

Intra-taste eigenvalue splittings of staggered, KW and BC fermions in 2D

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Lattice 2022 – 9 August 2022

Introduction: Minimally doubled fermions in 2D

In 2D staggered, Karsten-Wilczek and Borici-Creutz fermions are *minimally doubled*.

Taste splitting is cut-off effect, thus goes away in continuum limit, but *how exactly* ?

- Susskind (“staggered”) fermions

$$D_{\text{stag}}(x, y) = \sum_{\mu} \eta_{\mu}(x) \nabla_{\mu}(x, y) , \quad \nabla_{\mu}(x, y) = \frac{1}{2} \left[U_{\mu}(x) \delta_{x+\hat{\mu}, y} - U_{\mu}^{\dagger}(x-\hat{\mu}) \delta_{x-\hat{\mu}, y} \right]$$

- Karsten-Wilczek fermions

$$D_{\text{KW}}(x, y) = \sum_{\mu} \gamma_{\mu} \nabla_{\mu}(x, y) - i \frac{ar}{2} \gamma_d \sum_{i=1:d-1} \Delta_i(x, y) \quad \left(\begin{array}{c} \text{canon.} \\ r = 1 \end{array} \right)$$

- Borici-Creutz fermions

$$D_{\text{BC}}(x, y) = \sum_{\mu} \gamma_{\mu} \nabla_{\mu}(x, y) - i \frac{ar}{2} \sum_{\mu} \gamma'_{\mu} \Delta_{\mu}(x, y) \quad \left(\begin{array}{c} \text{canon.} \\ r = 1 \end{array} \right)$$

Schwinger Model: QED in 2D with any N_f

SM at $N_f=0$ simulated with Metropolis/overrelax/instanton-hit/parity-hit.
Topological charge autocorrelation time is $O(1)$ at any β [arXiv:1203.2560].

Wilson gauge action per site:

$$s_{\text{wil}}(x) = 1 - \text{Re}(U(x)) = 1 - \cos(\theta(x))$$

Plaquette at position $x = (x_1, x_2)$:

$$U(x) = U_1(x)U_2(x+e_1)U_1^\dagger(x+e_2)U_2^\dagger(x)$$

$$U(x) = \exp(i\theta(x))$$

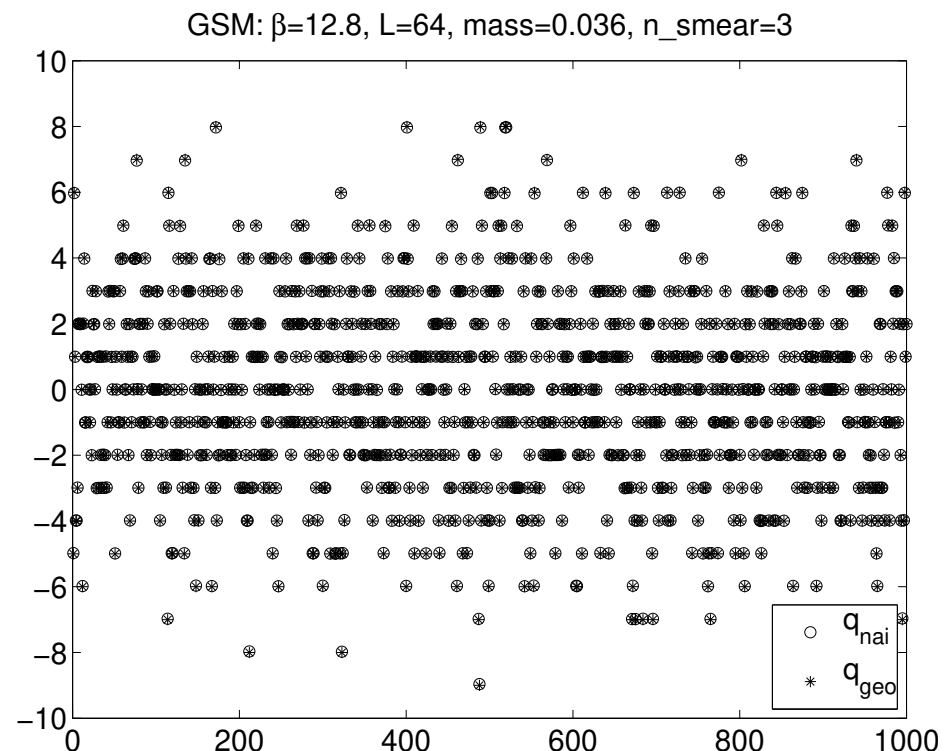
Two gluonic topological charges:

$$q_{\text{raw}}^{(n)} = \sum \sin(\theta^{(n)}(x))/(2\pi) \in \mathbf{R} \quad (\text{"fth}/Z")$$

$$q_{\text{geo}}^{(n)} = \sum \theta^{(n)}(x)/(2\pi) \in \mathbf{Z} \quad (\text{"geometric"})$$

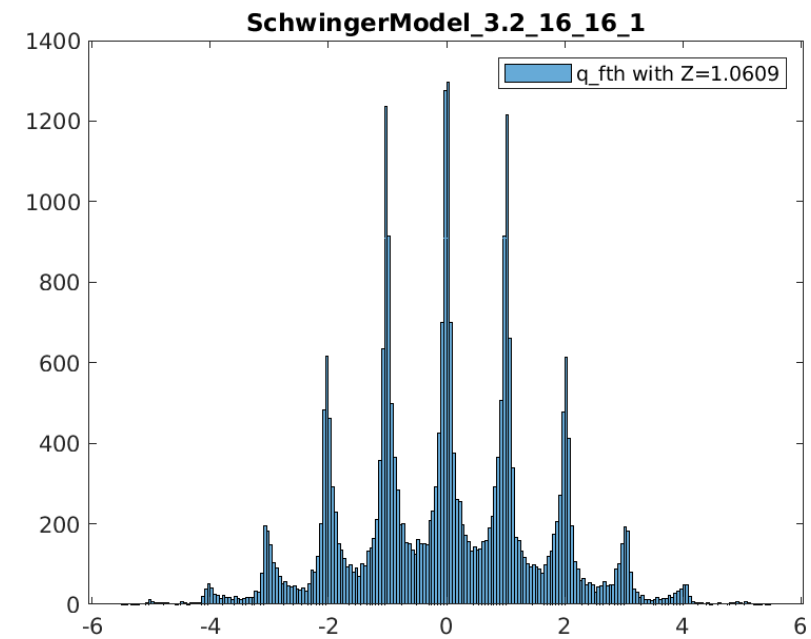
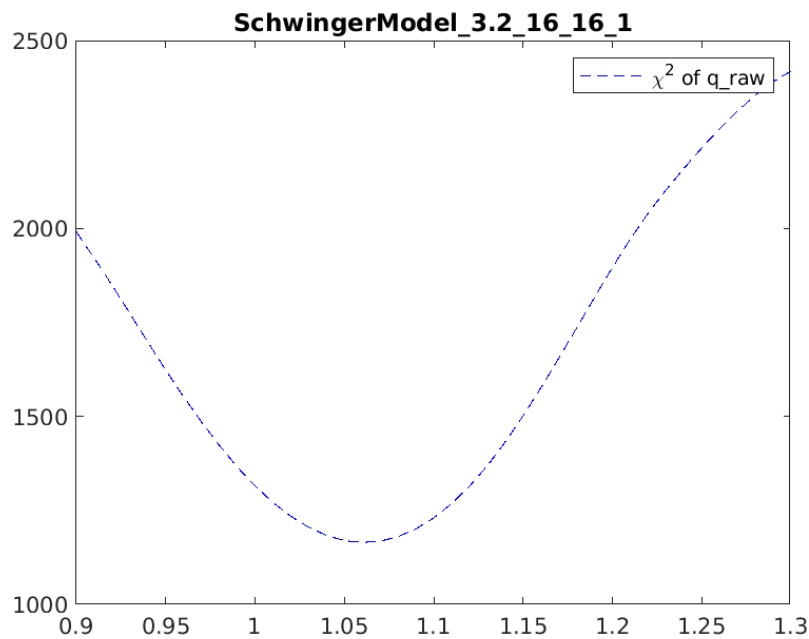
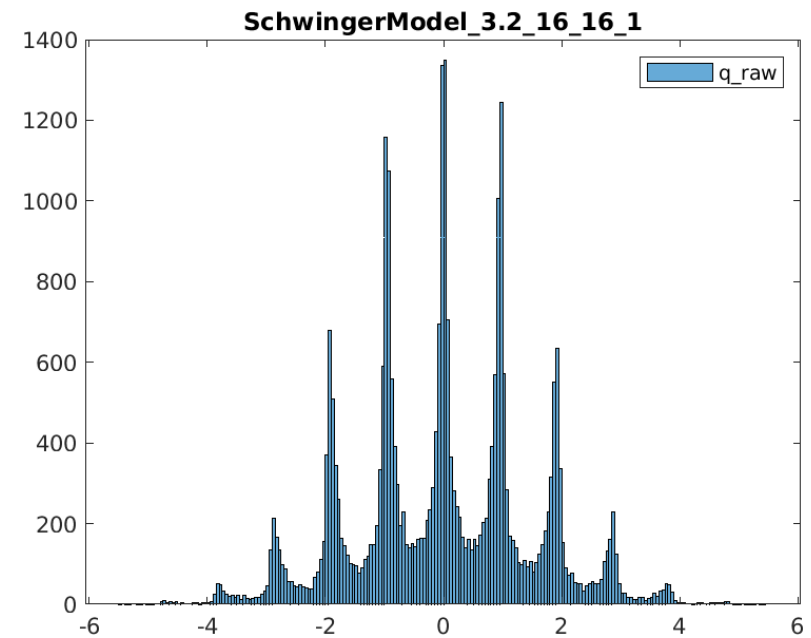
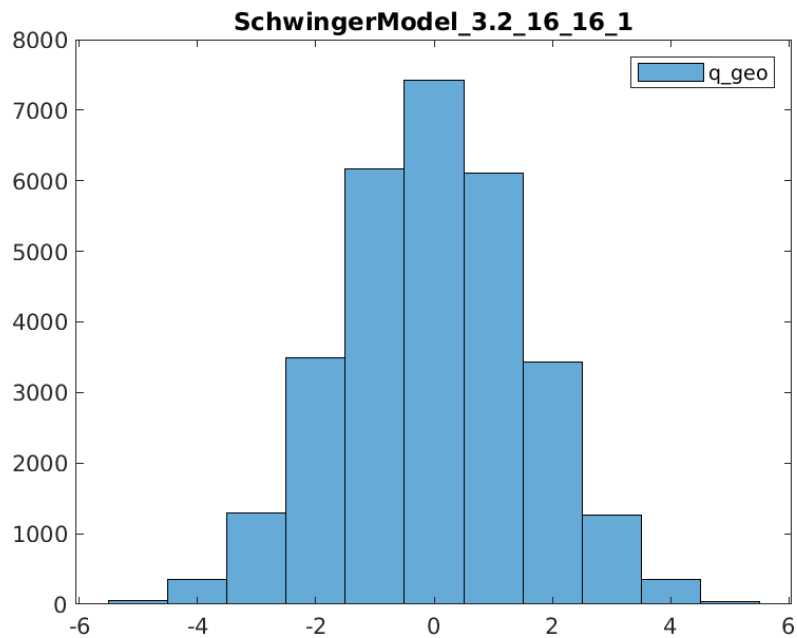
$\theta^{(n)}$ plaquette angle after n smearings

