen, and Francesco Sanfilippo, Chiral phase transition in two-flavor QCD from an imaginary chemical potential, Phys. Rev. D 90, 074030 (2014)

Motivation

- Study the Z_2 critical surface when $\mu_i \neq 0$
- Use N_f as a continuous parameter, as for $\mu_i = 0$
- Use the tricritical scaling to extrapolate to the chiral limit
- Compare the results with the ones at zero density

Strategy

- For a generic observable $\mathcal{O}$

$$\langle\mathcal{O}\rangle = \mathcal{Z}^{-1}\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S_g}e^{-S_f}e^{-S_{\mu_i}}$$

- Staggered fermions with parameters $\beta = 6/g^2$, N_f, am, $N_\tau = (a(\beta)T)^{-1}$, μ_i

- μ_i is set to a fraction of the RW one

$$\mu_i \approx 0.8\frac{\pi T}{3}$$

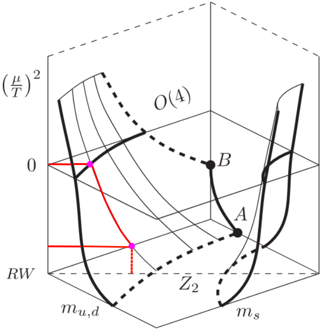


Figure: Position of the simulated μ_i for $N_f = 3$

Strategy

- Simulations with our public available **CL^{2}QCD** code and RHMC algorithm
- For fixed N_f and N_τ , the n^{th} standardized moment for a generic $\mathcal{O}$ is

$$B_n(\beta, am, N_\sigma) = \frac{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^n\rangle}{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^2\rangle^{\frac{n}{2}}}$$

- $B_3(\beta = \beta_c, am, N_\sigma) = 0$ gives β_c of the phase transition
- $B_4(\beta_c(am), am, N_\sigma)$ of the sampled distribution of $\langle\bar{\psi}\psi\rangle$ necessary to localize the am_c and the order of the lattice chiral phase transition

Strategy

- The universal infinite volume values for B_4 and the critical exponent ν are the following

	Crossover	1^{st} order	$3D$ Ising
B_4	3	1	1.604
ν	–	1/3	0.6301(4)

- On finite volumes the discontinuity in B_4 becomes smooth and for large enough N_σ^3 and $\beta \sim \beta_c$

$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c(am - am_c)N_\sigma^{1/\nu}$$

- Simulations for $N_f \in \{3.6, 4.0, 4.5, 5.0\}$
- For any different N_f, $N_\tau \in \{4, 6, 8\}$, with aspect ratios 2,3,4

Analysis and results

$$am(N_\tau, N_f, \mu_i) \approx \mathcal{B}_1(N_\tau, \mu_i)(N_f - N_f^{\mathbf{tricr}})^{5/2} + \mathcal{B}_2(N_\tau, \mu_i)(N_f - N_f^{\mathbf{tricr}})^{7/2}$$

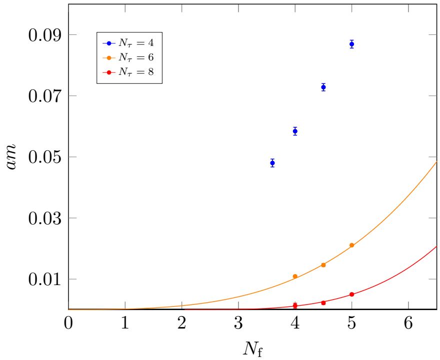


Figure: Preliminary results on the (am, N_f) plane

Analysis and results

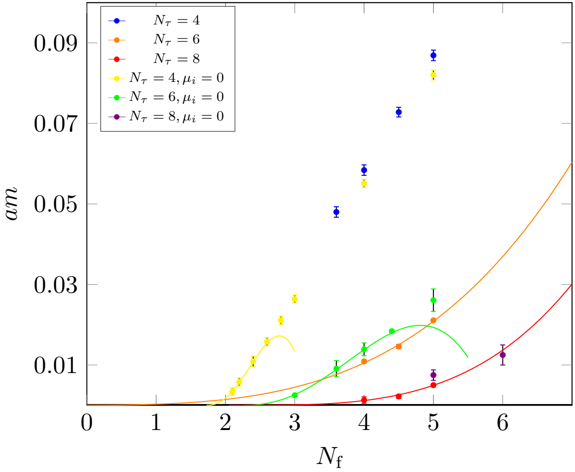


Figure: Comparison between the $\mu_i = 0$ and $\mu_i = 0.8\frac{\pi T}{3}$ results

