The glueball spectrum with $N_f = 4$ light fermions

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The Cyprus Institute
RESEARCH•TECHNOLOGY•INNOVATION

NIKOS

General considerations 1

- Recently for Pure Gauge:
  - Extracted the masses of lightest glueballs for all 5 irreducible reps (A_1, A_2, E, T_1, T_2) of cubic rotation group and $P = \pm, C = \pm$: $N \in [2, 12]$, improving older works: *M. Teper, 1987; Morningstar and Peardon, 1997; Lucini, Rago, and Rinaldi, 2010*
  - We extrapolated to the continuum limit
  - For enough $N \in [2, 12]$, that we extrapolated to the $N = \infty$ limit
  - We have identified continuum spins J for the lightest states
  - Control systematic errors: multiglueball states; di-torelons; topological freezing
- Publications:
  - SU(3): *AA and M. Teper, JHEP 11 (2020), 172, e-Print: 2007.06422 [hep-lat]*
  - SU(∞): *AA and M. Teper, JHEP 12 (2021), 082, e-Print: 2106.00364 [hep-lat]*

General considerations 2

- Glueball spectrum in SU(3) has a phenomenological importance

- What is the effect of the light dynamical quarks?
  - Very noisy correlators – requires lots of statistics
  - Extract the spectrum with $N_f = 4$ light quarks with $m_\pi \approx 250$ MeV
    - Ensembles with 20k configurations
  - Investigate the low-lying spectrum of A_1^{++} with $N_f = 2+1+1$ light quarks
  - Ideally, perform an extended investigation for $N_f = 2+1+1$ light quarks

- So far: *Chen, 2021 ($N_f = 2+1$, $m_\pi = 140$ MeV), Gregory, 2012 ($N_f = 2+1$, $m_\pi = 360$ MeV),*
- ***See also Davide Vadacchino's plenary:* A REVIEW ON GLUEBALL HUNTING**

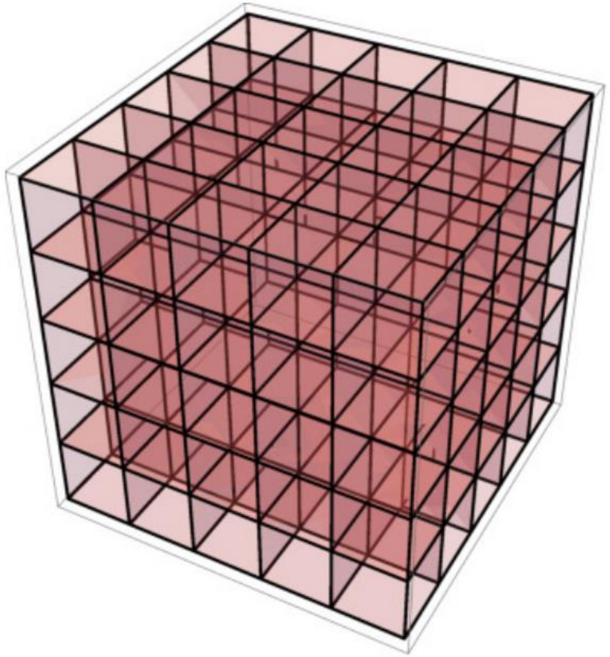
Lattice Setup - fermions

- We use ensembles with Clover improved Twisted mass fermions produced with
  - $N_f = 4$ degenerate light flavors at three different lattice spacings
  - $N_f = 2 + 1 + 1$ (2 degenerate light flavors) + strange + charm
- We use the Iwasaki improved action

$$S_G = \frac{\beta}{3} \sum_x \left(c_0 \sum_{\substack{\mu,\nu=1 \\ \mu<\nu}}^{4} \left[1 - \mathrm{Re\,Tr}\left(U^{1\times1}_{x,\mu\nu}\right)\right] + c_1 \sum_{\substack{\mu,\nu=1 \\ \mu\neq\nu}}^{4} \left[1 - \mathrm{Re\,Tr}\left(U^{1\times2}_{x,\mu\nu}\right)\right] \right)$$

- The fermionic sector is implemented using the twisted mass formulation of lattice QCD, which for two mass degenerate quarks takes the form

$$S^l_F = a^4 \sum_x \bar{\chi}^{(l)}(x) \left(D_W[U] + \frac{i}{4} c_{SW} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}[U] + m_{0,l} + i\mu_l \gamma_5 \tau^3 \right) \chi^{(l)}(x)$$

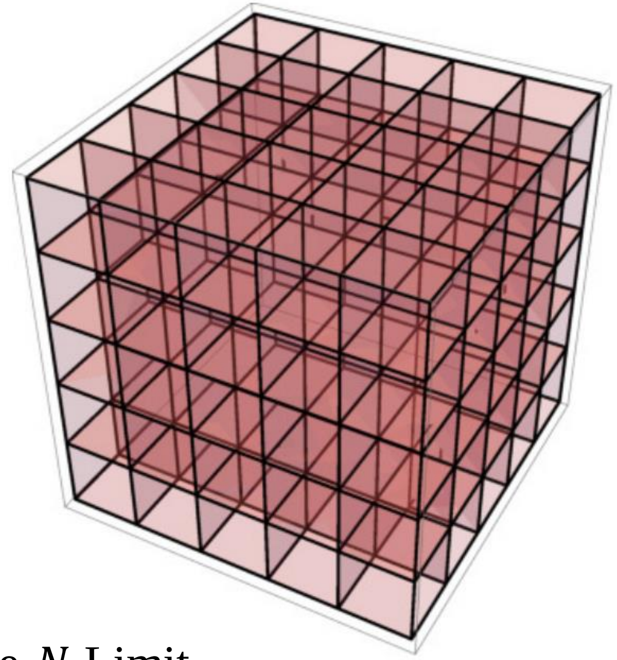


	β	c_{sw}	μ	L	am_{PS}	t_0/a^2
$N_F = 4$						
cA4.60.16	1.726	1.74	0.006	16	0.2313(23)	3.565(39)
cB4.06.16	1.778	1.69	0.006	16	0.2652(53)	4.947(62)
cB4.06.24	1.778	1.69	0.006	24	0.1580(8)	4.667(17)
cC4.05.24	1.836	1.6452	0.005	24	0.1546(20)	6.422(48)
$N_F = 2+1+1$						
cA211.53.24	1.726	1.74	0.005	24	0.1661(4)	2.342(6)
cA211.25.32	1.726	1.74	0.003	32	0.1253(1)	2.392(4)

Extended Twisted Mass Collaboration

Lattice Setup – Pure Gauge

- We generate Pure Gauge ensembles
  - For several values of the lattice spacing (β)
  - For $N = 2$ to $N = 12$
- We use the Wilson action



$$S_L = \beta \sum_p \{1 - \frac{1}{N} \mathrm{ReTr} U_p\}$$

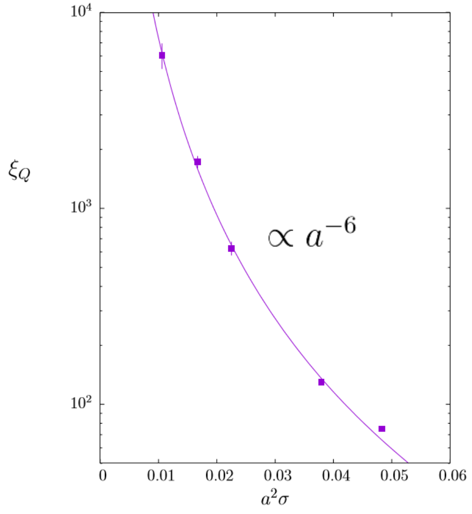
$$\beta = \frac{2N}{g^2}$$

- Topological Freezing

Correlation Length:

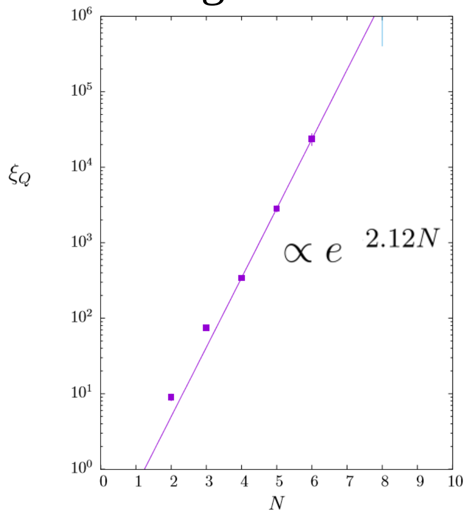
$\langle Q(is)Q(is+\xi_Q)\rangle / \langle Q^2 \rangle = e^{-1}$