$q_{\text{opt}}(x)$ is clover-leaf version of $q_{\text{raw}}(x)$



Operators use $n=0, 1, 3$ steps of $\rho=0.25$ stout-smearing [Morningstar Peardon 2003].

Schwinger Model: topological charge distributions



Schwinger Model: ensemble details

β	3.2	5.0	7.2	12.8	20.0	28.8	51.2	80.0
L/a	16	20	24	32	40	48	64	80
n_{stout}	0,1,3	0,1,3	0,1,3	0,1,3	0,1,3	0,1,3	0,1,3	0,1,3
$s_{\text{wils}}^{(0)}$	0.17625	0.10662	0.07230	0.03989	0.02533	0.01752	0.00981	0.00627
$p_{\text{inst.hit}}$	0.750(2)	0.737(2)	0.729(2)	0.725(2)	0.726(2)	0.722(2)	0.721(2)	0.721(1)

Table 1: Ensembles used in the “cut-off effect” study; they implement constant physical volume through $(L/a)^2/\beta = 80$. For every choice of $(\beta, L/a)$ three ensembles of 10 000 configurations are generated, to be used with 0, 1 or 3 steps of $\rho = 0.25$ stout smearing, respectively. The **analytic result** $s_{\text{wils}}^{(0)}$ is taken from [Elser:2001pe].

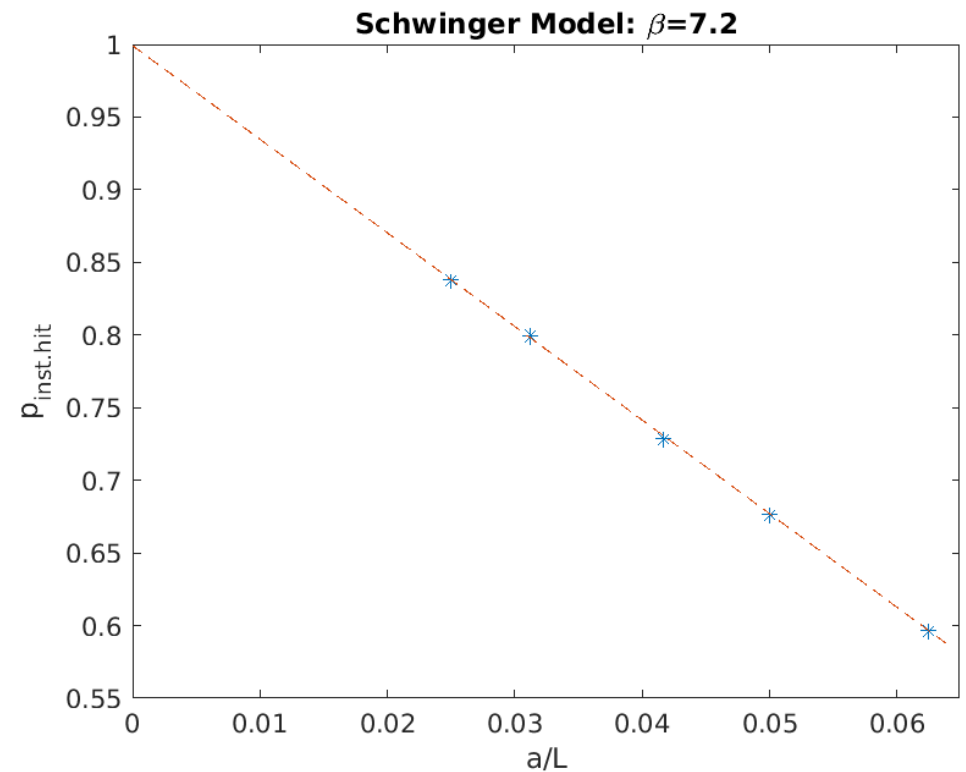
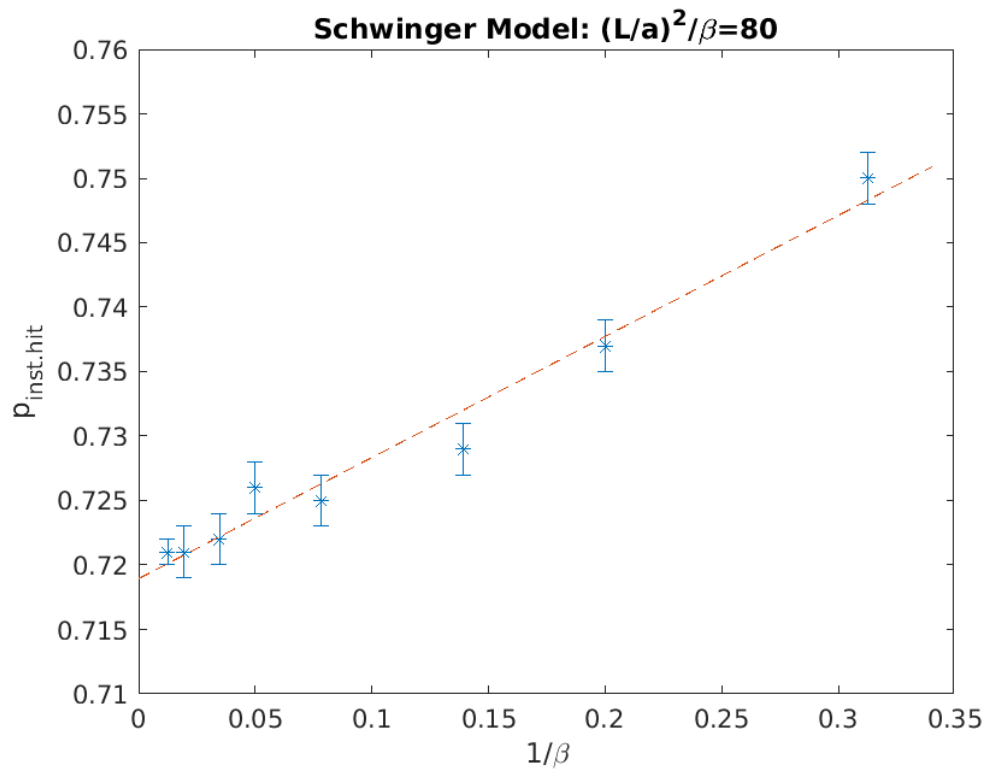
β	7.2	7.2	7.2	7.2	7.2
L/a	16	20	24	32	40
n_{stout}	1	1	1	1	1
$p_{\text{inst.hit}}$	0.597(2)	0.677(2)	0.729(2)	0.799(2)	0.838(2)

Table 2: Ensembles used in the “finite volume” study; each one contains 10 000 configurations and is used after a single step of $\rho = 0.25$ stout smearing. Also the **acceptance ratio of the instanton hit update** at the respective $(\beta, L/a)$ is given.

Schwinger Model: instanton hit updates

Updates: mixture of multi-hit Metropolis, overrelaxation, instanton-hit, parity-hit.

