Lattice calculation of glueball masses using the renormalized energy-momentum tensor with gradient flow

Keita Sakai and Shoichi Sasaki
Dept. of Phys., Tohoku Univ.

Lattice 2022, Aug. 8-13

Contents of this talk

What is the origin of the glueball masses?

in pure Yang-Mills theory,
breaking of scale invariance is induced by quantum effects

trace anomaly

classical

$$T_{\mu\mu} = 0$$

renormalization

quantized

$$T_{\mu\mu} \neq 0$$

purpose

to quantify the trace anomaly contribution
to the glueball masses by using lattice simulations

Trace anomaly

Decomposition of EMT $T_{\mu\nu} = \underbrace{\bar{T}_{\mu\nu}}_{\text{traceless}} + \underbrace{\hat{T}_{\mu\nu}}_{\text{trace} \neq 0}$

X.-D. Ji, Phys. Rev. Lett. 74, 1071 (1995).

Mass Decomposition

because $T_{00} = H,\quad H = \bar{H} + \hat{H} \;\Rightarrow\; M = \bar{M} + \hat{M}$

Trace anomaly is important to hadron mass generation

nucleon

$$M_N = \underbrace{M_q + M_g}_{\text{kinetic energy}} + \underset{\text{trace anomaly (gluon condensate)}}{M_a} + \overset{\text{quark condensate}}{M_m}$$

Trace anomaly

Decomposition of EMT

$$T_{\mu\nu} = \underbrace{\bar{T}_{\mu\nu}}_{\text{traceless}} + \underbrace{\hat{T}_{\mu\nu}}_{\text{trace} \neq 0}$$

X.-D. Ji, Phys. Rev. Lett. 74, 1071 (1995).

Mass Decomposition

because $T_{00} = H$, $H = \bar{H} + \hat{H}$ $\Rightarrow$ $M = \bar{M} + \hat{M}$

Trace anomaly is important to hadron mass generation

glueball

quark ~~condensate~~

$$M_G = \cancel{M_q} + \boxed{M_g + M_a} + \cancel{M_m}$$

kinetic energy

trace anomaly (gluon condensate)

Trace anomaly

Decomposition of EMT $T_{\mu\nu} = \underbrace{\bar{T}_{\mu\nu}}_{\text{traceless}} + \underbrace{\hat{T}_{\mu\nu}}_{\text{trace} \neq 0}$

X.-D. Ji, Phys. Rev. Lett. 74, 1071 (1995).

because $T_{00} = H, \quad H = \bar{H} + \hat{H}$ $\Rightarrow$ Mass Decomposition $M = \bar{M} + \hat{M}$

Trace anomaly is important to hadron mass generation

glueball

$$M_G = \underbrace{\cancel{M_q} + M_g}_{\text{kinetic energy}} + \boxed{M_g + M_a} + \cancel{M_m}$$

~~quark condensate~~ → M_m

trace anomaly(gluon condensate) → M_a

Renormalized energy-momentum tensor on the lattice is necessary to measure each contribution

Gradient flow

Discretization of space-time → Translational invariance ✗ → **It's difficult to construct EMT**

EMT can be constructed using the gradient flow

H. Suzuki, PTEP 2013, 083B03 (2013).

↓

Diffusing the gauge fields as a function of $\tau(\geq 0)$

Flow equation

M. Lüscher, JHEP. 1008, 071 (2010).

$$\partial_\tau B_\mu(\tau,x) = D_\nu G_{\nu\mu}(\tau,x),\ B_\mu(\tau=0,x) = A_\mu(x)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu],\ D_\mu = \partial_\mu + [B_\mu, \cdot\ ]$$

τ : diffusion time   diffusion radius $R_d = \sqrt{8\tau}$

No UV divergence in operators which are constructed using diffused gauge fields

Energy-momentum tensor (EMT)

by using flowed gauge fields at,

$G^a_{\mu\nu}$: **field strength**

$$U_{\mu\nu}(\tau,x) \equiv G^a_{\mu\rho}(\tau,x)G^a_{\nu\rho}(\tau,x) - \frac{1}{4}\delta_{\mu\nu}G^a_{\rho\sigma}(\tau,x)G^a_{\rho\sigma}(\tau,x)$$

$$E(\tau,x) \equiv \frac{1}{4}G^a_{\mu\nu}(\tau,x)G^a_{\mu\nu}(\tau,x)$$ **with flow time** τ

H. Suzuki, PTEP 2013, 083B03 (2013).

$$\left\{T_{\mu\nu}(x)\right\}_R = \lim_{\tau\to 0}\left\{\underbrace{\frac{1}{\alpha_U(\tau)}U_{\mu\nu}(\tau,x)}_{\text{traceless}} + \underbrace{\frac{1}{4\alpha_E(\tau)}\delta_{\mu\nu}\left(E(\tau,x) - \langle E(\tau,x)\rangle_0\right)}_{\text{trace anomaly}}\right\}$$

$\alpha_U(\tau)$, $\alpha_E(\tau)$: **perturbative coefficients**

M. Asakawa, et al. Phys. Rev. D 90, 011501 (2014).

$\tau \to 0$ **limit must be taken** → **on lattice, the minimal spatial distance is** a

diffusion radius of gradient flow $R_d = \sqrt{8\tau}$

on lattice

R_d **must be in the range of** $$a \ll R_d \ll (\text{box size})$$

Glueball two-point function and effective mass plot

Expectation value of operator $T_{\mu\nu}$ $\Rightarrow$ $\langle T_{\mu\nu} \rangle \simeq \dfrac{\langle G(t_{sep}) T_{\mu\nu}(t') G(0) \rangle}{\langle G(t_{sep}) G(0) \rangle}, \ (t_{sep} \gg t' \gg 0)$

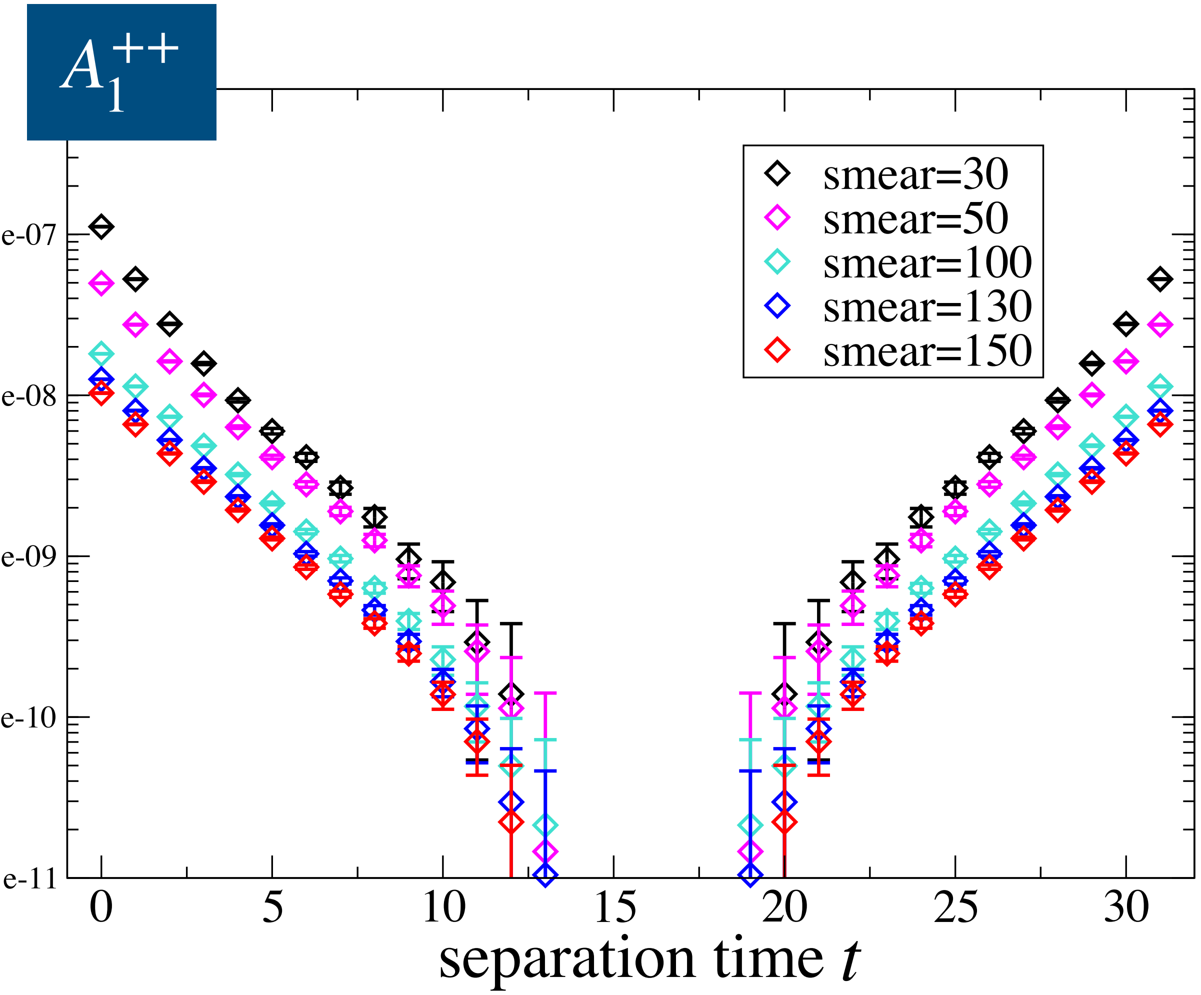
Precisions of 2pt. func. $\langle G(t_{sep}) G(0) \rangle$ **are very important**