Continuum Limit



AA and M. Teper, arXiv:2007.06422

Large-N Limit



AA and M. Teper, arXiv:2106.00364

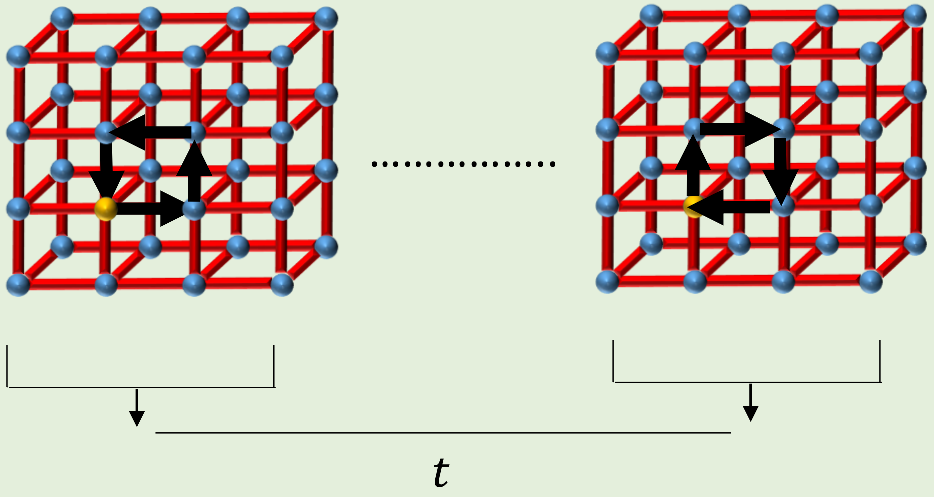
Quantum Numbers

- On the lattice, continuous rotational symmetry broken to the symmetry of the octahedral group
- Irreducible representations are A_1, A_2, E, T_1, T_2
- We use **gluonic** operators in irreducible representations of the octahedral group
- Near the continuum limit, full rotational symmetry is recovered
- Continuous spin obtained from the subduced representations of the rotation group $SO(3)$ restricted to the octahedral irreducible representations

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

- In addition, Parity and Charge Conjugation $P = \pm, C = \pm$

Correlation Function

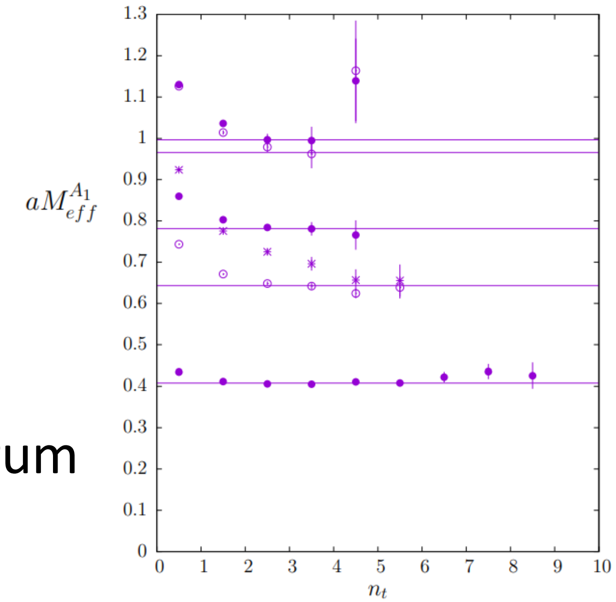


$$
\begin{aligned}
C(t) &= \langle \Phi^\dagger(t)\Phi(0)\rangle \\
&= \langle \Phi^\dagger(0)e^{-Ht}\Phi(0)\rangle \\
&= |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t} \\
&\quad + \sum_{n=1} |\langle n|\Phi(0)|vac\rangle|^2 e^{-E_n t} \\
&\xrightarrow{t\to\infty} |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t}
\end{aligned}
$$

- We calculate the effective energies

$$
\lim_{t\to\infty}\left[-\ln\left(\frac{C(t)}{C(t-a)}\right)\right] = aE_0
$$

Example of effective masses



- We use the variational calculation to extract the excitation spectrum

Extraction of the Excitation Spectrum

- Construct a large basis of Operators $\Phi_i : i = 1, 2, \ldots$ with **RIGHT QUANTUM NUMBERS**
- Calculate the correlation function (Matrix) $C_{ij}(t) = \langle \Phi_i^\dagger(t) \Phi_j(0) \rangle$
- Diagonalize the matrix $C^{-1}(t=0)C(t=ma)$
- Extract the eigenvectors
- Extract the correlator for each state $(\sim \mathrm{e}^{-E_n t})$
- By fitting the results, we extract the mass (energy) for each state

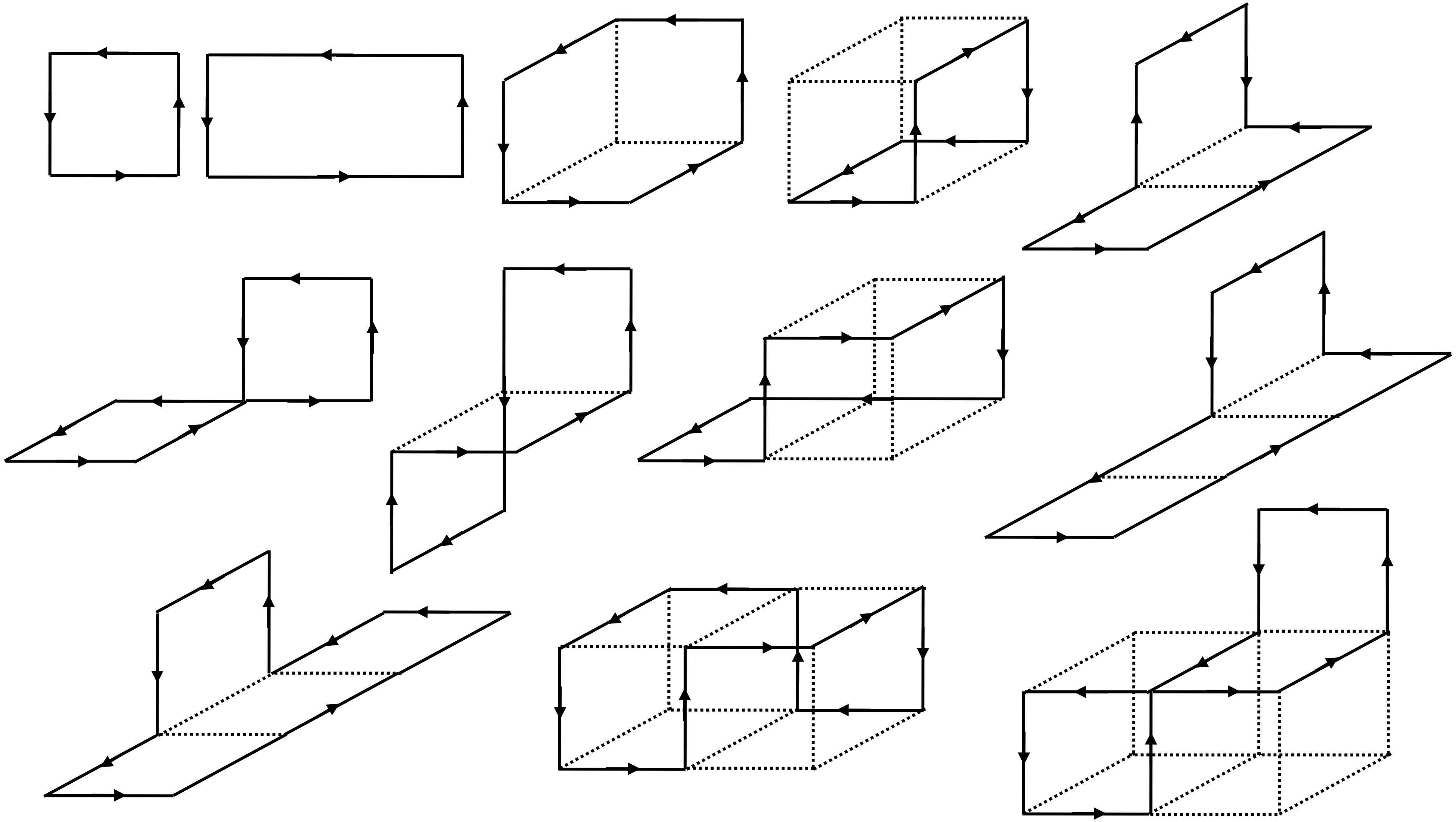
$$\begin{bmatrix} [A_1^{PC}] & 0 & 0 & 0 & 0 \\ 0 & [A_2^{PC}] & 0 & 0 & 0 \\ 0 & 0 & [E^{PC}] & 0 & 0 \\ 0 & 0 & 0 & [T_1^{PC}] & 0 \\ 0 & 0 & 0 & 0 & [T_2^{PC}] \end{bmatrix}$$