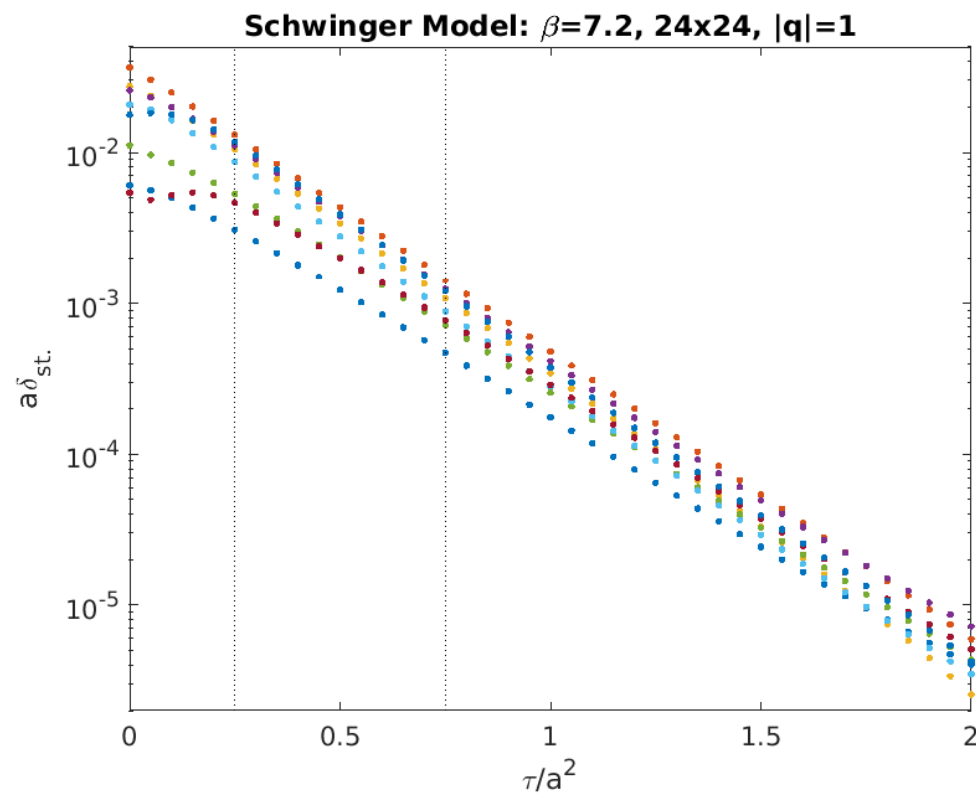
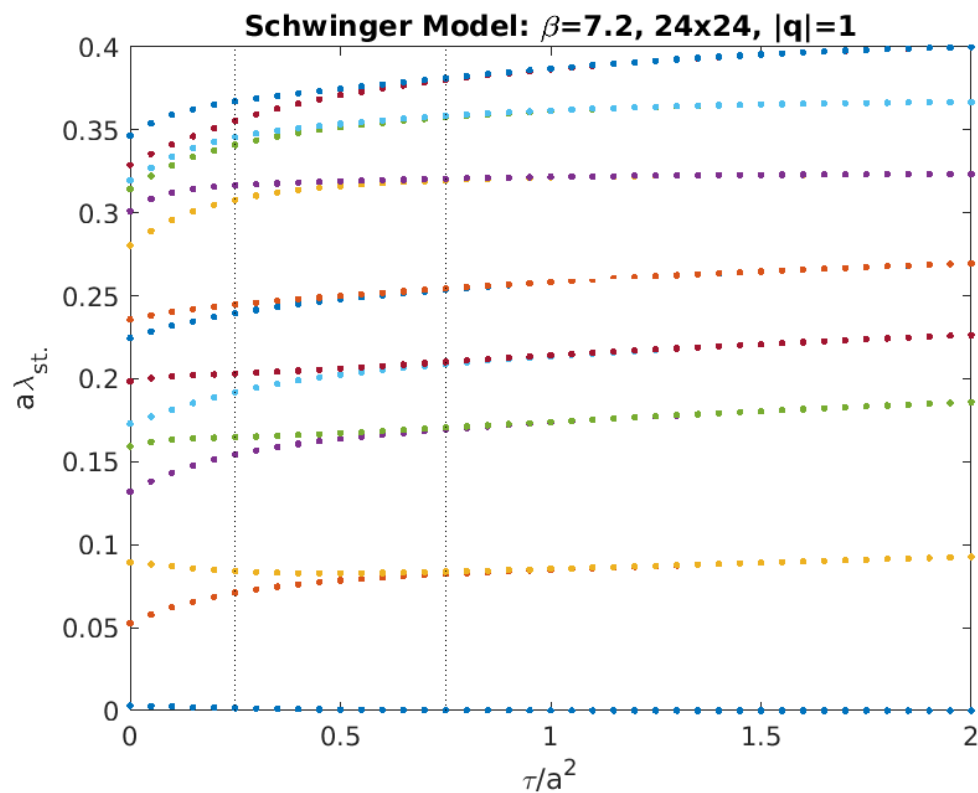
Instanton hit update: propose $U_\mu^{\text{new}}(x) = U_\mu^{\text{inst}}(x) \cdot U_\mu^{\text{old}}(x)$ with $q = \pm 1$ configuration $U_1^{\text{inst}}(x) = \exp(\mp 2\pi i x_2 / [L_1 L_2])$ and $U_2^{\text{inst}}(x) = \exp(\pm 2\pi i x_1 / L_1 \cdot \delta_{x_2, L_2})$ with $p = \frac{1}{2}$.



Instanton hit acceptance ratio in a fixed physical volume versus $1/\beta$ (from Tab. 1).
Instanton hit acceptance ratio versus a/L at fixed β (from Tab. 2).

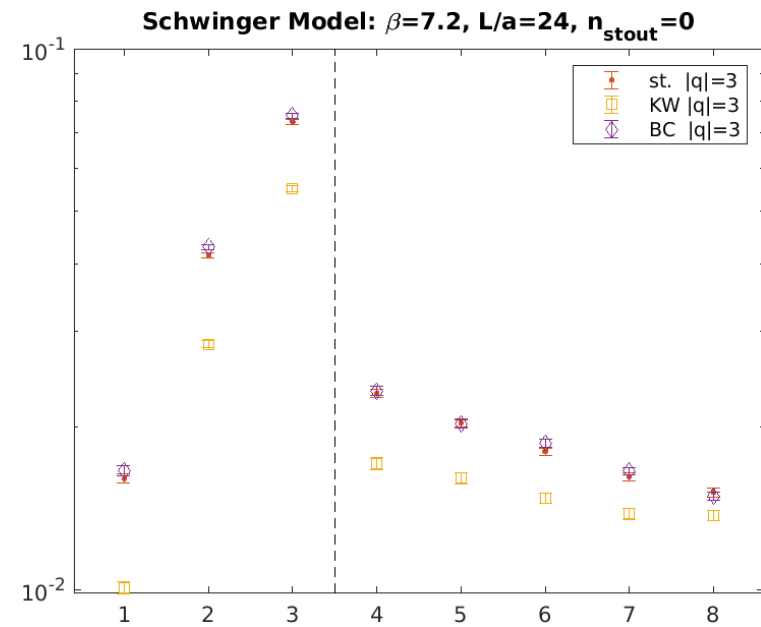
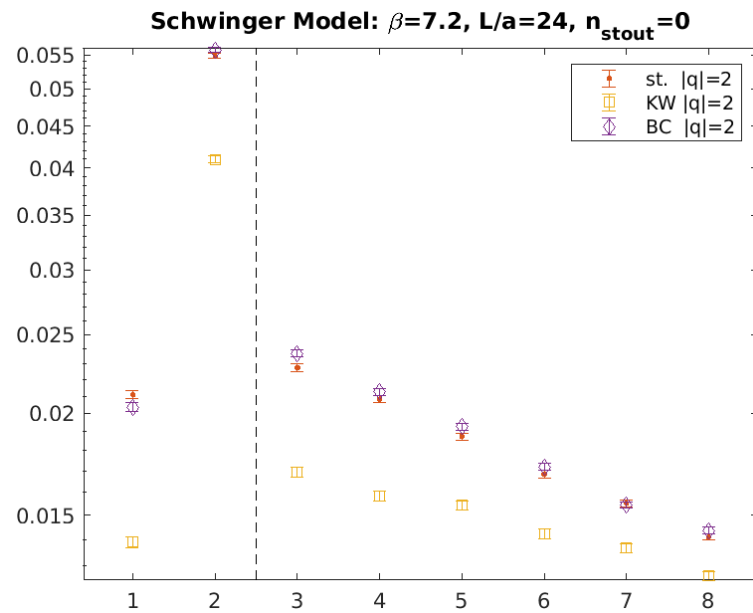
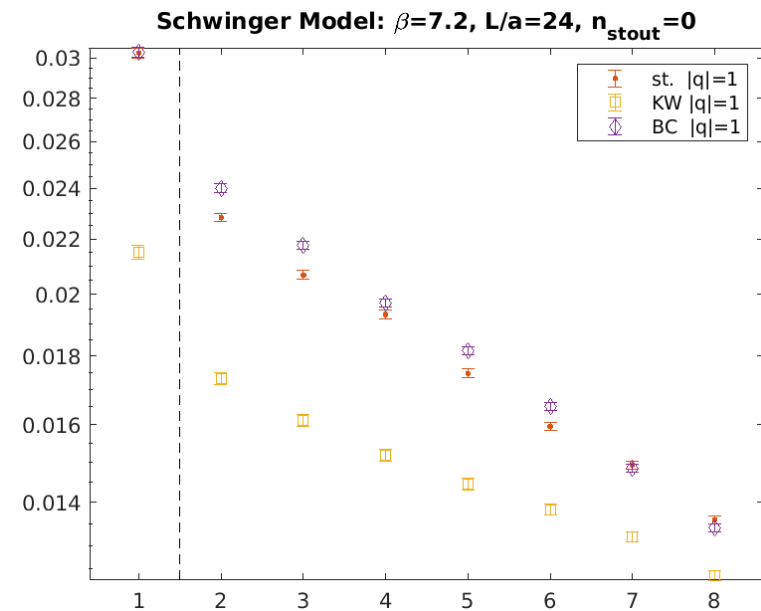
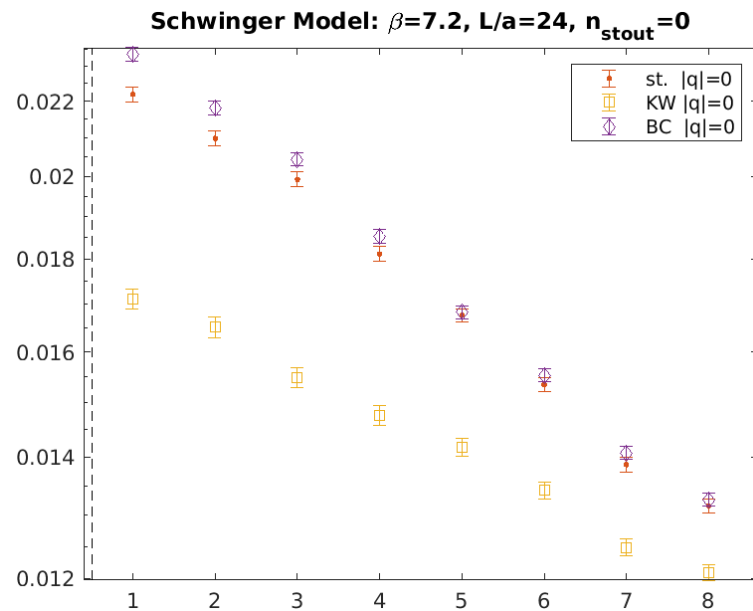
Taste splittings: $a\delta_{\text{stag}}$ under gradient flow

