Los Alamos NATIONAL LABORATORY

nEDM from the theta-term and chromoEDM operators

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Introduction

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 4 operators anomalously small.

- Dimension 4:
  - CP violating mass $m\bar{\psi}\gamma_5\psi$.
  - Topological charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\mathrm{EW}}/M_{\mathrm{BSM}}^2$:
  - Quark Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Quark Chromo-electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M_{\mathrm{BSM}}^2$:
  - Gluon Chromo-electric Dipole moment: $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction
States

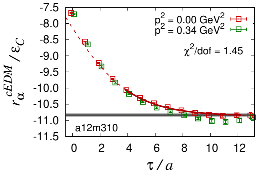
Asymptotic states are 'free' particles: always have a P symmetry.

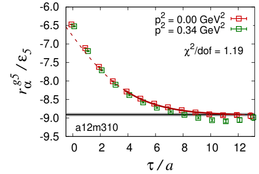
If interaction does not have these symmetries, the symmetry generator will be different on different asymptotic states.
Change the nucleon interpolating operator

$$\hat{N} \to e^{-i\alpha_N \gamma_5} \hat{N}$$

to get standard parity for nucleon.
α_N can be chosen real if interactions have PT symmetry.
α_N can be calculated in chiral perturbation theory and as

$$\lim_{\tau\to\infty} [r_\alpha(\tau) \equiv \frac{\Im \operatorname{Tr} \gamma_5 (1+\gamma_4) \langle N(\tau) \bar{N}(0) \rangle}{\Re \operatorname{Tr} (1+\gamma_4) \langle N(\tau) \bar{N}(0) \rangle}]$$





Introduction
Form Factors

Vector form-factors in standard representation for C, P, and T:

Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$\langle N|V_\mu(q)|N\rangle = \overline{u}_N \left[\gamma_\mu F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \frac{F_2(q^2)}{2m_N} + (2i\, m_N \gamma_5 q_\mu - \gamma_\mu \gamma_5 q^2)\frac{F_A(q^2)}{m_N^2} + \frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N}\right] u_N$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- F_A violates PT; F_3 violates CP.

Lattice Calculation

Lattice Calculation
Technique

nEDM due to the quark EDM operator proportional to the tensor charge.
The quark chromo-EDM operator can be handled by the Schwinger source method

$$\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu} \longrightarrow \not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})$$

The fermion determinant vanishes for the isovector case

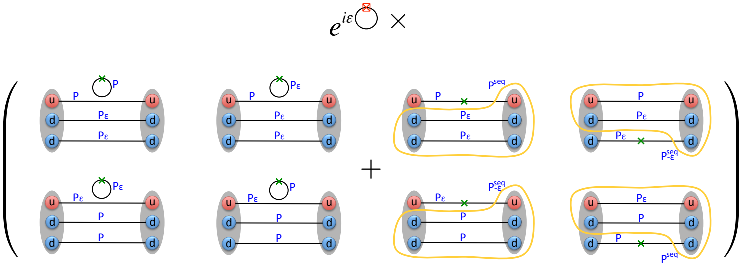
$$\frac{\det(\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu}))}{\det(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})}$$
$$= \exp \mathrm{Tr} \ln \left[1 + i\epsilon\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right]$$

Θ-term and the gluon chromo-EDM operators are 'reweighting factors'.

Quark Chromoelectric Dipole Moment

Quark Chromoelectric Dipole Moment

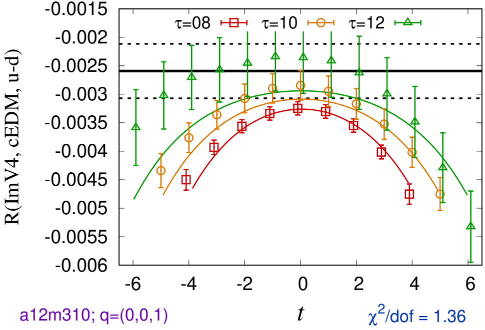
Three-point function

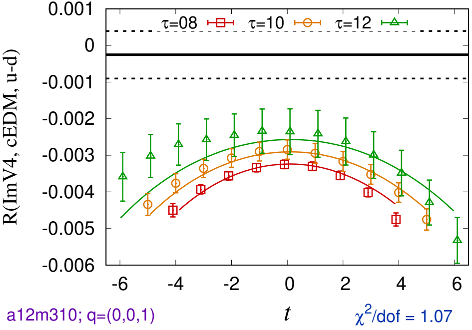


ϵ small to avoid multiple insertions.

