On the determination of the strong QCD coupling at the Z-pole with new gradient-flow based beta-function

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Outline

- Goal: High-precision determination of $\alpha_S(m_Z)$ at the Z-boson pole in massless $N_f = 3$ QCD with β-function computed using gradient flow based renormalization group of the lattice gauge field
- Methodology:
  - g^2 is defined in infinite volume, computed in both perturbative and hadronic regimes
  - connect the data with the 3-loop GF β-function and with scale setting parameter such as r_0
  - $r_0\,\Lambda_{\overline{MS}}$, equivalently $\alpha_S(m_Z)$, is obtained
- The methodology is applied on $N_f = 0$ and $N_f = 10$ to demonstrate its challenges and feasibility. They serve as pilot tests towards the application on $N_f = 3$ QCD

Determination of $\alpha_S(m_Z)$

- Goal: High precision determination of the strong coupling $\alpha_S(m_Z)$ at the Z-pole in QCD with massless $N_f = 3$ fermions.
- The determination of $\alpha_S(m_Z)$, equivalently $r_0\Lambda_{\overline{MS}}$, requires integration of the inverse β-function from perturbative regime to the scale of hadron physics at strong coupling.

$$r_0 \cdot \Lambda_{\overline{MS}} = (b_0\bar{g}^2)^{-b_1/2b_0^2} \cdot \exp(-1/2b_0\bar{g}^2) \cdot \exp\left(-\int_0^{\bar{g}} dx[1/\beta(x) + 1/b_0x^3 - b_1/b_0^2x]\right)$$

- The integration beyond the perturbative regime requires non-perturbative calculation of the β-function, which can be done on lattice

Determination of $\alpha_S(m_Z)$

- There are many ways to do such calculations [FLAG 2021]. One approach is to utilize step β-functions defined on lattices of finite physical volumes in Schrodinger functional boundary conditions, matching different schemes at different scale regimes.
- We propose to define a β-function over infinite physical volume as response to infinitesimal RG scale changes, using only gradient flow on the lattice. It is expected to be feasible for the entire nonperturbative integration range without matchings.

Determination of $\alpha_S(m_Z)$

- $g^2(t)$ is defined in infinite volume as a function of continuous gradient flow time t.
- Discretization schemes of RG flow with different combinations of action, flow and energy operator ($\propto g^2$) on the lattice: WSC,WSS,SSC,SSS
- The β-function is obtained from $g^2(t)$: $\beta(g^2(t)) = t \cdot dg^2/dt$ (note the sign convention kept from BSM studies)
- dg^2/dt is approximated numerically. Here we use five-point stencil in t

$$[-g^2(t+2\varepsilon)+8g^2(t+\varepsilon)-8g^2(t-\varepsilon)+g^2(t-2\varepsilon)]/(12\varepsilon) = dg^2/dt + O(\varepsilon^4)$$

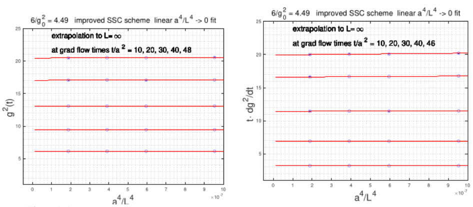
  (cross-checked by other approximations such as spline-based derivative and modified Akima interpolation method)
- Originally a similar approach was introduced in our previous study of the near-conformal $N_f = 2$ sextet model reaching the chiral limit from small fermion mass (m) deformations.[Fodor et al, 2018]. Our current approach here is to obtain infinite-volume β-function in the massless theories.

Determination of $\alpha_S(m_Z)$

- There are a few ways of taking the infinite volume limit and continuum limits depending on which parameter is kept fixed in the limits
- One variant is as follows:
  - $g_L^2(t/a^2)$ and $\beta_L \equiv t \cdot dg_L^2/dt$ are measured on ensembles in a set of $\{6/g_0^2,\ L/a\}$ (at $m = 0$ when $N_f > 0$).
  - At each $6/g_0^2$ and each t/a^2, the infinite-volume limit ($a^4/L^4 \to 0$) of $\{g_L^2, \beta_L\}$ is taken (denoted here by $\{g^2(t/a^2), \beta(t/a^2)\}$)
  - A set of target values of infinite-volume continuum coupling $\{g^2_{\text{target}}\}$ within the range of integration is chosen. At each target value of g^2_{target} , the corresponding $\{\beta, t/a^2\}|_{g^2=g^2_{\text{target}}}$ is obtained by interpolating in t/a^2 in each $\{6/g_0^2\}$
  - At each g^2_{target}, the continuum limit $a^2/t \to 0$ of β is taken.
- The following tests are done on symanzik-improved gauge action with 4-step $\rho = 0.12$ stout-smeared staggered fermions in periodic boundary conditions for gauge links and anti-periodic boundary conditions for fermions when $N_f > 0$.