Eigenvalues $a\lambda_1, \dots, a\lambda_{15}$ of D_{stag}/i on a $|q|=1$ configuration at $(\beta, L/a) = (7.2, 24)$ versus gradient flow time τ/a^2 . Note that λ_1 pairs with $-\lambda_1$, while $\lambda_2 \simeq \lambda_3$ pair, and so on. Splittings defined with proper pairing: $\delta_1 = 2\lambda_1$, $\delta_2 = \lambda_3 - \lambda_2$, ... for $|q| = 1$.

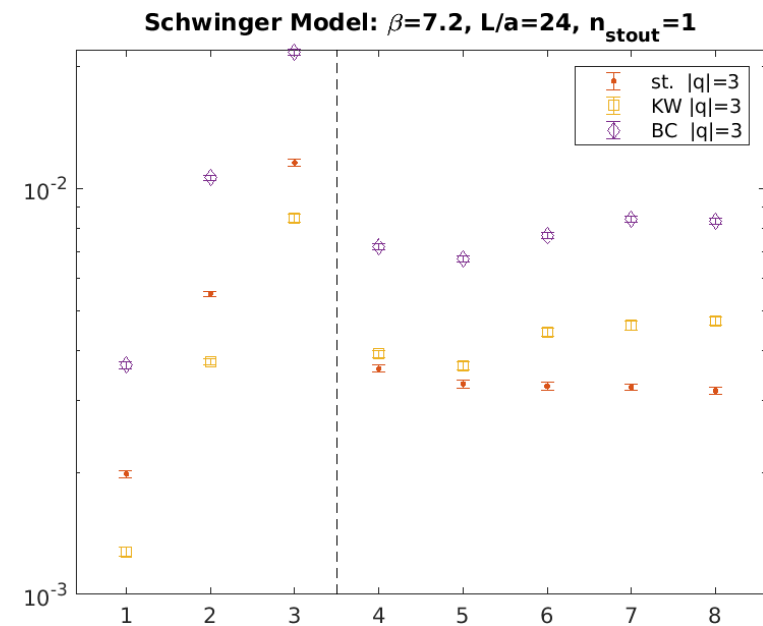
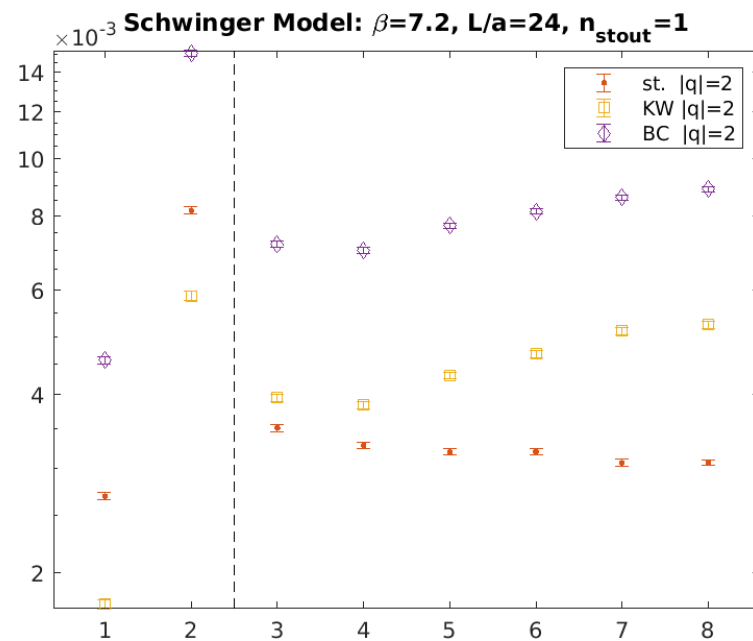
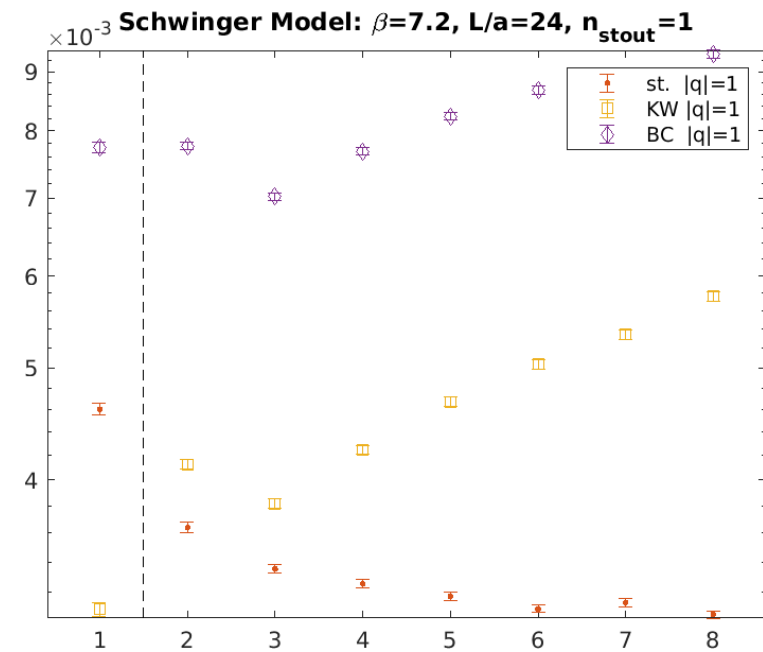
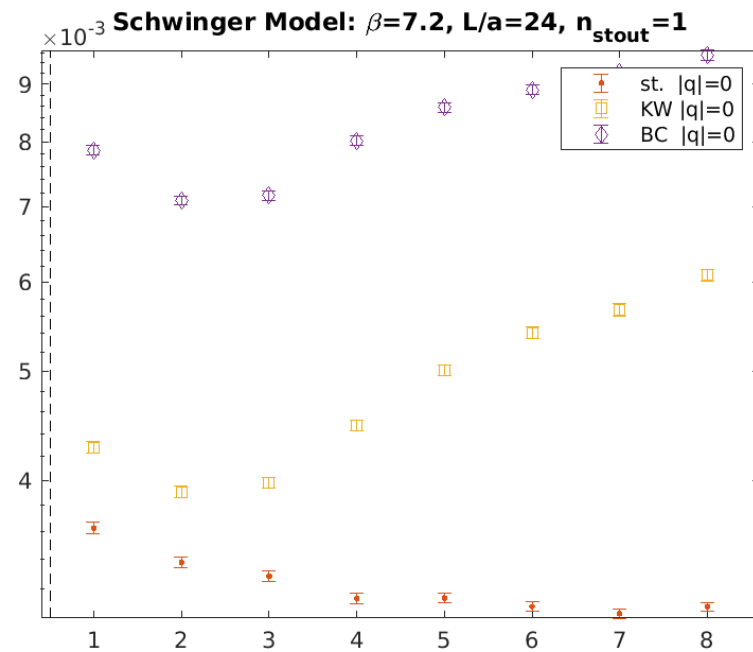


Main investigation carried out with $n = 0, 1, 3$ steps of $\rho = 0.25$ stout smearing, paramount to $\tau/a^2 = 0, 0.25, 0.75$ (up to small discretization effects).