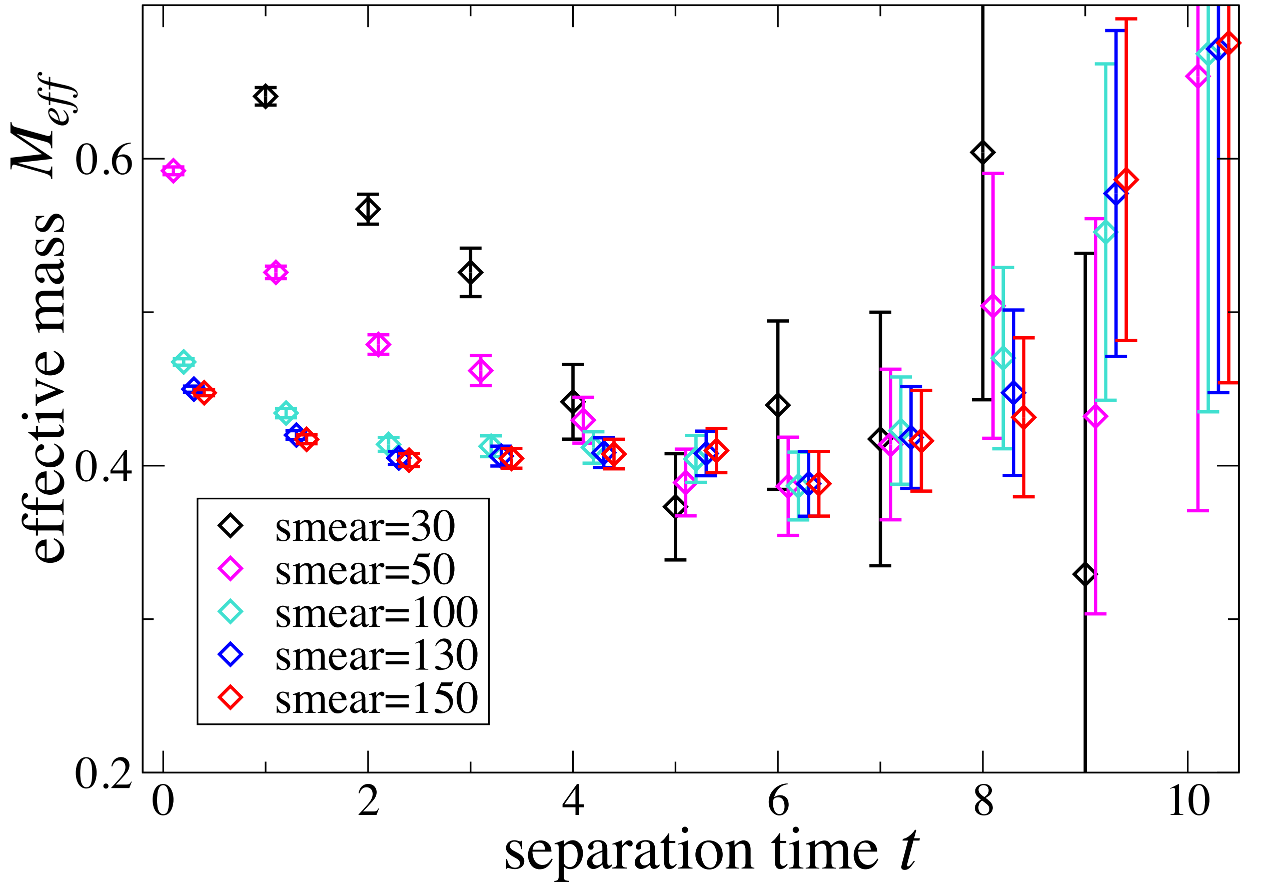
Stout smearing[1] is used to construct extended glueball operators[2]

[1] C. Morningstar and M. Peardon, Phys. Rev. D 69, 054501 (2004)

[2] K. S and S. Sasaki, PoS(LATTICE2021)333.

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84





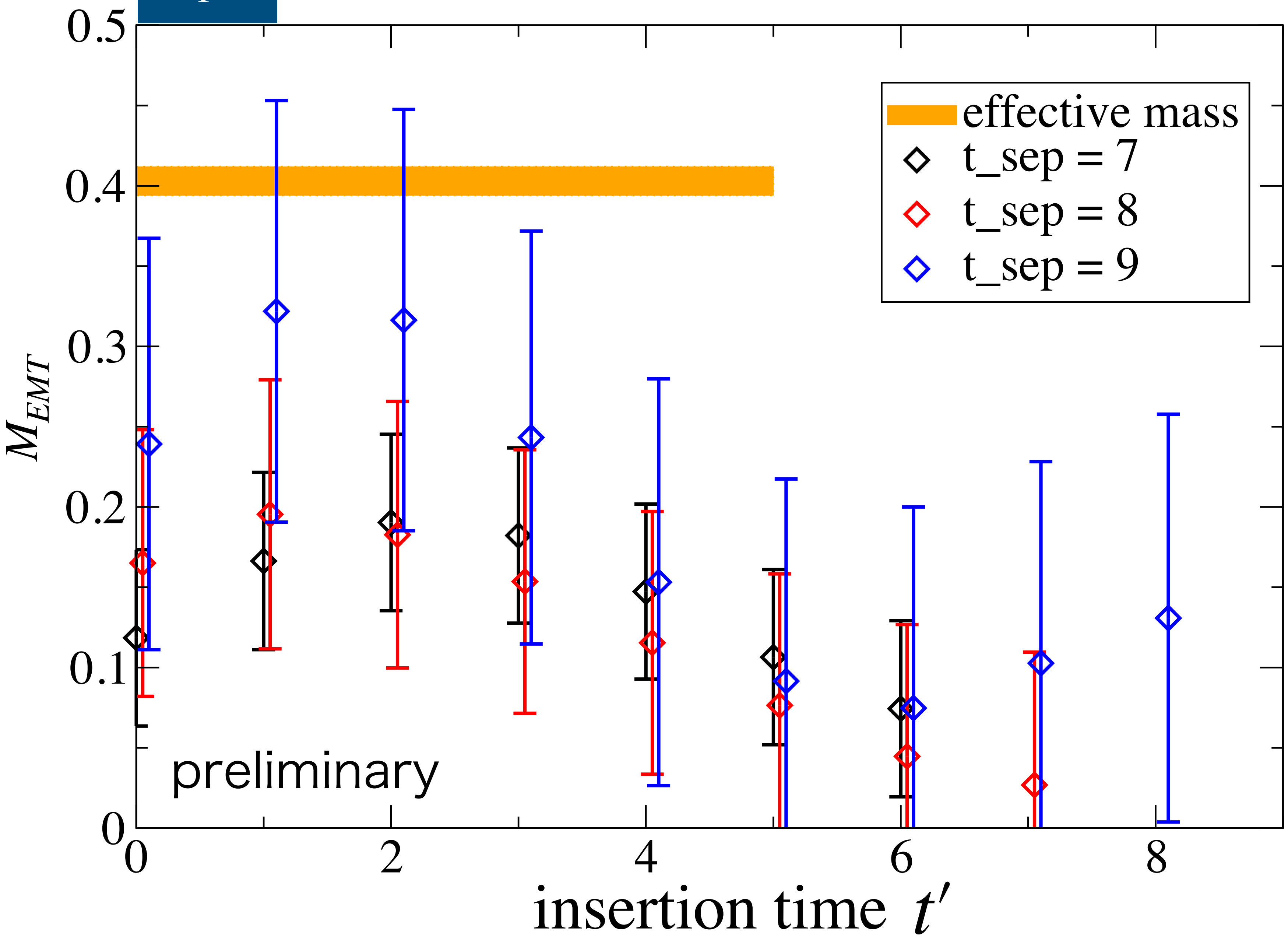
Glueball mass calculated from EMT operator

$$\langle T_{00} \rangle \simeq \frac{\langle G(t_{sep}) T_{00}(t') G(0) \rangle}{\langle G(t_{sep}) G(0) \rangle}, \ (t_{sep} \gg t' \gg 0)$$

$T_{00}(t')$ **given at** $\tau = 2.5$

$\beta(= 6/g^2)$	$L^3 \times T$	Meas.	$1/a[\mathrm{GeV}]$
6.4	$32^3 \times 32$	12000	3.84

A_1^{++}



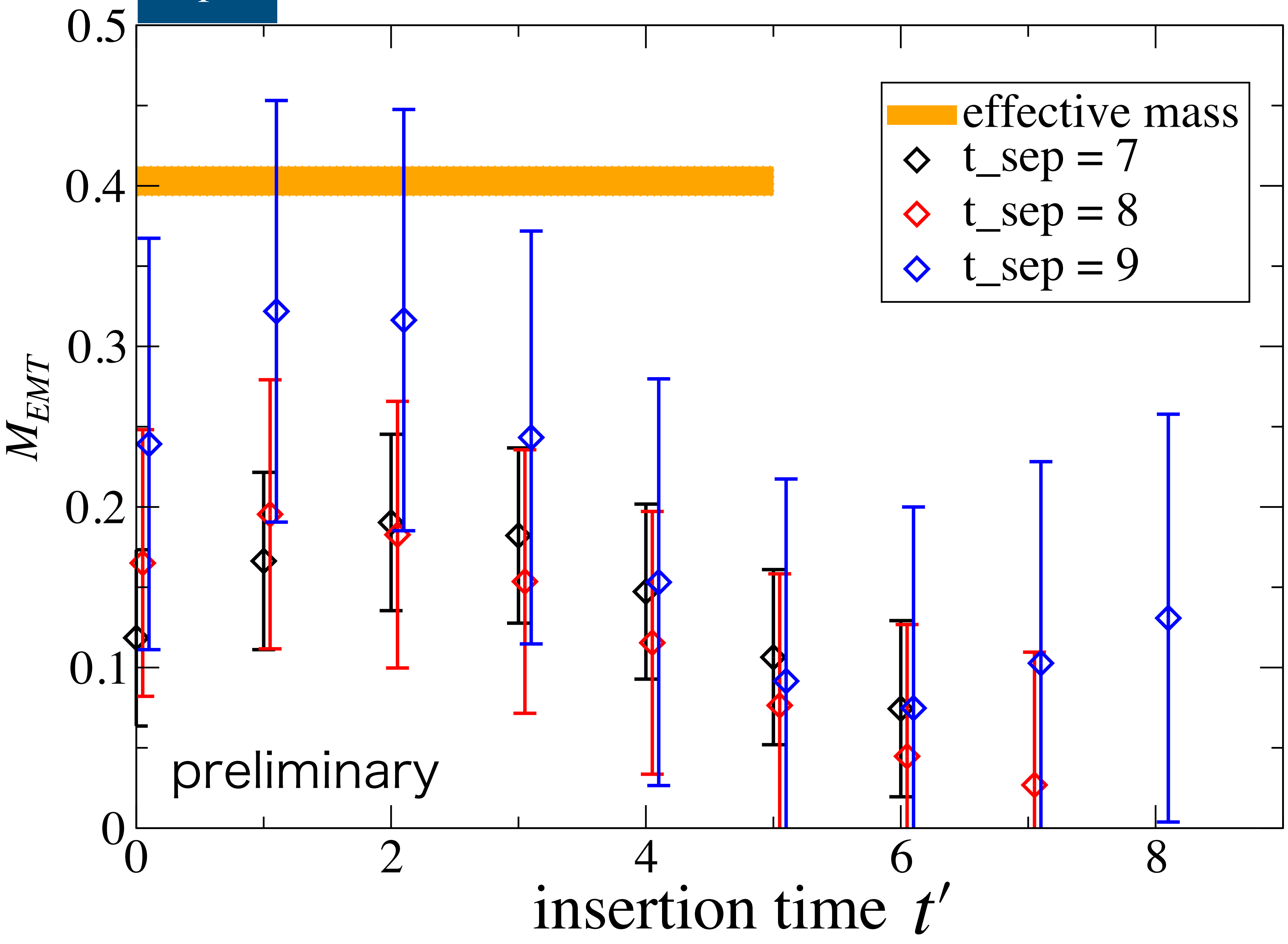
Glueball mass calculated from EMT operator

$$\langle T_{00} \rangle \simeq \frac{\langle G(t_{sep}) T_{00}(t') G(0) \rangle}{\langle G(t_{sep}) G(0) \rangle}, \; (t_{sep} \gg t' \gg 0)$$

$T_{00}(t')$ **given at** $\tau = 2.5$

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a[\mathrm{GeV}]$
6.4	$32^3 \times 32$	12000	3.84