- This is the so called Generalized Eigenvalue Problem (GEVP)

M. Lüscher and U. Wolff, Nucl. Phys. B339 (1990) 222–252.

Lattice Operators

- Operators have been constructed by linear combinations of:



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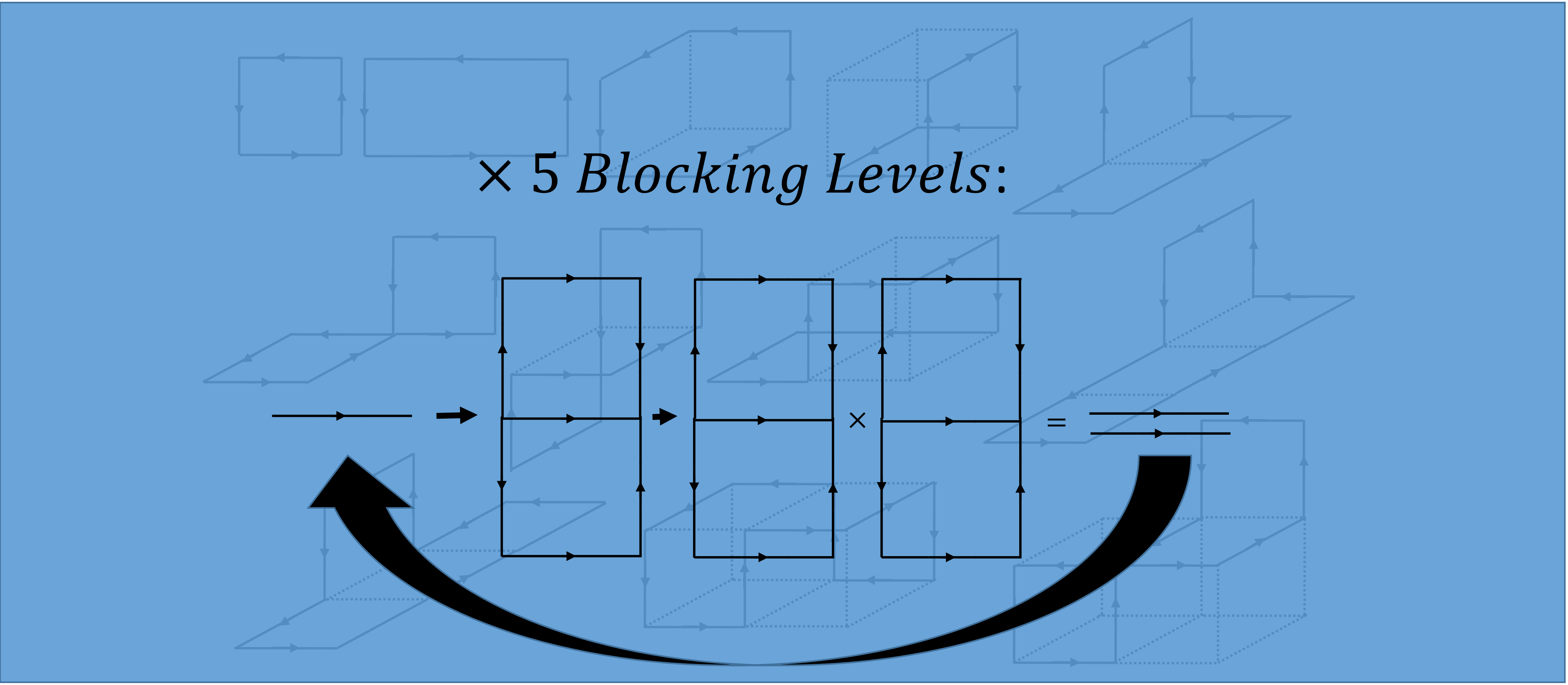
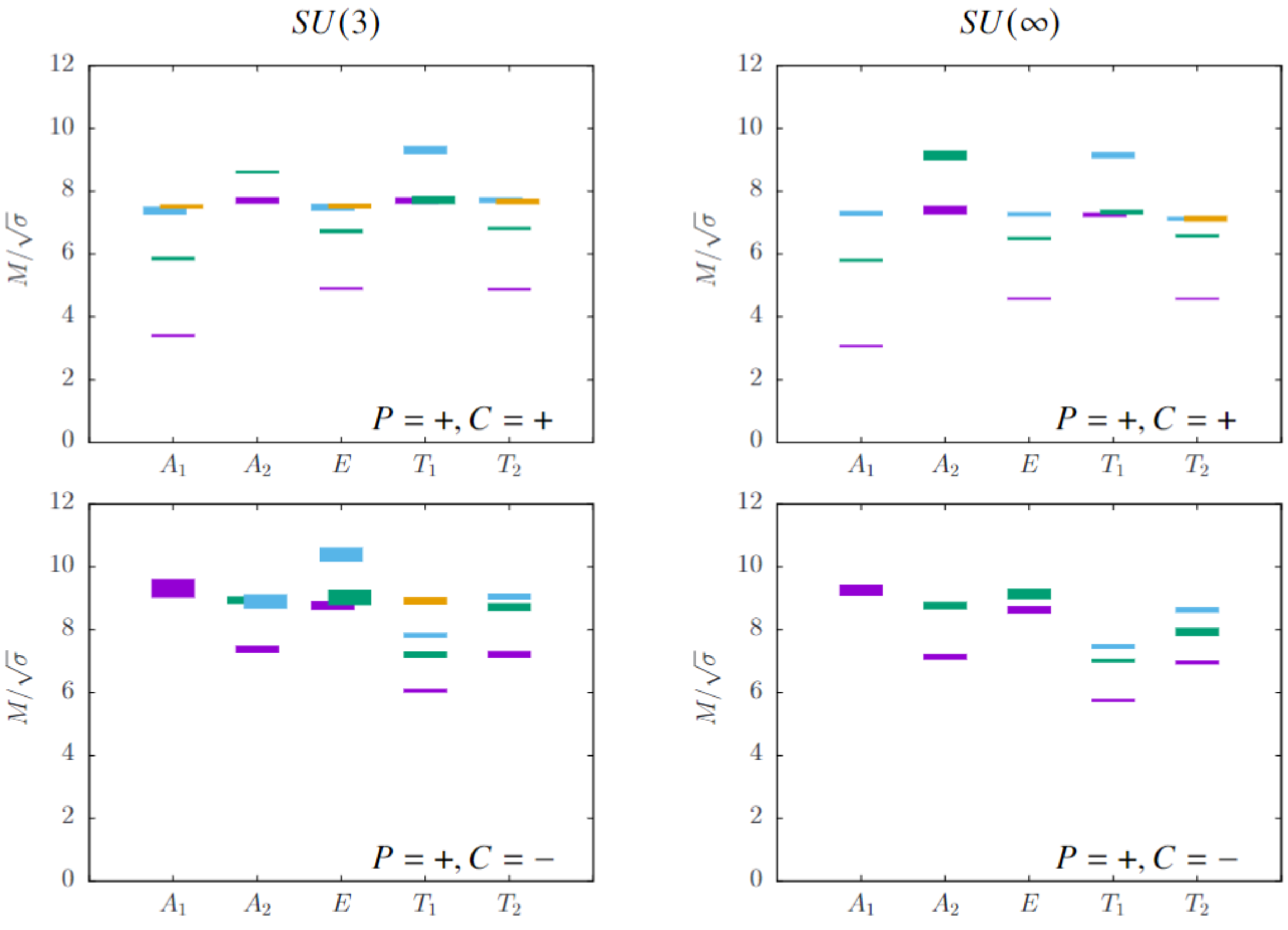


