Isothermal and isentropic speed of sound in (2+1)-flavor QCD at non-zero baryon chemical potential

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Lattice2022, 11 Aug 2022

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CRC–TR 211
Strong-interaction matter under extreme conditions

Motivation and strategy

$$c_T^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_T \qquad\qquad c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{s/n_B}$$

Can use c_s^2 to probe QCD phase diagram:

- Related to cooling/expansion rate
- Minimum/"softest point" may indicate long-lived fireball

Look at c_X^2 at finite μ/T. Commonly played game:

1. Write P/T^4 as **Taylor expansion** in μ/T
2. Derive other quantities from P/T^4 using **thermodynamics**
3. **Measure** Taylor coefficients on lattice
4. Compare against **HRG** at low T

More strategy

For convenience, $\hat{X} \equiv XT^{-k}$ with k s.t. $\hat{X}$ dimensionless (e.g. $\hat{\mu} = \mu/T$)

Dealing with 3 chemical potentials $\hat{\mu}_B$, $\hat{\mu}_Q$, $\hat{\mu}_S$

To make contact with T-$\hat{\mu}_B$ plane, need to eliminate 2 independent variables

Can **impose external constraint**, e.g.

1. $\hat{\mu}_Q = \hat{\mu}_S = 0$
2. $n_S = 0$, $n_Q/n_B = 0.4$ (RHIC-like)
3. $n_S = 0$, $n_Q/n_B = 0.5$ (isospin-symmetric)

and think of **expansions in $\hat{\mu}_B$ only**

Some context and setup

Related studies from the past, for instance:

- Lattice $\hat{\mu}_B = 0$[1,2]
- Lattice $\hat{\mu}_B > 0$[3,4]

We expand on our previous $N_f = 2 + 1$ HISQ studies:

- **Increased statistics** $N_\tau = 8,\ 12$ by more than factor 10 ($\lesssim 1.5$M per β)
  - Overlap with configurations in charm study (Sipaz, Tue 16:30)
- Added $N_\tau = 16$ allowing **continuum extrapolation**
- Taylor series up to 8^{th} **order**; converges well at least for $\hat{\mu}_B < 1.5$[5,6]

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[1] A. Bazavov et al., Phys. Rev. D, 90, 094503 (2014).
[2] S. Borsányi et al., Physics Letters B, 730, 99–104 (2014).
[3] A. Bazavov et al., Phys. Rev. D, 95.5, 054504 (2017).
[4] S. Borsanyi et al., J. High Energ. Phys. 2018.10, 205 (2018).
[5] D. Bollweg et al., Phys. Rev. D, 105.7, 074511 (2022).
[6] Strictly speaking, reliability range depends on observable and temperature. (Jishnu)

Expansion coefficients and lattice extraction

After imposing our constraint:

$$\Delta X(T,\hat{\mu}_B) \equiv X(T,\hat{\mu}_B) - X(T,0)$$

$$\Delta\hat{p} = \sum_{k\ \text{even}} P_k\,\hat{\mu}_B^k$$

$$\Delta\hat{n}_B = \sum_{k\ \text{odd}} N_k\,\hat{\mu}_B^k$$

$$\Delta\hat{s} = \sum_{k\ \text{even}} \sigma_k\,\hat{\mu}_B^k$$

(Jishnu, next talk)

Lattice extraction:

$$Z = \int \mathcal{D}U \prod_f (\det D_f)^{1/4}\, e^{-S_G}$$

$$\langle X\rangle = \frac{1}{Z}\int \mathcal{D}U \prod_f (\det D_f)^{1/4}\, e^{-S_G}\, X.$$

For example

$$\frac{1}{V}\partial_{\hat{\mu}_B}\log Z = \frac{1}{4V}\sum_f \operatorname{tr} D_f^{-1}\frac{\partial}{\partial\hat{\mu}_B}D_f,$$

unbiased estimates with 500+ random vectors.

