Probing the singularities of the Landau-gauge gluon and ghost propagators with rational approximants

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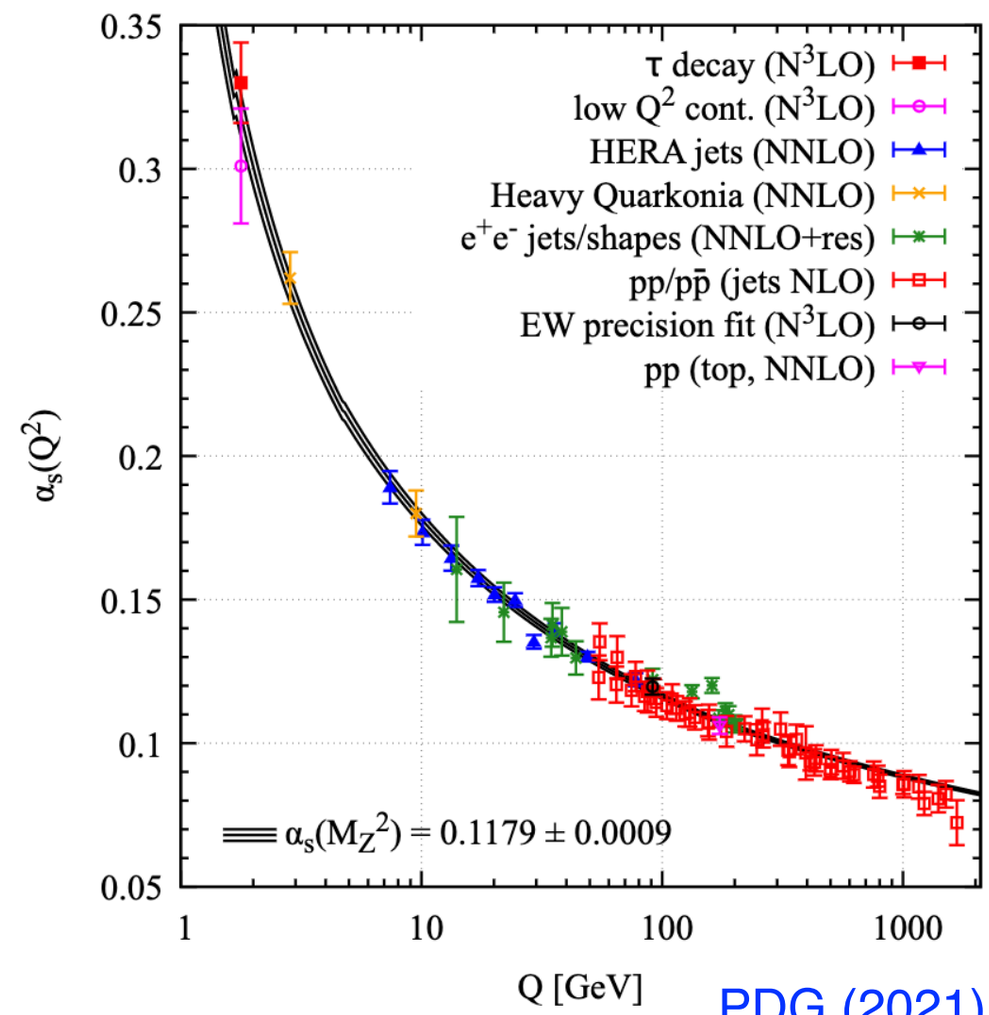
Introduction

Confinement

- Free quarks or gluons have never been observed
- Strong coupling is larger for low energies
- Confinement not well understood

important to know the ghost and gluon propagators in the infrared limit



PDG (2021)

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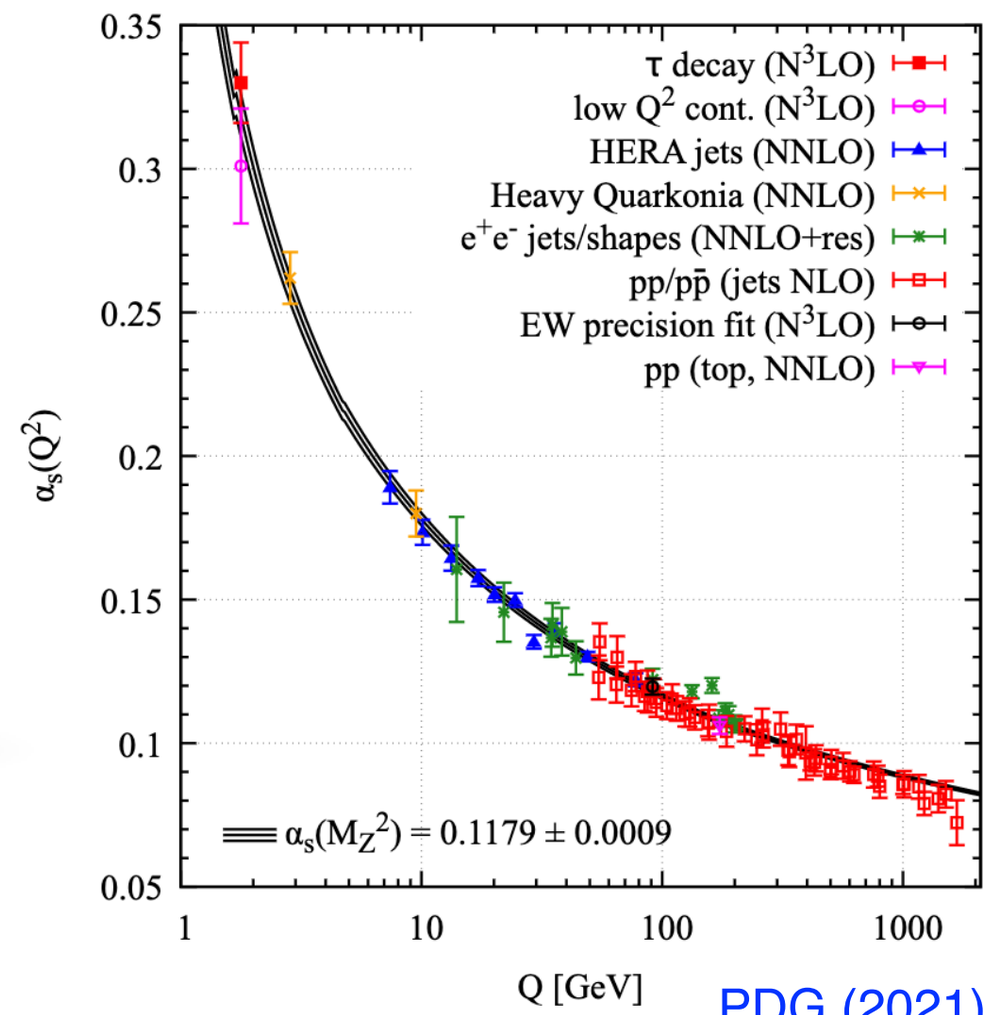


PDG (2021)

- Theoretical studies
  - Gribov-Zwanziger, scaling solution, etc.
  - Refined Gribov-Zwanziger, decoupling solution, etc.

lattice data in 3d and 4d

$$D(0) \neq 0 \quad \text{and} \quad G(p^2 \approx 0) \sim 1/p^2$$

Cucchieri, Mendes (2007)
Cucchieri, Mendes (2008)
Bogolubsky, *et al* (2009)

Objective

Objective

Study the analytic structure of the four-dimensional SU(2) Landau-gauge gluon and ghost propagators in the infrared regime

Model-Independent Method

Rational approximants as fitting functions

Method

- Similar method applied recently:
  - Falcão, Oliveira, Silva (2020) arXiv:2008.02614 [hep-lat]
  - Oliveira, Falcão, Silva (2021) arXiv:2111.04320 [hep-lat]

- **Main differences:**

This work	Previous work
SU(2) data Cucchieri, Mendes (2007, 2008, 2009) Cucchieri, *et al* (2012, 2016)	SU(3) data Bicudo, *et al* (2015) Duarte, Oliveira, Silva (2016) Dudal, Oliveira, Silva (2018)
propagation of all the errors considering the correlations	propagation of the errors not fully discussed

limit the number of parameters of the fit

Padé Approximants

Padé Approximants

$$P_N^M(z) = \frac{Q_M(z)}{R_N(z)} = \frac{a_0 + a_1 z + \ldots + a_M z^M}{1 + b_1 z + \ldots + b_N z^N}$$

Model independent

- Canonical Padé:
  - determination of the PA coefficients through matching
  - PA reproduces the first $M+N+1$ Taylor series coefficients of $f(z)$
- Padé as fitting function to data
  - already applied in other particle physics problems successfully
- Partial Padés and D-log Padés also used as fitting functions

Advantages

- Systematic and model-independent method
- Capable of reproducing singularities — poles
- Can mimic the existence of branch cuts — accumulation of poles and zeros

Lattice Data

Lattice Data

- Lattice data discussed before in references:
  - Cucchieri, Mendes (2007) arXiv:0710.0412 [hep-lat]
  - Cucchieri, Mendes (2008) arXiv:0712.3517 [hep-lat], arXiv:0804.2371 [hep-lat]
  - Cucchieri, Mendes (2009) arXiv:1001.2584 [hep-lat]
  - Cucchieri, Dudal, Mendes, Vandersickel (2012) arXiv:1111.2327 [hep-lat]
  - Cucchieri, Dudal, Mendes, Vandersickel (2016) arXiv:1602.01646 [hep-lat]

