Study of $SU(2)$ gauge theories with multiple Higgs fields in different representations

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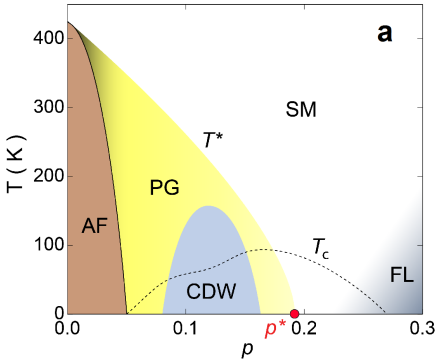
Outline

- $SU(2)$ gauge theory with 4 adjoint Higgs in 3d
  - Motivation – Cuprate superconductors
  - Preliminary results
- $SU(2)$ gauge theory with 2 fundamental Higgs in 4d
  - Motivation – 2-Higgs Doublet Model (2HDM)
  - Preliminary results

Motivation



[C. Proust and L. Taillefer, Annu. Rev. Condens. Matter Phys. 2019. 10:409–29]

- Cuprates show superconductivity exceeding 150 $K \sim$ half of room temperature
- Large magnetic field suppresses superconductivity
- Underlying states still not well understood
- Attempts to explain the critical point and surrounding phases as fluctuations of spin density wave order (SDW)

Proposal

Electron spin magnetic moment in spacetime dependent rotating reference frame $\boldsymbol{\sigma} \cdot \boldsymbol{S}_i = R_i \boldsymbol{\sigma} R_i^\dagger \cdot \boldsymbol{H}_i$

$\boldsymbol{H}_i$ transform under adjoint representation of $SU(2)$

Parametrize : $\boldsymbol{H}_i = \mathrm{Re}\left[\boldsymbol{\mathcal{H}}_x e^{i\boldsymbol{K}_x \cdot \boldsymbol{r}_i} + \boldsymbol{\mathcal{H}}_y e^{i\boldsymbol{K}_y \cdot \boldsymbol{r}_i}\right]$

$\mathcal{L}_{\mathcal{H}} = \frac{1}{4g^2} \boldsymbol{F}_{\mu\nu} \cdot \boldsymbol{F}_{\mu\nu} + |\partial_\mu \boldsymbol{\mathcal{H}}_x - \boldsymbol{A}_\mu \times \boldsymbol{\mathcal{H}}_x|^2 + |\partial_\mu \boldsymbol{\mathcal{H}}_y - \boldsymbol{A}_\mu \times \boldsymbol{\mathcal{H}}_y|^2 + V(\boldsymbol{\mathcal{H}}_{x,y})$

$V(\boldsymbol{\mathcal{H}}_{x,y}) =$
$s\left(\boldsymbol{\mathcal{H}}_x^* \cdot \boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^* \cdot \boldsymbol{\mathcal{H}}_y\right) + u_0\left(\boldsymbol{\mathcal{H}}_x^* \cdot \boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^* \cdot \boldsymbol{\mathcal{H}}_y\right)^2 + \frac{u_1}{4}\phi^2 + \frac{u_2}{2}\left(|\Phi_x|^2 + |\Phi_y|^2\right) + u_3\left(|\Phi_+|^2 + |\Phi_-|^2\right)$

$(u_1 = u_2 = u_3 \rightarrow$ global $O(4)$ symmetry)

[S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy Rev B 99, 054516 (2019)]

$\boldsymbol{\mathcal{H}}_x, \boldsymbol{\mathcal{H}}_y$: complex fields
$\rightarrow$ 4 adjoint Higgs

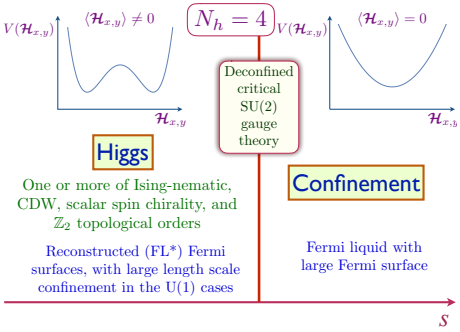
Gauge-invariant bilinears

$\phi = |\boldsymbol{\mathcal{H}}_x|^2 - |\boldsymbol{\mathcal{H}}_y|^2\,,$

$\Phi_x = \boldsymbol{\mathcal{H}}_x \cdot \boldsymbol{\mathcal{H}}_x\,, \Phi_y = \boldsymbol{\mathcal{H}}_y \cdot \boldsymbol{\mathcal{H}}_y\,,$

