Nonperturbative Matching of Hamiltonian and Lagrangian Simulations

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1. Motivation

- Testing universality: Lagrangian and Hamiltonian simulations should lead to the same result
- Matching: continuum limit in time direction on the Lagrangian side
- Measurement of the renormalized anisotropy of a lattice, needed for nonperturbative matching

2. Setup

- Anisotropic Wilson-Action: $S = \beta \frac{1}{\xi_{\text{input}}} \sum_{r,\nu} P_{0\nu} + \beta \xi_{\text{input}} \sum_{r,\nu>\mu>0} P_{\mu\nu}$
- Anisotropy $\xi_{\text{ren}} = \frac{a_t}{a_s}$ on a $L^2 \times L/\xi_{\text{input}}$ lattice, take limit $\xi_{\text{ren}} \to 0$
- $\xi_{\text{input}} \neq \xi_{\text{ren}} \Rightarrow$ determine ξ_{ren} with static potential , in $(2+1)$D: $V(r) = a + \sigma \cdot r + b\ln(r)$
- U(1) configurations generated with Metropolis-Hastings MCMC

3. Normal or Sideways Potential

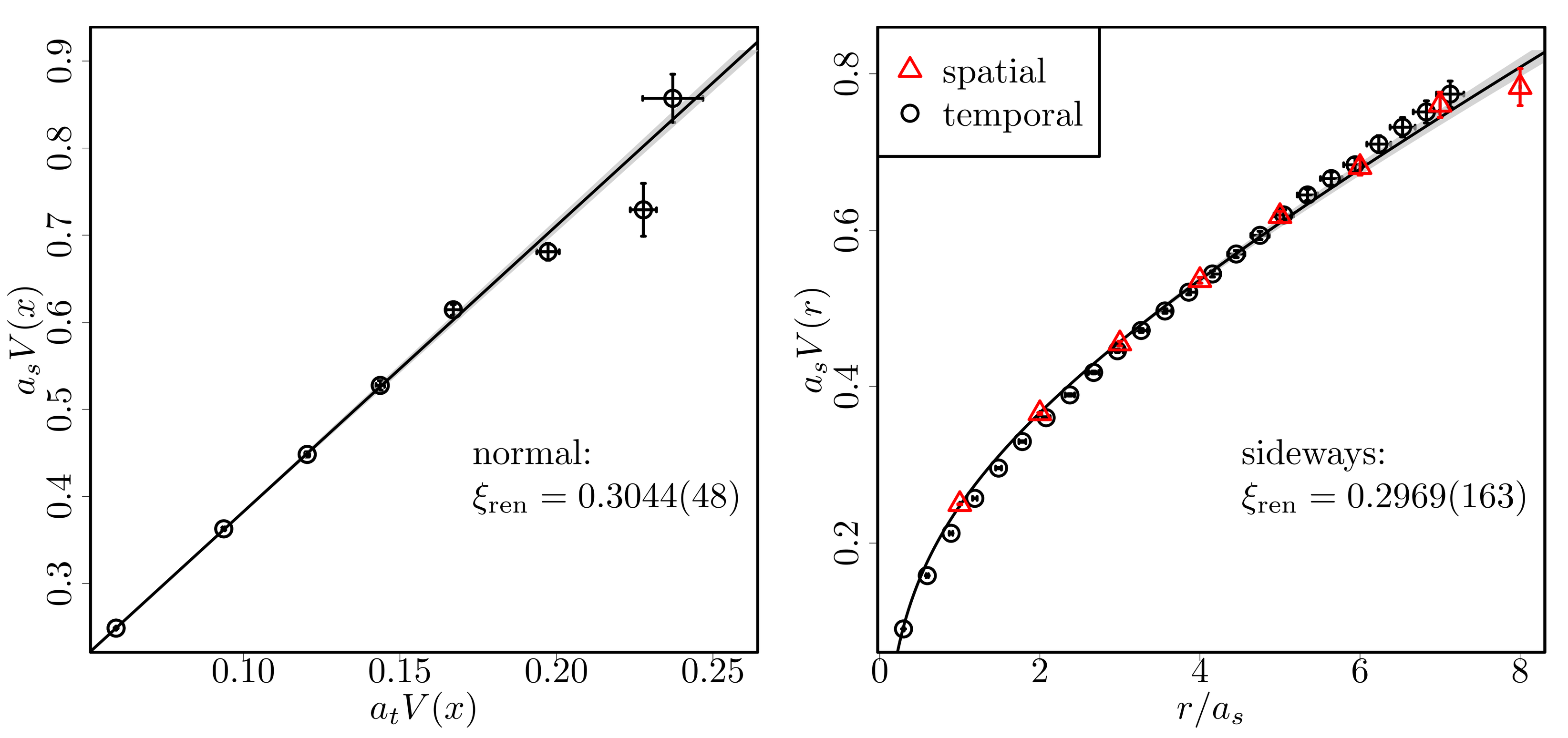

Figure 1: Fit of potentials at $\beta = 1.7, L = 16, \xi_{\text{input}} = \frac{1}{3}$