- Four-dimensional SU(2) Landau-gauge propagators

- Large lattice volume: $V = n^4 = 128^4$

physical volume: $(27 \text{ fermi})^4$

$\mathbf{p}_{\min} \approx 46$ **MeV**

Lattice Data

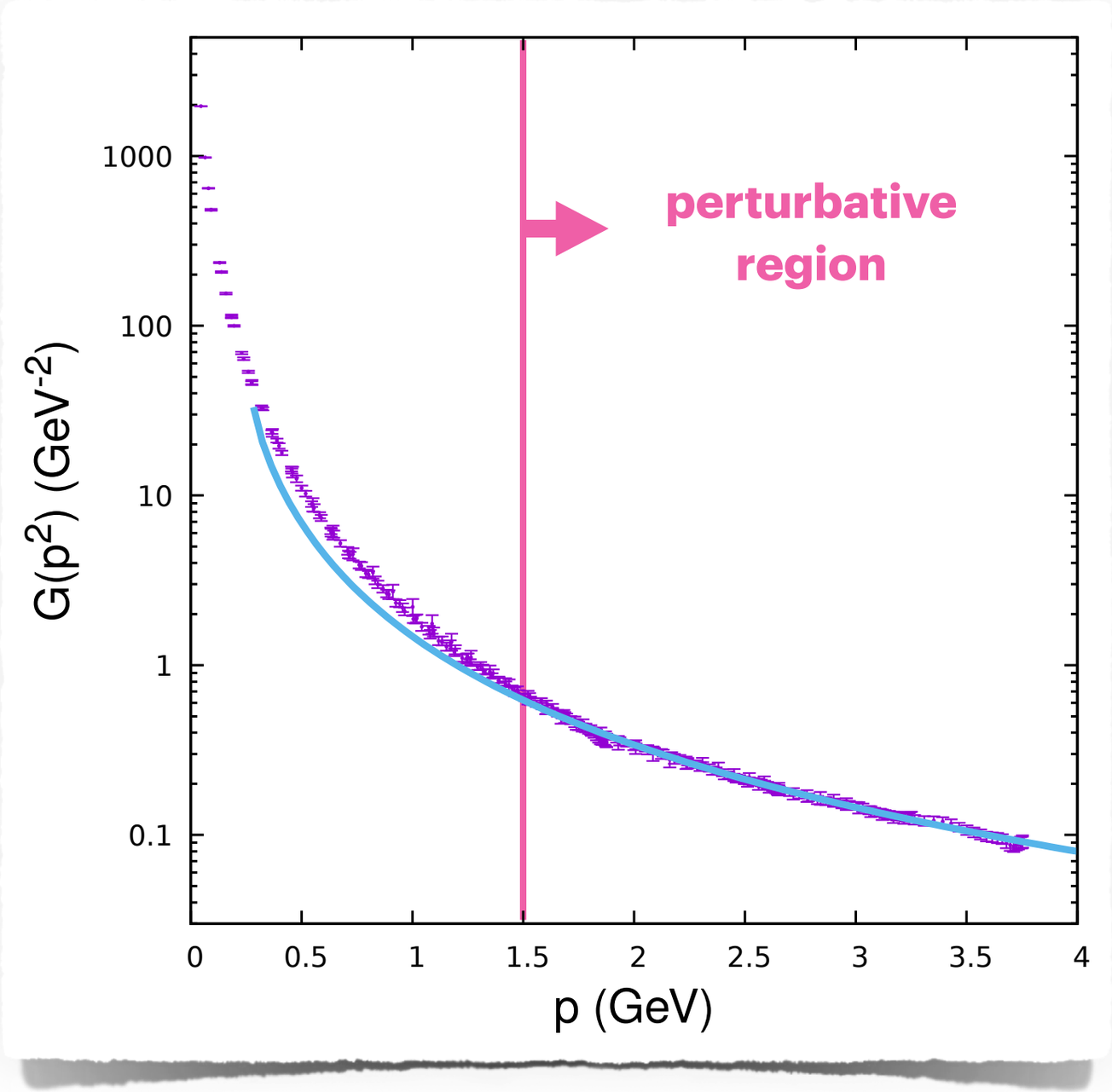
Ghost Propagator

- As a function of (unimproved) lattice momenta

$$p^2 = \sum_\mu p_\mu^2$$

- 21 configurations

- Fit restricted to the region $\sqrt{p^2} \leq 3.12\ \mathrm{GeV}$ — 220 points



Lattice Data

Gluon Propagator

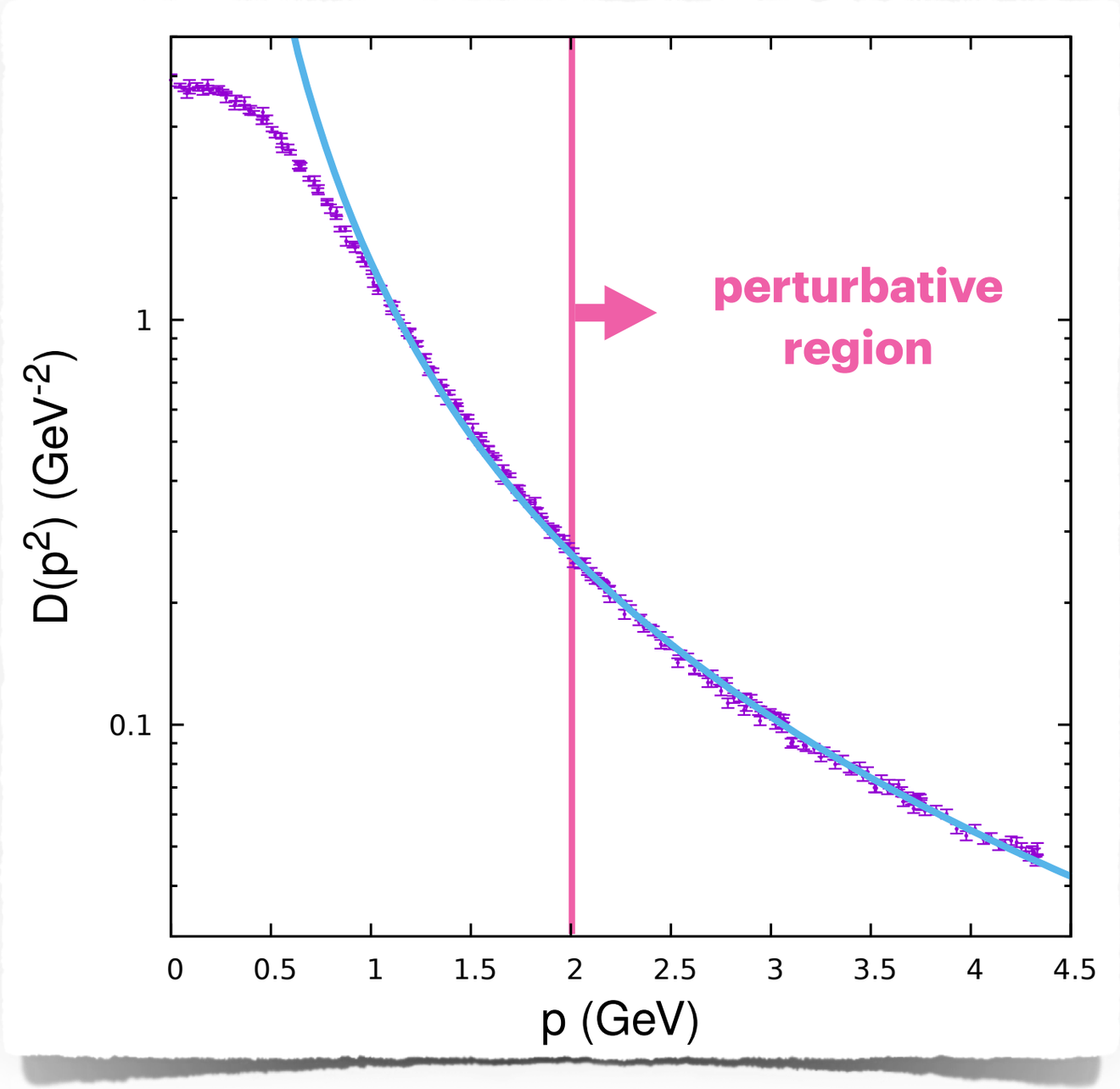
- As a function of improved momenta

$$p^2 = \sum_\mu p_\mu^2 + \frac{1}{12}\sum_\mu p_\mu^4$$

Ma (1999)
Cucchieri, Dudal, Mendes, Vandersickel (2012)

- 168 configurations

- Fit restricted to the region $\sqrt{p^2} \leq 2.4\ \mathrm{GeV}$ — 160 points



Results for the Gluon Propagator

Fitting Method

- SU(2) gluon and ghost propagators lattice data fitted by Padés

- PA parameters: χ^2 minimization considering all correlations

- Different methods for error propagation — all consistent
  - Hessian matrix;
  - Monte Carlo propagation;
  - $\Delta\chi^2$;
  - linear error propagation.

- Fit quality: χ^2/dof and p-value

PAs to Gluon Propagator Data

Example

- Correlation between the data points is negligible

- Example: P_2^1 — first of the sequence P_{k+1}^k

$$P_2^1(p^2) = \frac{a_0 + a_1\, p^2}{1 + b_1\, p^2 + b_2\, p^4}$$

$\chi^2/\text{dof} = 1.28$

$p-\text{value} = 0.010$

$$a_0 = 3.82 \pm 0.02\ \text{GeV}^{-2}$$
$$a_1 = 1.21 \pm 0.06\ \text{GeV}^{-4}$$
$$b_1 = 1.18 \pm 0.02\ \text{GeV}^{-2}$$
$$b_2 = 1.65 \pm 0.05\ \text{GeV}^{-4}$$