A_1^{++}



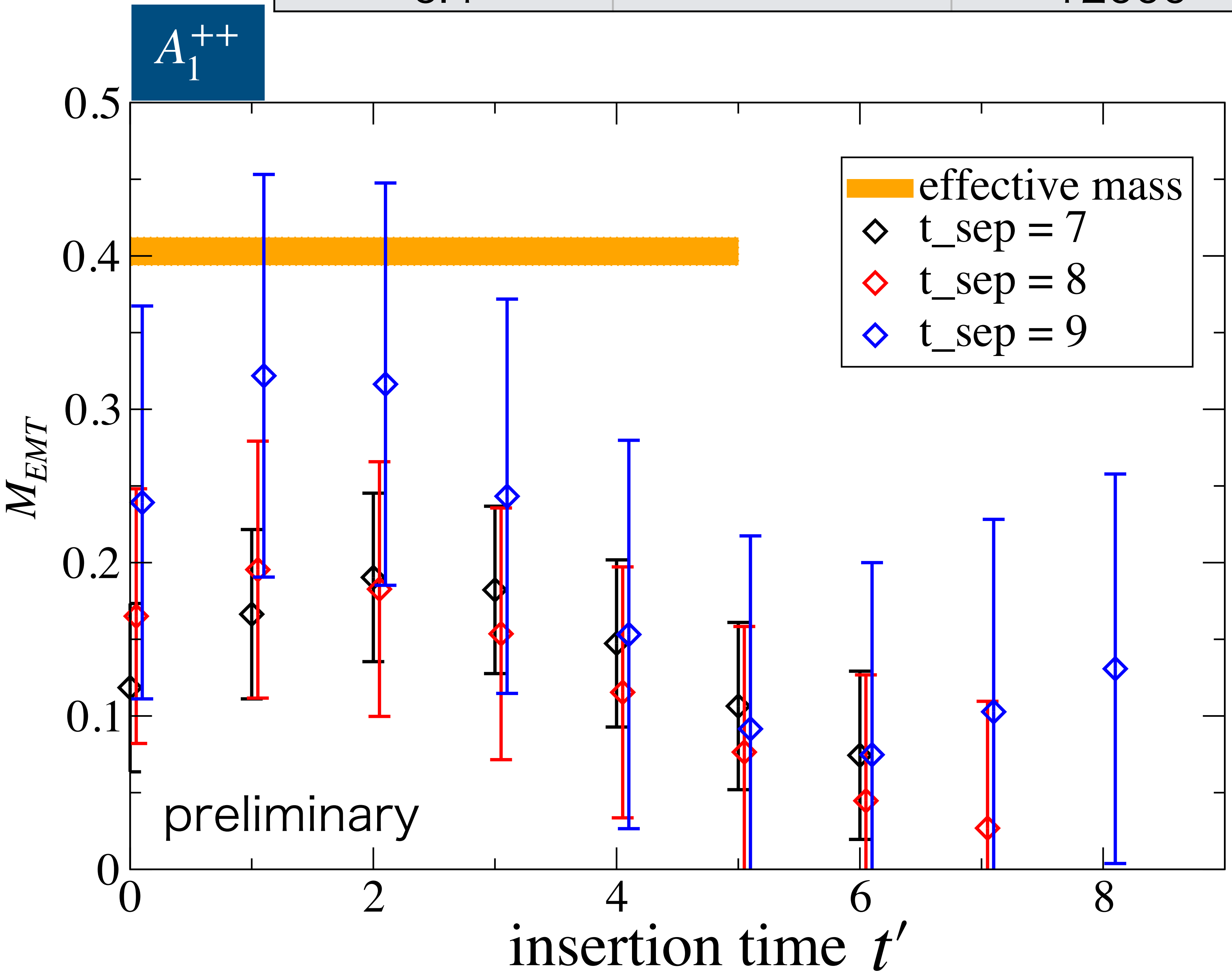
It shows a consistency with effective mass

Glueball mass calculated from EMT operator

$$\langle T_{00} \rangle \simeq \frac{\langle G(t_{sep}) T_{00}(t') G(0) \rangle}{\langle G(t_{sep}) G(0) \rangle}, \ (t_{sep} \gg t' \gg 0)$$

$T_{00}(t')$ **given at** $\tau = 2.5$

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a[\mathrm{GeV}]$
6.4	$32^3 \times 32$	12000	3.84



It shows a consistency with effective mass

$t_{sep} = 9$ **possibly isn't long enough to reach saturation, though** $t_{sep} = 10$ **doens't have enough precision**

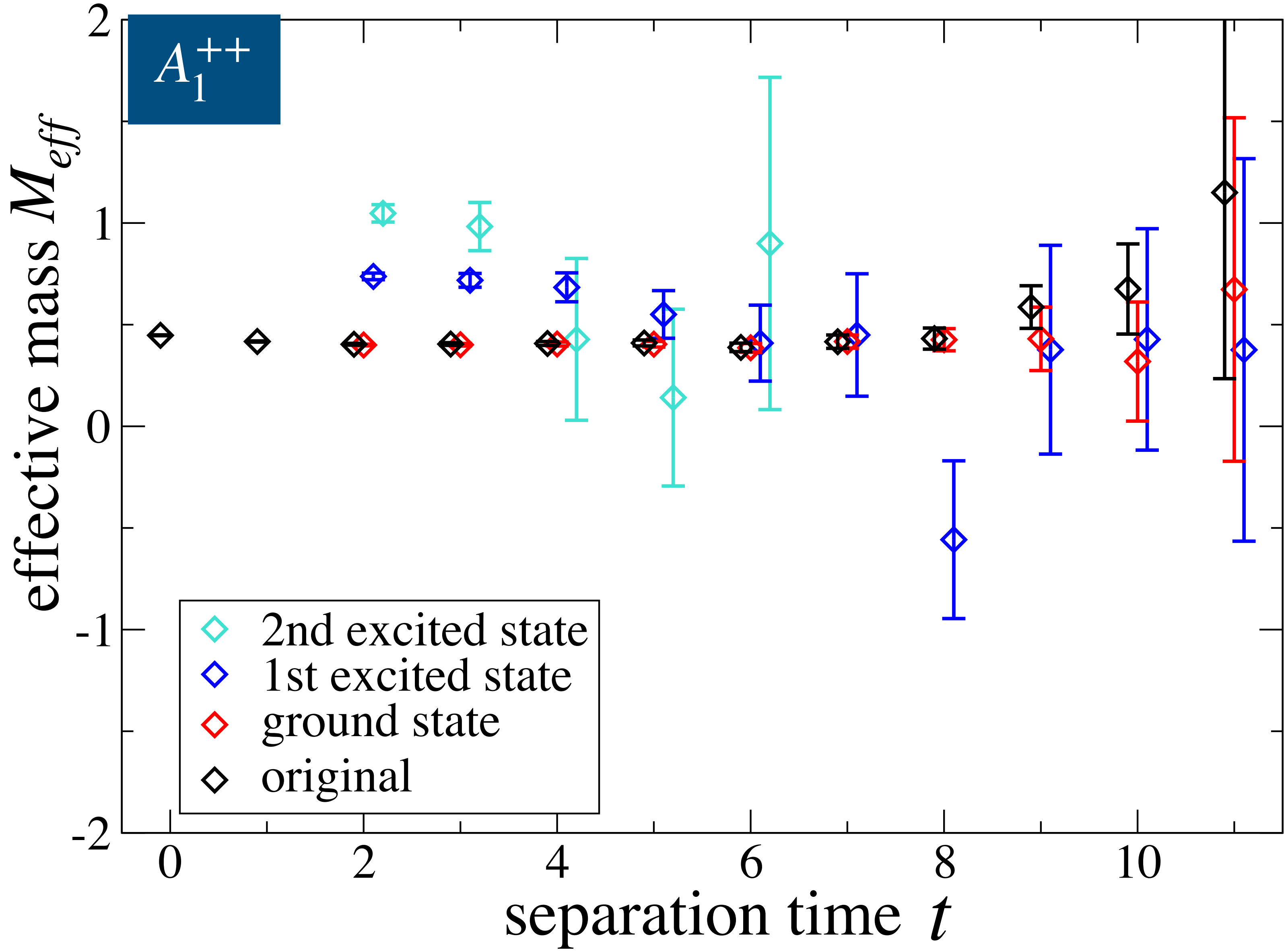
variational method

After a lot of smearing steps, shapes of operators lose its anisotropy

➡ Data in different smearing steps are used to get transfer matrix

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

Diagonalization of 5x5 transfer matrix(smearing step=30, 50, 100, 130, 150)



1st excited state can be extracted

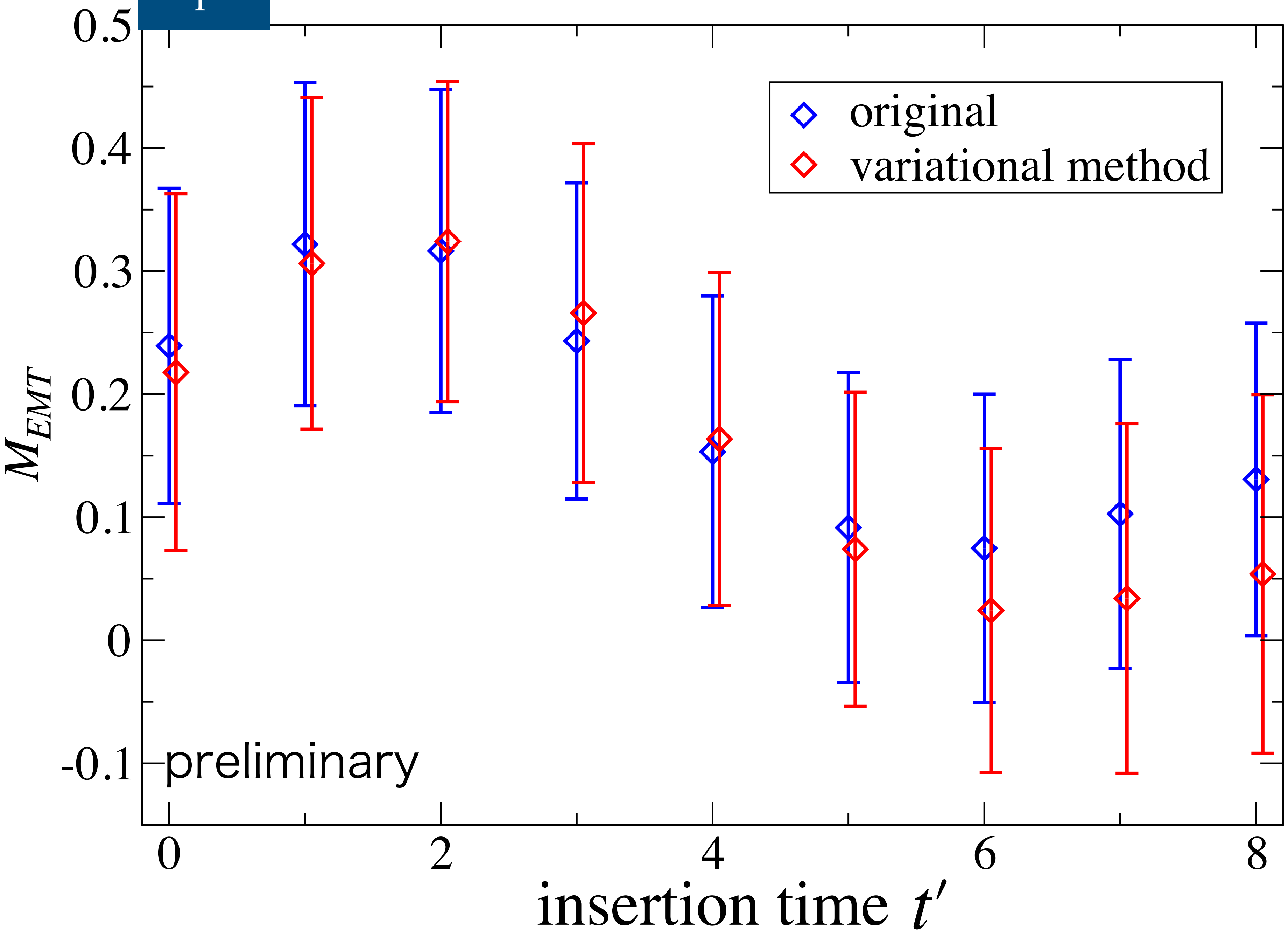
no large effects on the ground state

Variational method on three-point function

3pt. func. using glueball operator (optimized to ground state)