Recent results on Pure Gauge Theory



AA and M. Teper, JHEP 11 (2020) 172

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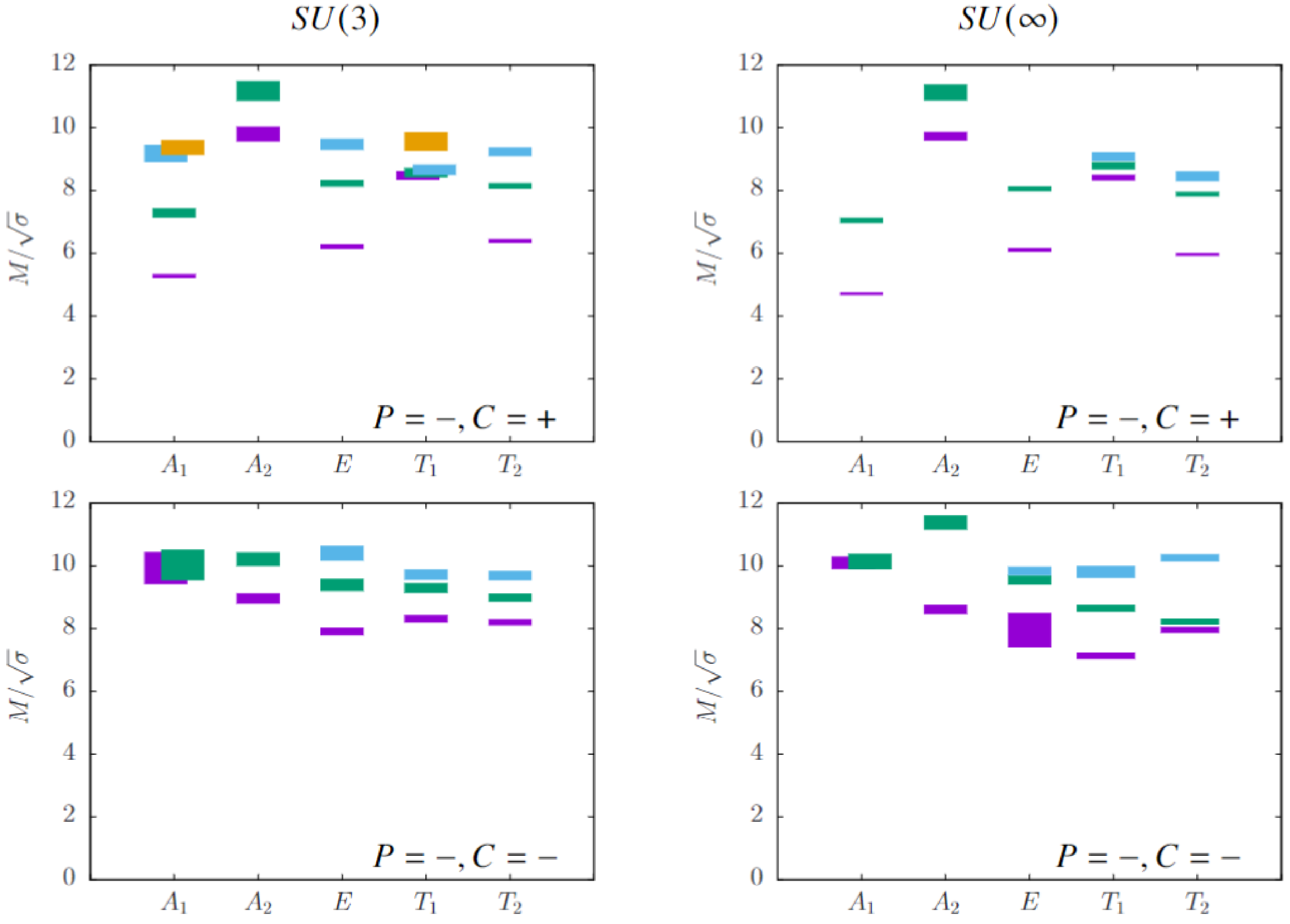

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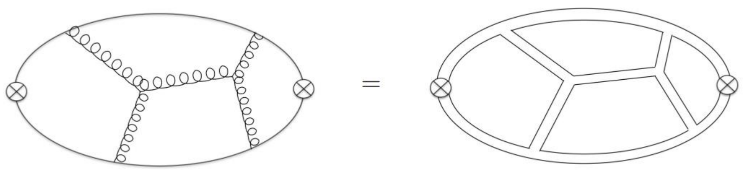
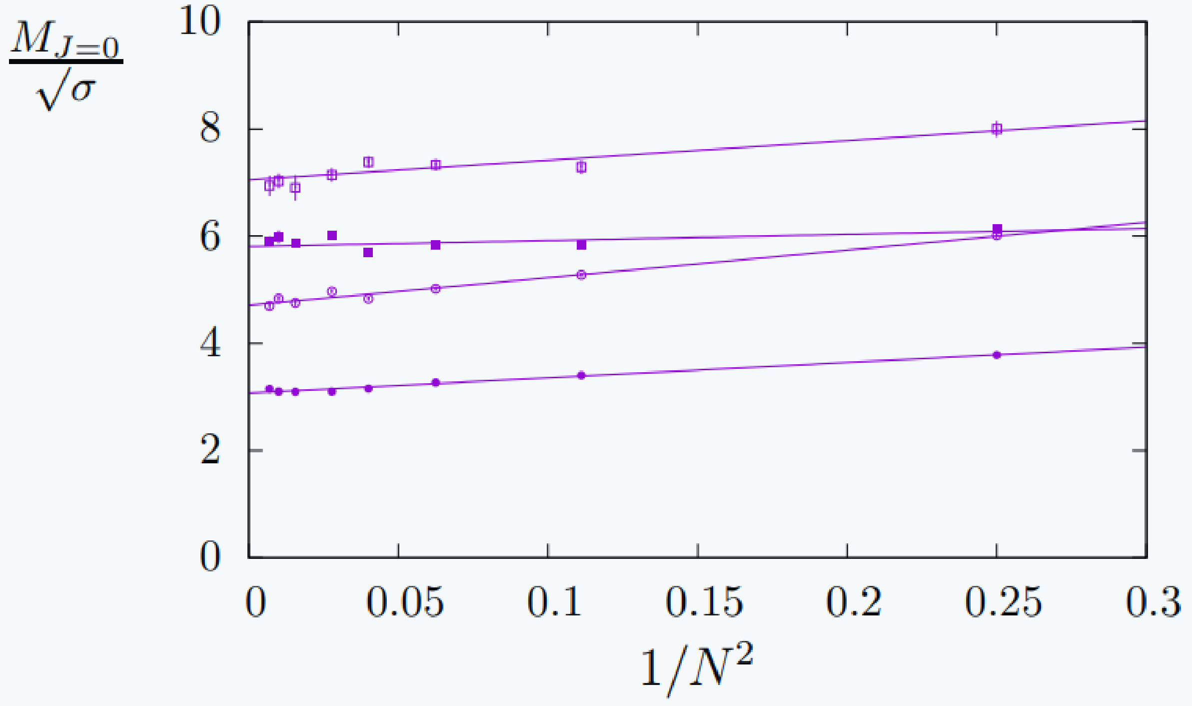
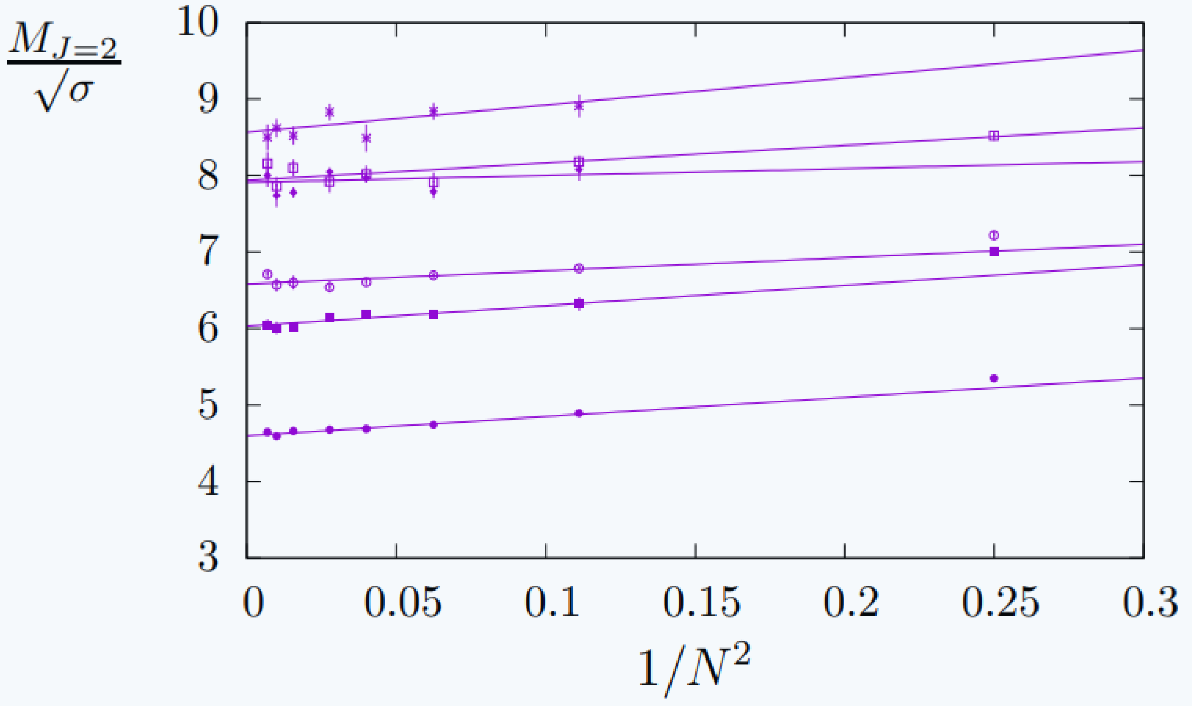
Results for Planar Limit