$\Phi_+ = \boldsymbol{\mathcal{H}}_x \cdot \boldsymbol{\mathcal{H}}_y\,, \Phi_- = \boldsymbol{\mathcal{H}}_x \cdot \boldsymbol{\mathcal{H}}_y^*$

Predictions and Previous work



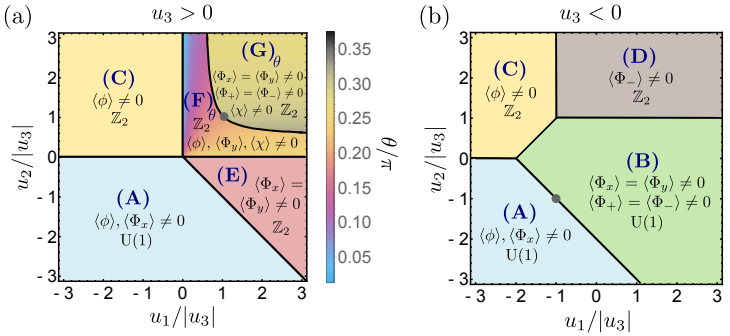
[S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy. Rev. B 99, 054516 (2019)]

- Higgs phase hosts different broken symmetries which resemble hole-doped cuprates
- $SU(2)$ broken to either $U(1)$ or $Z(2)$
- Recent numerical studies with the $O(4)$ symmetric potential finds the two patterns of symmetry breaking

  [H. D. Scammell, K. Patekar, M. S. Scheurer, S. Sachdev, Phy. Rev. B 101, 205124 (2020)]

  [C. Bonati, A. Franchi, A. Pelissetto, E. Vicari, Phy. Rev. B 104, 115166 (2021)]

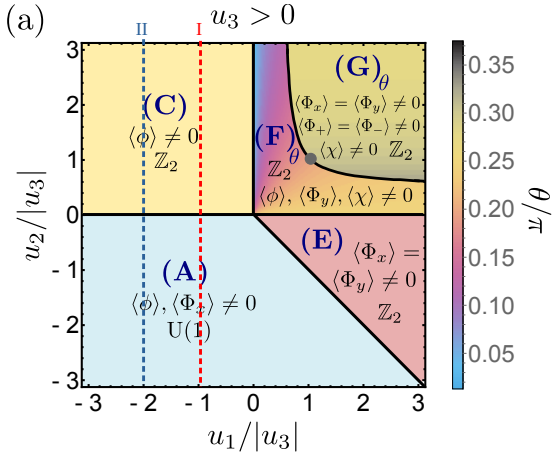
Mean Field phase diagram

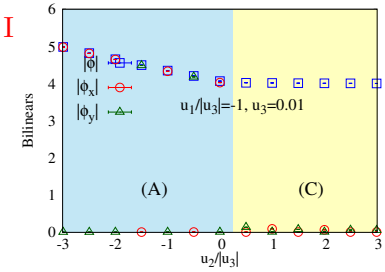


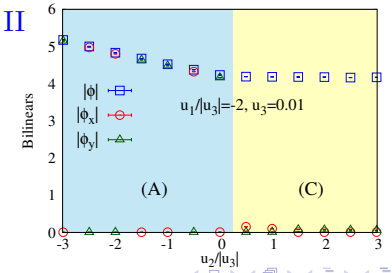
[S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy. Rev. B 99, 054516 (2019)]

- first-time numerical study of complete phase diagram
- using Hybrid Monte Carlo Algorithm
- CPU code with MPI parallelization
- preliminary study on 12^3 lattices, s chosen negative and u_0 positive enough to be in the broken phase

Preliminary results $u_3 > 0, u_1/|u_3| = -1, -2$







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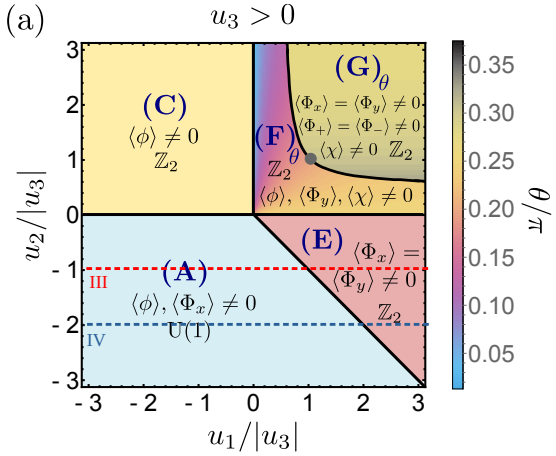