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a[\mathrm{GeV}]$
6.4	$32^3 \times 32$	12000	3.84

A_1^{++}



No large effects on the ground state, as is in 2pt. func. result

flow time

$\tau = 2.5$

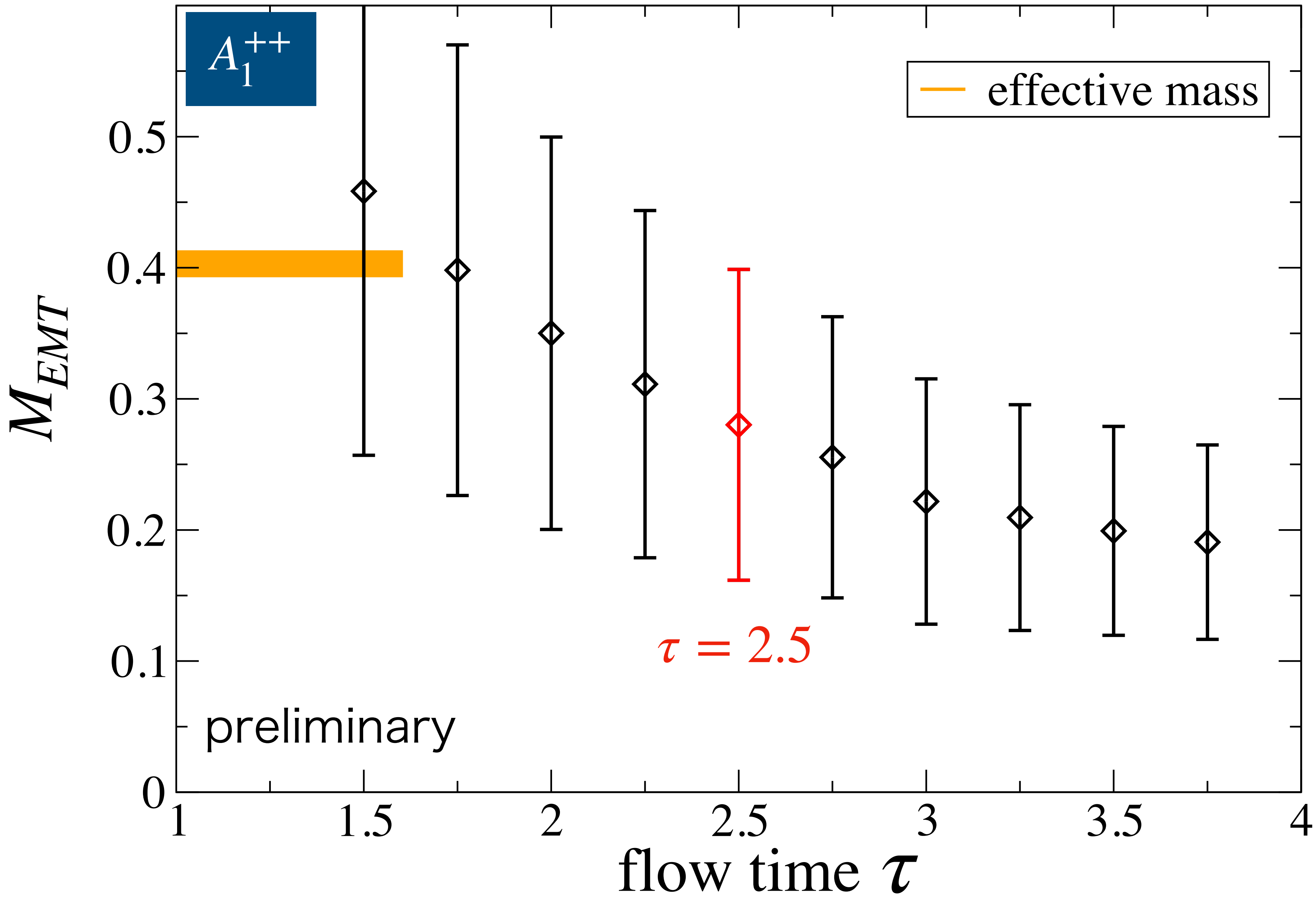
Flow time dependence

To get correct EMT, flow time $\tau \to 0$ limit has to be taken eventually

➡ It needs to be done after the continuum limit

H. Suzuki, PTEP 2013, 083B03 (2013)

Flow time dependence becomes mild as $\tau \to$ large

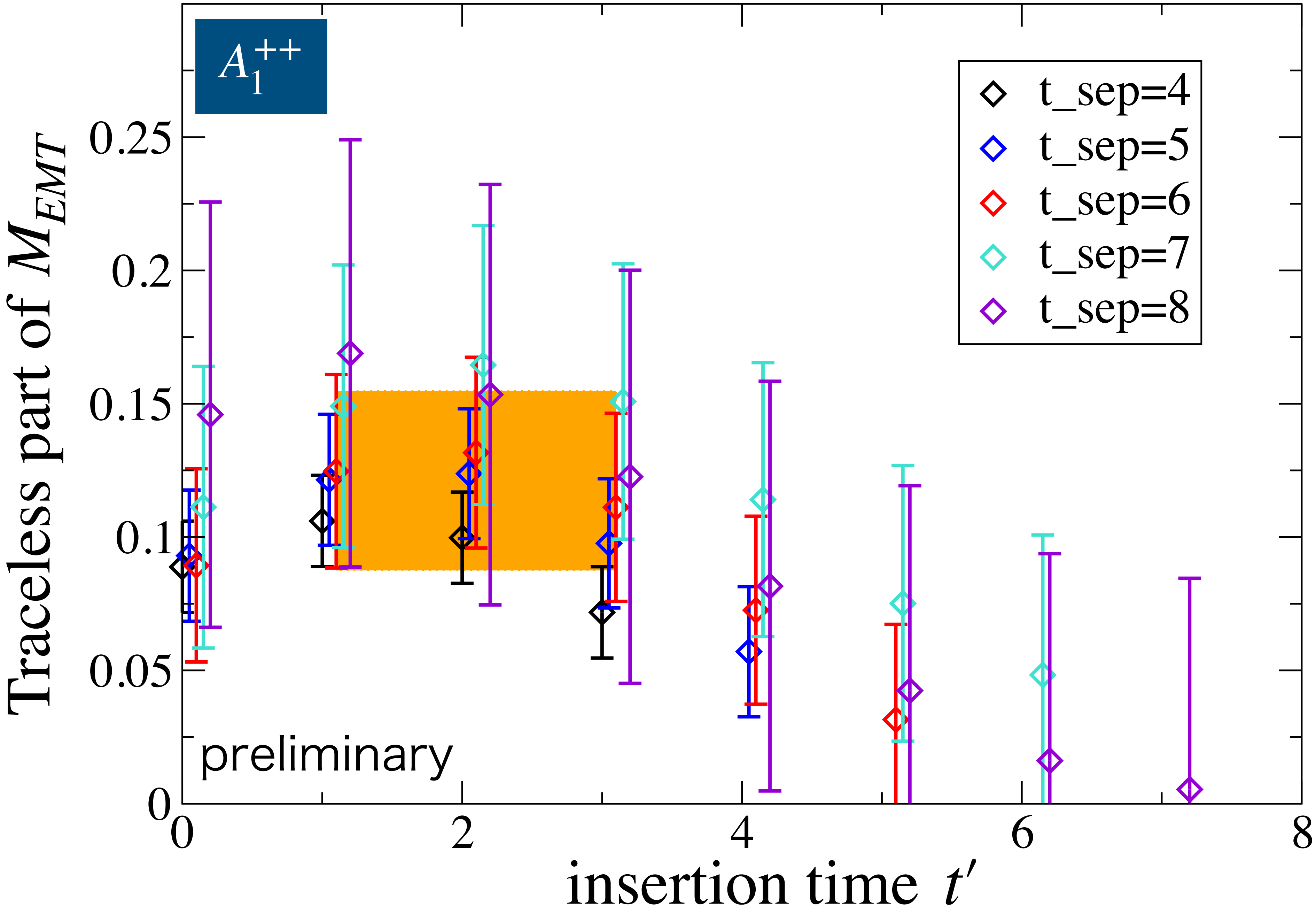


Traceless part

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

Behavior of traceless part

$$M_{EMT} = M_{traceless} + M_{anomaly}$$

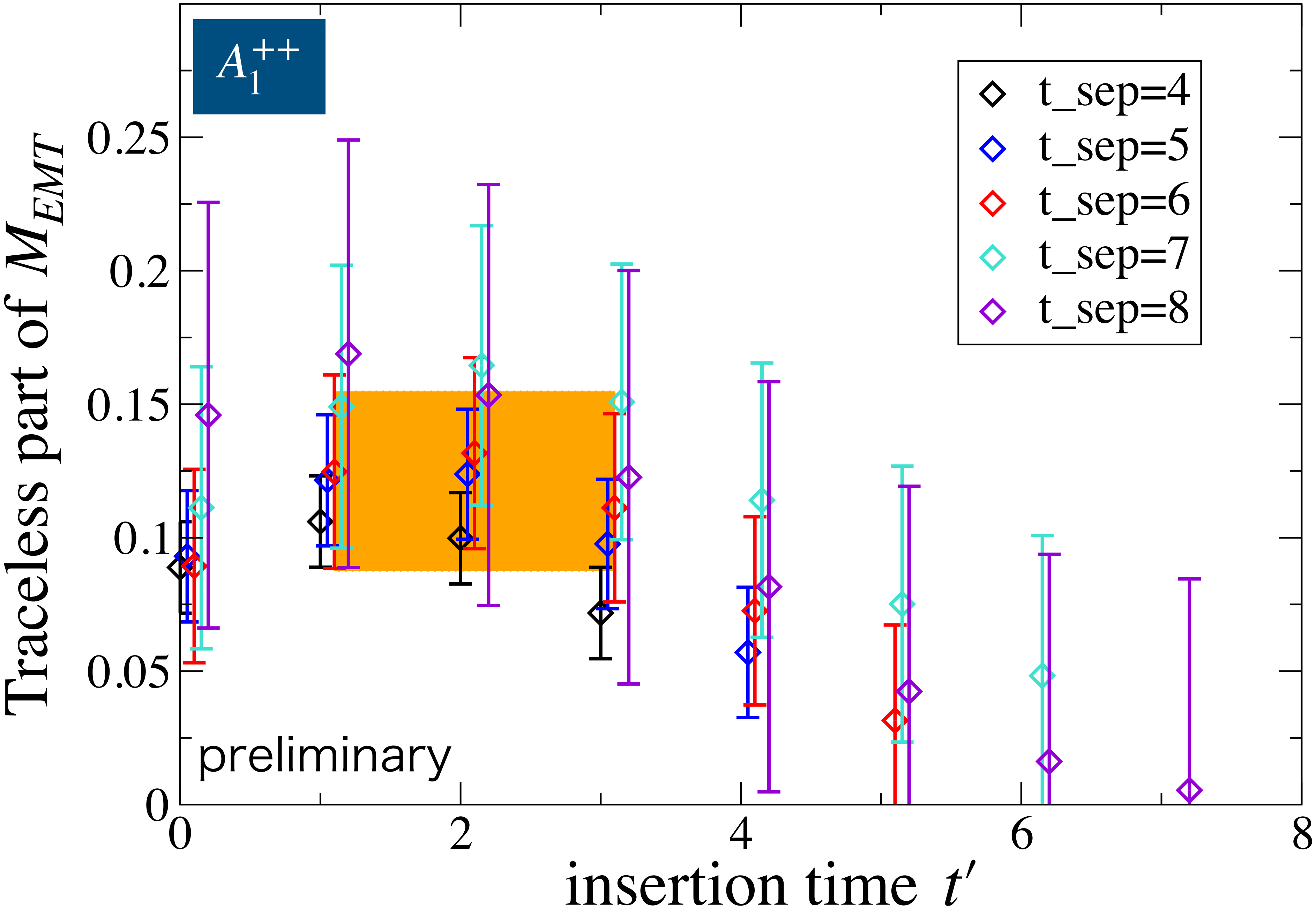


Traceless part