Calculating c_X^2 on lattice, $\hat{\mu}_Q = \hat{\mu}_S = 0$

Again, after constraining,

$$X' \equiv T\frac{\partial X}{\partial T},$$

$$c_T^{-2} - 3 = \frac{P_2'}{P_2} + \sum_{k \text{ even}} S_k\,\hat{\mu}_B^k.$$

Every term in S_k contains factors like $\left(\frac{P_m}{P_2}\right)'$, with m even. This structure hints at $\hat{\mu}_B$ independence in HRG.

$$\hat{\epsilon} = 3\hat{p} + T\frac{\partial \hat{p}}{\partial T}$$

At fixed s/n_B:

$$c_s^2 \approx \frac{\sigma_0}{\hat{C}_V}\left(1 + v_2\,\hat{\mu}_B^2 + v_4\,\hat{\mu}_B^4\right)$$

with

$$\hat{C}_V \equiv \frac{C_V}{T^3}\bigg|_{\hat{\mu}_B=0} = 3\sigma_0 + \sigma_0'$$

and $v_k(N_i, \sigma_m, \sigma_n', \hat{C}_V)$.

$$\hat{s} = \hat{\epsilon} + \hat{p} - \hat{\mu}_B\hat{n}_B - \hat{\mu}_Q\hat{n}_Q - \hat{\mu}_S\hat{n}_S$$

Hadron resonance gas: the basics

Non-interacting, quantum, relativistic gas eventually gives (single species)

$$\frac{P}{T}=\frac{m^2gT}{2\pi^2}\sum_{k=1}^{\infty}\frac{\eta^{k+1}z^k}{k^2}K_2\left(\frac{mk}{T}\right), \qquad z\equiv e^{\hat{\mu}_B B+\hat{\mu}_Q Q+\hat{\mu}_S S},$$

with K_2 modified Bessel function 2^{nd} kind. HRG:

- Assume such gas where hadrons and resonances only d.o.f.
- Hence valid up to $\sim T_{\text{pc}}$
- Sum over all such states, each with g_i, m_i, etc.
- K_2 exponentially suppressed, so can keep few terms

HRG at $\hat{\mu}_Q = \hat{\mu}_S = 0$

Schematically,

$$\begin{aligned}
\hat{p} &= f_M(T) + f_B(T)\cosh(\hat{\mu}_B) \\
\hat{\epsilon} &= 3f_M(T) + f'_M(T) + (3f_B(T) + f'_B(T))\cosh(\hat{\mu}_B) \\
\hat{n}_B &= f_B(T)\sinh(\hat{\mu}_B)
\end{aligned}$$

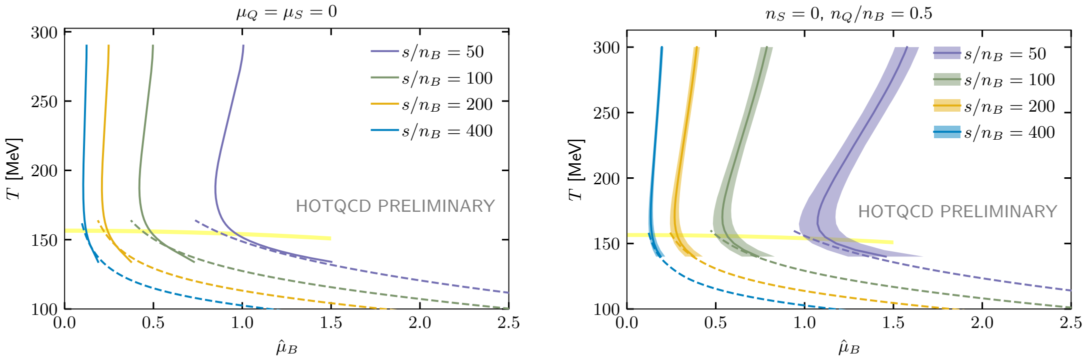
with $f_M(T)$ mesonic and $f_B(T)$ baryonic contributions, respectively.

$$c_T^2 = \left(\frac{\partial p}{\partial \hat{\mu}_B}\right)\left(\frac{\partial \epsilon}{\partial \hat{\mu}_B}\right)^{-1} = \frac{1}{3 + f'_B(T)/f_B(T)}$$

c_s^2 is more involved:

$$c_s^2 = \frac{\hat{n}_B^2\left(3\hat{s} + T\frac{\partial \hat{s}}{\partial T}\right) - 2\hat{s}\hat{n}_B\frac{\partial \hat{s}}{\partial \hat{\mu}_B} + \hat{s}^2\frac{\partial \hat{n}_B}{\partial \hat{\mu}_B}}{(\hat{\epsilon} + \hat{p})\left(\left(3\hat{s} + T\frac{\partial \hat{s}}{\partial T}\right)\frac{\partial \hat{n}_B}{\partial \hat{\mu}_B} - \frac{\partial \hat{s}}{\partial \hat{\mu}_B}\frac{\partial \hat{s}}{\partial \hat{\mu}_B}\right)}$$

