Massless Schwinger model with a 4-fermi-interaction at topological angle $\theta = \pi$

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The system in standard representation

Partition sum

$$Z = \int \mathcal{D}[A]\, B_{\beta,\theta}[A] \int \mathcal{D}[\overline{\psi}, \psi]\, e^{-S_F[\overline{\psi},\psi,A]}$$

Villain Boltzmann factor with topological term

$$B_{\beta,\theta}[A] = \sum_{\{n\}} \exp\left(-\frac{\beta}{2} \sum_x \Big((\mathrm{d}A)_x + 2\pi n_x \Big)^2 - i\theta \underbrace{\sum_x n_x}_{Q \in \mathbb{Z}} \right)$$

Fermionic action

$$S_F\left[\overline{\psi}, \psi, A\right] = \frac{1}{2} \sum_{x,\mu} \gamma_{x,\mu} \Big(e^{iA_{x,\mu}} \overline{\psi}_x \psi_{x+\hat{\mu}} - e^{-iA_{x,\mu}} \overline{\psi}_{x+\hat{\mu}} \psi_x \Big) - \frac{J}{4} \sum_{x,\mu} \overline{\psi}_x \psi_x \overline{\psi}_{x+\hat{\mu}} \psi_{x+\hat{\mu}}$$

Charge conjugation symmetry

$$Z = \int \mathcal{D}[A]\, B_{\beta,\theta}[A] \int \mathcal{D}[\overline{\psi}, \psi]\, e^{-S_F[\overline{\psi},\psi,A]}$$

Charge conjugation: $A_{x,\mu} \longrightarrow -A_{x,\mu}$ $\quad n_x \longrightarrow -n_x$

$\psi \longleftrightarrow \overline{\psi}$ $\quad S_F \longrightarrow S_F$

In general: $B_{\beta,\theta} \not\longrightarrow B_{\beta,\theta}$ $\quad Z \not\longrightarrow Z$

For $\theta = 0$ and $\theta = \pi$: $B_{\beta,\theta} \longrightarrow B_{\beta,\theta}$ $\quad Z \longrightarrow Z$ Exact symmetry!

$$\boxed{\textbf{Is the } \theta = \pi \textbf{ symmetry broken for specific values of the quartic coupling } J\textbf{?}}$$

System in dual-formulation

Complex action problem in standard representation: $B_{\beta,\theta}[A] = \sum K\, e^{i\theta Q}$

Exact dual-representation solves the complex action problem:

$$Z = \left(\frac{1}{\sqrt{32\pi\beta}}\right)^V \sum_{\{l,d,p\}} C[l,d,p] \;\; (1+J)^{\sum_{x,\mu} d_{x,\mu}} \;\; e^{-\frac{1}{2\beta}\sum_x \left(p_x + \frac{\theta}{2\pi}\right)^2}$$

where

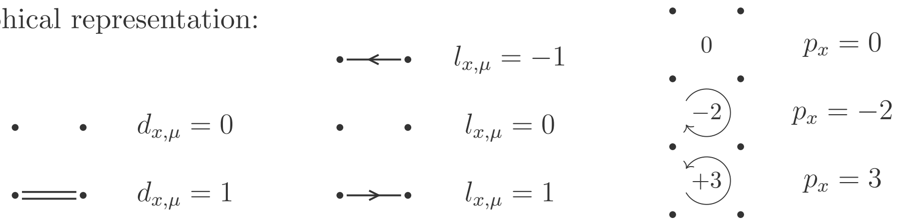
$$\begin{array}{lll} l_{x,\mu} & \in \{-1,0,1\} & \ldots\ \textit{loop variables} \text{ living on links} \\ d_{x,\mu} & \in \{0,1\} & \ldots\ \textit{dimer variables} \text{ living on links} \\ p_x & \in \mathbb{Z} & \ldots\ \textit{plaquette occupation numbers} \text{ living on plaquettes} \end{array}$$

Charge conjugation:

$$l_{x,\mu} \longrightarrow -\,l_{x,\mu} \qquad d_{x,\mu} \longrightarrow d_{x,\mu} \qquad p_x \to \begin{cases} -p_x & \text{for } \theta = 0 \\ -p_x - 1 & \text{for } \theta = \pi \end{cases}$$

Graphical representation and constraints

Graphical representation:

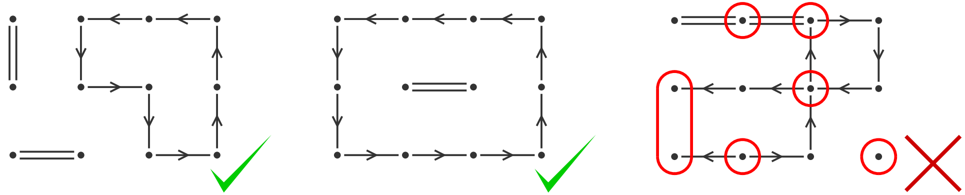


Constraints:

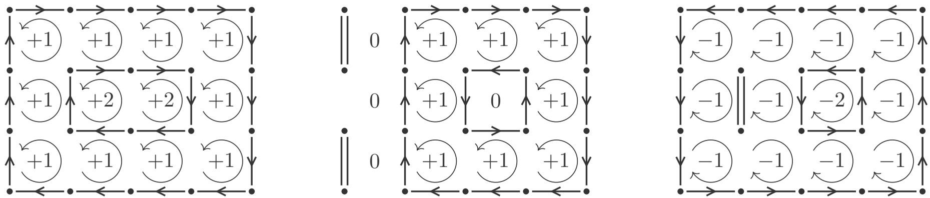
- Every site must be either:
  - endpoint of a single dimer or
  - run through (in and out) by a single loop
- Plaquette occupation numbers must compensate flux of loops

Graphical representation and constraints

Dimers/loops:

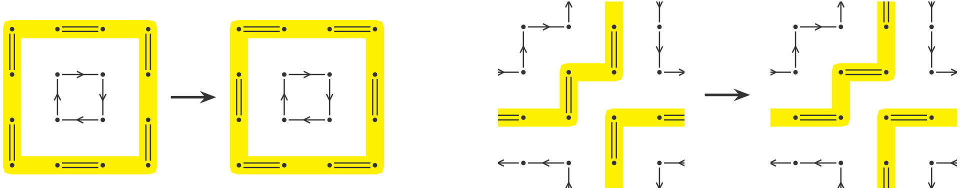


Plaquette occupation numbers:

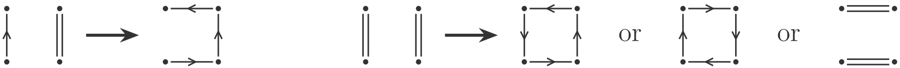


Update schemes

- Constant shift of all plaquette occupation numbers
- Worms of dimer chains with inversion of dimer occupation



- Local Transformations on plaquettes to shrink or expand loops



Observables and their dual representation

Topological charge:

$$\langle q \rangle = -\frac{1}{V}\frac{\partial}{\partial\theta}\ln Z = \frac{1}{V}\left\langle \sum_x n_x \right\rangle$$

$$\longrightarrow \frac{1}{2\pi\beta V}\left\langle \sum_x \left(p_x + \frac{\theta}{2\pi}\right)\right\rangle$$

$$\chi_q = \frac{\partial}{\partial\theta}\langle q \rangle = \frac{1}{V}\left\langle \left[\sum_x n_x - \left\langle \sum_x n_x \right\rangle\right]^2 \right\rangle$$

$$\longrightarrow \frac{1}{4\pi^2\beta}\left[1 - \frac{1}{V\beta}\left\langle \left[\sum_x \left(p_x + \frac{\theta}{2\pi}\right) - \left\langle \sum_x \left(p_x + \frac{\theta}{2\pi}\right)\right\rangle\right]^2 \right\rangle\right]$$

Observables and their dual representation

Expectation value of the gauge action density:

$$\langle F^2 \rangle = -\frac{2}{V}\frac{\partial}{\partial\beta}\ln Z = \frac{1}{V}\left\langle \sum_x \Big((\mathrm{d}A)_x + 2\pi n_x\Big)^2 \right\rangle$$

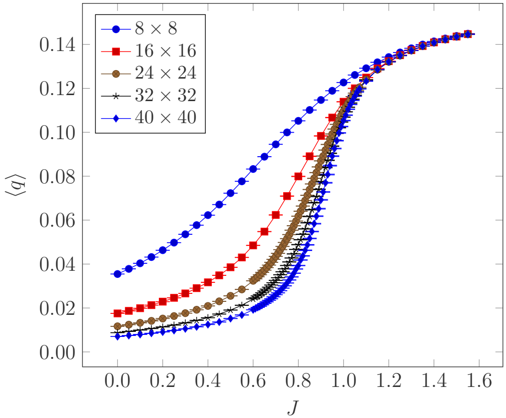
$$\longrightarrow \frac{1}{\beta} - \frac{1}{V\beta^2}\left\langle \sum_x \left(p_x + \frac{\theta}{2\pi}\right)^2 \right\rangle$$