$\beta(= 6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

Behavior of traceless part

$$M_{EMT} = M_{traceless} + M_{anomaly}$$



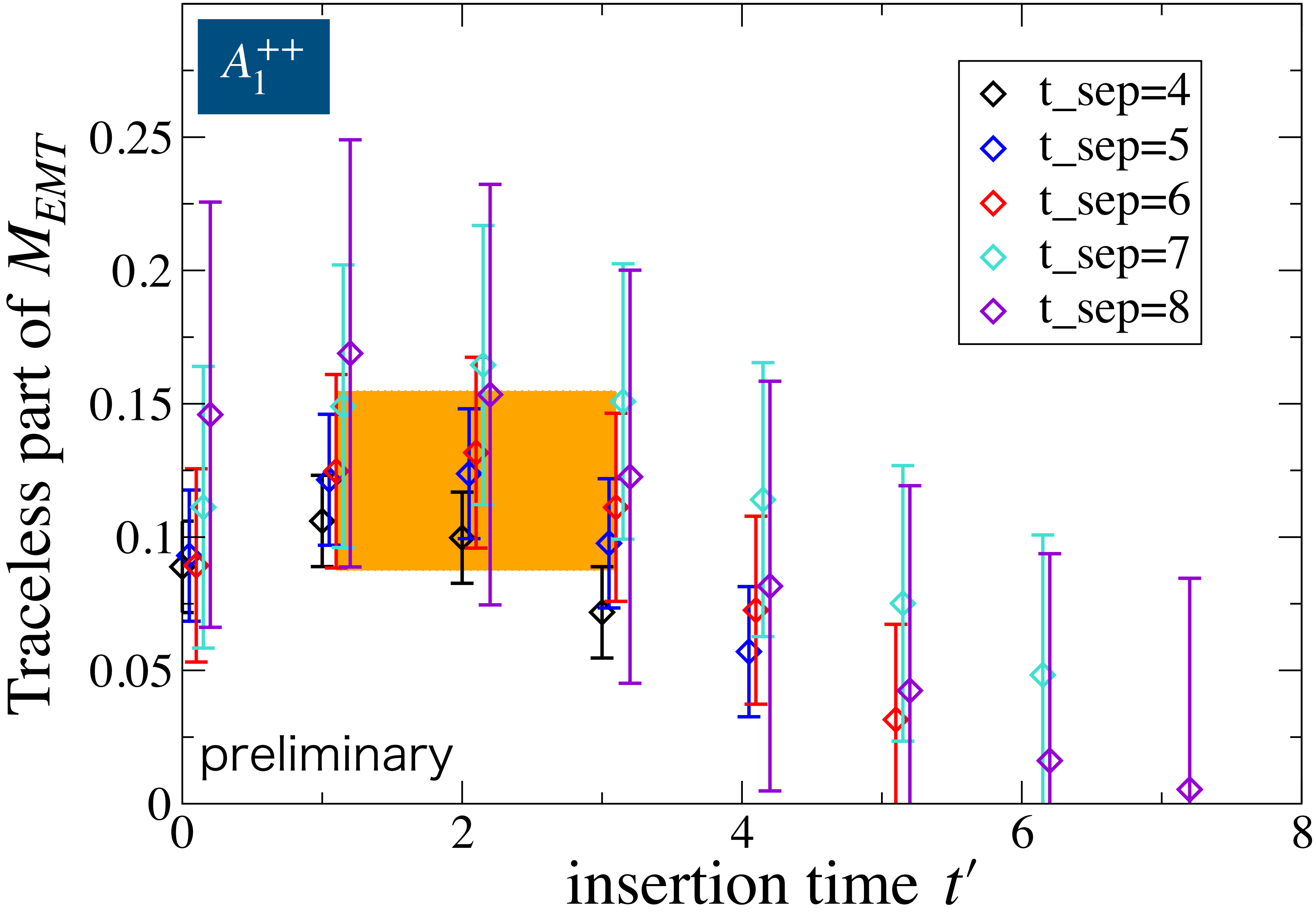
Saturation around $t_{sep} = 6$

Traceless part

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

Behavior of traceless part

$$M_{EMT} = M_{traceless} + M_{anomaly}$$



Saturation around $t_{sep} = 6$

By fitting data,

$$M_{traceless} = 0.12(3)$$

in lattice units

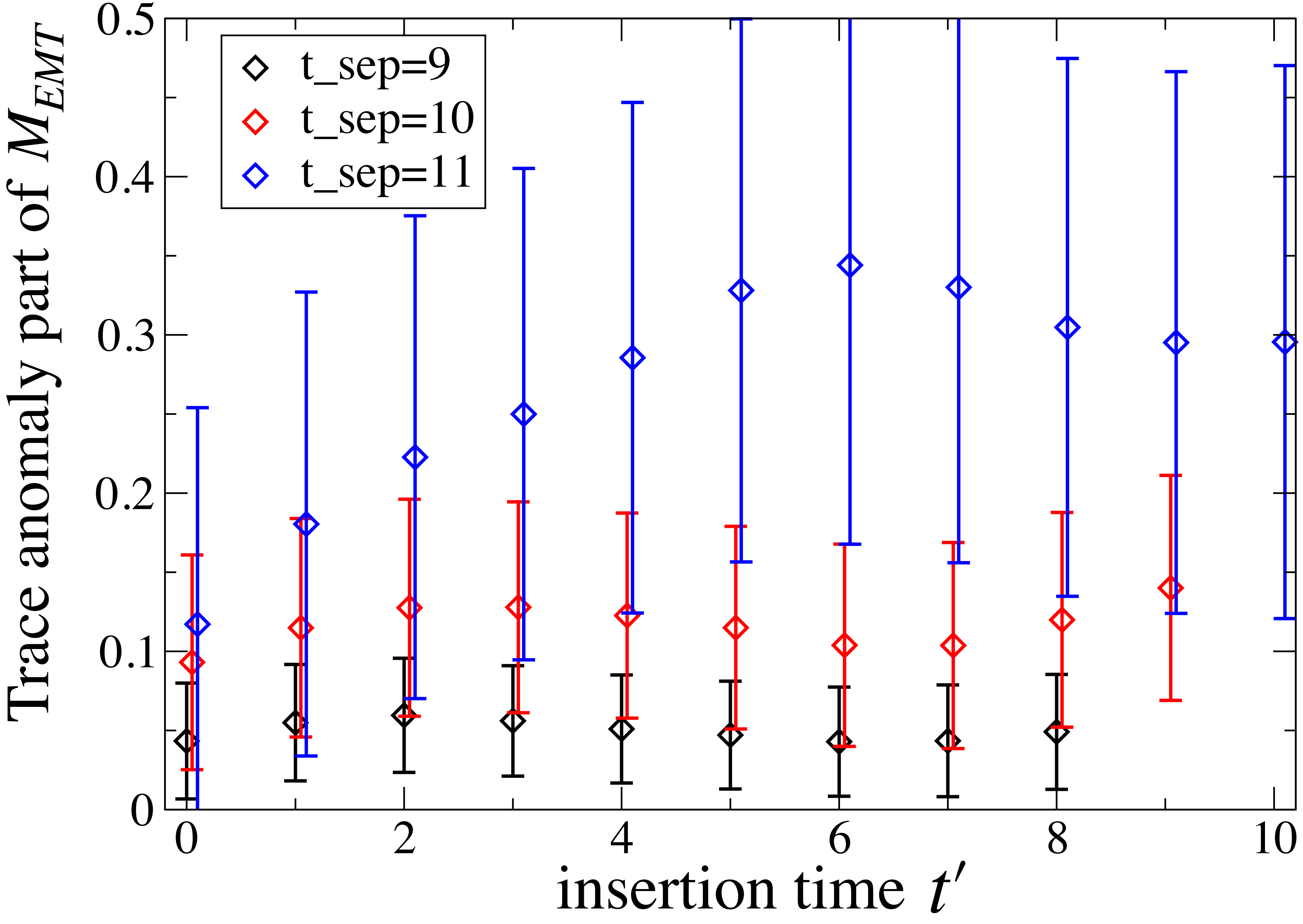
Trace anomaly part

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

$$\langle T_{00} \rangle = \frac{\langle G(t_{sep}) T_{00}(t') G(0) \rangle}{\langle G(t_{sep}) G(0) \rangle}, \; (t_{sep} - t' \to \infty, \, t' \to \infty)$$