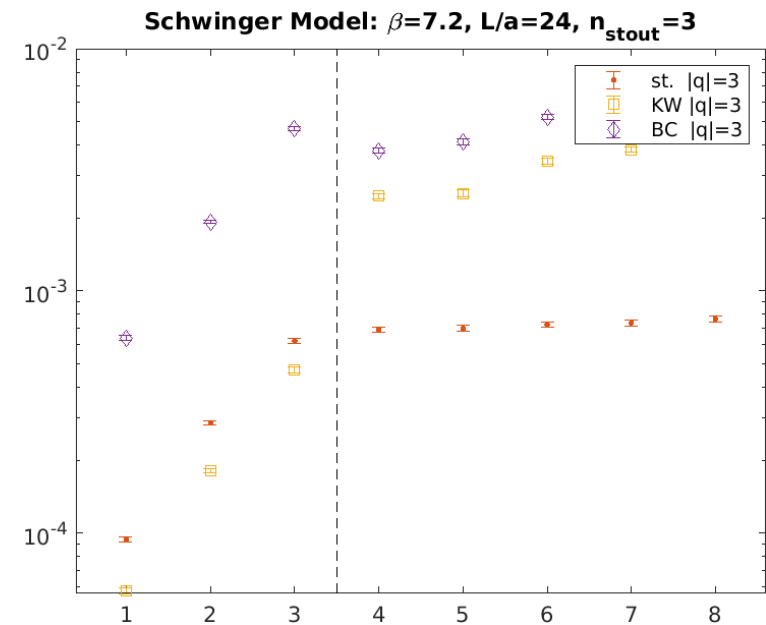
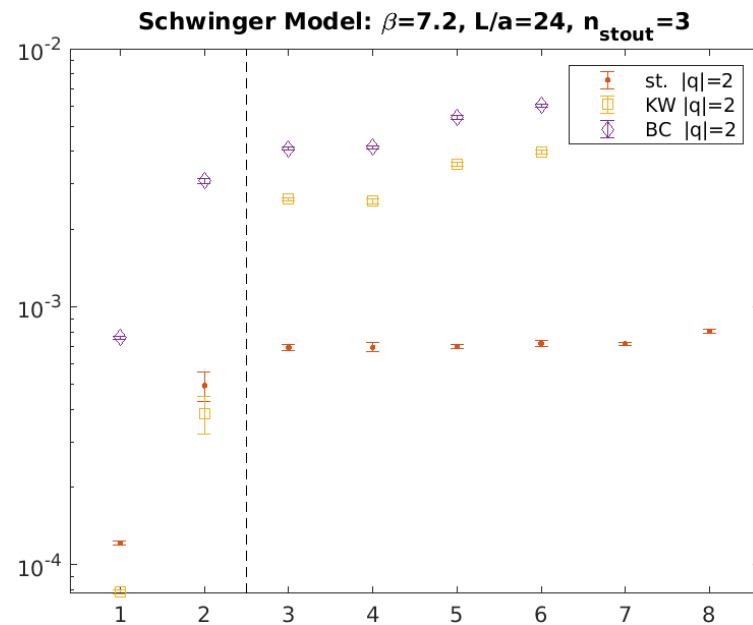
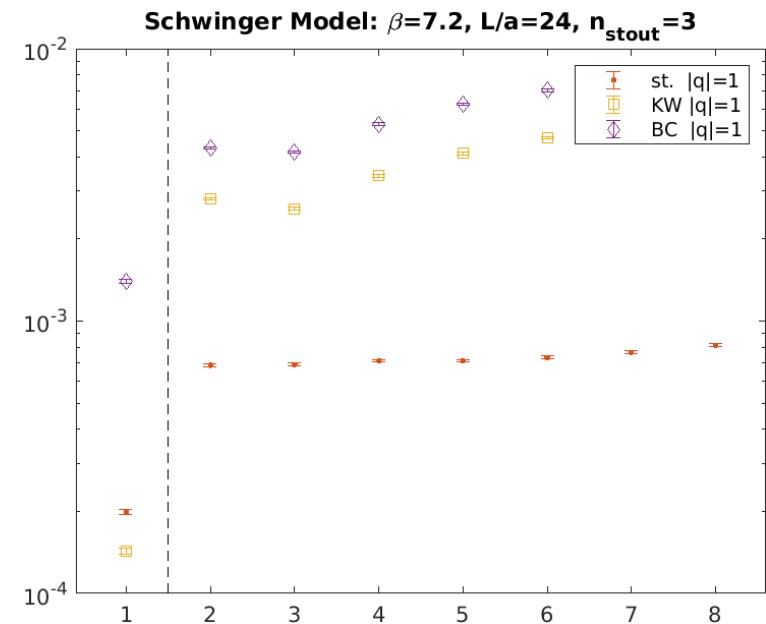
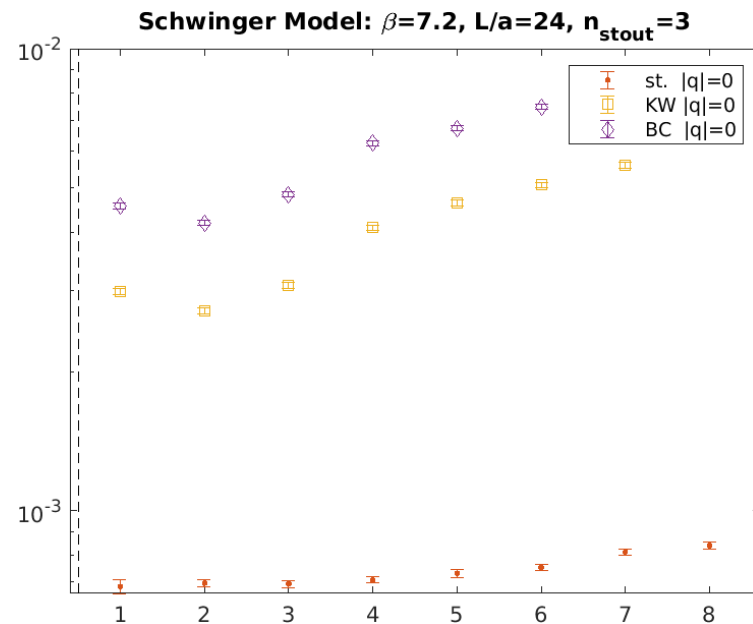
Taste splittings: $a\delta$ on “central ensemble” with $n_{\text{stout}} = 0$



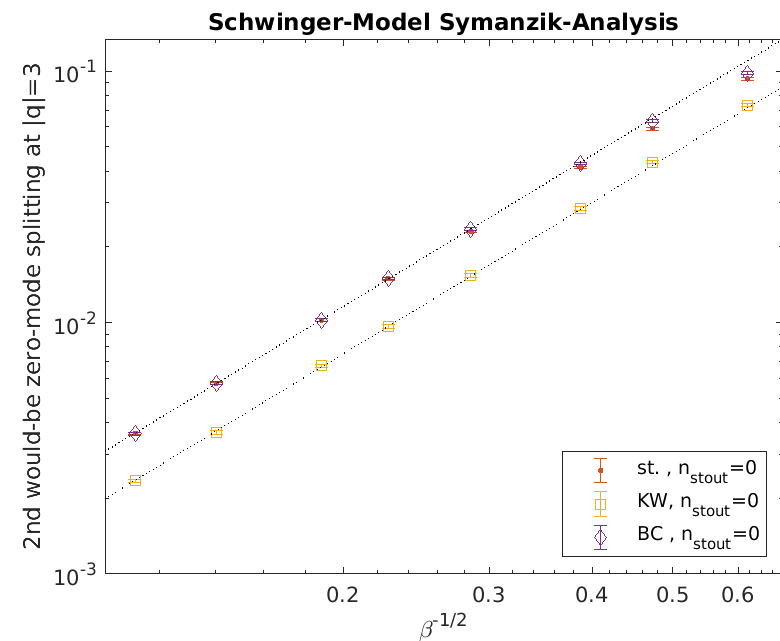
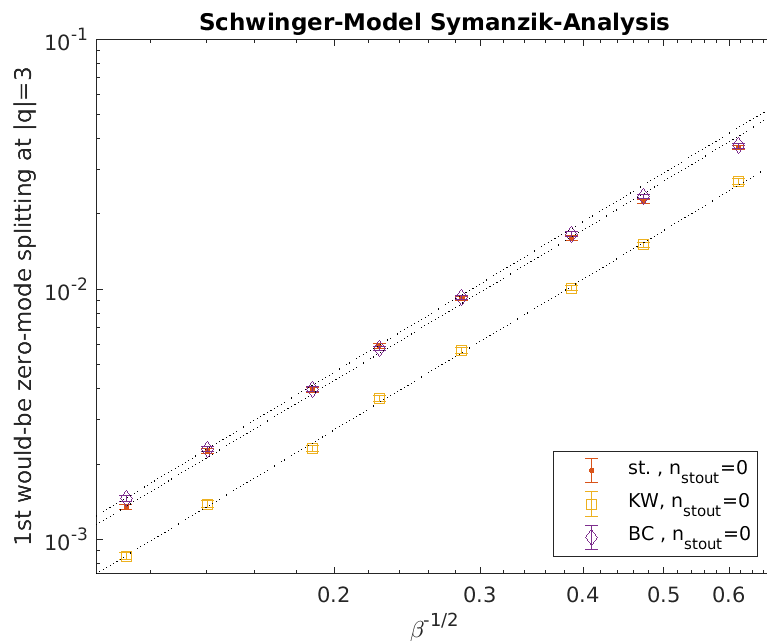
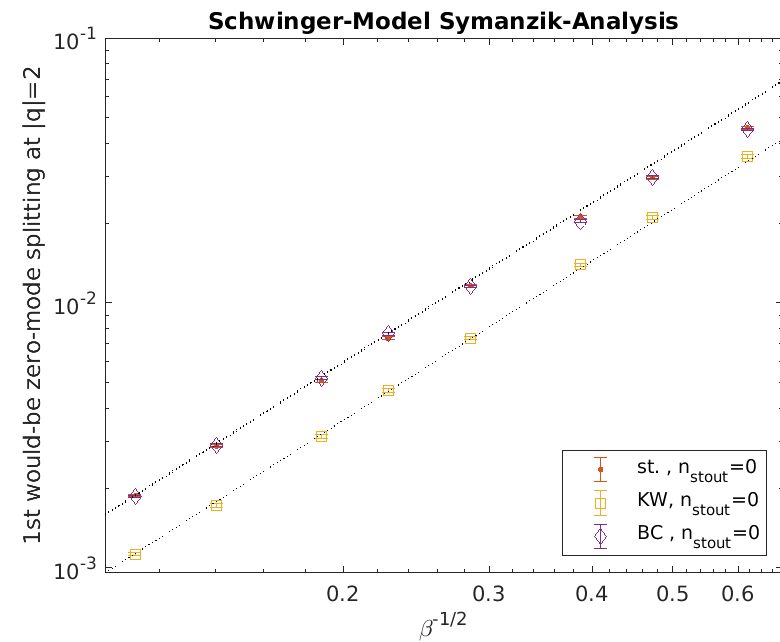
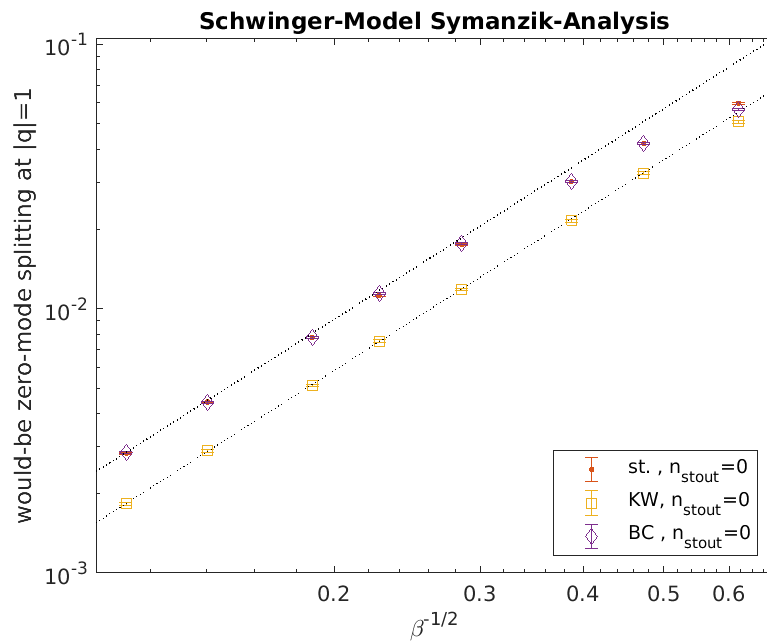
Taste splittings: $a\delta$ on “central ensemble” with $n_{\text{stout}} = 1$