- Extrapolations to the Large-N limit



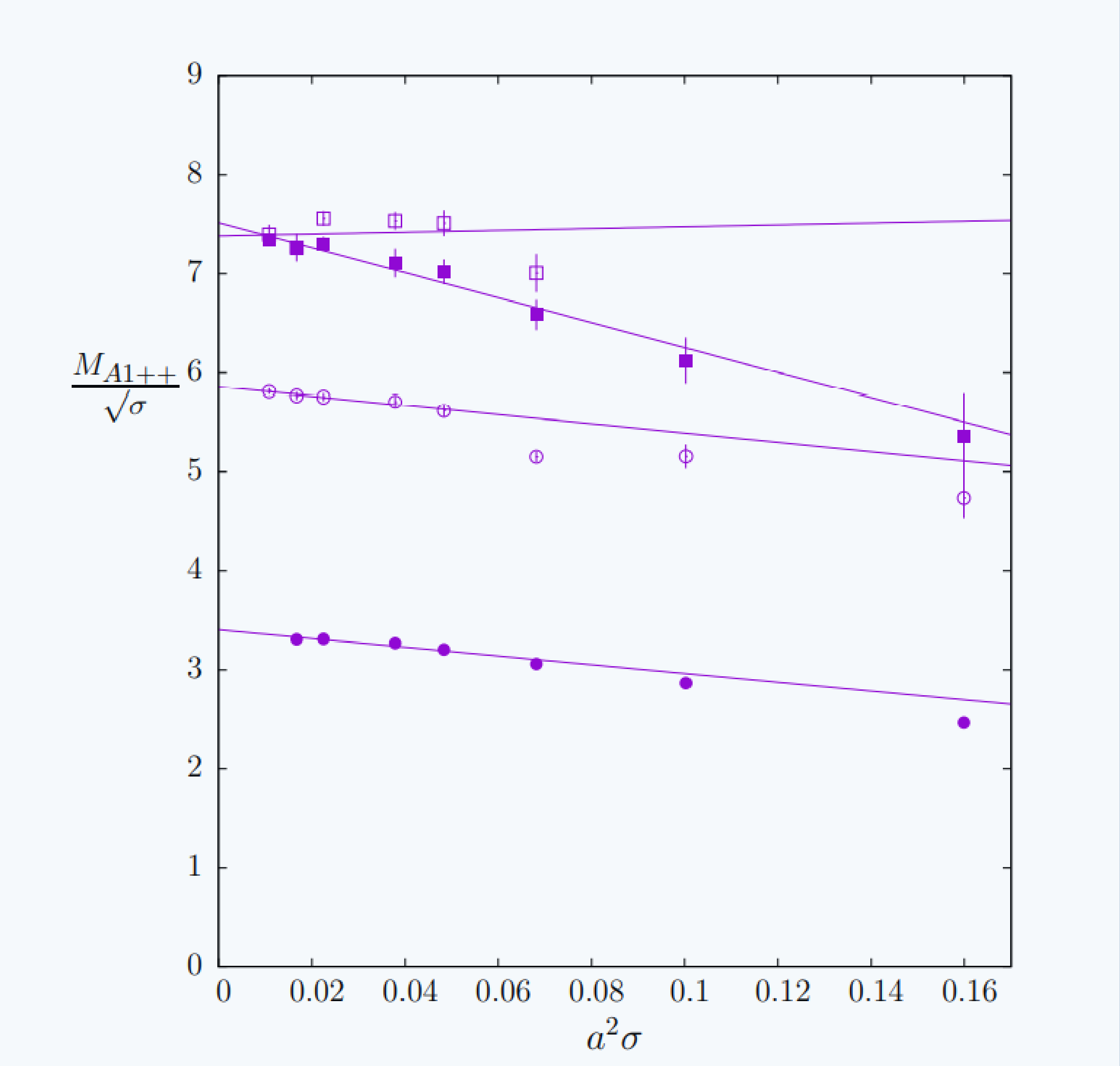


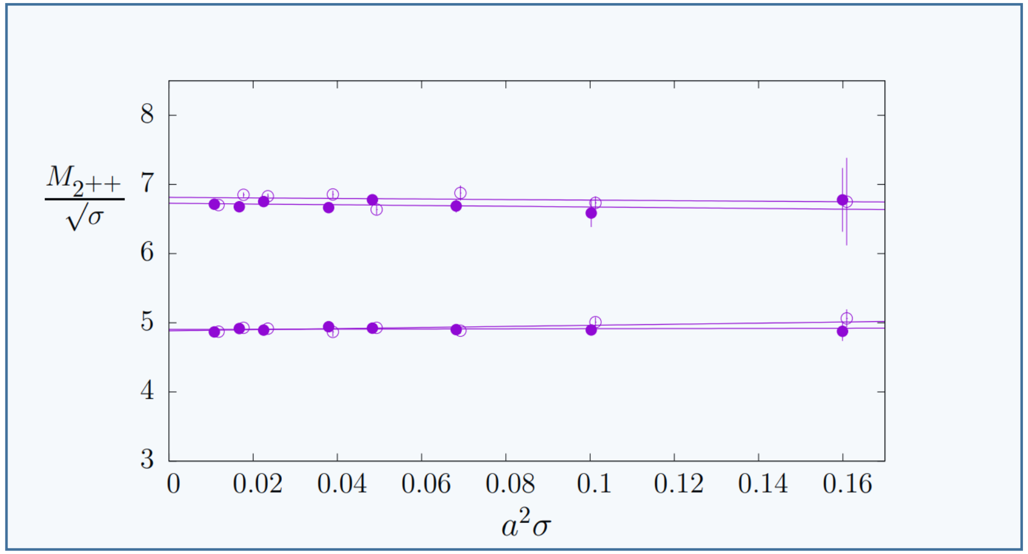
J^{PC}= 0^{++} ground (•) and first excited (■); 0^{-+} ground (◦) and first excited (□). With extrapolations to $N = \infty$ from $N = 2 - 12$.

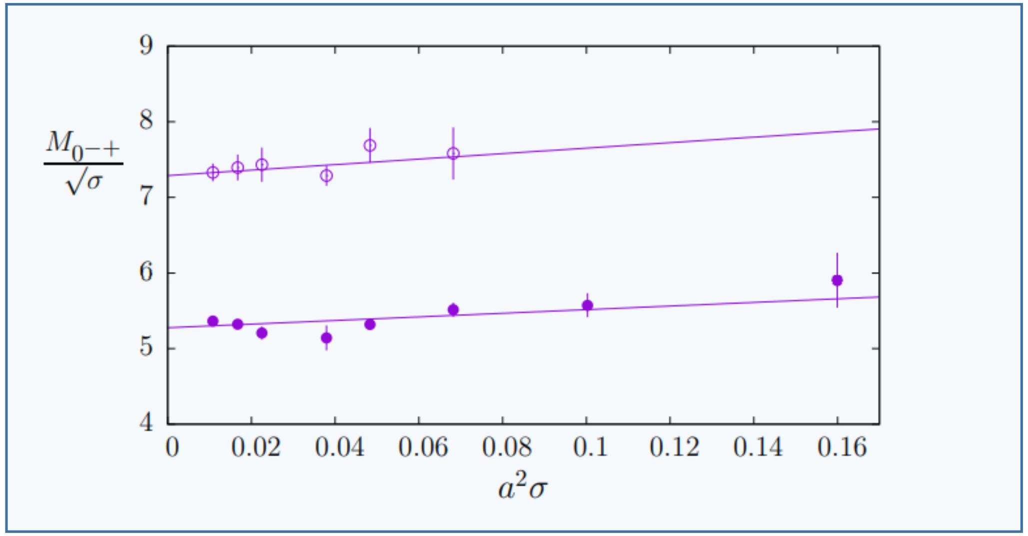


J^{PC}= 2^{++}ground (•) and first excited (◦) tensors; 2^{-+} ground (■) and first excited (□) pseudotensors; lightest 2^{+-} (∗) and the lightest 2^{--} (♦).