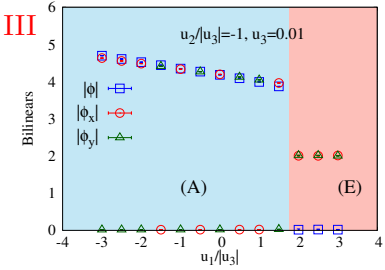

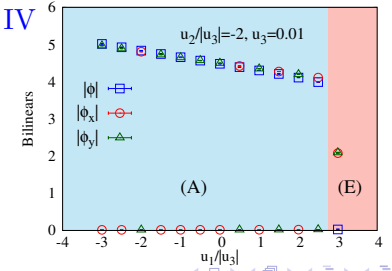

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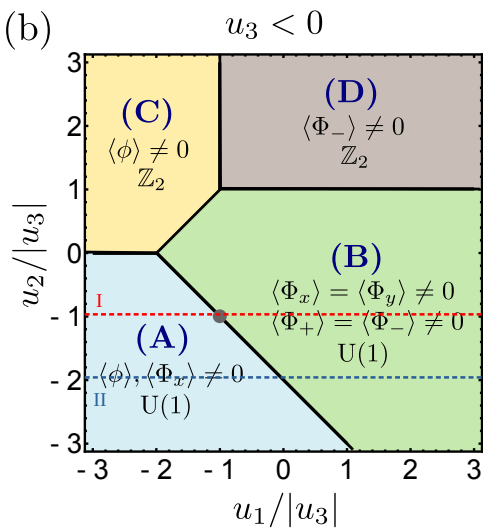
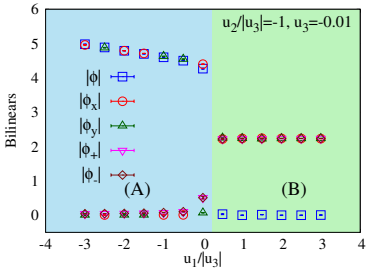
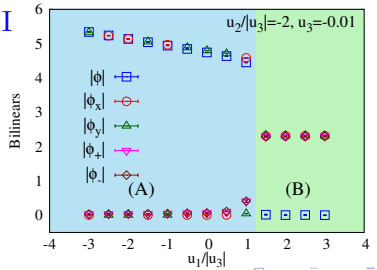
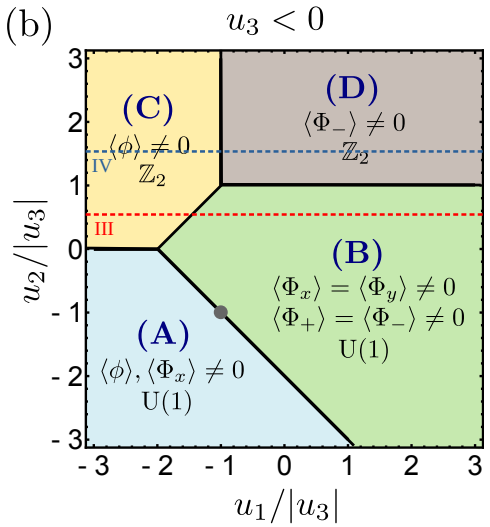
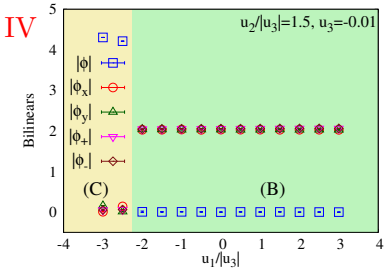