Quark Chromoelectric Dipole Moment
Excited state fits

The excited state spectrum cannot be determined from 3-pt functions.
Systematic errors from possible enhancement of light $N\pi$ states in the 3-pt functions





Quark Chromoelectric Dipole Moment
Power divergence

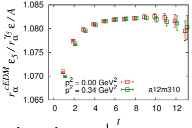
Quark chromo-EDM operator is power-divergent:

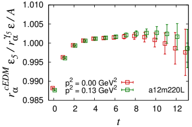
$$\tilde{C} = i\bar{\psi}\Sigma^{\mu\nu}\gamma_5 G_{\mu\nu} T^a \psi - i\frac{A}{a^2}\bar{\psi}\gamma_5 T^a \psi$$

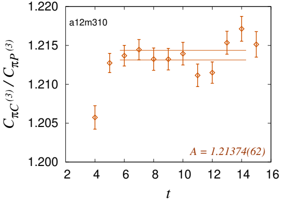
Demanding $\langle\Omega|\,\tilde{C}\,|\pi\rangle = 0$ fixes A:

Leading order χPT:

$$\alpha_N(\tilde{C}) = 0 \implies \frac{1}{A}\frac{\alpha_N(C)}{\alpha_N(\gamma_5)}) = 1$$







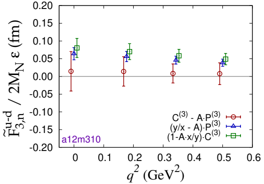
Ensemble	c_{SW}	a (fm)	t-range	A
a12m310	1.05094	0.1207(11)	6–14	1.21374(62)
a12m220L	1.05091	0.1189(09)	7–14	1.21800(33)
a09m310	1.04243	0.0888(08)	8–22	0.99621(30)
a06m310	1.03493	0.0582(04)	14–30	0.77917(24)

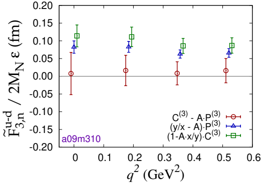
Quark Chromoelectric Dipole Moment

Multiplicative renormalization

Isovector pseudoscalar can be rotated away up to $O(a)$ effects!
We can determine the $O(a)$ effects non-perturbatively:

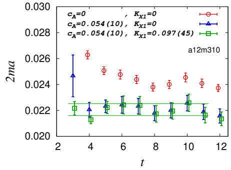
$$\frac{\langle \pi \left[a\partial_\mu A^\mu - \bar{c}_A a^2 \partial^P + \bar{K}(a^2 C - AP) \right] \rangle}{\langle \pi P \rangle} = 2\bar{m}a(1 + O(a^2))$$





So, on-shell zero-momentum

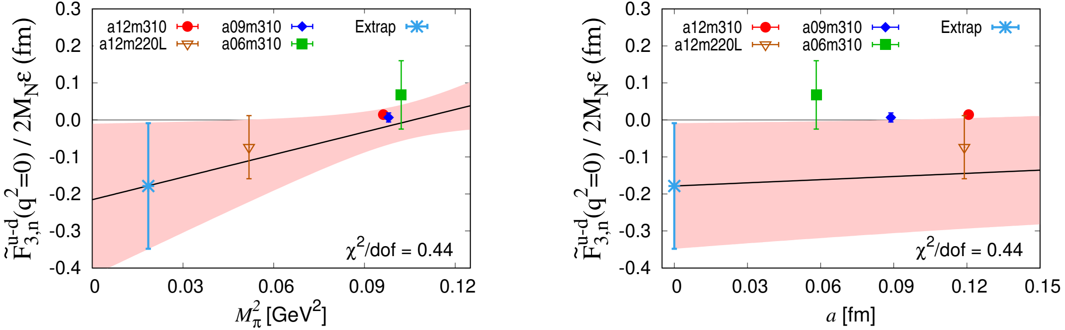
$$\text{M.E. of } P = \text{M.E. of } \frac{x \equiv a^2 \bar{K}}{y \equiv 2\bar{m}a + A\bar{K}} C\,.$$