$\lim_{t\to\infty} \frac{W(x,t+1)}{W(x,t)} = \exp(-a_t V(x[a_s]))$

fit $a_s V(x[a_s]) = \frac{1}{\xi_{\text{ren}}} a_t V(x[a_s]) + se$

Sensitive to values at small x

$\lim_{x\to\infty} \frac{W(x+1,t)}{W(x,t)} = \exp(-a_s V(t[a_t]))$

Linear interpolation of $a_s V(t[a_t])$

$\forall y$ demand $V(1/\xi_{\text{ren}} \cdot y[a_s]) = V(t[a_t])$

Sensitive to fluctuations at large y

4. Naive Continuum Limit

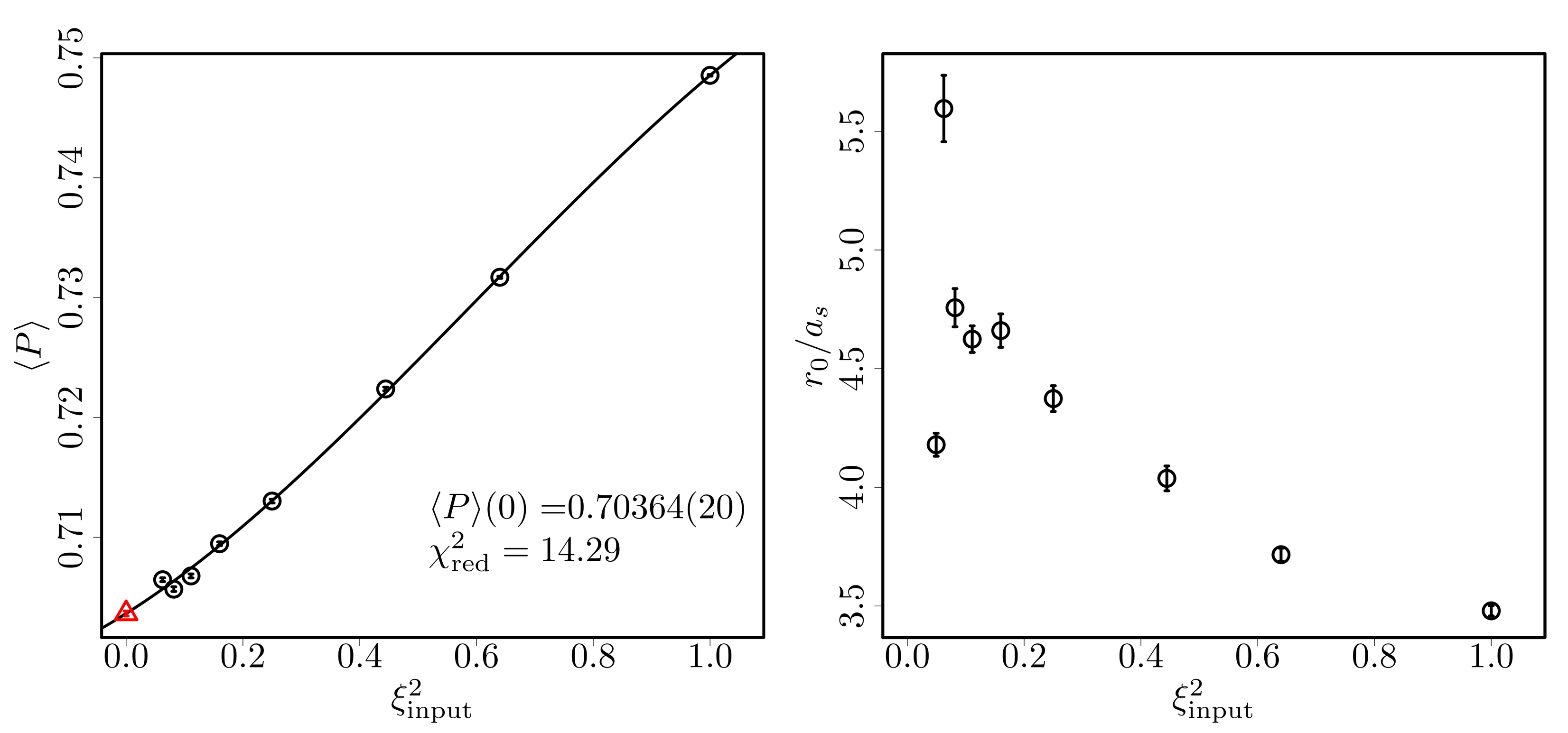

Figure 2: Naive continuum limit at $\beta = 1.7, L = 16$ for the plaquette and the Sommer parameter