Analysis and results

$$\beta_c(N_\tau, N_f(N_\tau), am) \approx \beta_{\mathbf{tricr}}(N_\tau) + \mathcal{A}_1(N_\tau)(am)^{2/5} + \mathcal{A}_2(N_\tau)(am)^{4/5}$$

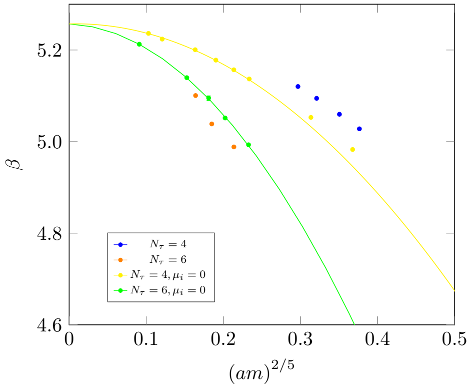


Figure: Preliminary results on the (β, am) plane compared to $\mu_i = 0$

Analysis and results

$$am(N_\tau, N_f, \mu_i)^{2/5} \approx \mathcal{A}_1(N_f, \mu_i)(aT - aT_{\mathbf{tricr}}) + \mathcal{A}_2(N_f, \mu_i)(aT - aT_{\mathbf{tricr}})^2$$

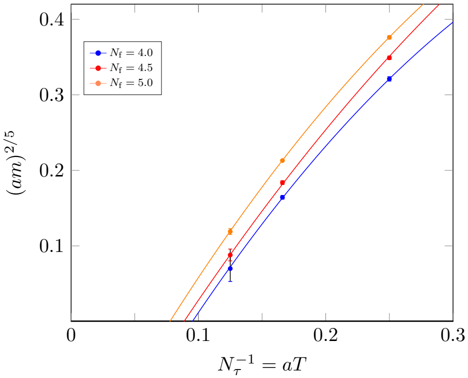


Figure: Preliminary results on the (am, aT) plane

Analysis and results

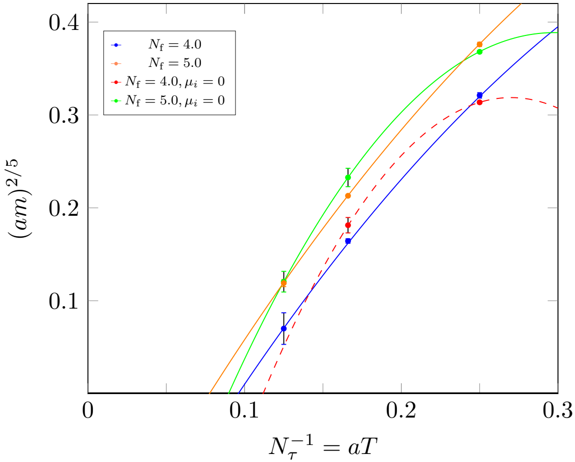


Figure: Comparison between the $\mu_i = 0$ and $\mu_i = 0.8\frac{\pi T}{3}$ results

Conclusions and further steps

- Zero density results show a 2^{nd} order chiral phase transition in the chiral limit for $N_f \in [2, 6]$
- The preliminary results for $\mu_i = 0.8\frac{\pi T}{3}$ suggest a similar trend of the data as for $\mu_i = 0$, in all of the possible planes
- A tricritical scaling is expected in both cases when the extrapolation towards the chiral limit is performed
- Possibly no μ_i-dependence of the order of the chiral phase transition?

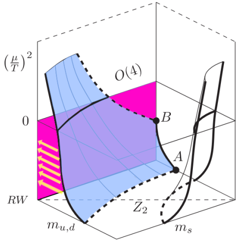


Figure: Possible scenario displaying a 2^{nd} order chiral phase transition in the chiral limit independent on μ_i

Thanks you all for your attention!