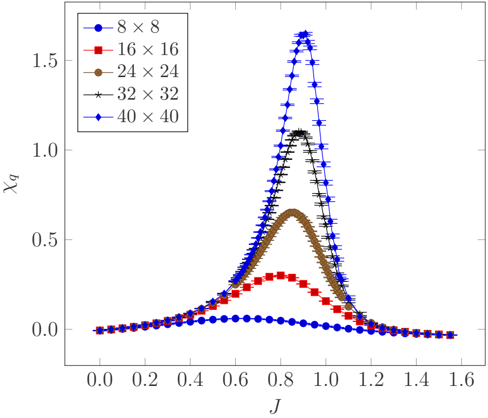
$$\chi_{F^2} = -2\frac{\partial}{\beta}\langle F^2 \rangle = \frac{1}{V}\left\langle \left[\sum_x \Big((\mathrm{d}A)_x + 2\pi n_x\Big)^2 - \left\langle \sum_x \Big((\mathrm{d}A)_x + 2\pi n_x\Big)^2 \right\rangle\right]^2 \right\rangle$$

$$\longrightarrow \frac{1}{2V\beta^4}\left\langle \left[\sum_x \left(p_x + \frac{\theta}{2\pi}\right)^2 - \left\langle \sum_x \left(p_x + \frac{\theta}{2\pi}\right)^2 \right\rangle\right]^2 \right\rangle + \frac{4}{\beta}\langle F^2 \rangle - \frac{2}{\beta^2}$$

Results

Topological charge $\langle q \rangle$ with corresponding susceptibility χ_q at $\beta = 0.5, \theta = \pi$:

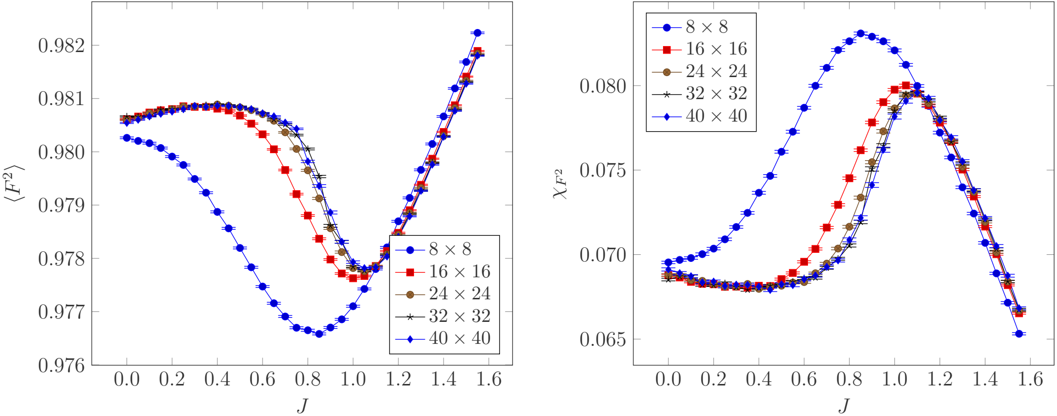




Spontaneous breaking of charge conjugation symmetry at $J_c \approx 0.9$

Results

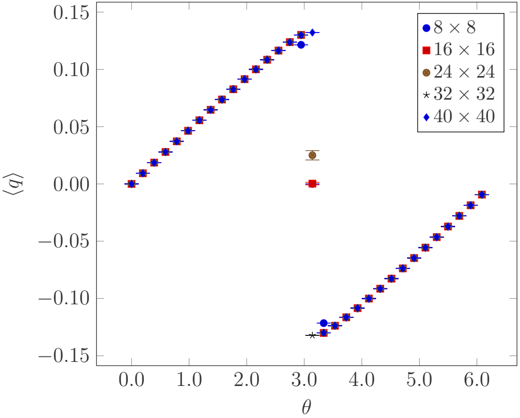
Expectation value of the gauge action density $\langle F^2 \rangle$ with corresponding susceptibility χ_{F^2} at $\beta = 0.5, \theta = \pi$:



Even observables do not signal breaking of charge conjugation symmetry

Results

Topological charge $\langle q \rangle$ at $\beta = 0.5, J = 1.2$:



In the broken phase we observe a first order transition as a function of θ

Summary and outlook

- We find spontaneous symmetry breaking at a critical coupling $J_c \approx 0.9$
- Transition is expected in the Ising universality class
- We are currently studying Binder cumulants and use finite size scaling to determine the critical exponents
- Additional runs at higher β are in preparation

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