Results for SU(3) Pure Gauge, continuum extrapolations







Asymptotic behaviour: $\sim a^2 \left[\frac{1}{-\log(a\Lambda)}\right]^{\hat{\gamma}_1}$, *Husung et al, Eur.Phys.J.C 80 (2020) 3, 200*

Topological Charge and scale setting

- We calculate the topological

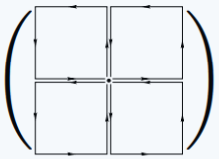
$$\mathcal{Q} = \int d^4x q(x)$$

- With topological charge density:

$$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left\{G_{\mu\nu}G_{\rho\sigma}\right\}$$

- We use the Clover definition:

$$C_{\mu\nu}\ \ (x) = \frac{1}{4}\mathrm{Im}\left(\quad\right)$$



- We smooth the UV fluctuations using the Wilson Flow.

- We solve the evolution equations:

$$\begin{aligned} \dot{V}_\mu\left(x,\tau\right) &= -g_0^2\left[\partial_{x,\mu}S_G(V(\tau))\right]V_\mu\left(x,\tau\right) \\ V_\mu\left(x,0\right) &= U_\mu\left(x\right)\ , \end{aligned}$$

- With link derivative defined as:

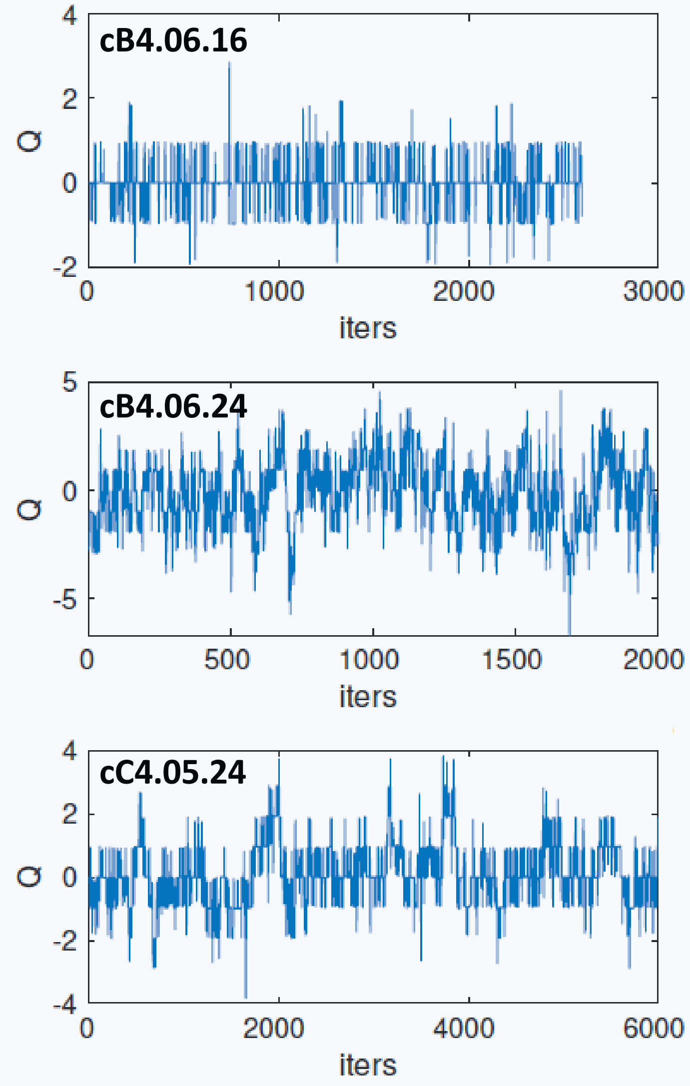
$$\begin{aligned} \partial_{x,\mu}S_G(U) &= i\sum_a T^a \frac{\mathrm{d}}{\mathrm{d}s}S_G\left(e^{isY^a}U\right)\Big|_{s=0} \\ &\equiv i\sum_a T^a \partial_{x,\mu}^{(a)}S_G(U)\,, \end{aligned}$$

- We can define a scale parameter t_0

$$F(t) = t^2\langle E(t)\rangle \ \text{ where } E(t) = \frac{1}{4}B_{\mu\nu}^2(t)$$

$$F(t)|_{t=t_0(c)} = c$$

- With $c = 0.3$.