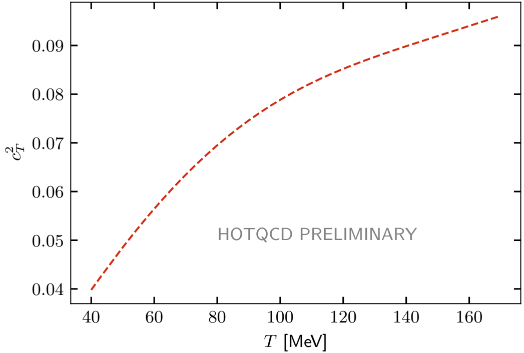
Lines of constant s/n_B

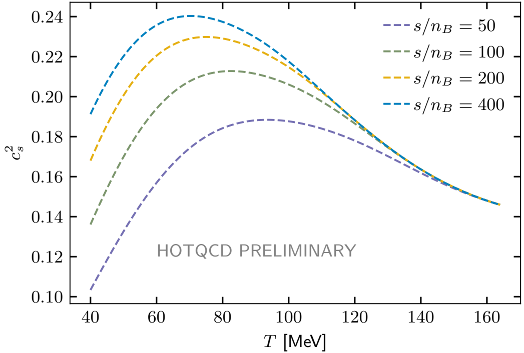


$$\text{Lattice:}\quad \hat{\mu}_B = \sum_{n=1}^{\infty} h_{2n-1}(N_i, \sigma_m) \left(\frac{s}{n_B}\right)^{1-2n}$$

Good agreement between lattice and HRG below $T_{\rm pc}$.

Results: HRG c_X^2 for $\hat{\mu}_Q = \hat{\mu}_S = 0$

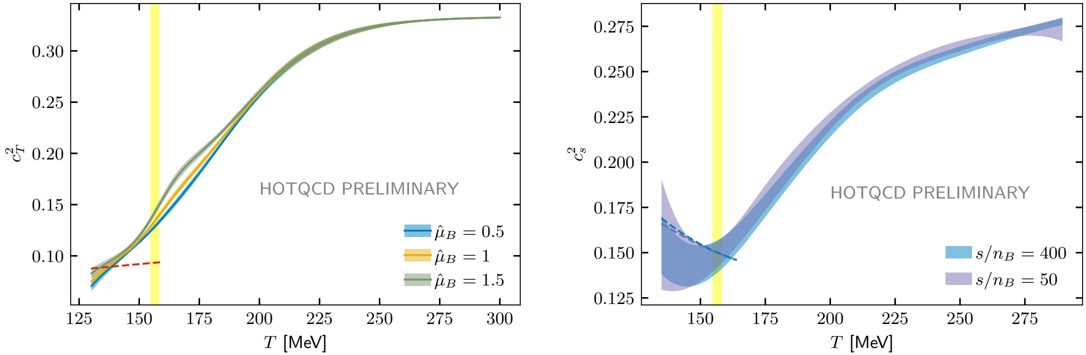




$$\text{HRG:}\quad c_T^2 = \left(\frac{\partial p}{\partial \hat{\mu}_B}\right)\left(\frac{\partial \epsilon}{\partial \hat{\mu}_B}\right)^{-1} = \frac{1}{3 + f_B'(T)/f_B(T)}$$

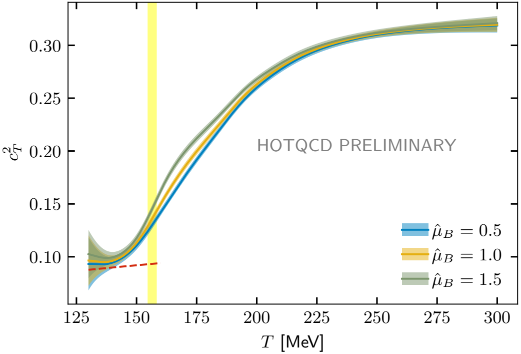
Meson-independence of c_T^2 eliminates the bump.

Results: Lattice c_X^2 for $\hat{\mu}_Q = \hat{\mu}_S = 0$



Dip in c_s^2 in vicinity of $T_{\rm pc}$.
Dependence of c_s^2 on s/n_B is at most mild.

Results: Lattice c_T^2 for $n_Q/n_B = 0.5$



$$c_T^{-2} - 3 = \frac{P_2'}{P_2} + \sum_{k \text{ even}} S_k \, \hat{\mu}_B^k .$$

Conclusions

- Dip in c_s^2 can be attributed to mesonic contribution in HRG
- Further calculations at $n_Q/n_B = 0.5$ ongoing
- May be interesting to look into scaling behavior near chiral transition, sensitivity not expected except for very low m_l
- We are working on other EoS observables at finite μ (Jishnu)

Thanks for your attention.