Test on $N_f = 0$

- $L/a = \{32, 36, 40, 48, 64\}$, $6/g_0^2 = \{4 \text{ to } 8.6\}$



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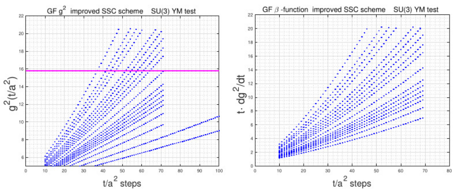

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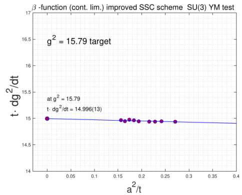
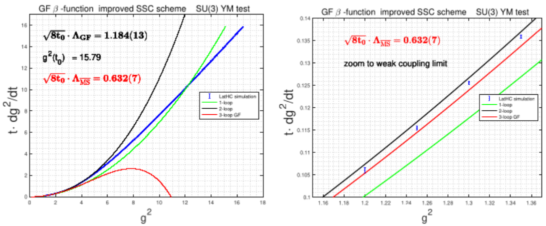


$\Rightarrow$

Test on $N_f = 0$



$$\sqrt{8t_0} \cdot \Lambda_{GF} = (b_0 \bar{g}^2)^{-b_1/2b_0^2} \cdot \exp(-1/2b_0\bar{g}^2) \cdot \exp\left(-\int_0^{\bar{g}} dx[-1/\beta(x) + 1/b_0x^3 - b_1/b_0^2 x]\right)$$

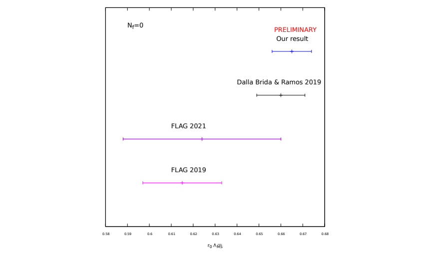
- Integration broken into two parts ($\bar{g}^2(t_0) \equiv 15.79$) :
  - $x^2 = 0$ to 1.2: three-loop value of β-function
  - $x^2 = 1.2$ to $\bar{g}^2(t_0)$: numerical integration based on spline fit to the data
- $\Lambda_{GF}/\Lambda_{\overline{MS}} = 1.873$[Harlander et al, 2016; Artz et al, 2019]

Test on $N_f = 0$

- Same approach is taken in ongoing work in [Hasenfratz et al, Phys.Rev.D 101 (2020) 3, 034514 ; Peterson et al , PoS LATTICE2021 (2022) 174]
(Talks on Wed : O. Witzel 14:00, C. Peterson 16:30, C. Monahan 16:50)
- Our result has high accuracy comparable with the work [Dalla Brida & Ramos, 2019] of ALPHA collaboration using step function, also agree in values
- Our PRELIMINARY result: $\sqrt{8t_0} \cdot \Lambda_{\overline{MS}} = 0.632(7)$
- Dalla Brida & Ramos: $0.6227(98)$
- Our results are preliminary. We plan to
  - check consistency across different discretization schemes
  - properly take topological effects into account

Test on $N_f = 0$

- Using $\sqrt{8t_0}/r_0 = 0.948(7)$[Luescher, 2014], our result converts into: $r_0 \cdot \Lambda_{\overline{MS}} = 0.665(9)$
- Dalla Brida & Ramos (converted differently) : $0.660(11)$
- World average significantly lower:
  FLAG 2019: $r_0 \cdot \Lambda_{\overline{MS}} = 0.615(18)$
  FLAG 2021 (including ALPHA's result): $r_0 \cdot \Lambda_{\overline{MS}} = 0.624(36)$



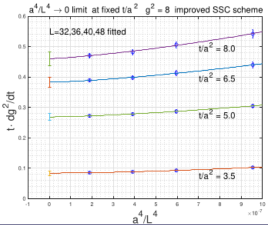
- This recent unresolved issue in the $N_f = 0$ model justifies the need of an independent determination of $r_0 \cdot \Lambda_{\overline{MS}}$ in $N_f = 3$ QCD as well