Ensemble	fit-range c_A	fit-range $\bar{K}_{X1}$	χ^2/d.o.f c_A	χ^2/d.o.f $\bar{K}_{X1}$	c_A	$\bar{K}_{X1}$	$2\bar{m}a$	$\frac{2\bar{m}a}{K_{X1}}$	$\frac{2\bar{m}a}{2ma + AK_{X1}}$
a12m310	4–11	3–11	0.66	0.88	0.054(10)	0.097(45)	0.02205(46)	0.23(10)	0.158(58)
a12m220L	4–11	3–11	2.08	3.09	0.0342(77)	0.183(35)	0.01152(21)	0.063(12)	0.0491(86)
a09m310	5–15	4–15	0.99	1.09	0.0277(40)	0.047(15)	0.01684(15)	0.35(11)	0.263(61)
a06m310	6–20	5–20	0.29	1.53	0.0093(17)	0.0272(60)	0.010460(37)	0.385(87)	0.331(50)

Quark Chromoelectric Dipole Moment

Results



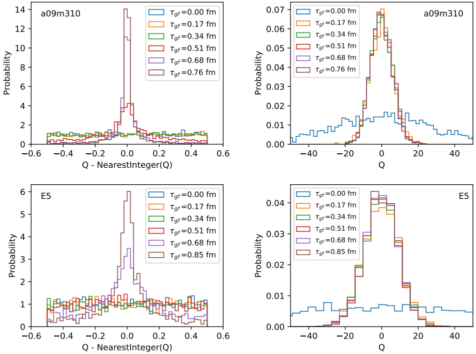
Preliminary data for power-divergent-subtracted bare operator $\tilde{C}$.

QCD Topological term

QCD Topological term
Comparing clover and HISQ sea

- Previous calculation
  - Fermion: Clover (valence) on HISQ (sea)
  - Lattice spacings: 0.057 fm – 0.151 fm
  - Pion mass: 128 MeV – 320 MeV
  - Number of configurations: 550 – 2200

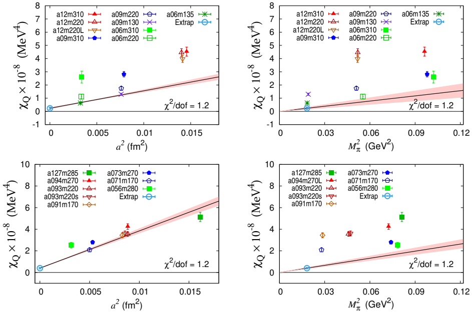
- New calculation
  - Fermion: Clover (valence) on Clover (sea)
  - Lattice spacings: 0.056 fm – 0.127 fm
  - Pion mass: 167 MeV – 285 MeV
  - Number of configurations: 810 – 2100