Refined Gribov-Zwanziger

non-trivial correlations

	a_1	b_1	b_2
a_0	-0.426	0.821	-0.436
a_1	$-$	-0.586	0.995
b_1	$-$	$-$	-0.632

PAs to Gluon Propagator Data

Example

- Correlation between the data points is negligible

- Example: P_2^1 — first of the sequence P_{k+1}^k

$$P_2^1(p^2) = \frac{a_0 + a_1\, p^2}{1 + b_1\, p^2 + b_2\, p^4}$$

pole $p^2 = (-0.36 \pm 0.02) \pm (0.690 \pm 0.005)i \ \mathrm{GeV}^2$

zero $p^2 = (-3.2 \pm 0.2) \ \mathrm{GeV}^2$

Taylor series coefficients

$c_0 = 3.82 \pm 0.02 \ \mathrm{GeV}^{-2}$

$c_1 = -3.3 \pm 0.1 \ \mathrm{GeV}^{-4}$

$c_2 = -2.4 \pm 0.4 \ \mathrm{GeV}^{-6}$

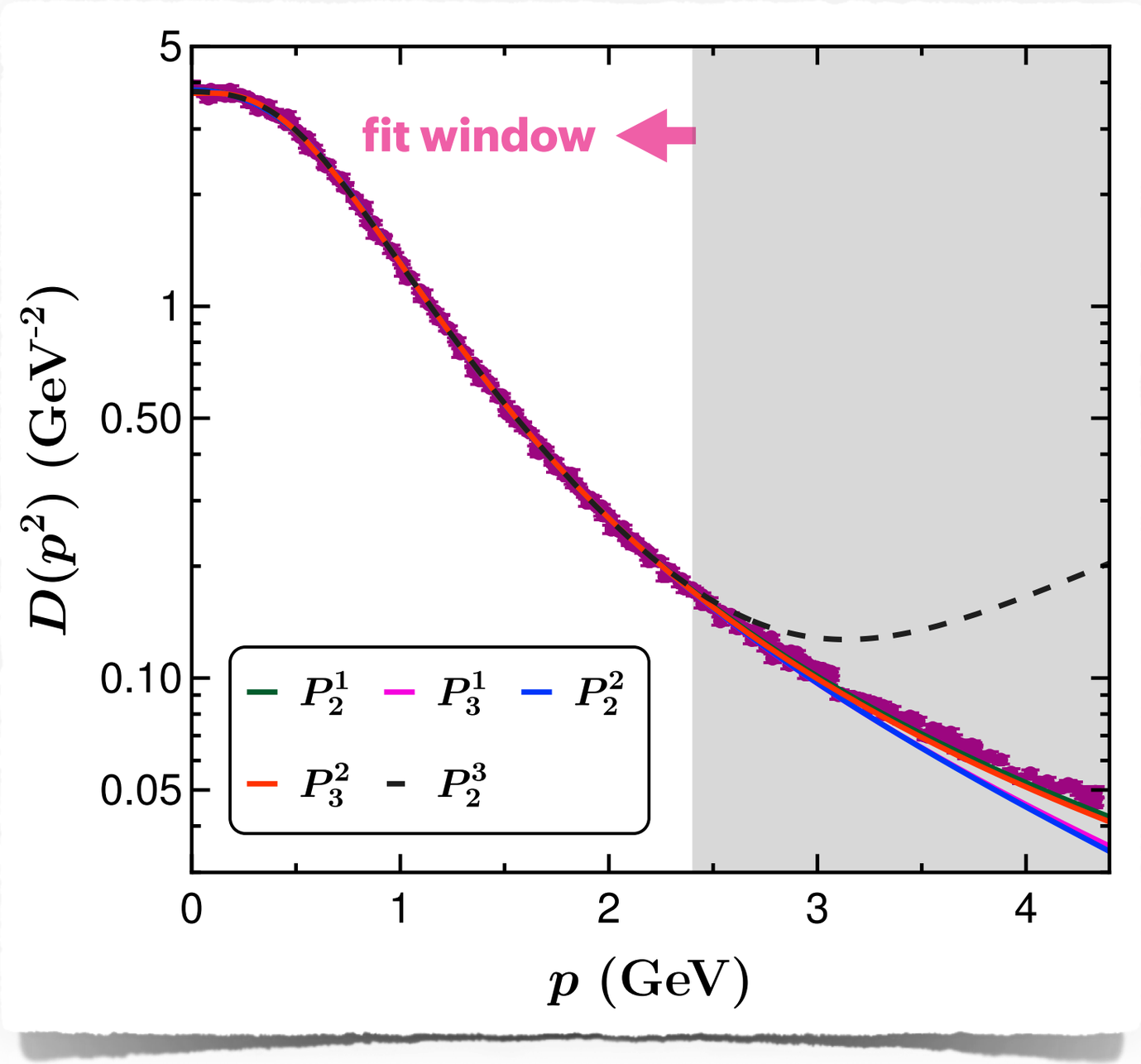
$c_3 = 8.3 \pm 0.4 \ \mathrm{GeV}^{-8}$

$c_4 = -5.9 \pm 0.6 \ \mathrm{GeV}^{-10}$

$c_5 = -7 \pm 2 \ \mathrm{GeV}^{-12}$

PAs to Gluon Propagator Data

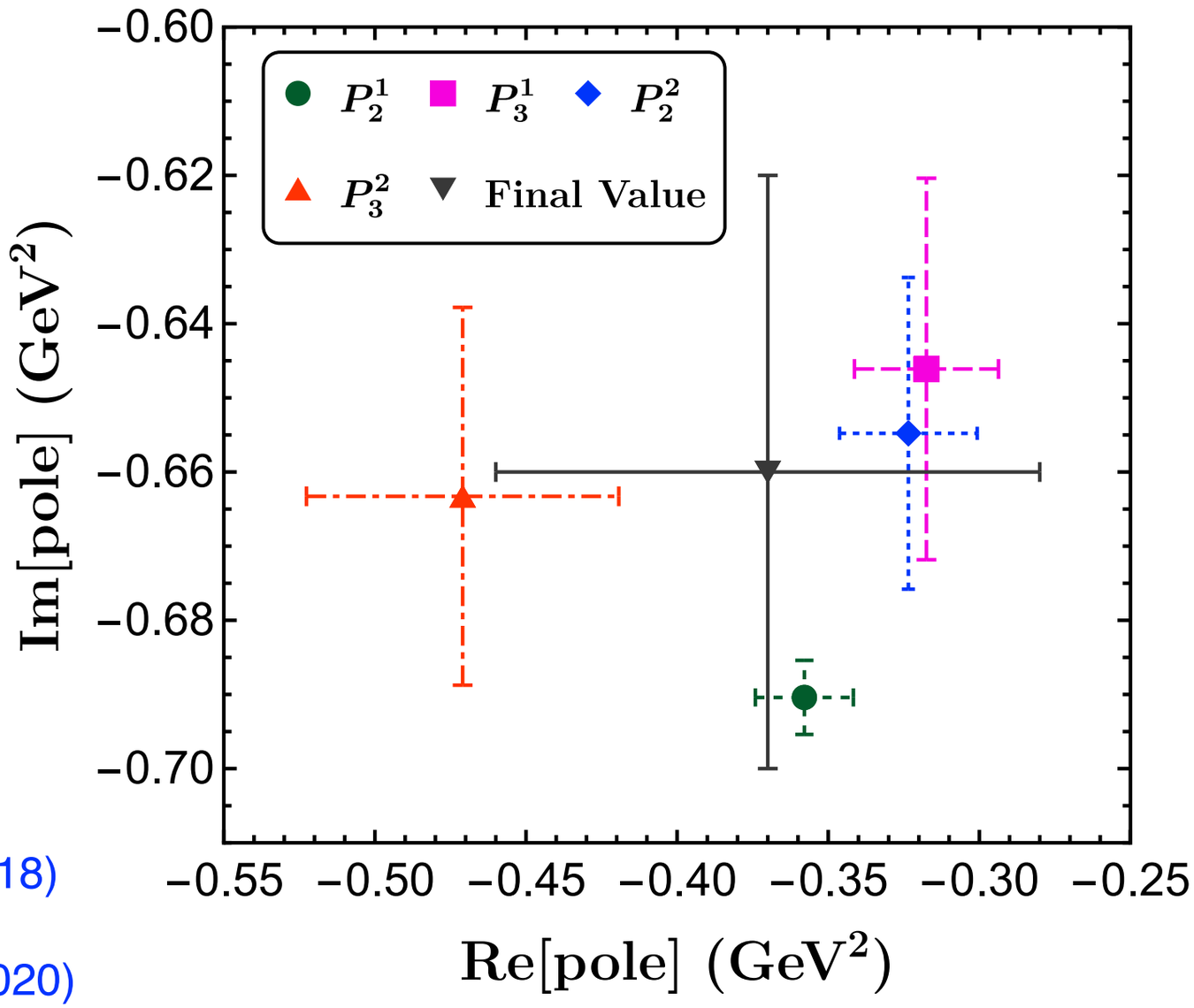
Boito, Cucchieri, London, Mendes
(in preparation)



- Errors increase considerably for PAs with 6 or more parameters
- P_2^3 — behavior at high energies: $a_4\, p^2$ with $a_4 > 0$
- Consistent pair of complex poles
- Consistent zero on the negative real axis of p^2

PAs to Gluon Propagator Data

Predicted Pole



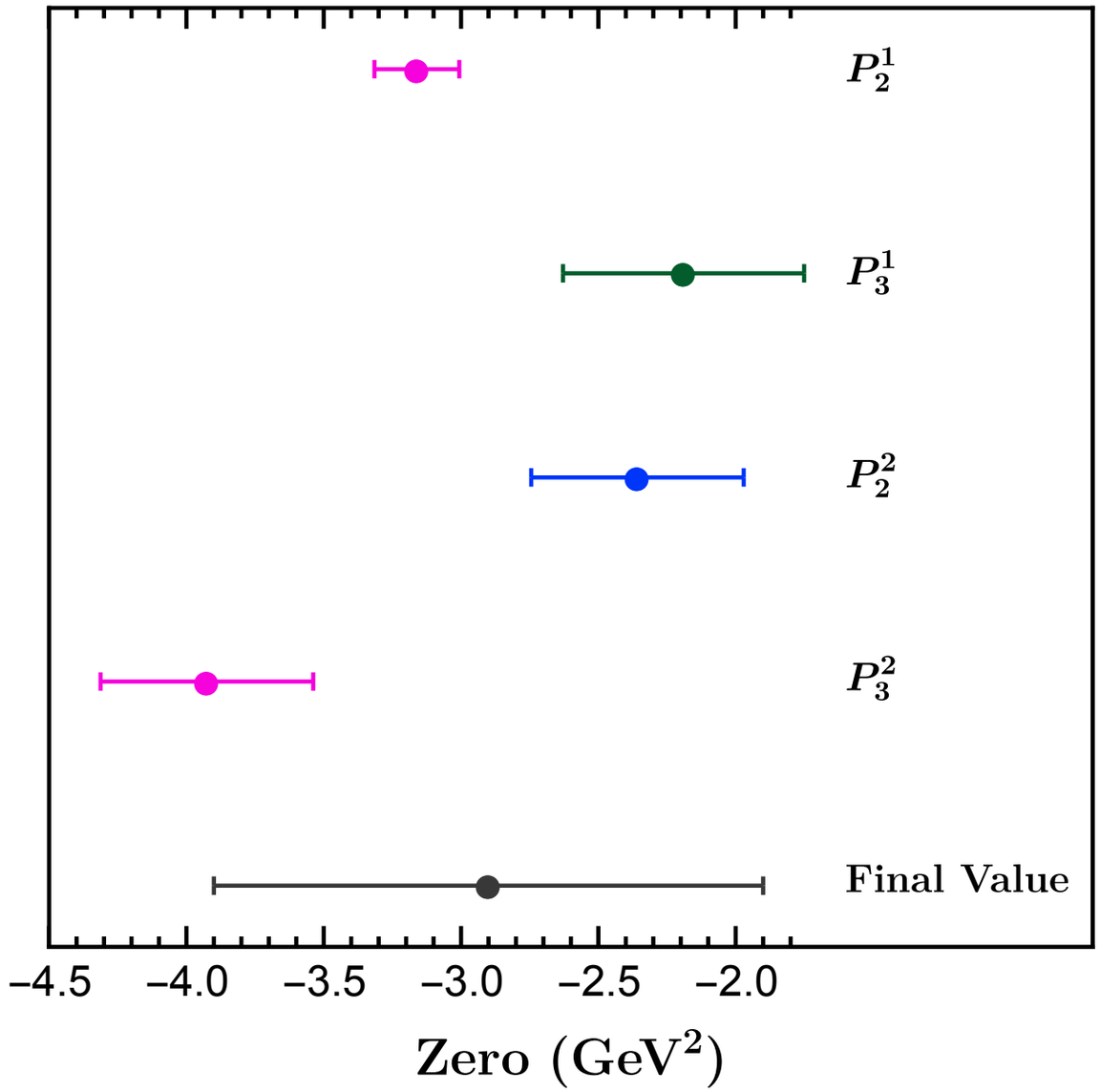
compatible with other estimates

Cucchieri, *et al* (2012)
Dudal, Oliveira, Silva (2018)
Binosi, Tripolt (2020)
Falcão, Oliveira, Silva (2020)

$$p^2_{\text{pole}} = [(-0.37 \pm 0.05_{\text{stat}} \pm 0.08_{\text{sys}}) \pm i\,(0.66 \pm 0.03_{\text{stat}} \pm 0.02_{\text{sys}})]\ \text{GeV}^2$$