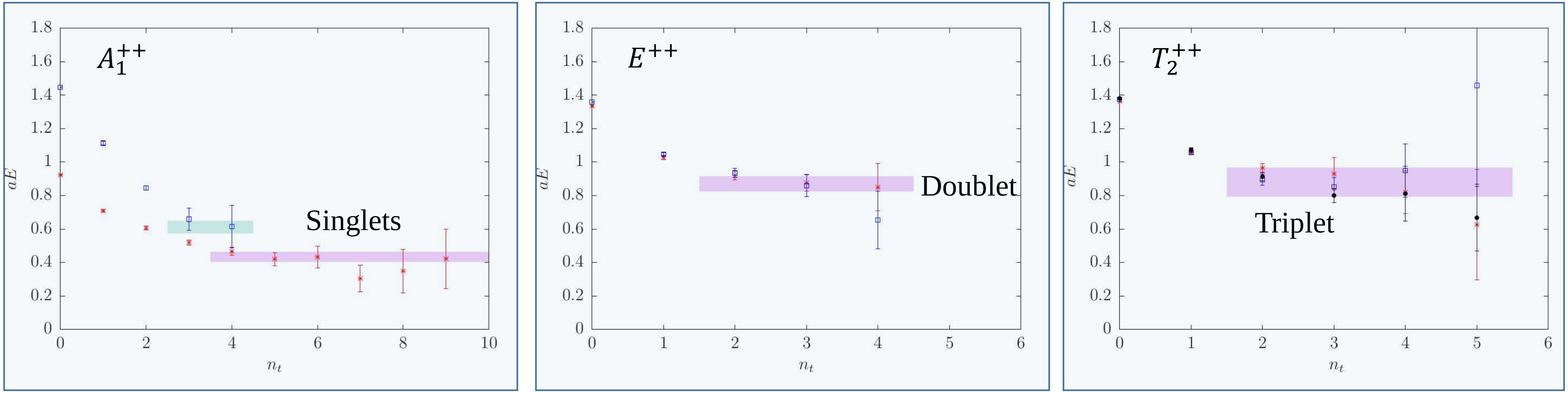
Effective Masses

- Correlation functions of specific operators used for extracting the spectrum

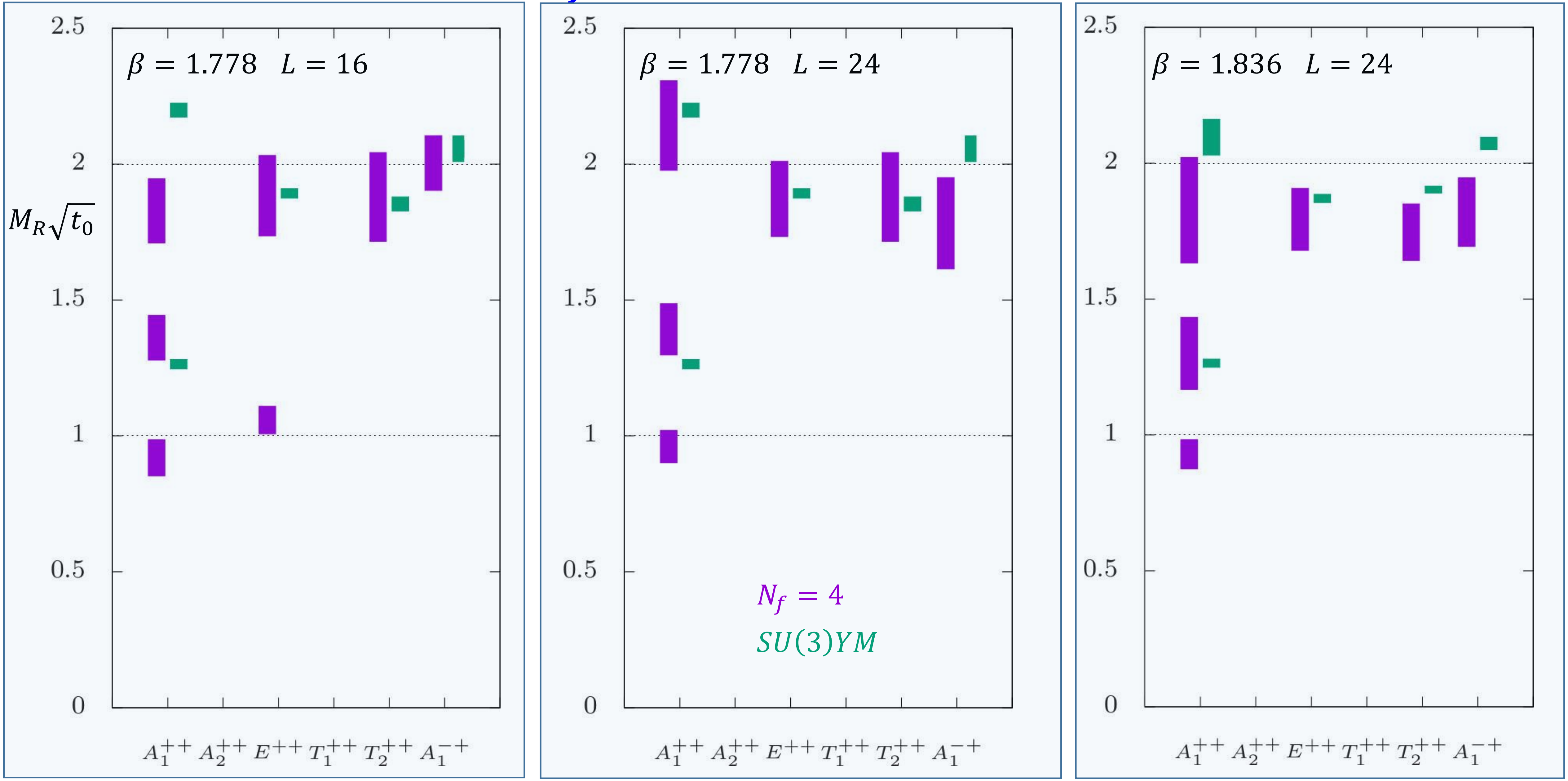
$$\begin{aligned} C(t) &= \langle \Phi^\dagger(t)\Phi(0)\rangle = \langle \Phi^\dagger(0)e^{-Ht}\Phi(0)\rangle \\ &= |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t} + \sum_{n=1} |\langle n|\Phi(0)|vac\rangle|^2 e^{-E_n t} \stackrel{t\to\infty}{\longrightarrow} |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t} \end{aligned}$$

- We calculate the effective energy $\lim_{t\to\infty}\left[-\ln\left(\frac{C(t)}{C(t-a)}\right)\right] = aE_0$

- Effective Energies for **cB4.06.24**



The $N_f = 4$ Glueball Spectrum



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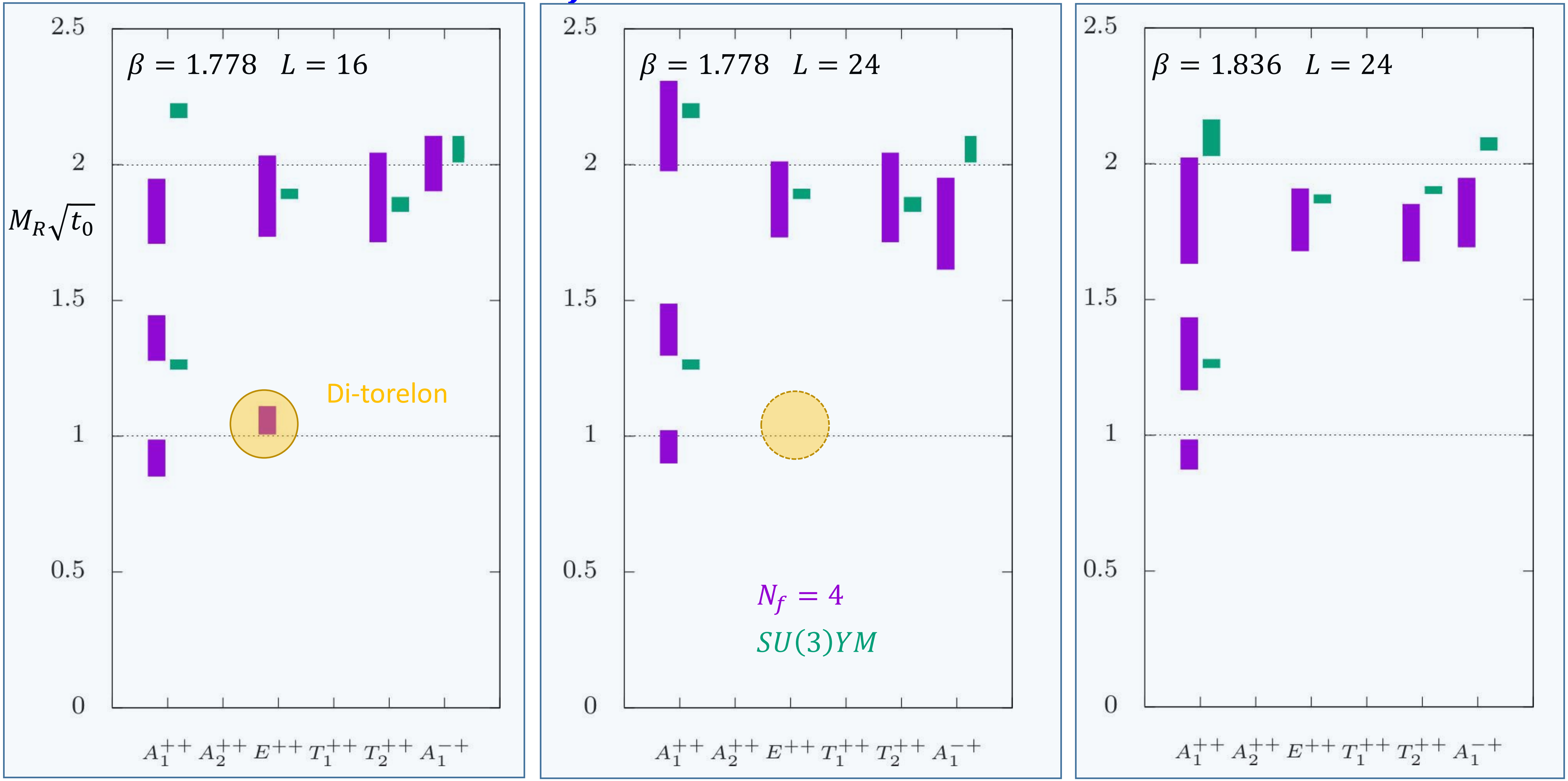