A_1^{++}

preliminary



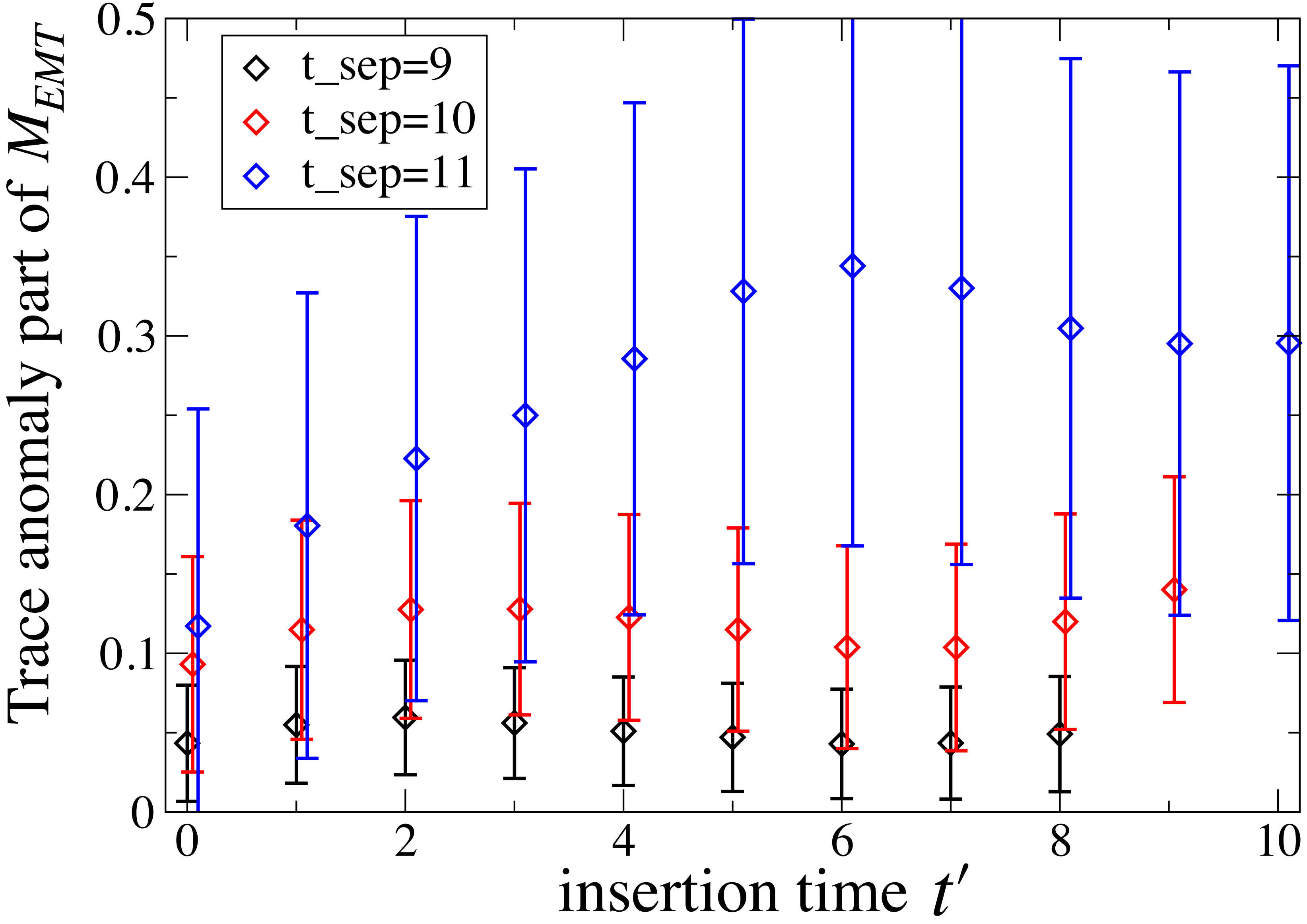
Trace anomaly part

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a[\mathrm{GeV}]$
6.4	$32^3 \times 32$	12000	3.84

$$\langle T_{00}^{anomaly} \rangle = \frac{\langle G(t_{sep}) T_{00}^{anomaly}(t') G(0) \rangle}{\langle G(t_{sep}) G(0) \rangle}, \ (t_{sep} - t' \to \infty, \ t' \to \infty)$$

A_1^{++}

preliminary



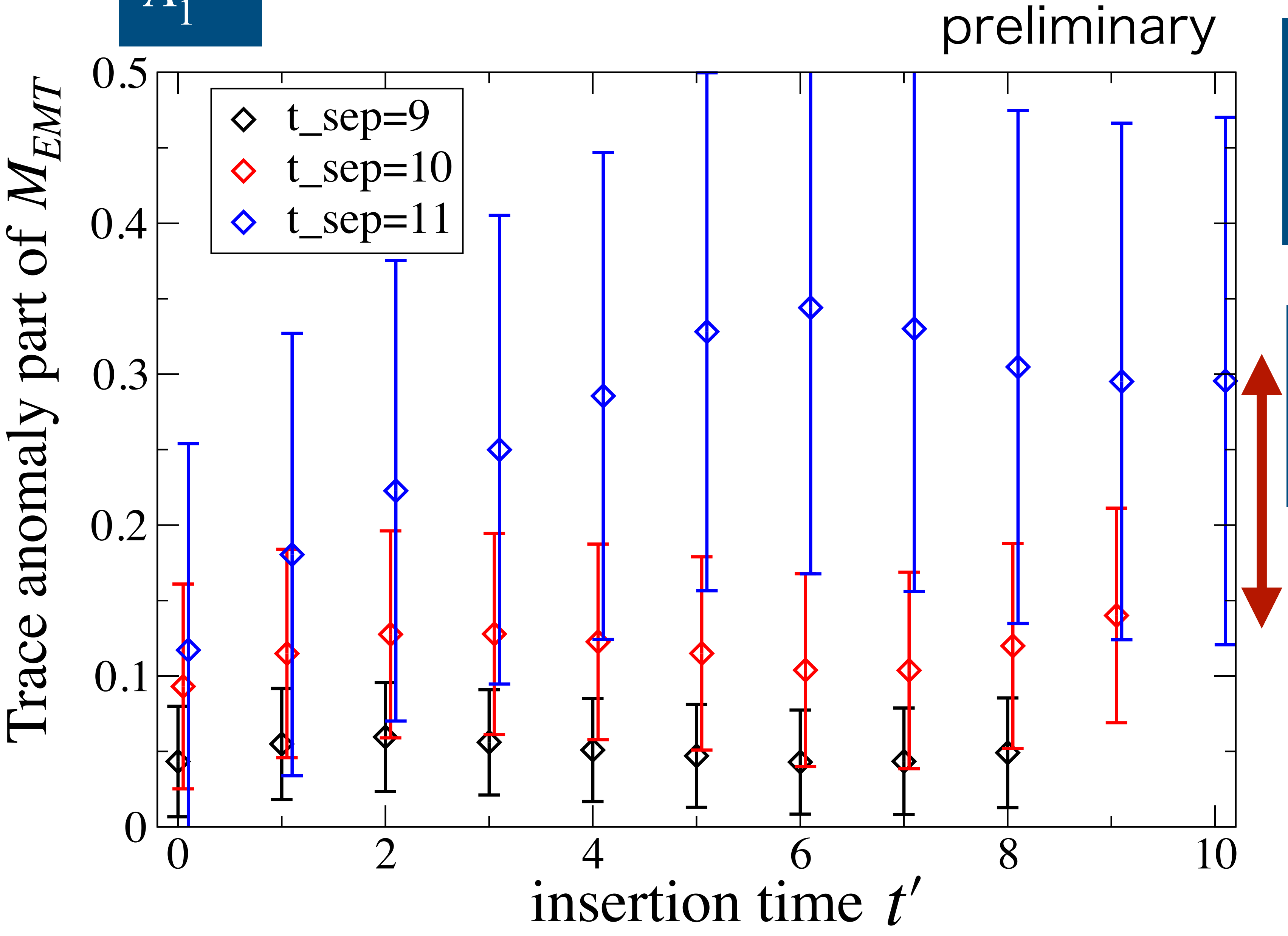
$t_{sep} = 11$ **isn't long enough**

Trace anomaly part

$\beta(=6/g^2)$	$L^3\times T$	Meas.	$1/a[\mathrm{GeV}]$
6.4	$32^3\times 32$	12000	3.84

$$\langle T_{00}^{anomaly}\rangle = \frac{\langle G(t_{sep})T_{00}^{anomaly}(t')G(0)\rangle}{\langle G(t_{sep})G(0)\rangle},\ (t_{sep}-t'\to\infty,\ t'\to\infty)$$

A_1^{++}



$t_{sep}=11$ **isn't long enough**

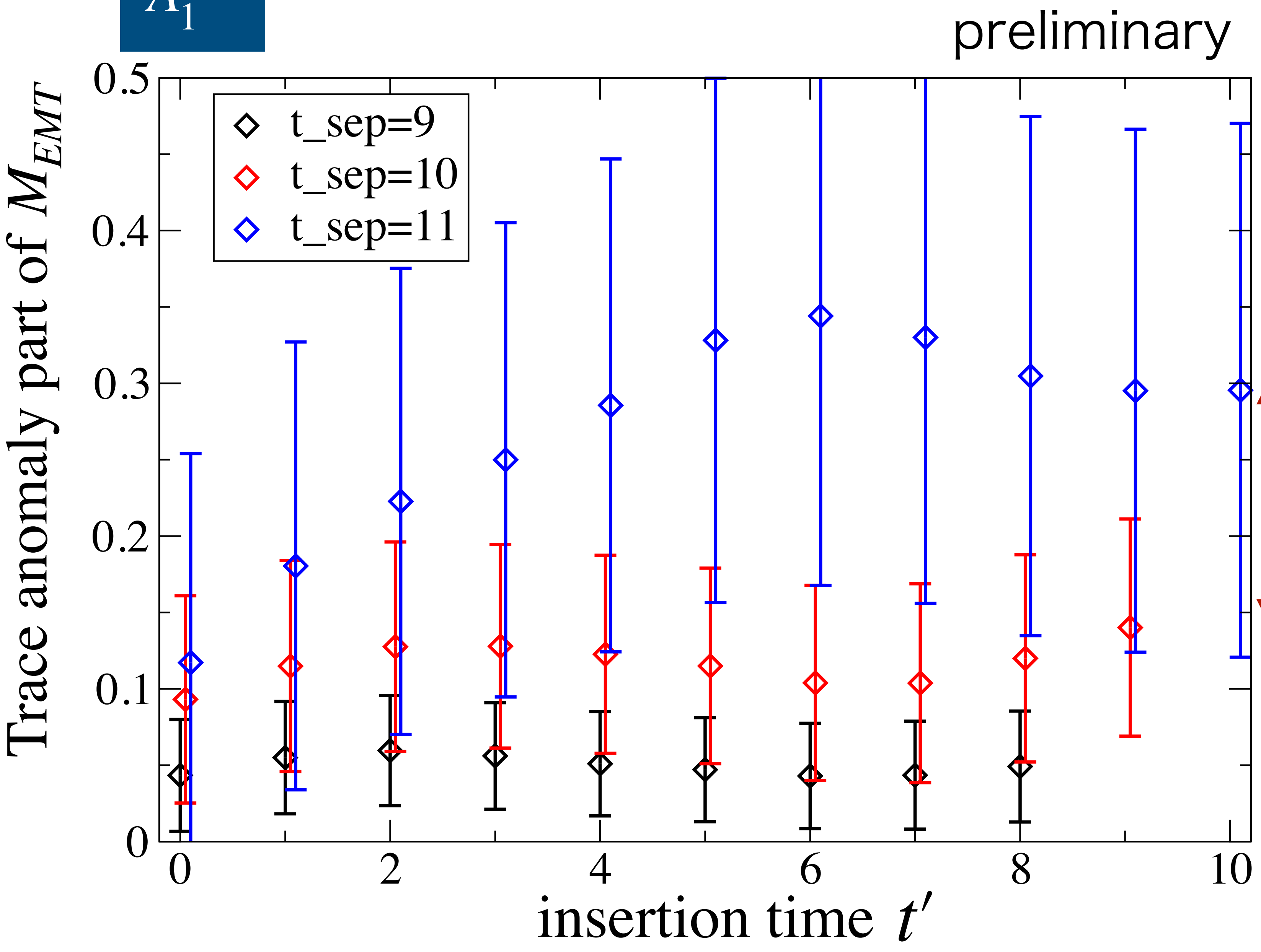
Big change between $t_{sep}=10$ and 11

Trace anomaly part

$\beta(=6/g^2)$	$L^3\times T$	Meas.	$1/a[\mathrm{GeV}]$
6.4	$32^3\times 32$	12000	3.84

$$\langle T_{00}^{anomaly}\rangle = \frac{\langle G(t_{sep})T_{00}^{anomaly}(t')G(0)\rangle}{\langle G(t_{sep})G(0)\rangle},\ (t_{sep}-t'\to\infty,\ t'\to\infty)$$

A_1^{++}



$t_{sep}=11$ isn't long enough?