PAs to Gluon Propagator Data

Predicted Zero



$$p^2_{\text{zero}} = (-2.9 \pm 0.4_{\text{stat}} \pm 0.9_{\text{sys}})\ \text{GeV}^2$$

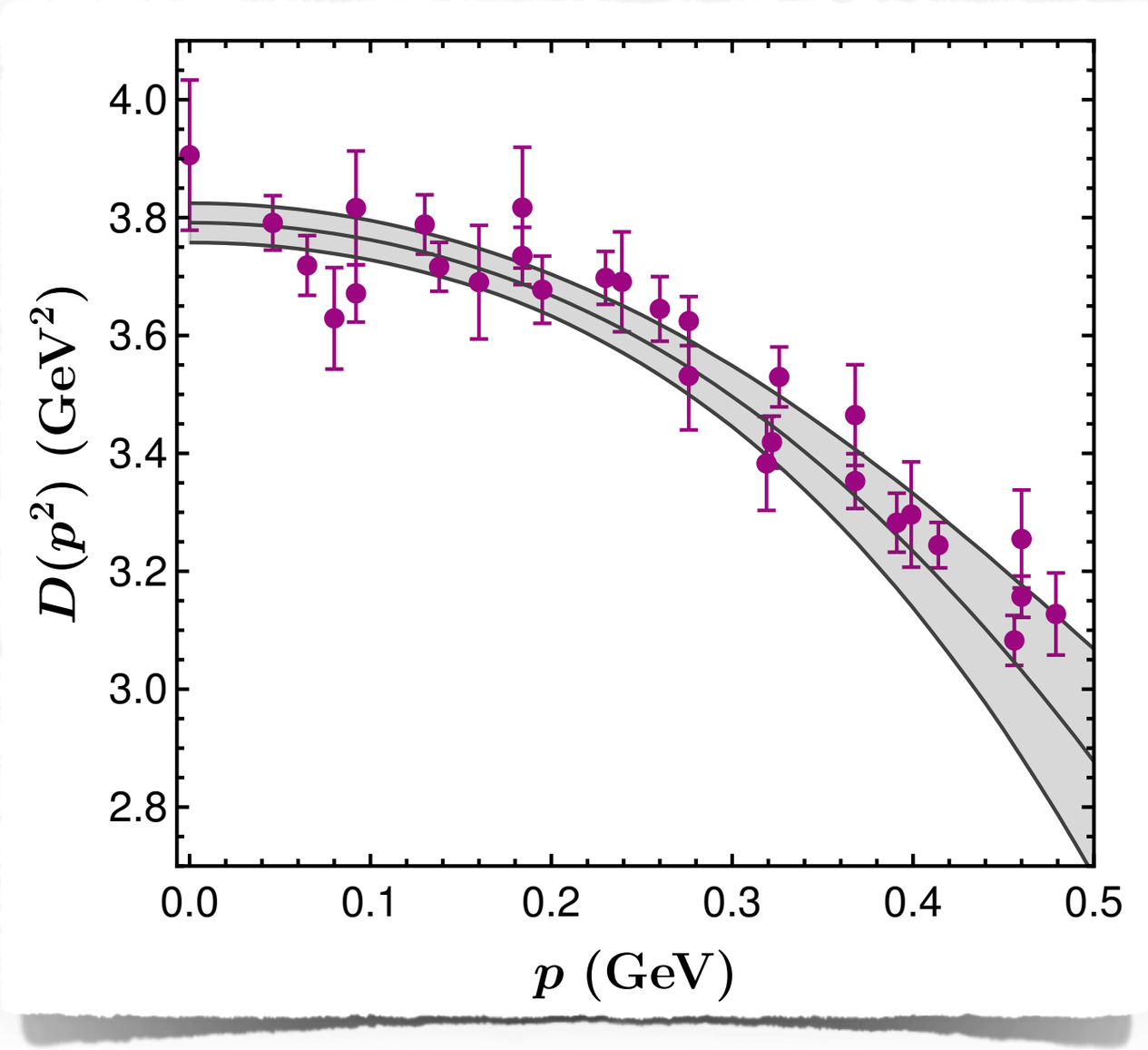
PAs to Gluon Propagator Data

Predicted Taylor Series Coefficients

$$D(p^2) = c_0 + c_1\, p^2 + c_2\, p^4 + \\ + c_3\, p^6 + c_4\, p^8 + \cdots$$

PAs with relative uncertainty smaller than 25%

c_0	$(3.79 \pm 0.03_{\text{stat}} \pm 0.04_{\text{sys}})\ \text{GeV}^{-2}$
c_1	$(-2.9 \pm 0.4_{\text{stat}} \pm 0.6_{\text{sys}})\ \text{GeV}^{-4}$
c_2	$(-5 \pm 2_{\text{stat}} \pm 3_{\text{sys}})\ \text{GeV}^{-6}$
c_3	$(9.4 \pm 1.0_{\text{stat}} \pm 0.9_{\text{sys}})\ \text{GeV}^{-8}$
c_4	$(-5.7 \pm 0.9_{\text{stat}} \pm 0.2_{\text{sys}})\ \text{GeV}^{-10}$



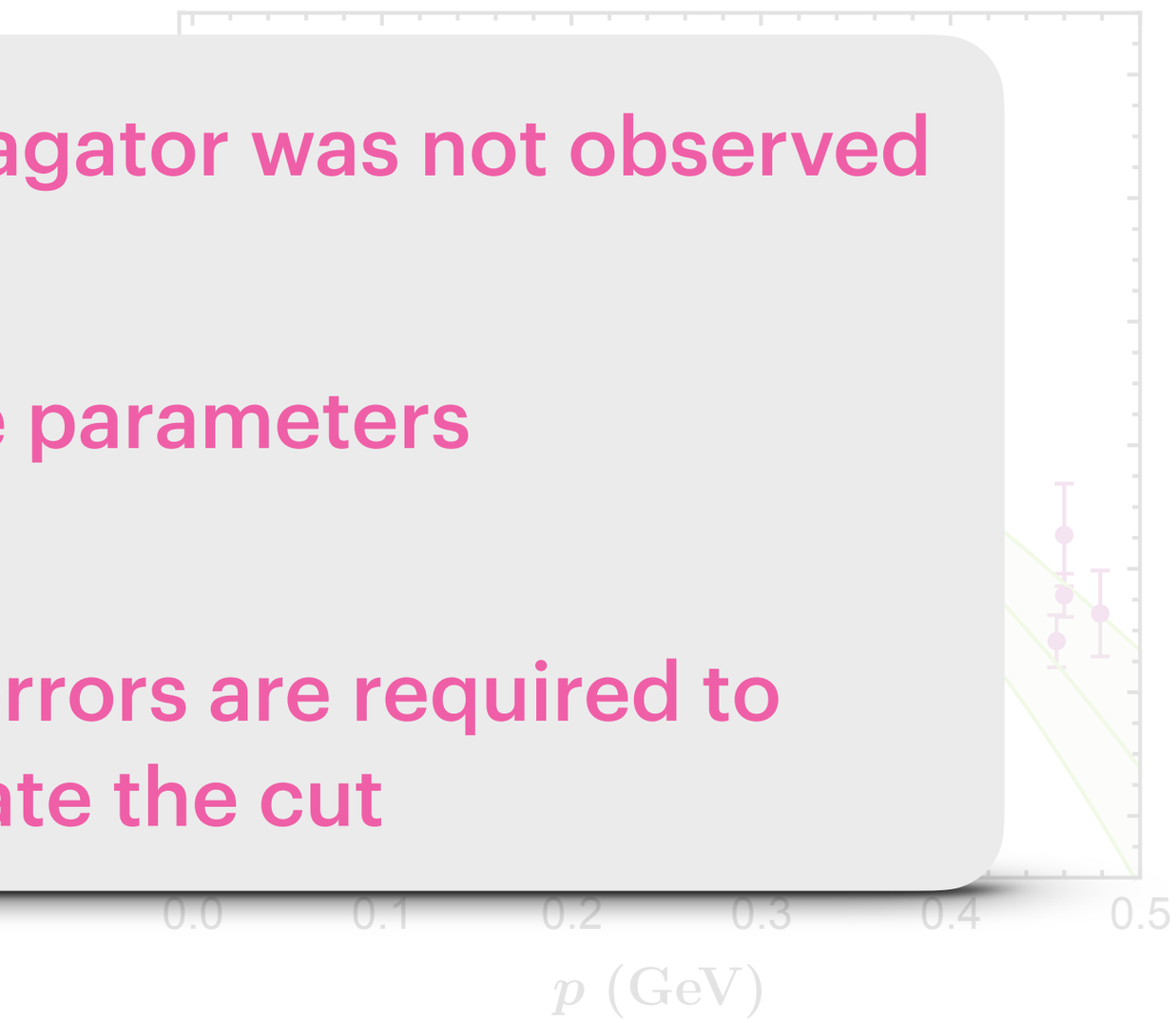
PAs to Gluon Propagator Data

Predicted Taylor Series Coefficients

$D(p^2) = c_0 + c_1 p^2 + c_2 p^4 +$

PA

c_0	
c_1	
c_2	$(-5 \pm 2_{\text{stat}} \pm 3_{\text{sys}})\ \text{GeV}^{-6}$
c_3	$(9.4 \pm 1.0_{\text{stat}} \pm 0.9_{\text{sys}})\ \text{GeV}^{-8}$
c_4	$(-5.7 \pm 0.9_{\text{stat}} \pm 0.2_{\text{sys}})\ \text{GeV}^{-10}$



Cut for the gluon propagator was not observed

Need more parameters

Data with smaller errors are required to investigate the cut

Results for the Ghost Propagator

PAs to Ghost Propagator Data

Partial Padé Approximants

- Correlation between data points large — small eigenvalues — hard to invert
- **Diagonal fits** Boito *et al* (2011) — arXiv:1110.1127 [hep-ph]
  - diagonal covariance matrix
  - all correlations in error propagation
  - first order linear error propagation
  - Q^2/dof not for fit quality

PAs to Ghost Propagator Data

Partial Padé Approximants

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  - diagonal covariance matrix
  - all correlations in error propagation
  - first order linear error propagation
  - Q^2/dof not for fit quality
- Fit parameters are very large
- **Pole close to the origin**
- Impose the simple pole at $p^2 = 0\ \mathrm{GeV}^2$
- Partial Padé approximant (PPA):