Determination of $\alpha_S(m_Z)$

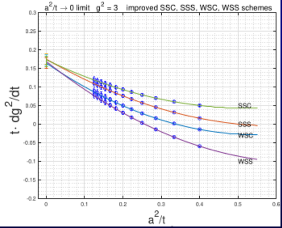
- A dense scan of $6/g_0^2$ is costly for fermionic simulations $\Rightarrow$
- We use another variant:
  - $g_L^2(t/a)$ and $\beta_L \equiv t \cdot dg_L^2/dt$ are measured on ensembles in a set of $\{6/g_0^2,\ L/a\}$ generated at $m = 0$.
  - A set of target values of infinite-volume continuum coupling $\{g^2_{\text{target}}\}$ within the range of integration is chosen. At each g^2_{target}, a set of target values of $\{t_{\text{target}}/a^2\}|_{g_L^2 = g^2_{\text{target}}}$ is chosen. β_L is obtained for each combination of $\{g^2_{\text{target}}, t_{\text{target}}/a^2, L/a\}$ by interpolating among measurements in $\{6/g_0^2\}$
  - At each t_{target}/a^2, the infinite-volume limit ($a^4/L^4 \to 0$) of β_L is taken while holding $\{g^2_{\text{target}}, t_{\text{target}}/a^2\}$ fixed. (denoted by $\beta(t/a^2)$)
  - At each g^2_{target}, the continuum limit $a^2/t \to 0$ of β is taken.
- This approach was first tested earlier in the $N_f = 2$ QCD [Hasenfratz et al, Phys.Rev.D 101 (2020) 3, 034514 ; Peterson et al , PoS LATTICE2021 (2022) 174] and $N_f = 12$ models [Fodor et al, PoS LATTICE2019 (2019) 121]

Test on $N_f = 10$

- $L/a = \{32, 36, 40, 48\}$,
  $6/g_0^2 = \{2.6, 2.7, \ldots, 4.1, 4.5, 5.0, 6.0 \ldots 8.0\}$
- For each $\{L/a, 6/g_0^2\}$, β at each target $\{g^2, t/a^2\}$ is obtained with fourth order polynomial interpolation



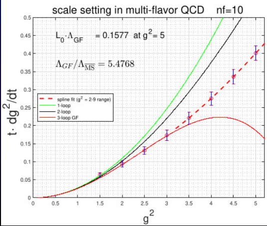
$\Rightarrow$



$\Rightarrow$

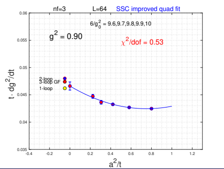
Test on $N_f = 10$

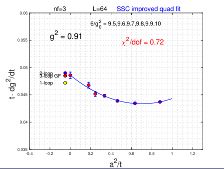
- Integration broken into two parts (we only integrate up to $\bar{g}^2(L_0^2/8) \equiv 5$ since data in confining regime is not available) :
  - $x^2 = 0$ to 3: three-loop value of β-function
  - $x^2 = 3$ to $\bar{g}^2(L_0^2/8)$: numerical integration based on spline fit to data
- $L_0 \cdot \Lambda_{GF} = 0.1577$ ($\approx 5\%$ error)
- $\Lambda_{GF}/\Lambda_{\overline{MS}} = 5.4768$ (1-loop) [Harlander et al, 2016; Artz et al, 2019]
- We successfully expressed $\Lambda_{\overline{MS}}$ of the theory in terms of a scale set implicitly at flow time $L_0^2/8$.
- If data in the confining regime is generated, $L_0^2/8$ can be set as t_0 and $r_0 \cdot \Lambda_{\overline{MS}}$ can be obtained. This holds promise for the challenge to apply the method to the strong coupling for $N_f = 3$ QCD.

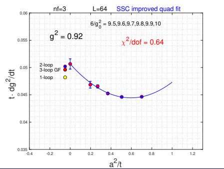


Outlook

- We would apply one or more of the variants of the method on massless $N_f = 3$ QCD to obtain $r_0 \cdot \Lambda_{\overline{MS}}$
- Some encouraging progress has been made : (PRELIMINARY, no infinite volume extrapolations)







Outlook

- We would apply one or more of the variants of the method on massless $N_f = 3$ QCD to obtain $r_0 \cdot \Lambda_{\overline{MS}}$
- Some encouraging progress has been made : (preliminary, no infinite volume extrapolations)