Big change between $t_{sep}=10$ and 11

2pt. func. doesn't have enough precision?

Improvement of long separation time behavior

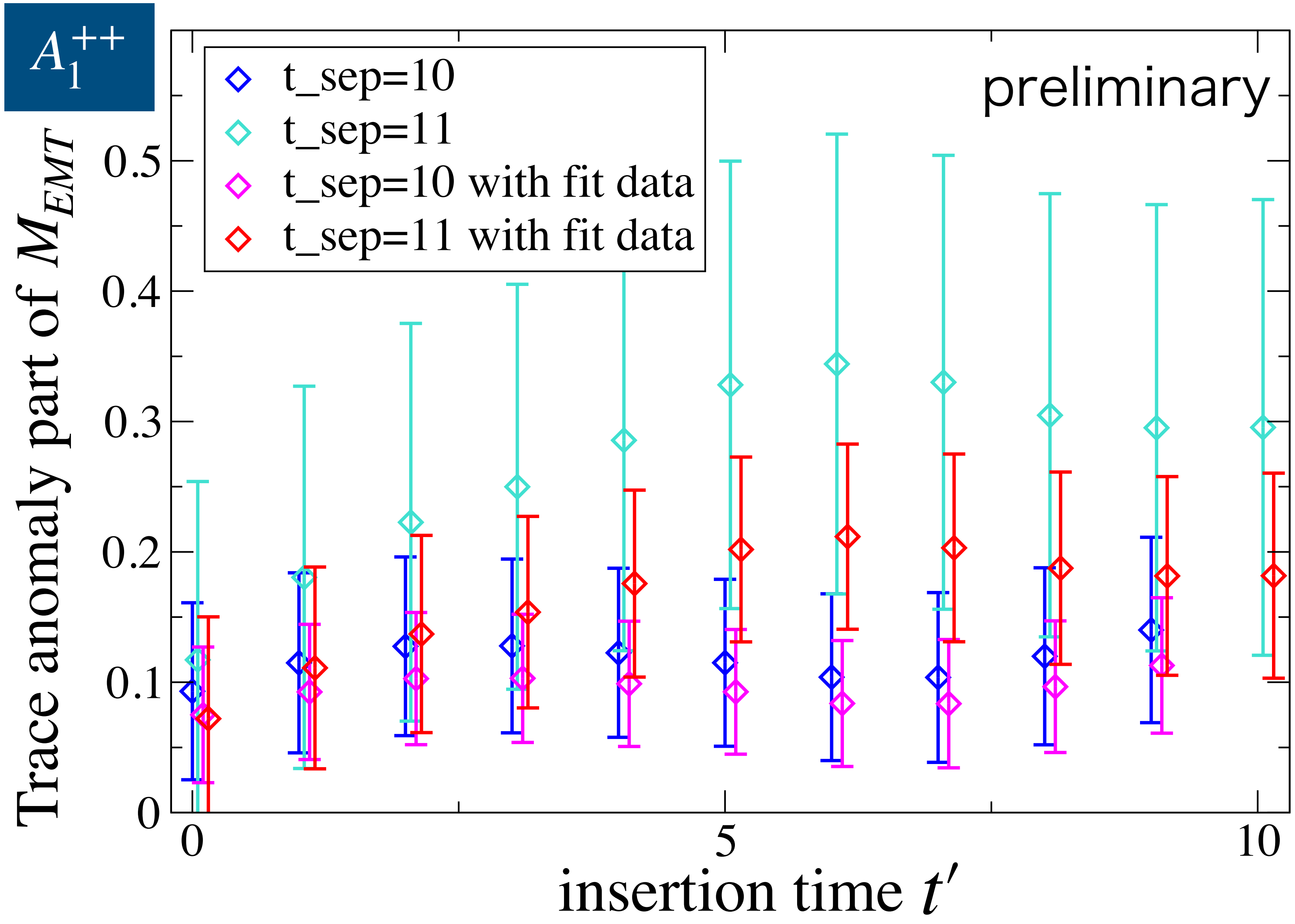
$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

$$\langle G(t_{sep})G(0)\rangle = \sum_i |\langle i|G|0\rangle|^2 e^{-E_i t_{sep}} \sim Ae^{-Mt_{sep}} \quad (t_{sep} \gg 0)$$

A : **const.**

1. **Getting A and M from fitting of 2pt. func. data**

2. **Using $A_{fit}e^{-M_{fit}t}$ func as 2pt. func. in calculations of $\langle T_{00}\rangle$**



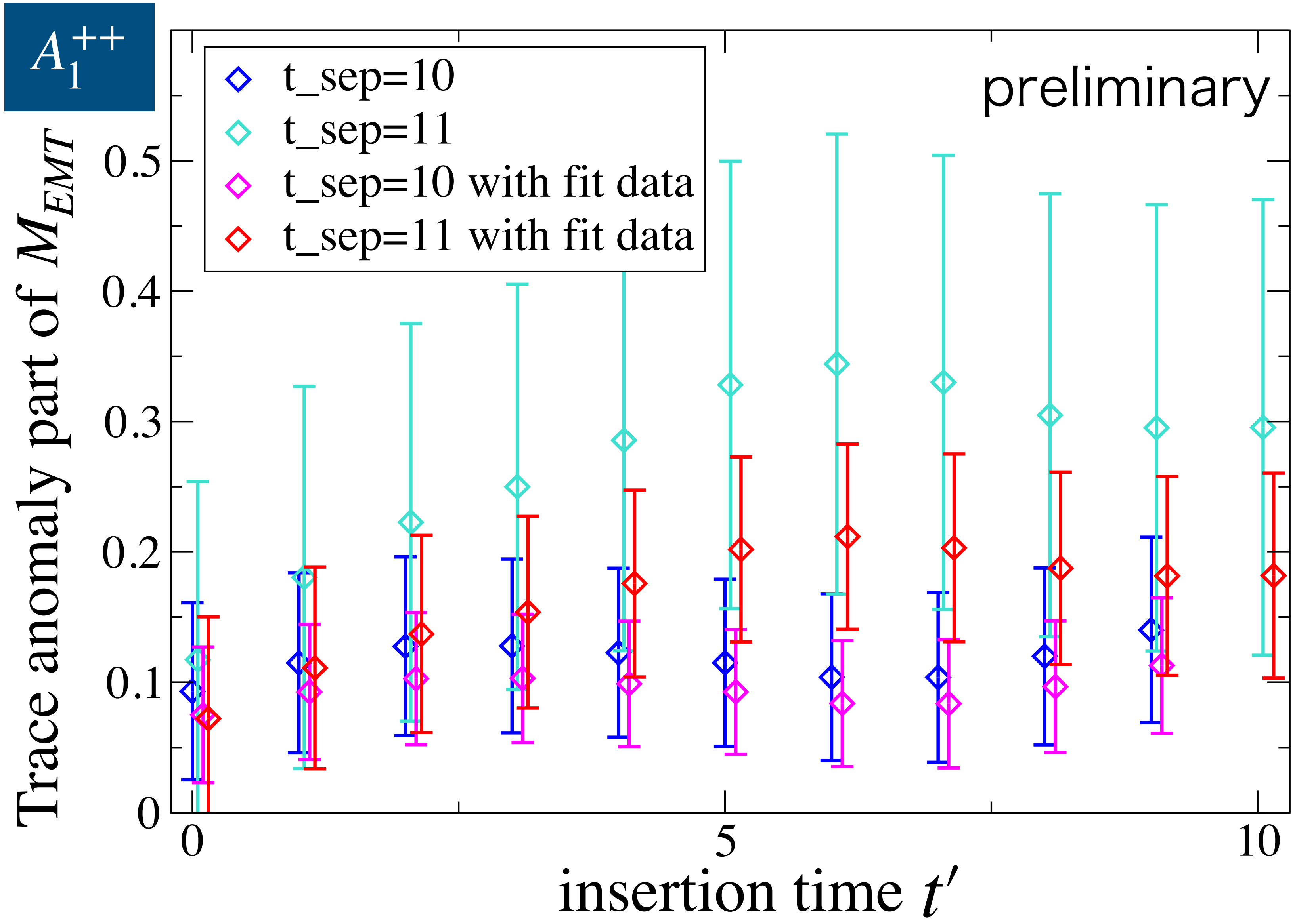
$$M_{EMT} = M_{traceless} + M_{anomaly}$$

Improvement of long separation time behavior

$\beta(=6/g^2)$	$L^3\times T$	Meas.	$1/a$[GeV]
6.4	$32^3\times 32$	12000	3.84

$$\langle G(t_{sep})G(0)\rangle = \sum_i |\langle i|G|0\rangle|^2 e^{-E_i t_{sep}} \sim Ae^{-Mt_{sep}} \quad (t_{sep}\gg 0)$$ A : **const.**

1. **Getting A and M from fitting of 2pt. func. data**
2. **Using $A_{fit}e^{-M_{fit}t}$ func as 2pt. func. in calculations of $\langle T_{00}\rangle$**



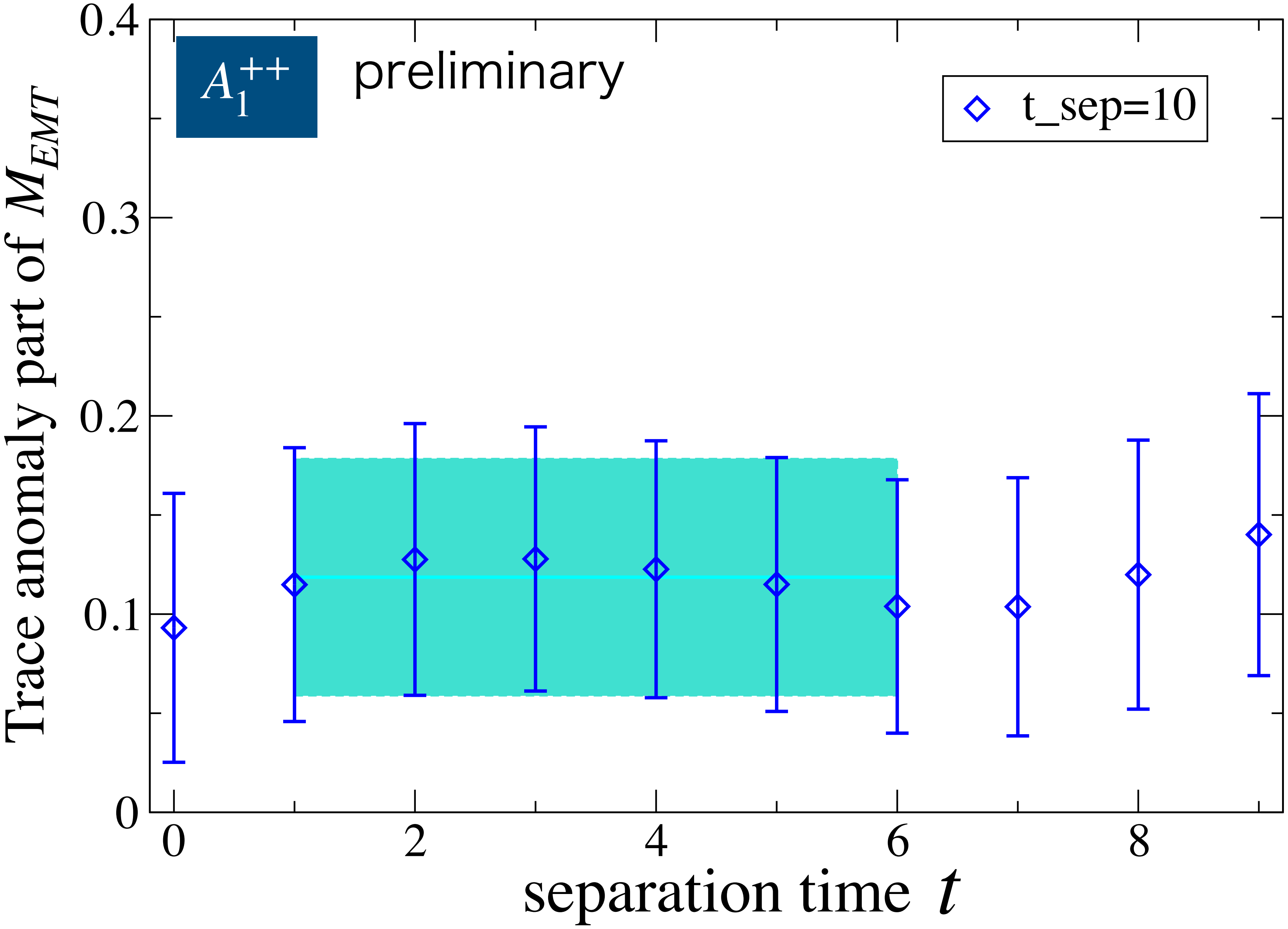
Saturation around $t_{sep}=10$ is confirmed