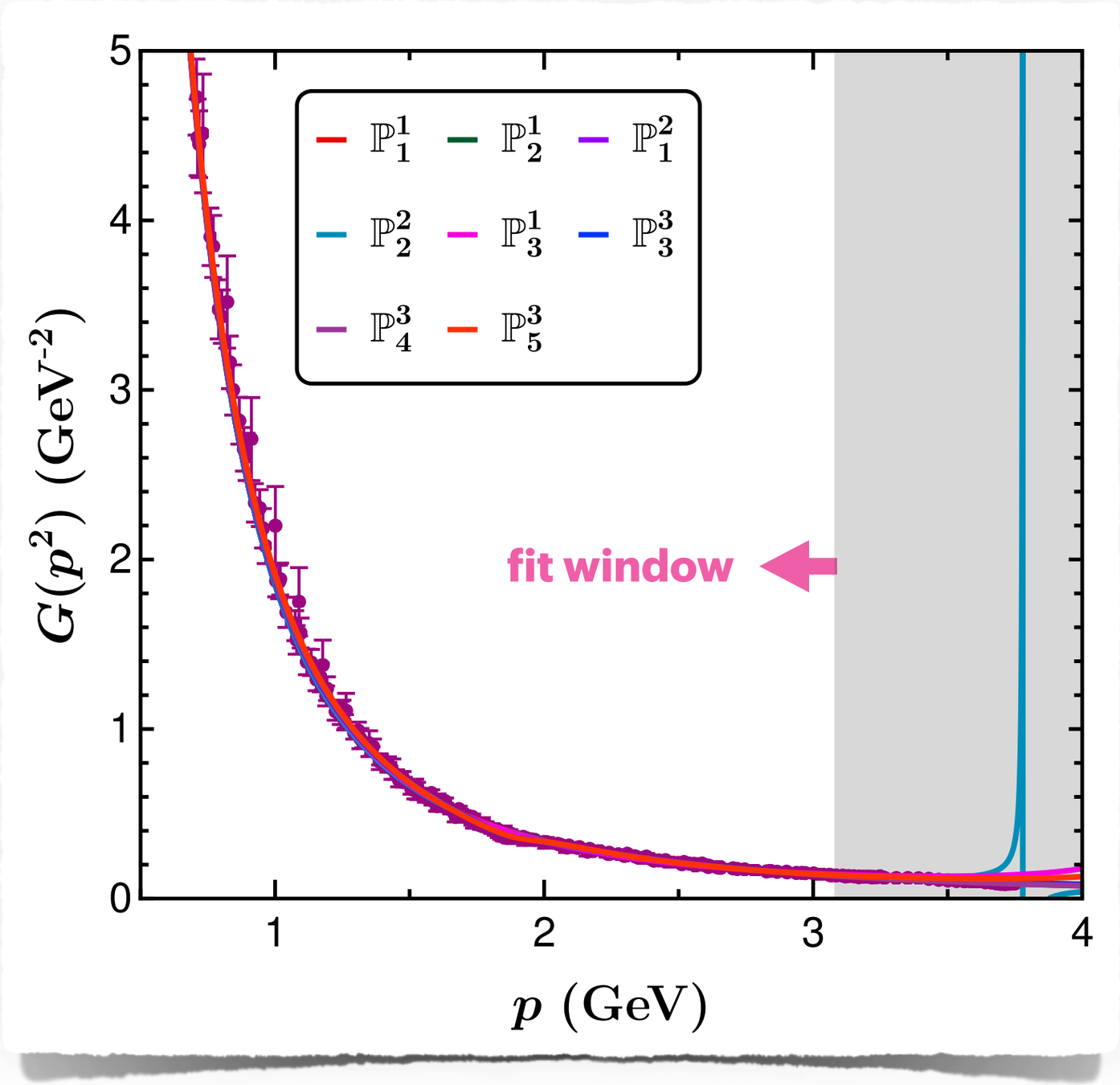
$$\mathbb{P}_N^M(p^2) = \frac{Q_M(p^2)}{R_N(p^2)\,p^2}$$

Baker, Graves-Morris (1986)
Boito, Masjuan, Oliani (2018)

PA	pole (GeV^2)	Q^2/dof
P_2^1	-1.27×10^{-10}	0.65
P_2^2	-6.98×10^{-14}	1.38
P_3^1	-2.48×10^{-11}	0.60
P_2^3	-1.17×10^{-12}	1.22
P_4^3	-1.24×10^{-10}	0.33

PPAs to Ghost Propagator Data

Boito, Cucchieri, London, Mendes
(in preparation)



- Errors increase considerably for PPAs with 8 or more parameters
- Non-physical pole on the positive real axis of p^2 almost cancelled by zero — Froissart doublets

Baker, Graves-Morris (1986)
Masjuan (2010)

PPAs to Ghost Propagator Data

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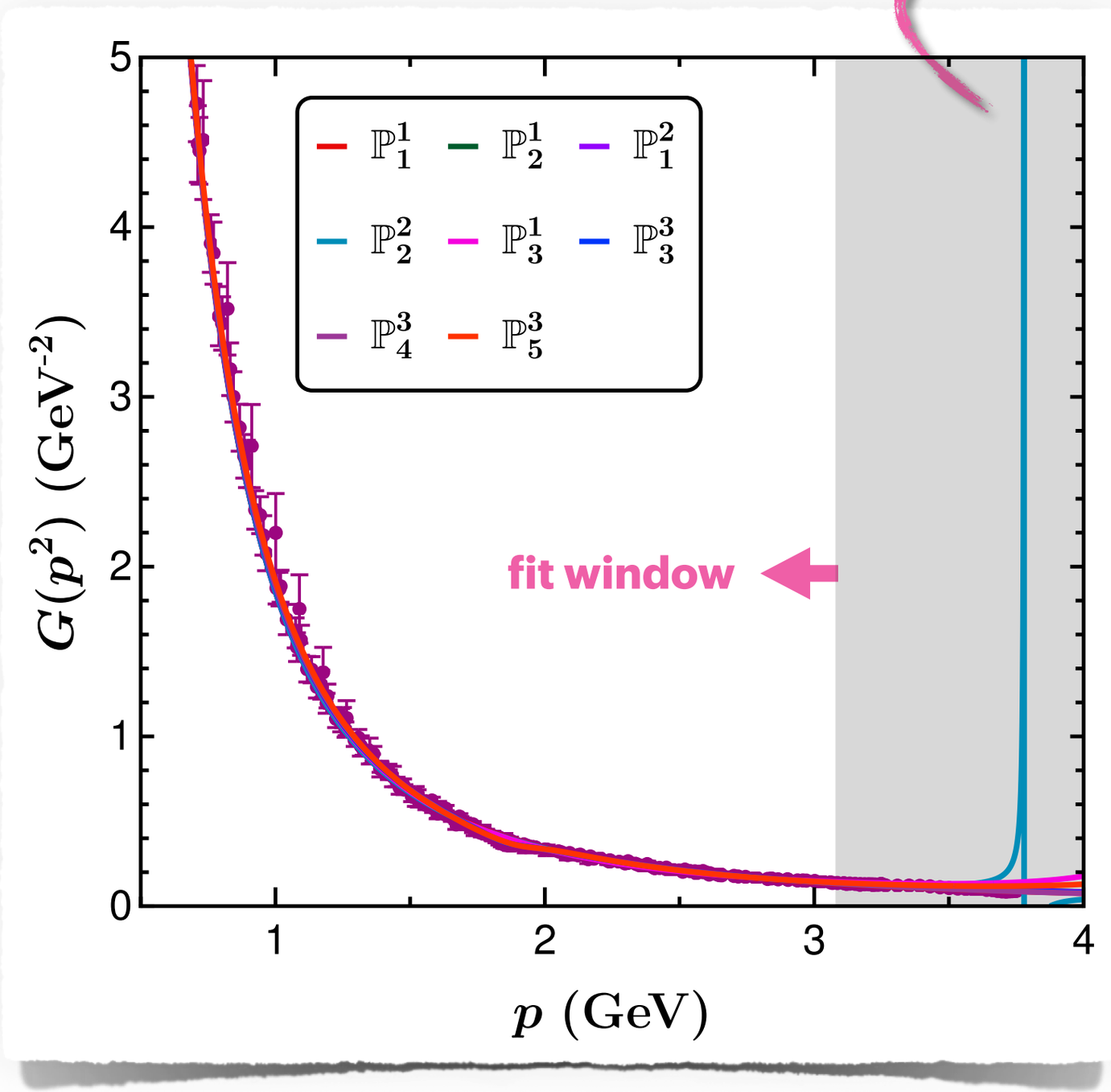