Taste splittings: $a\delta$ on “central ensemble” with $n_{\text{stout}} = 3$

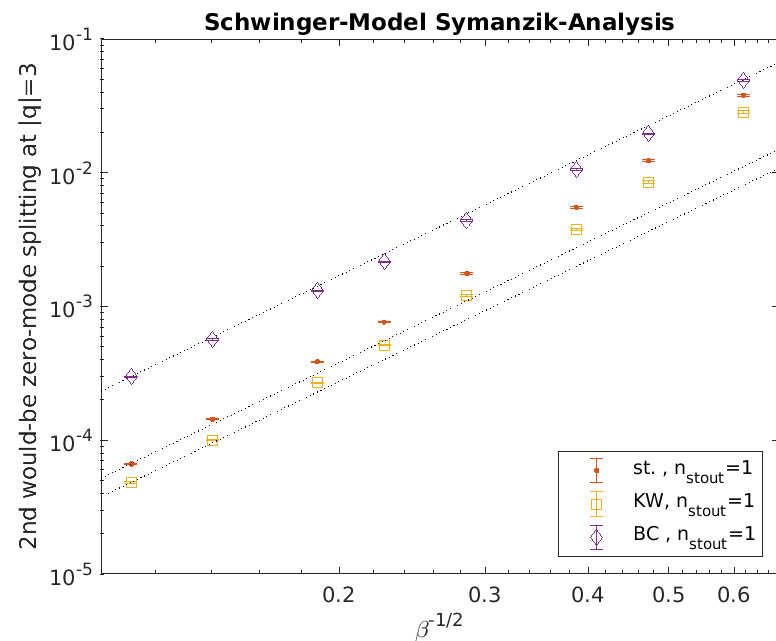
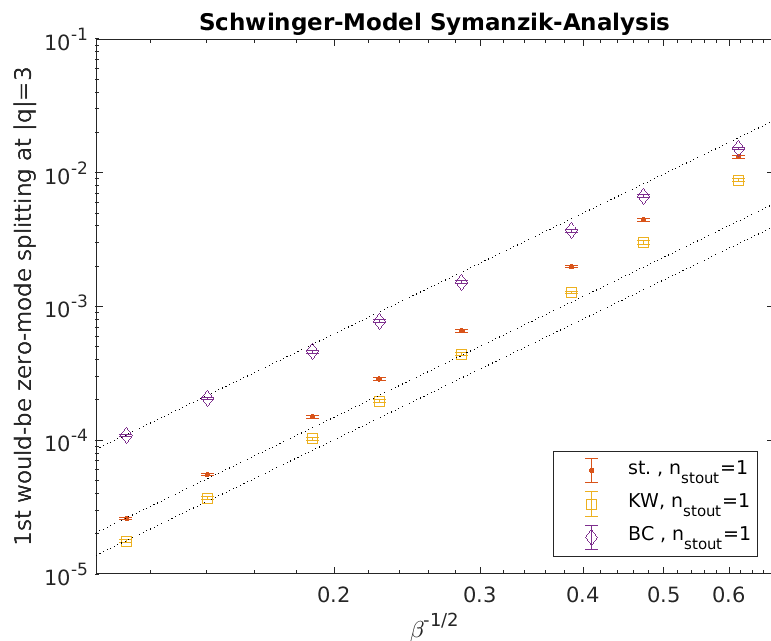
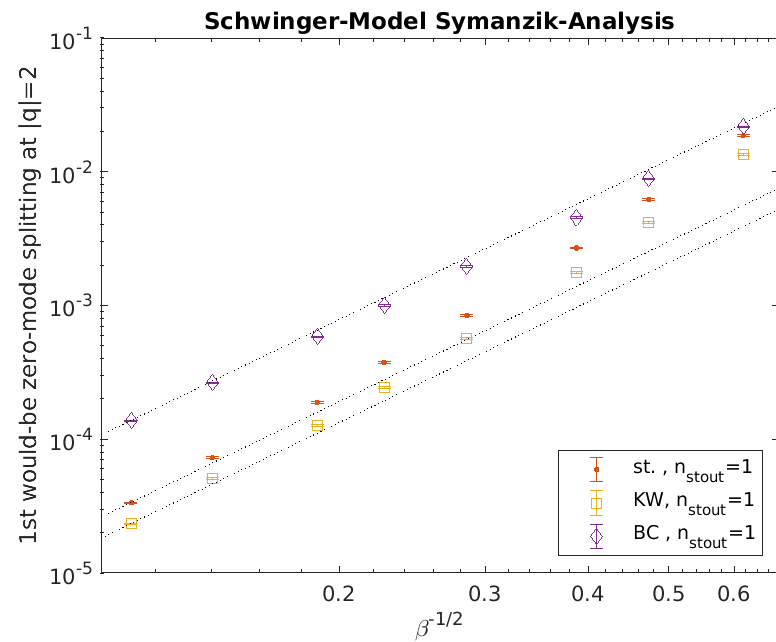
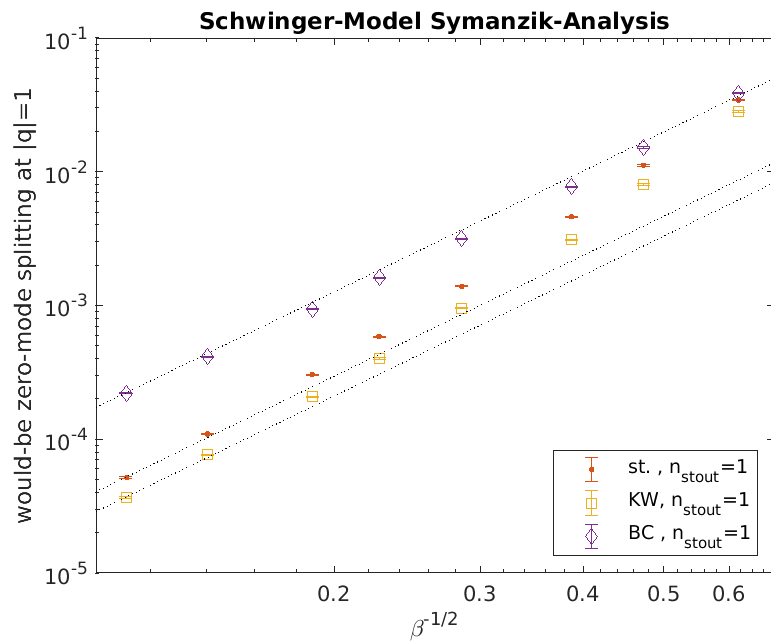