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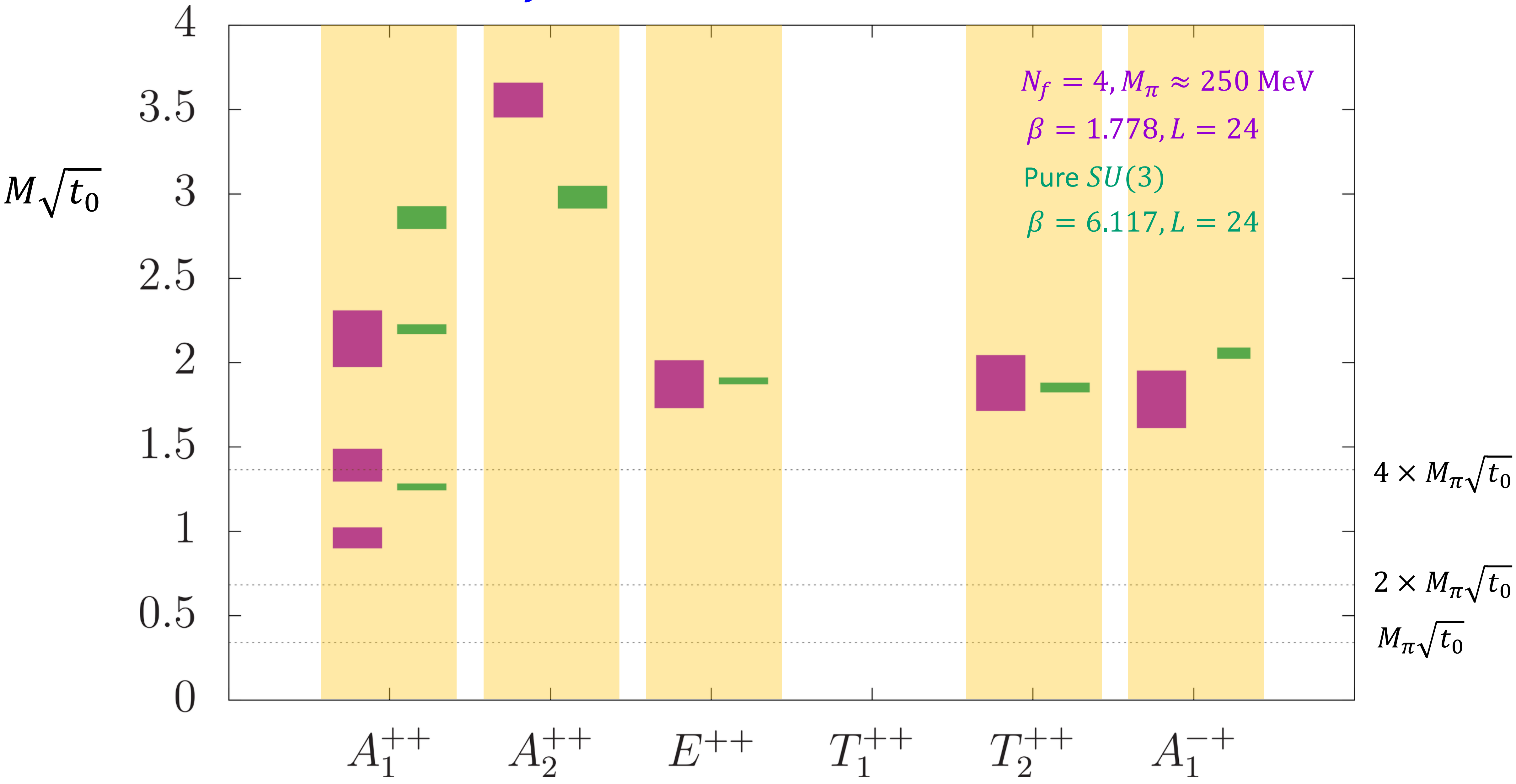
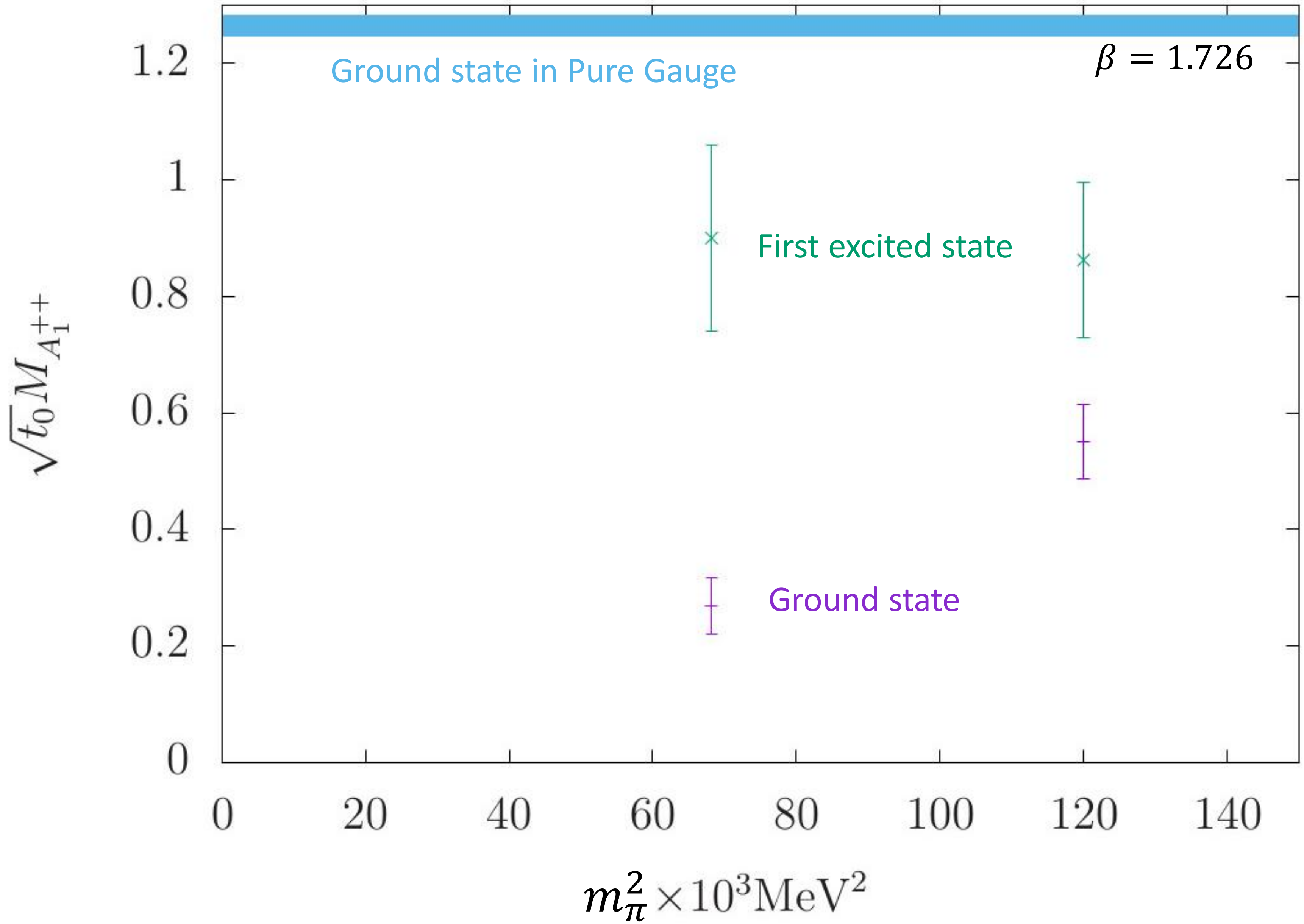


The A_1^{++} channel with $N_f = 2 + 1 + 1$ fermions

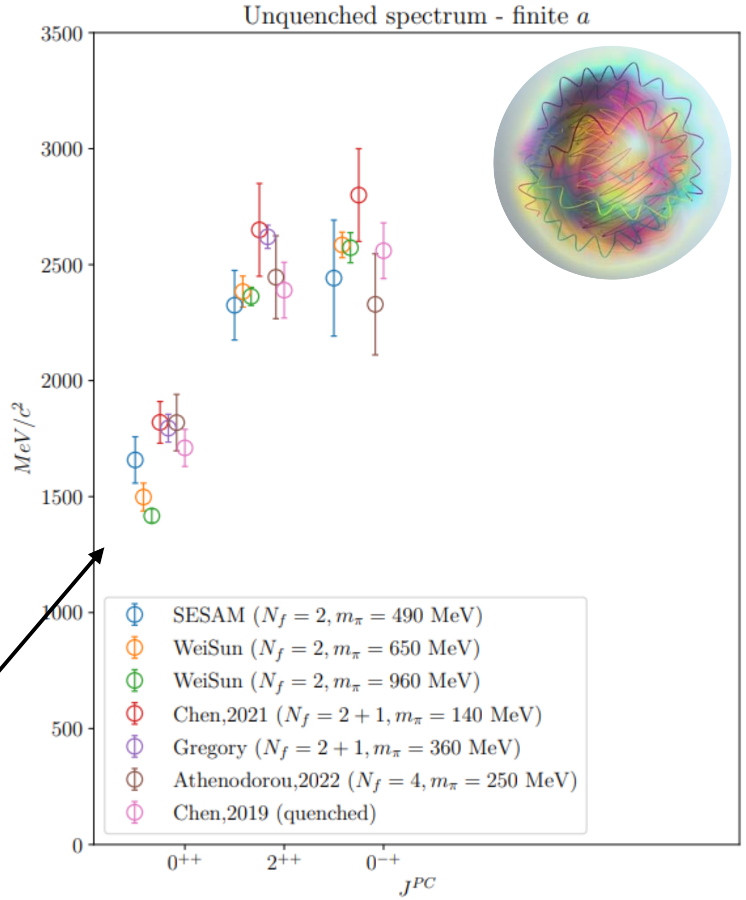


Observations

- Observations for Pure Gauge:
  - $N = 3$ 'close to' $N = \infty$: modest $O(1/N^2)$ correction suffices
  - J^{PC}= 0^{++} scalar is the lightest glueball
  - J^{PC}= 2^{++} next with mass $\sim 1.5 \times 0^{++}$
  - J^{PC}= 0^{-+} mass is next, very close to 2^{++}
  - J^{PC}= 1^{+-} is next, very close to first excited 0^{++}
  - Other $C = -$ states are much heavier

- Observations for QCD with light dynamical quarks
  - A_1^{++} includes an additional state
  - A_1^{++} ground state depends strongly on m_π
  - J^{PC}= 2^{++} ground state is consistent with Pure Gauge $SU(3)$
  - J^{PC}= 0^{-+} ground state is very close to 2^{++}

- The glueball masses are affected negligibly by dynamical quarks



From Vadacchino's plenary


Thanks for your attention!!!
30
36