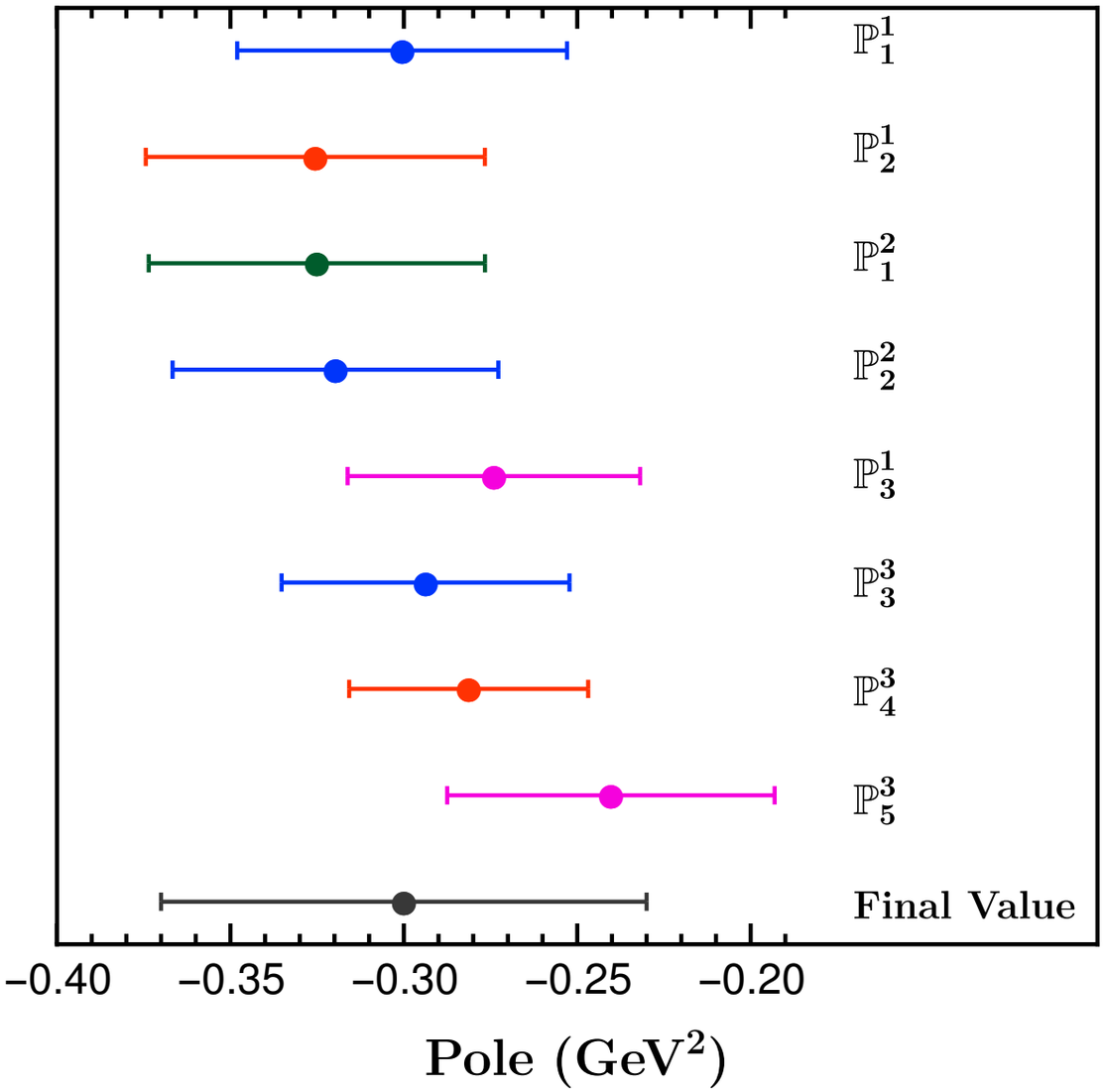
non-physical artifact

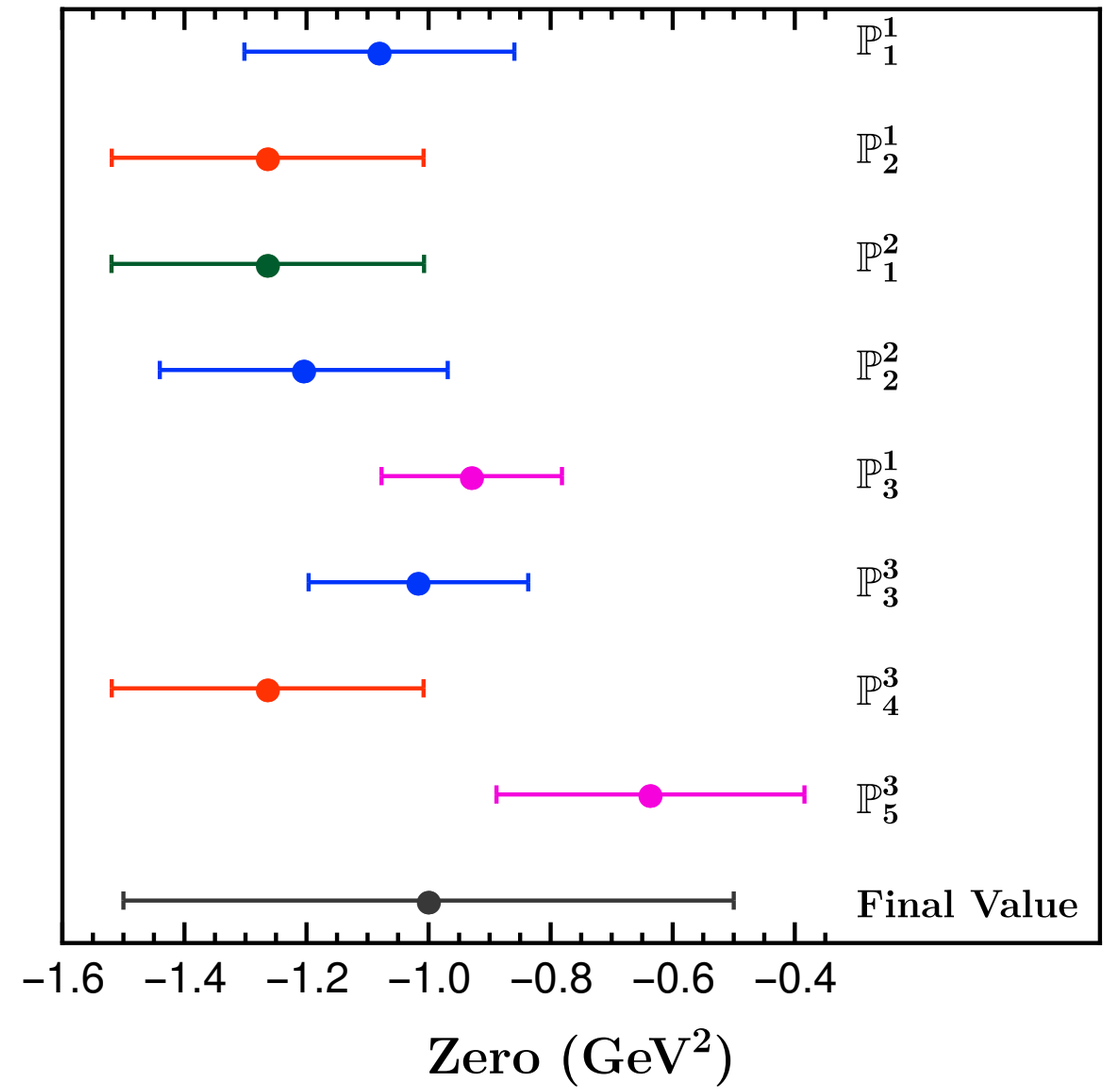
- Errors increase considerably for PPAs with 8 or more parameters
- Non-physical pole on the positive real axis of p^2 almost cancelled by zero — Froissart doublets

  Baker, Graves-Morris (1986)
  Masjuan (2010)
- Consistent pole on the negative real axis of p^2
- Consistent zero on the negative real axis of p^2

PPAs to Ghost Propagator Data

Predicted Pole and Zero





$$p^2_{\text{pole}} = (-0.30 \pm 0.05_{\text{stat}} \pm 0.05_{\text{sys}})\ \text{GeV}^2$$

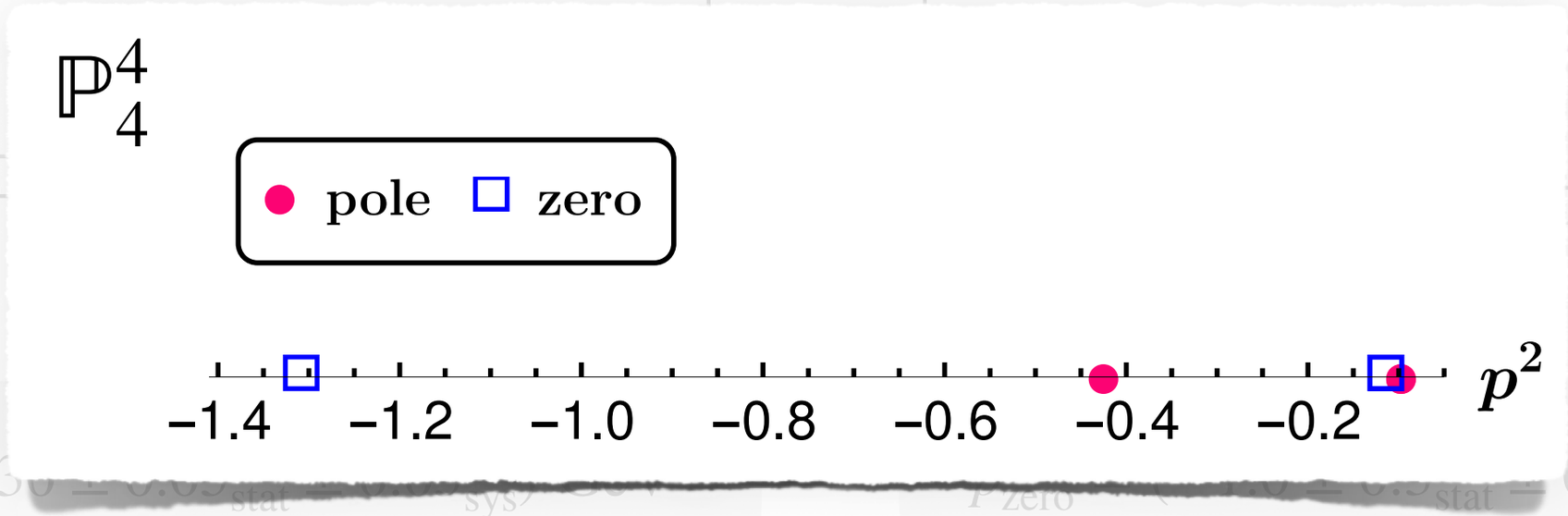
$$p^2_{\text{zero}} = (-1.0 \pm 0.3_{\text{stat}} \pm 0.4_{\text{sys}})\ \text{GeV}^2$$

PPAs to Ghost Propagator Data

Predicted Pole and Zero

Possible cut on the negative real axis of p^2 ?

PPAs of higher orders have interleaved poles and zeros



D-log Padé Approximants

D-log Padés

- Check if the predicted pole and zero for the ghost propagator are a possible cut

- Useful for functions with cuts or branch points

Baker, Graves-Morris (1986)
Boito, Masjuan, Oliani (2018)

$$f(z) = \frac{A(z)}{(\mu - z)^{\gamma}} + B(z) \qquad F(z) = \frac{\mathrm{d}}{\mathrm{d}z} \ln f(z) \approx \frac{\gamma}{\mu - z}$$

cut becomes a single pole

- D-Log Padé to $f(z)$

$$\mathrm{Dlog}_N^M(z) = f_{\mathrm{norm}}(0) \exp\left[\int \mathrm{d}z' \; \bar{P}_N^M(z')\right]$$