Taste splittings: Symanzik scaling test for $n_{\text{stout}} = 0$



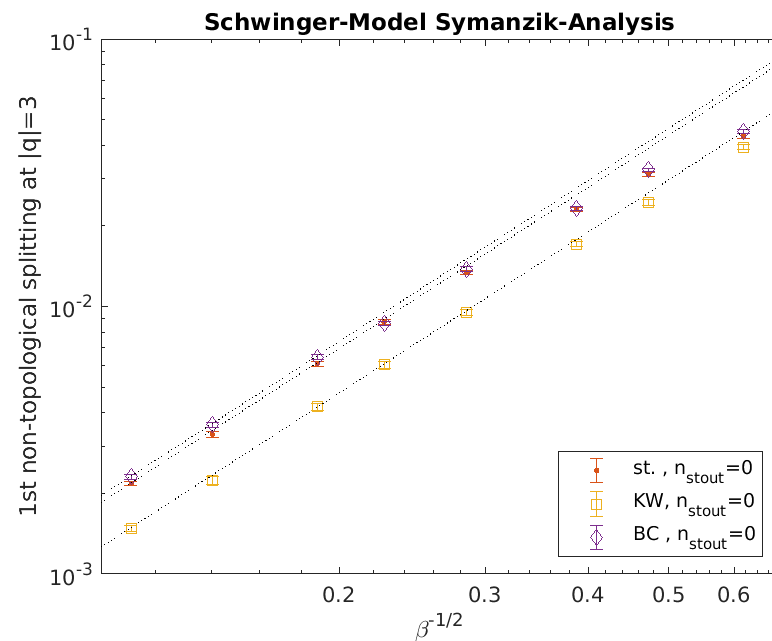
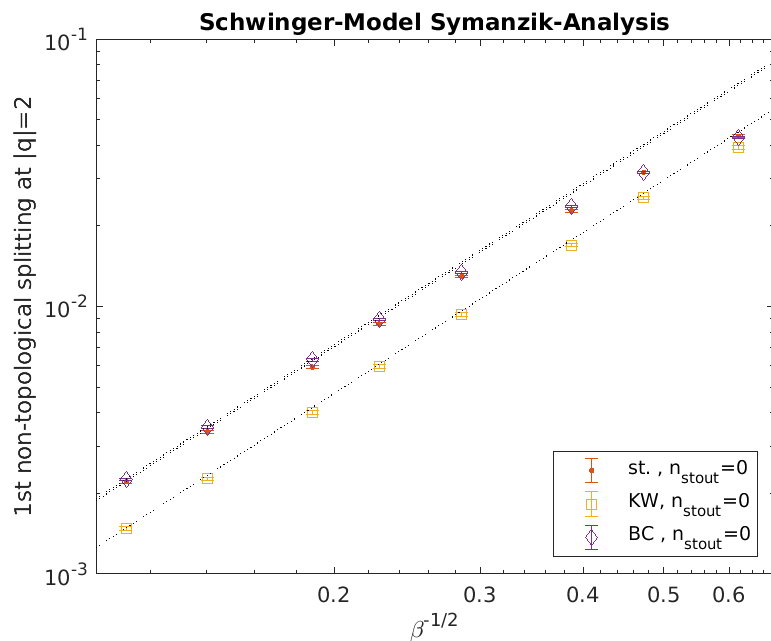
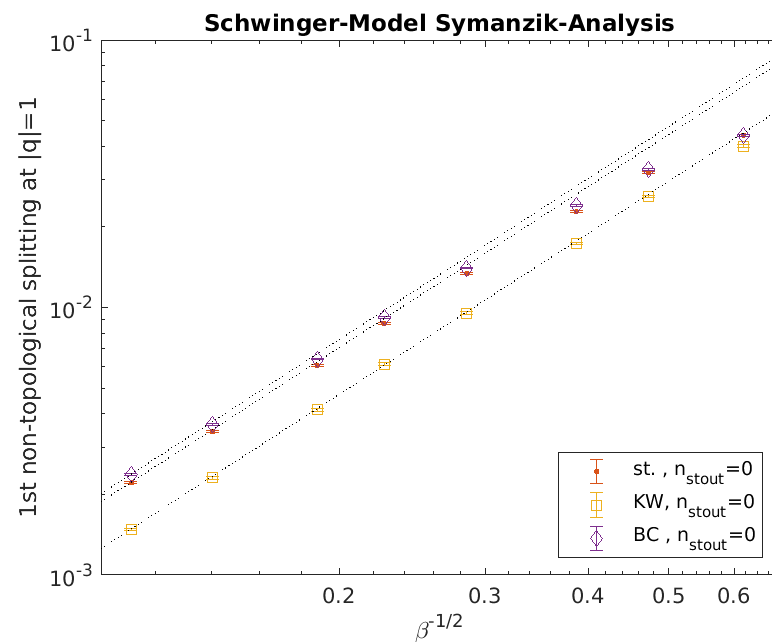
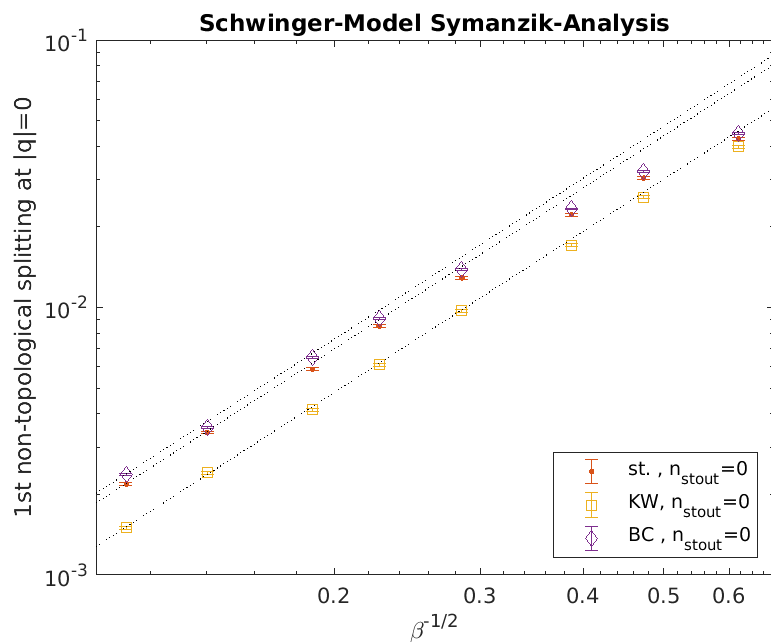
all a^2

Taste splittings: Symanzik scaling test for $n_{\text{stout}} = 1$



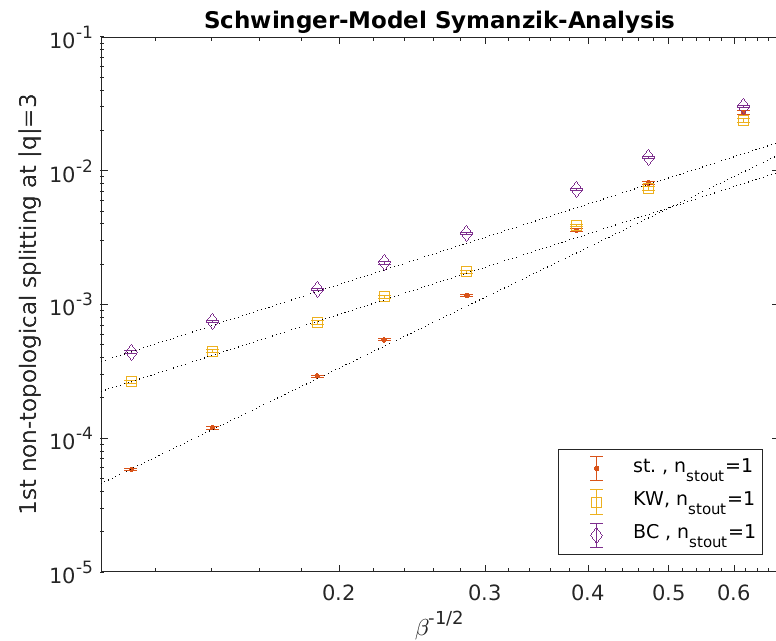
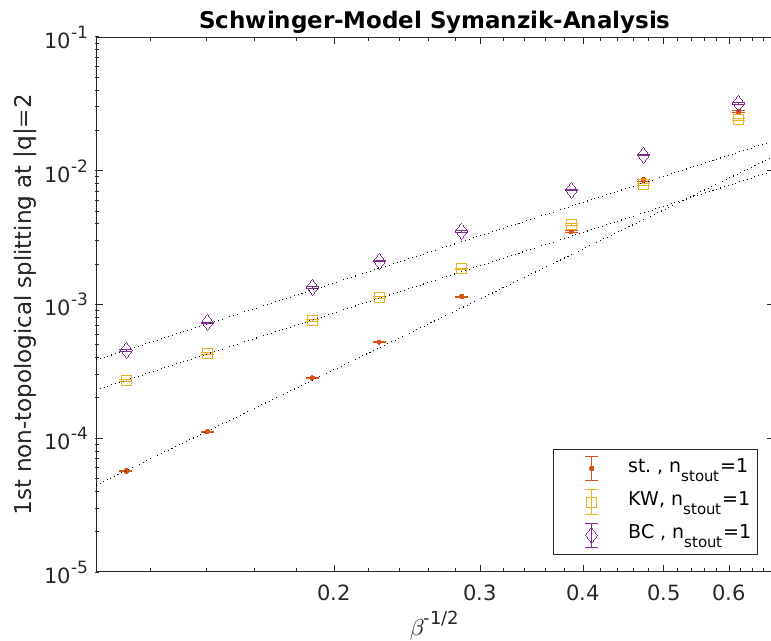
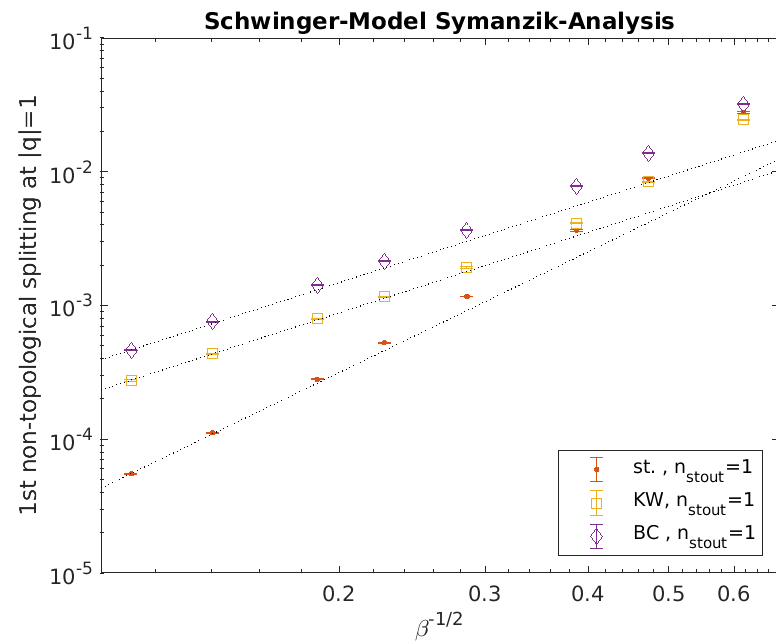
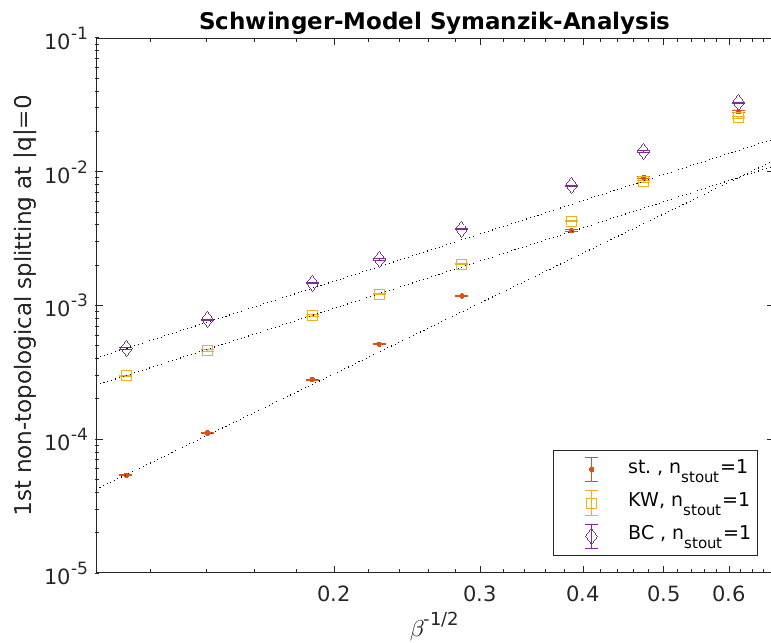
all a^3