- β =const., $\xi_{\text{input}} \to 0$, cubic fit of $\langle P\rangle$ in ξ^2_{input}
- $\langle P\rangle$ not monotonous, spatial lattice size changes $\Rightarrow$ Renormalization of (β, ξ) needed
- U(1): Analytical or perturbative $(\beta,\xi)_{\text{ren}}(\xi_{\text{input}})$ not known
- Keep a_s fixed with Sommer parameter r_0: $-r^2 \frac{\mathrm{d}}{\mathrm{d}r} V(r)|_{r=r_0} = c = -1.65$, choice of c: QCD + logarithm

5. Renormalized Continuum Limit

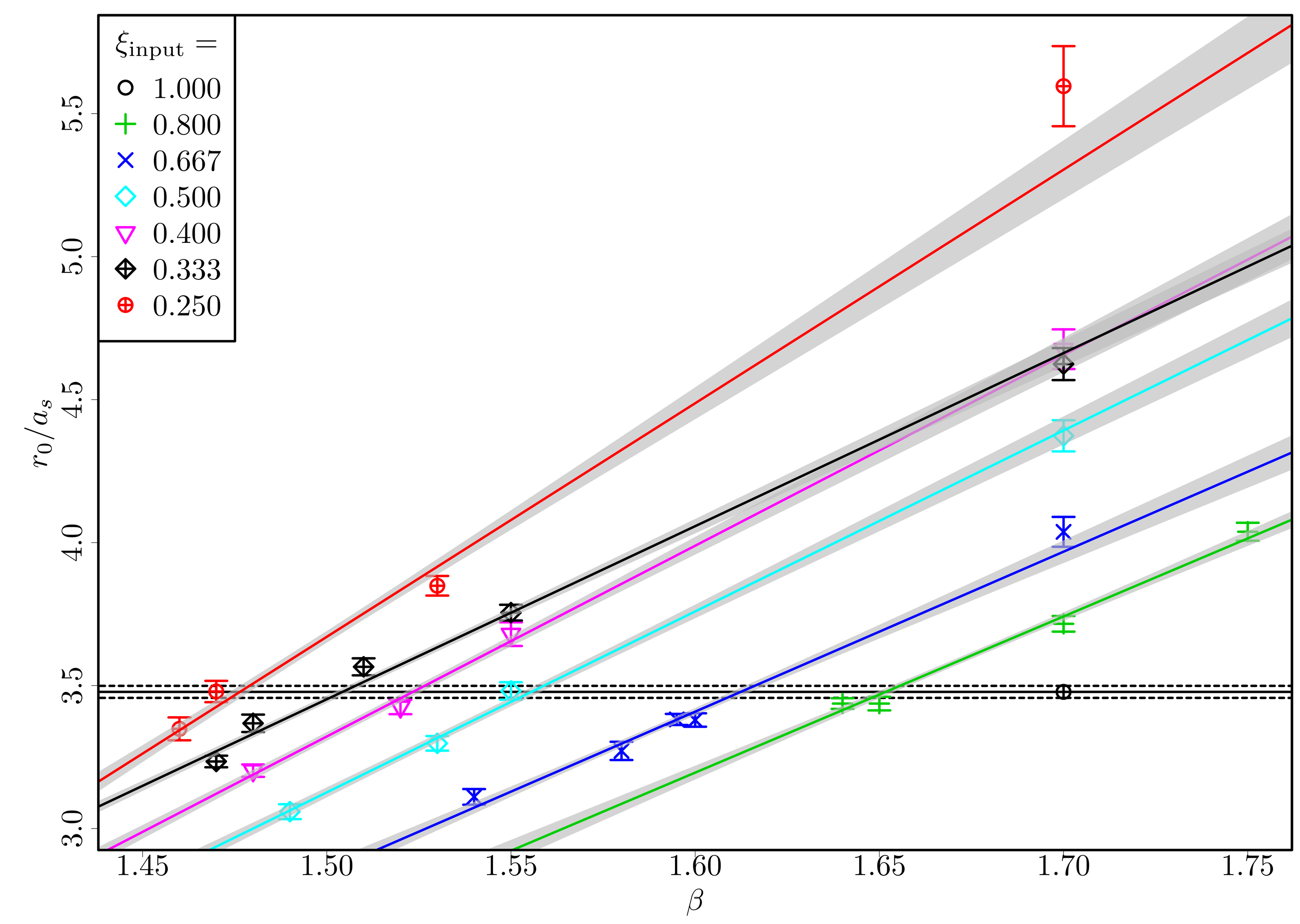

Figure 3: Determining β numerically such that $r_0/a_s(\beta,\xi)$ is constant, normal potentials