QCD Topological term
Topological Susceptibility

$$\chi_Q = [66(9)(4)\ \mathrm{MeV}]^4 \qquad \text{Clover-on-HISQ}$$

$$\chi_Q = [79.5(3.0)\ \mathrm{MeV}]^4 \qquad \text{Clover-on-Clover}$$



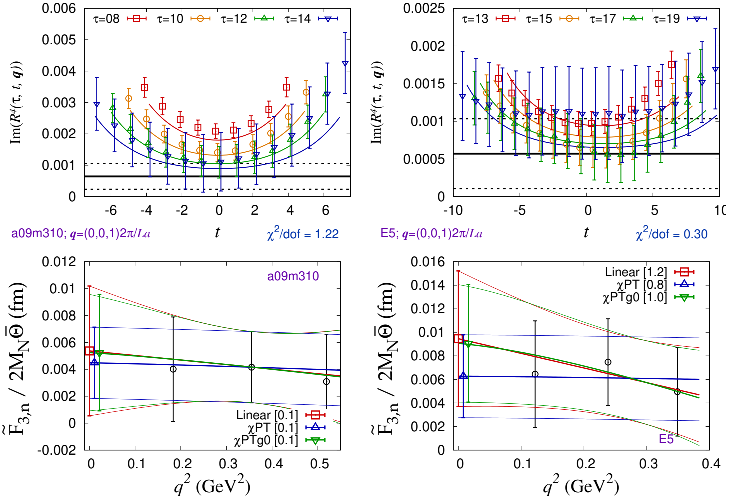
Expectation from χPT

$$\frac{1}{\chi_Q} = \frac{1}{\chi_Q^{\text{quench.}}} + \frac{4}{M_\pi^2 F_\pi^2}\left(1 - \frac{M_\pi^2}{3M_\eta^2}\right)^{-1}$$

$$\chi_Q = [79\ \mathrm{MeV}]^4$$

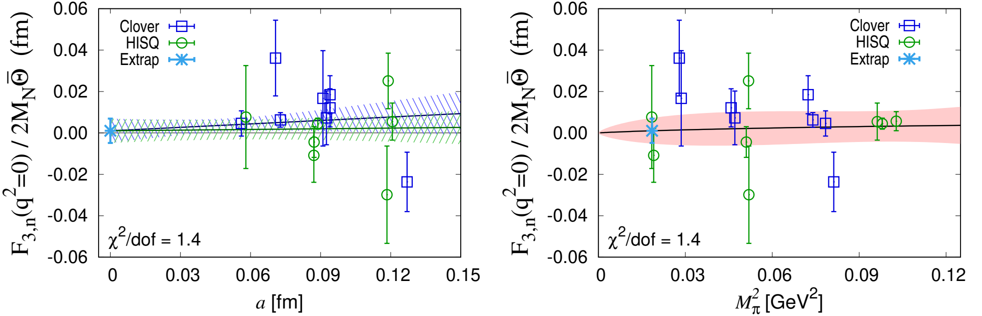
QCD Topological term

ESC and Q^2 extrapolation



QCD Topological term

Simultaneous fit



- Extrapolation to continuum $a \to 0$ and physical pion mass $M_\pi \to 135\mathrm{MeV}$
- Simultaneous fit to Clover-on-HISQ and Clover-on-Clover results

$$d_N = c_1 M_\pi^2 + c_2 M_\pi^2 \log\left(\frac{M_\pi^2}{M_N^2}\right) + c_3^{\mathrm{HISQ}} a + c_3^{\mathrm{Clover}} a = 0.0010(59)$$

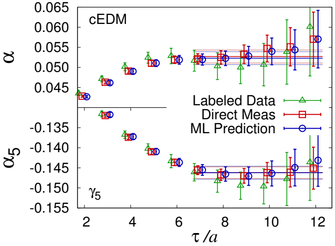
PRELIMINARY; statistical error only.

Future

Future

Next steps in progress

- Compare with Gradient-flow to control mixing.
  - In Gradient Flow scheme, smearing cEDM allows smooth $a \to 0$ limit at fixed physical smearing t.
  - Continuum calculation mixes matrix elements of cEDM and $t^{-1}\bar{\psi}\gamma_5\psi$ to obtain cEDM in other schemes.
  - Mixing with quark EDM.
- Use machine learning to find correlated observables, and reduce variance.



- CP violating phase $\alpha_N = 0$ without CP violation.
- Training on 70 confs, bias correction on 50 confs, prediction for 400 confs.
- ML algorithm is able to learn the computation of CPV observables from CP conserving one