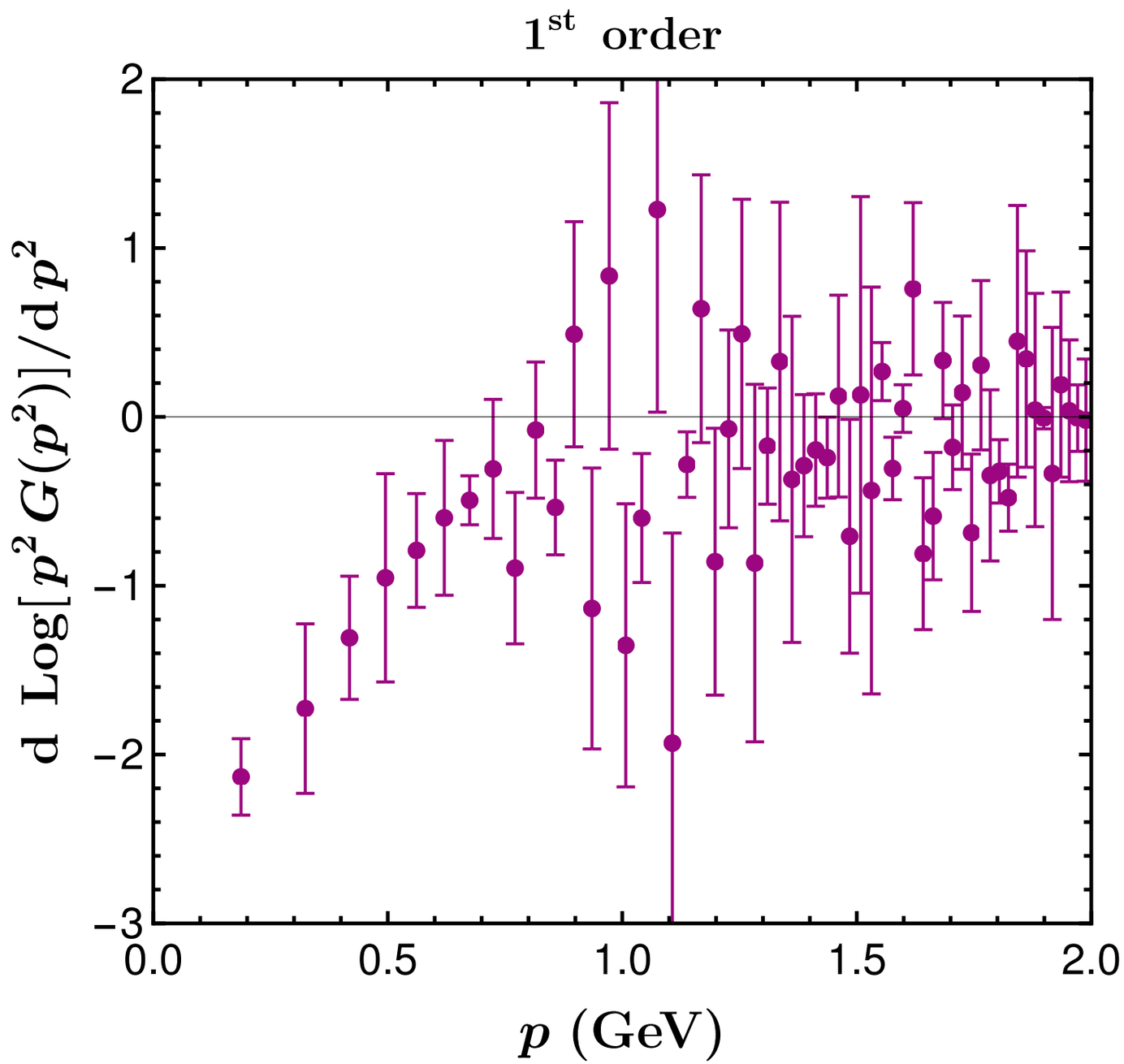
PA applied to $\mathrm{F(z)}$

- Need lattice data for $F(z)$

D-log Padés

Ghost Propagator Data

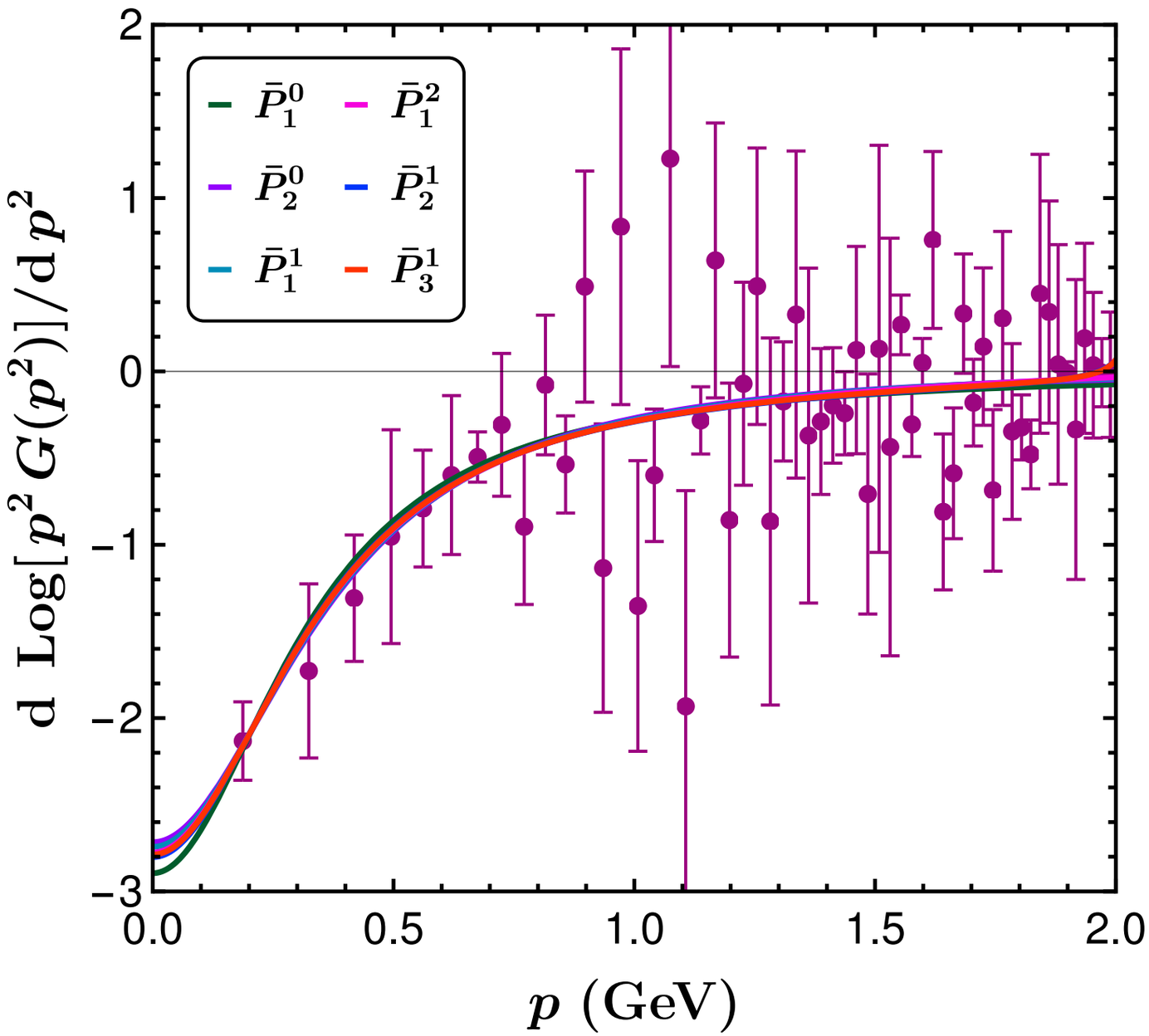
numerical derivative of logarithm of ghost propagator lattice data after linear interpolation



D-log Padés

Ghost Propagator Data

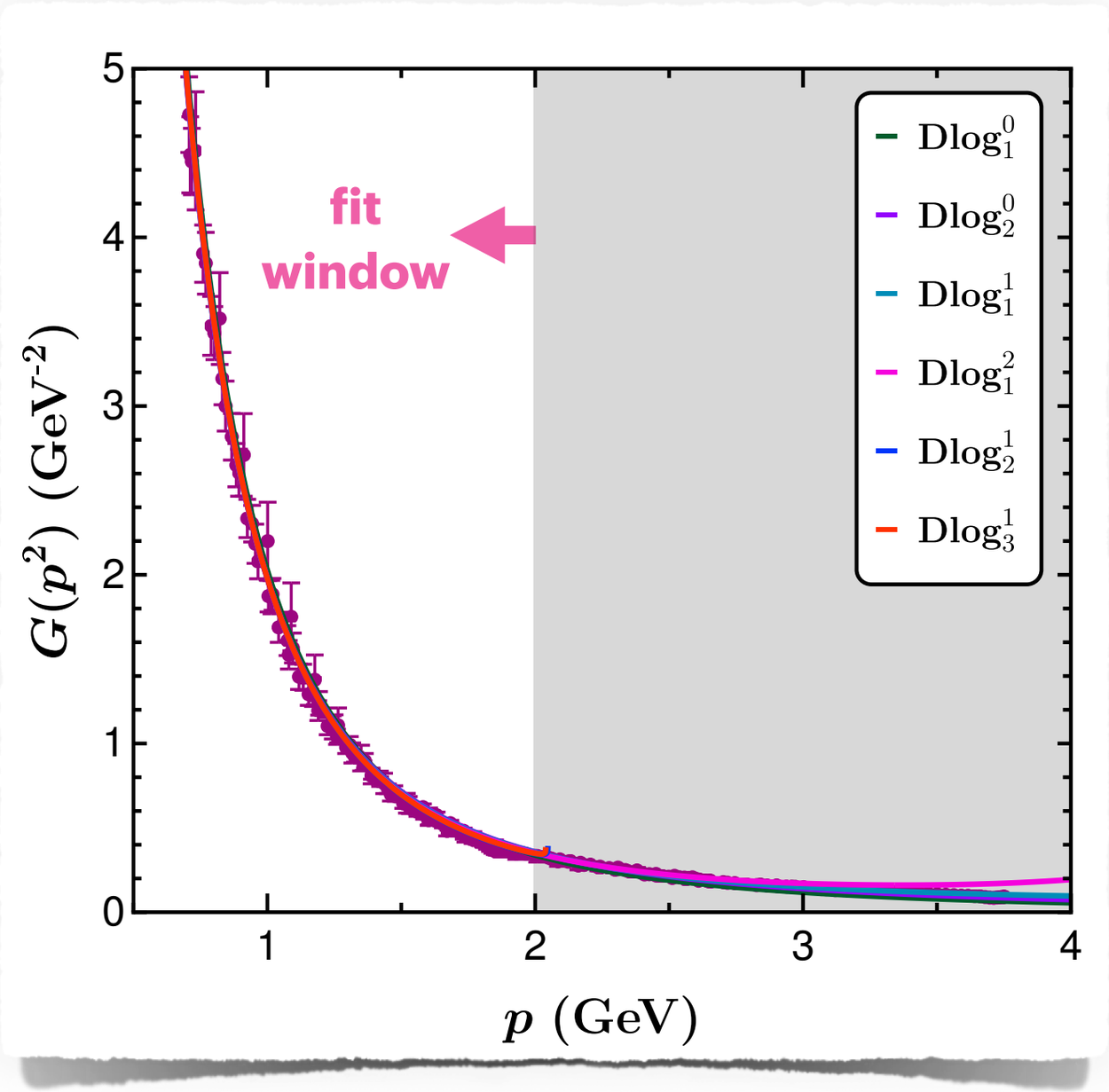
numerical derivative of logarithm of ghost propagator lattice data after linear interpolation



D-log Padés

Ghost Propagator Data — Results

Boito, Cucchieri, London, Mendes
(in preparation)



$$\mathrm{Dlog}(p^2) \propto \frac{1}{(p^2 - p_c)^{\gamma}}$$

D-Log PA	p_c (GeV^2)	γ
Dlog_1^0	-0.11 ± 0.03	0.31 ± 0.05
Dlog_2^0	-0.14 ± 0.07	0.4 ± 0.2
Dlog_1^1	-0.13 ± 0.05	0.4 ± 0.1
Dlog_1^2	-0.12 ± 0.09	0.3 ± 0.2
Dlog_2^1	-0.12 ± 0.04	0.3 ± 0.2
Dlog_3^1	-0.12 ± 0.08	0.3 ± 0.2