$$M_{EMT} = M_{traceless} + M_{anomaly}$$

Trace anomaly part

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

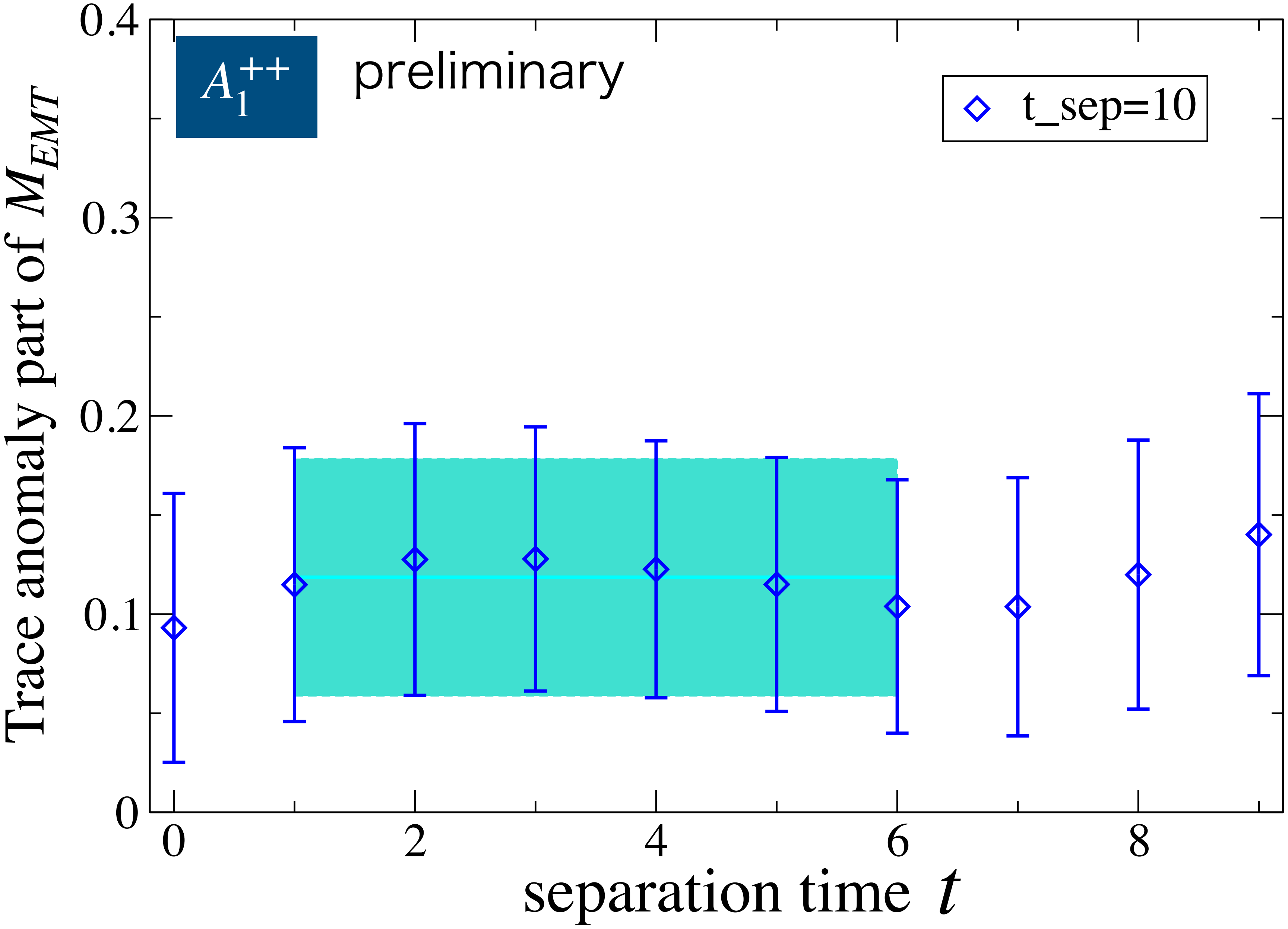
Behavior of trace anomaly part



Trace anomaly part

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

Behavior of trace anomaly part



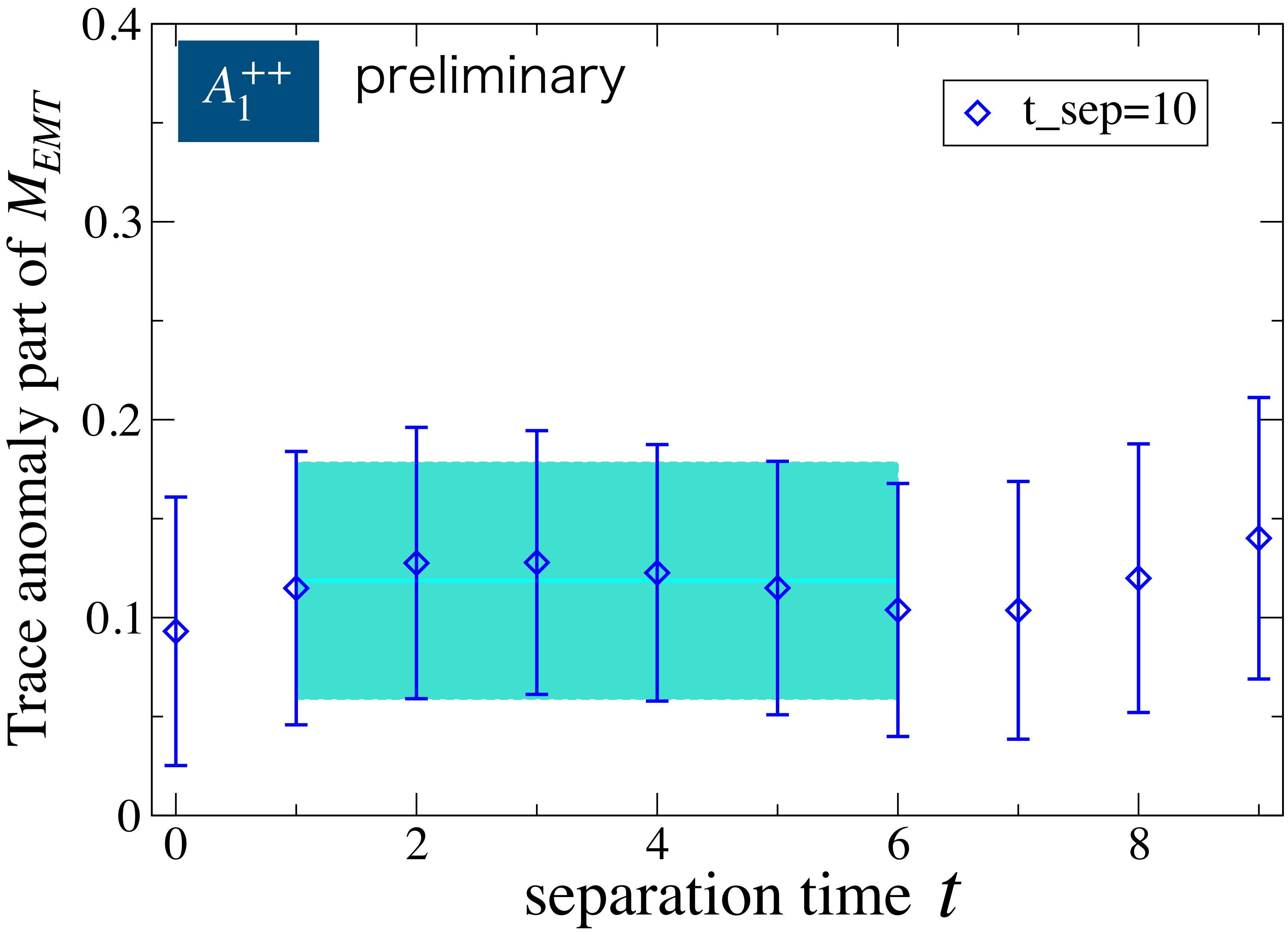
The contribution of anomaly part can be found by fitting

at $t_{sep} = 10$

Trace anomaly part

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	$1/a$[GeV]
6.4	$32^3 \times 32$	12000	3.84

Behavior of trace anomaly part



The contribution of anomaly part can be found by fitting at $t_{sep} = 10$

By fitting data,

$$M_{anomaly} = 0.12(6)$$

in lattice units

Mass decomposition of glueball

A sum of the traceless and trace anomaly part gives

$$M_{traceless} + M_{anomaly} = 0.24(7)$$ at $\tau = 2.5$

whose value agrees with the full EMT result $M_{EMT} = 0.28(12)$.

In comparison to the effective mass,

$$M_{EMT}/M_{eff} = 0.60(17)$$

This could approach to 1 with $\tau \to 0$ limit

By taking the ratio of $M_{traceless}/M_{anomaly} = 1.0(6)$

$$\boxed{M_{EMT} = M_{traceless}(50 \pm 20\%) + M_{anomaly}(50 \pm 29\%) \text{ at } \tau = 2.5}$$

We confirm that trace anomaly certainly plays an important role on glueball mass generation

Summary and future work

Summary

- We have studied trace anomaly contribution to glueball mass.
- The renormalization energy-momentum tensor (EMT) operator is constructed by using the gradient flow.
- Glueball mass M_{EMT} are calculated by using EMT operator through the ratio of 3pt. and 2pt. functions.
- We found that $M_{EMT}/M_{eff} = 0.60(17)$ at $\tau = 2.5$ of which value approaches unity toward $\tau \to 0$.
- The trace anomaly certainly contributes the mass of glueball as $M_{EMT} = M_{anomaly}(50 \pm 20\%) + M_{traceless}(50 \pm 29\%)$

Future work

- Taking continuum limit $a \to 0$ (in progress)
- Taking flow time $\tau \to 0$ (after continuum limit)
- Analysis of 2^{++} state glueball