Taste splittings: Symanzik scaling test for $n_{\text{stout}} = 0$



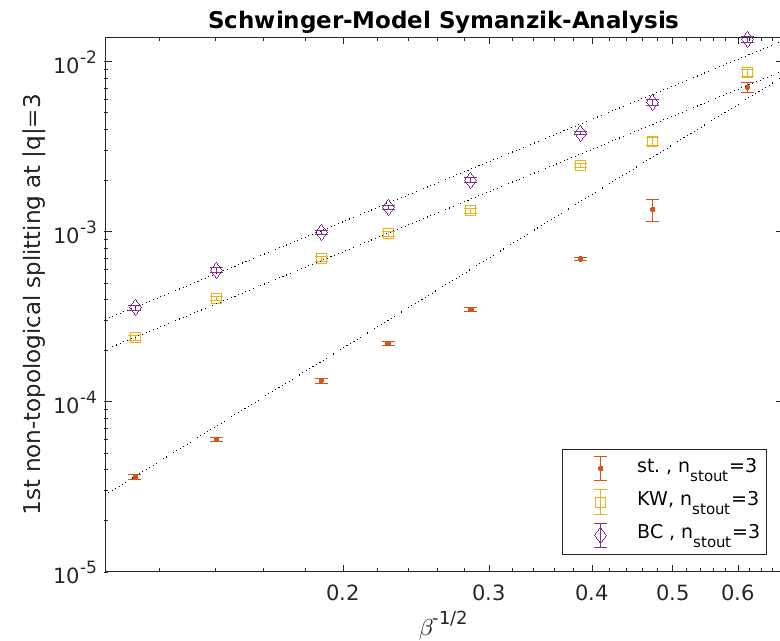
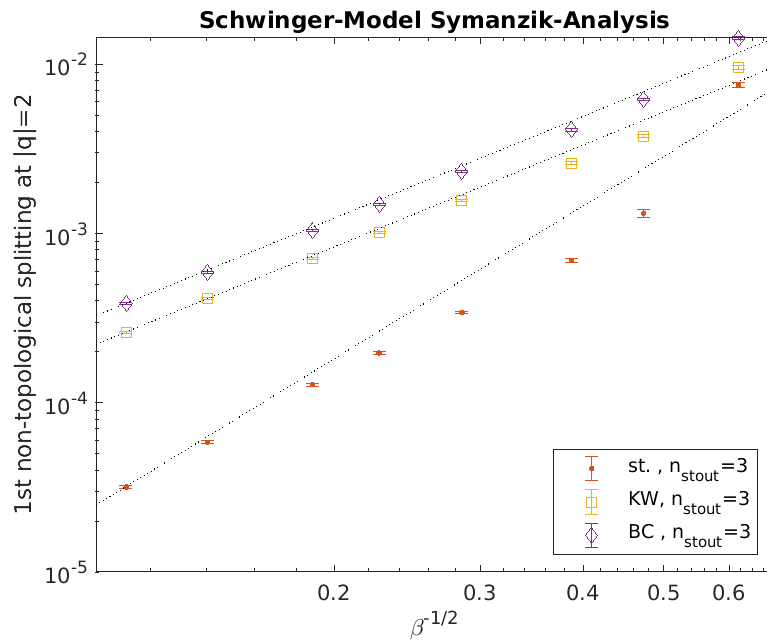
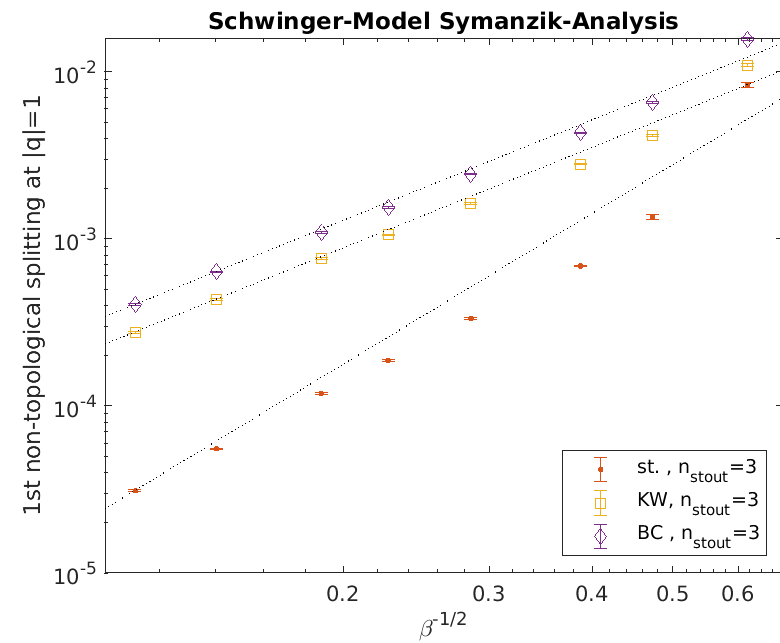
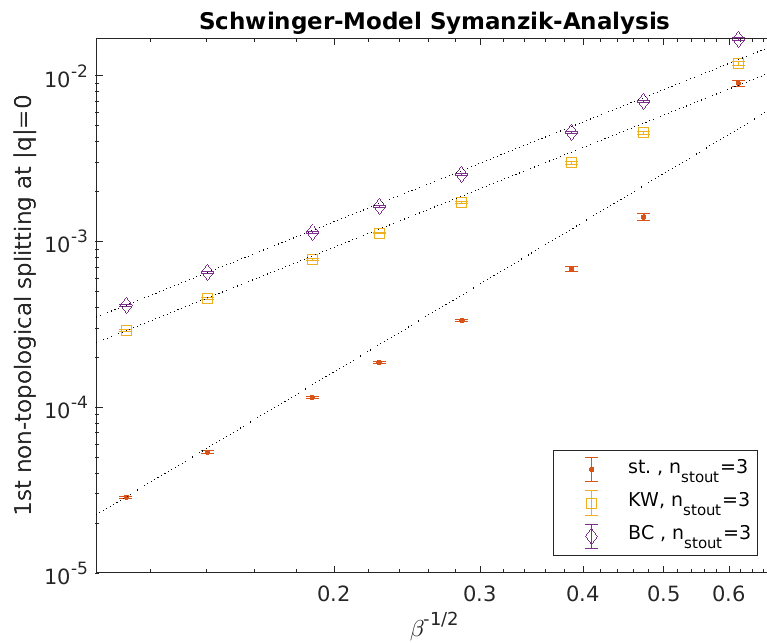
all a^2

Taste splittings: Symanzik scaling test for $n_{\text{stout}} = 1$



$a^{2,3}$

Taste splittings: Symanzik scaling test for $n_{\text{stout}} = 3$



$a^{2,3}$

Summary

In 2D all of $D_{\text{stag}}, D_{\text{KW}}, D_{\text{BC}}$ are minimally doubled, since on any $U_{|q|=1}$ one finds

- 2 would-be zero-modes of D_{stag} with *opposite chiralities* (invisible to ϵ)
- 2 would-be zero-modes of D_{KW} with *opposite chiralities* (invisible to γ_5)
- 2 would-be zero-modes of D_{BC} with *opposite chiralities* (invisible to γ_5)
- 1 would-be zero-mode of D_{wils} with *correct chirality*

but in all cases appropriate chirality operators can be defined [arXiv:2203.15699].

In quenched SM intra-taste splittings $\delta_{\text{stag}}, \delta_{\text{KW}}, \delta_{\text{BC}}$ are measured over a wide range of β , considering would-be-zero modes and non-topological modes separately.

Power $\#$ in Symanzik scaling law $\delta \propto a^\#$ depends on would-be zero mode (“wbz”) versus non-topological mode (“ntm”) and/or smearing level (which is disturbing):

	$n_{\text{stout}} = 0$		$n_{\text{stout}} = 1, 3$	
	wbz	ntm	wbz	ntm
$\delta_{\text{stag}} \propto a^\#$	1	1	2	2
$\delta_{\text{KW}} \propto a^\#$	1	1	2	1
$\delta_{\text{BC}} \propto a^\#$	1	1	2	1

Any clue ?

Please let
us know !