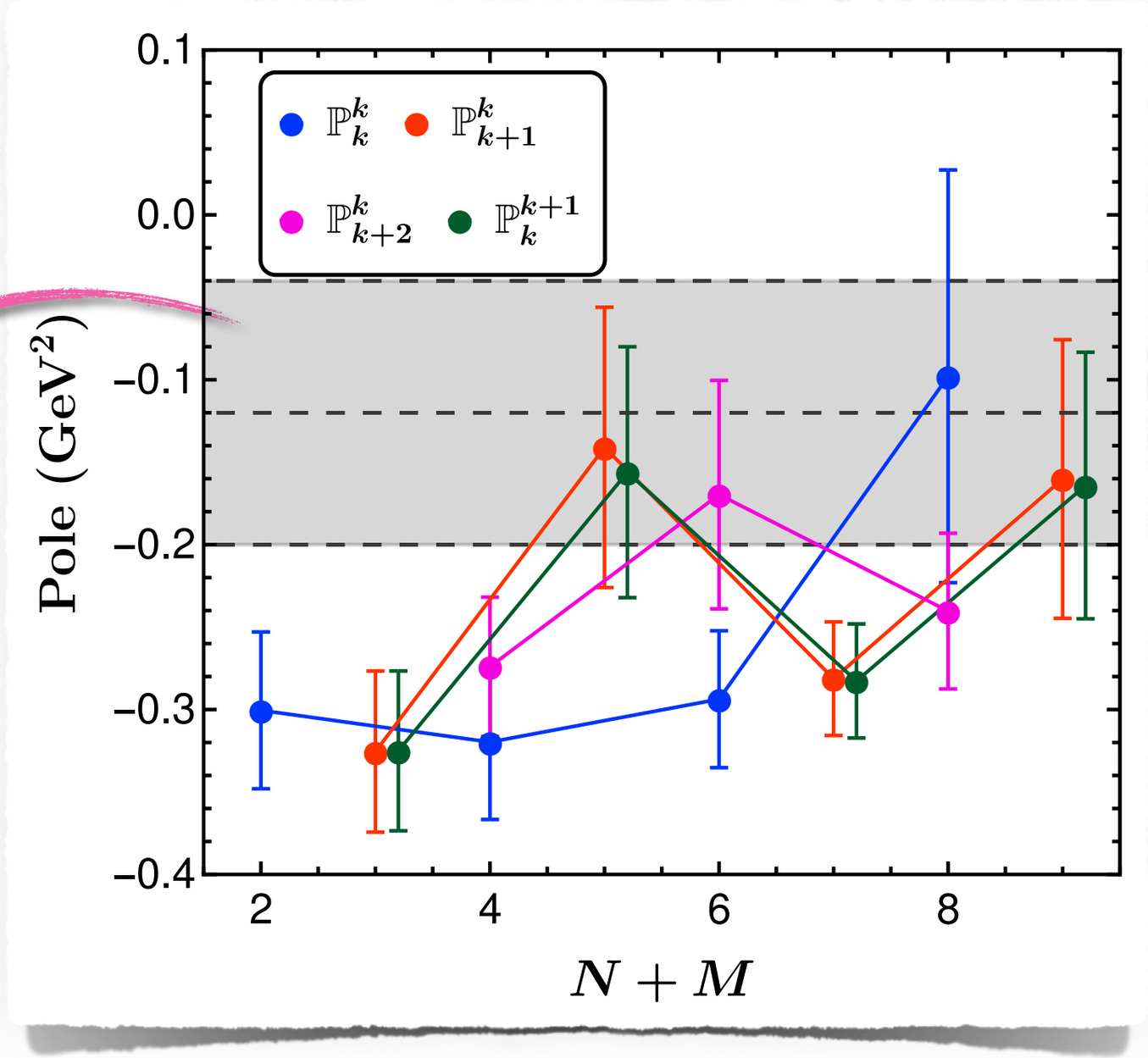
Indication of a possible cut on the negative real axis

D-log Padés

Ghost Propagator Data — Results

comparison of the first pole of PPAs to the value predicted by D-log Padés

$$p_c^2 = (-0.12 \pm 0.08_{\text{sys}} \pm 0.02_{\text{stat}}) \quad \text{GeV}^2$$



Suggested evidence of a cut on the negative real axis

Comparison

This work — SU(2)

- Gluon propagator:

$$p^2_{\text{pole}} = [(-0.37 \pm 0.09) \pm i\,(0.66 \pm 0.04)]\ \text{GeV}^2$$

- Ghost propagator:

simple pole at $p^2 = 0$

$$p_c^2 = (-0.12 \pm 0.08)\ \ \text{GeV}^2$$

Coimbra group — SU(3)

Falcão, Oliveira, Silva (2020)
Oliveira, Falcão, Silva (2021)

- Gluon propagator:

$$\begin{cases} p^2_{\text{pole}} = [-0.28(6) \pm i\,0.4(1)]\ \text{GeV}^2 \\ p^2_{\text{pole}} = [-0.19(4) \pm i\,0.4(1)]\ \text{GeV}^2 \end{cases}$$

- Ghost propagator:

simple pole at $p^2 = 0$

$$p_c^2 \sim -\,0.1\ \text{GeV}^2$$

SU(2) and SU(3) — similar results

Conclusions

Conclusions

- Model-independent method to study analytic structure of propagators

- Predictions of singularities of the SU(2) Landau-gauge ghost and gluon propagators

Gluon Propagator	Ghost Propagator
$p^2_{\text{pole}} = [(-0.37 \pm 0.09) \pm i\,(0.66 \pm 0.04)]\ \text{GeV}^2$	simple pole at $p^2 = 0$
$p^2_{\text{zero}} = (-2.9 \pm 1.0)\ \text{GeV}^2$	possible cut on $p_c^2 \approx -0.12\ \ \text{GeV}^2$

- Calculation of Taylor series near $p^2 \approx 0$

- D-Log Padés good alternative for gluon propagator data in the future (smaller errors)

Thank you for your attention!

CNPq

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Backup

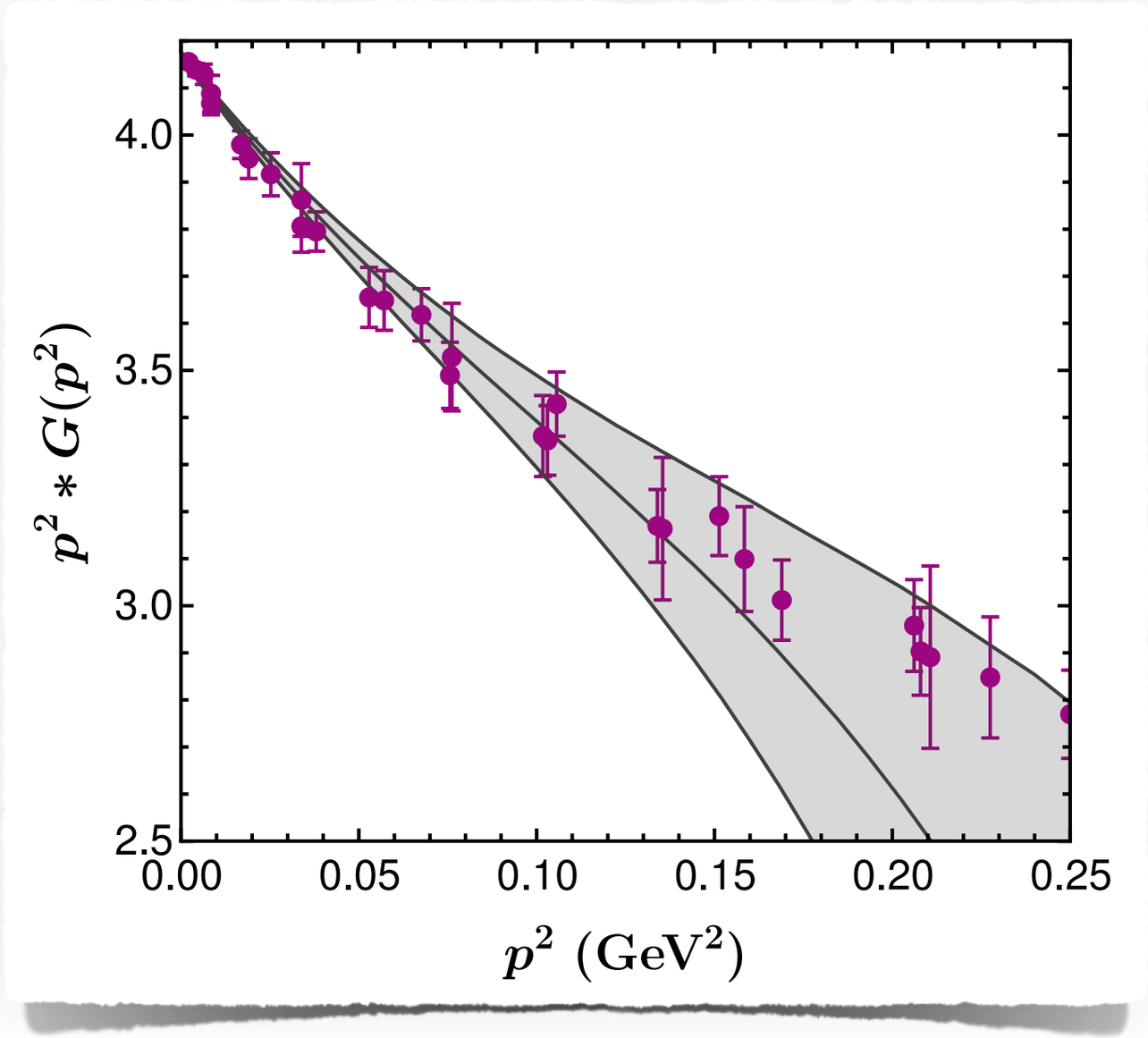
PPAs to Ghost Propagator Data

Predicted Taylor Series Coefficients

$$p^2\,G(p^2) = r_0 + r_1\,p^2 + r_2\,p^4 + \\ + r_3\,p^6 + r_4\,p^8 + \cdots$$

PPAs with relative uncertainty smaller than 45%

r_0	$4.17 \pm 0.01_{\text{stat}}$
r_1	$-9.7 \pm 1.0_{\text{stat}} \pm 0.5_{\text{sys}}\ \text{GeV}^{-2}$
r_2	$33 \pm 10_{\text{stat}} \pm 5_{\text{sys}}\ \text{GeV}^{-4}$
r_3	$-110 \pm 58_{\text{stat}} \pm 25_{\text{sys}}\ \text{GeV}^{-4}$



D-log Padés

Data

logarithm of the data

numerical derivative using finite differences

$$f'_{(1)}(p_i^2) \approx \frac{f(p_{i+1}^2) - f(p_{i-1}^2)}{2\,\Delta p^2}$$

equally spaced data points

linear interpolation

calculating all introduced correlations