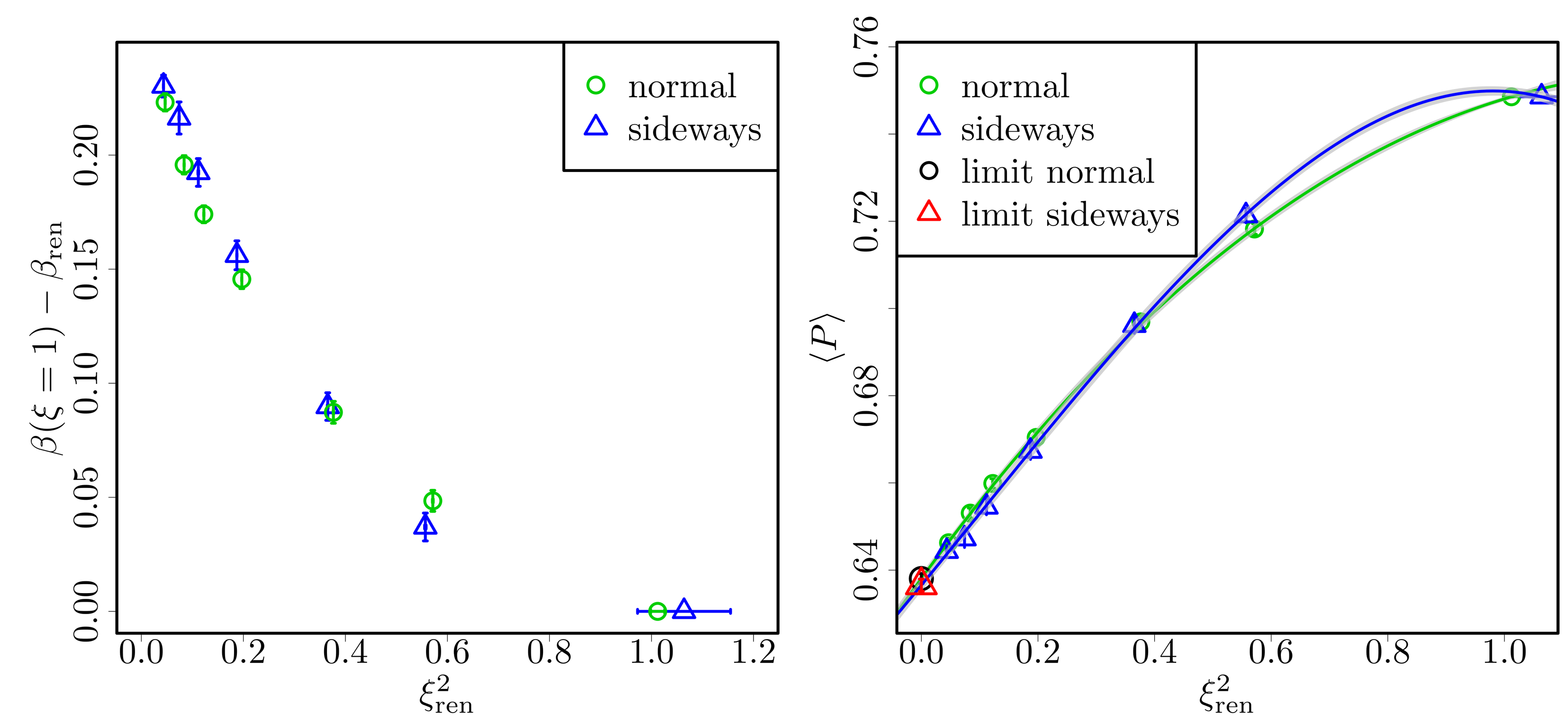

Figure 4: Change needed in β_{input} and renormalized continuum limit of the plaquette

- For each ξ_{input}, take measurements at several β and determine $r_0(\beta), \langle P\rangle(\beta), \xi_{\text{ren}}(\beta)$
- With linear interpolation, get $\beta_{\text{ren}} : r_0(\beta_{\text{ren}}) = r_0(\xi = 1)$
- Continuum limit in plaquette: cubic fit to $\langle P\rangle_{\beta_{\text{ren}}}$ in $\xi^2_{\text{ren},\beta_{\text{ren}}}$
- normal: $\langle P\rangle(0) = 0.6381(12)$, $\chi^2_{\text{red}} = 0.27$, sideways: $\langle P\rangle(0) = 0.6365(14)$, $\chi^2_{\text{red}} = 0.34$

6. Outlook

- Small volume limit to match volume in Hamiltonian computations
- Determine ξ_{ren} with the gradient flow also at smaller β
- Other observables for matching, e. g. slope at $V(1), V(\sqrt{2})$

References

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