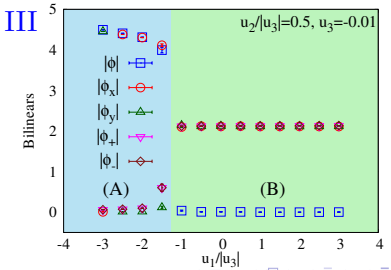




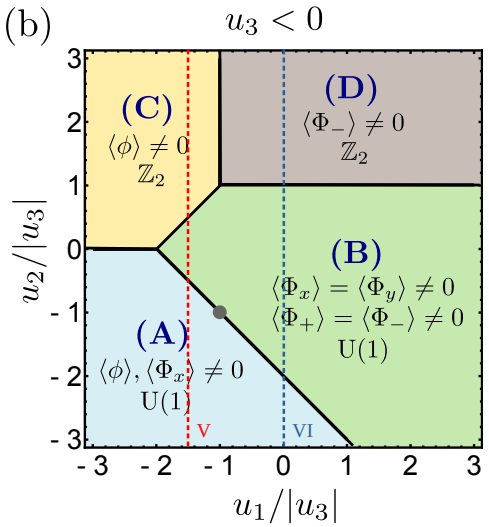
Preliminary results $u_3 < 0, u_2/|u_3| = 0.5, 1.5$

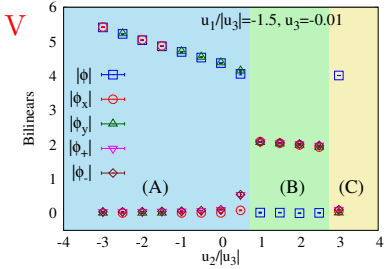


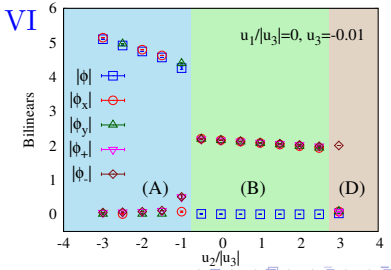




Preliminary results $u_3 < 0, u_1/|u_3| = -1.5, 0$







$SU(2)$ gauge theory with 2 fundamental Higgs in 4d

- Matter-antimatter asymmetry – first-order Electroweak Phase Transition (EWPT) not possible in SM
- Minimal extension to SM – a second Higgs doublet producing extra scalars

Goals

- Scan the zero-temperature phase diagram and look for critical points
- Set the scale with physical Higgs and W boson masses
- Study the particle spectrum in the Higgs phase
- Finite temperature EWPT

Lattice action

Quaternion representation
$\hat{\Phi}_n(x) = \frac{1}{\sqrt{2}} \sum_{\alpha=0}^{3} \theta_\alpha \hat{\phi}_\alpha^{(n)}(x)$
$\theta_0 = \mathbf{1}_{2\times 2}, \theta_i = \sigma_i$

Action with real coupling constants

$$S_{\text{2HDM}} = S_{\text{YM}} + \sum_x \sum_{n=1}^{2} \Bigg\{ \sum_\mu -2k_n \,\text{Tr}\left(\hat{\Phi}_n^\dagger(x) U_\mu(x) \hat{\Phi}_n(x+\hat{\mu})\right)$$
$$+ \,\text{Tr}\left(\hat{\Phi}_n^\dagger(x)\hat{\Phi}_n(x)\right) + \eta_n \left[\,\text{Tr}\left(\hat{\Phi}_n^\dagger(x)\hat{\Phi}_n(x)\right) - 1\right]^2 \Bigg\} + 2\mu^2 \,\text{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right)$$
$$+ \xi_2 \,\text{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right)^2 + \xi_1 \,\text{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_1(x)\right) \text{Tr}\left(\hat{\Phi}_2^\dagger(x)\hat{\Phi}_2(x)\right)$$
$$+ 2\,\text{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right) \left[\xi_3 \,\text{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_1(x)\right) + \xi_4 \,\text{Tr}\left(\hat{\Phi}_2^\dagger(x)\hat{\Phi}_2(x)\right)\right],$$

Observables / Simulation

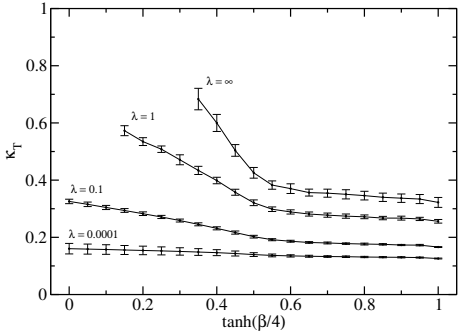
Observables:

- Plaquette $P = \frac{1}{12V} \sum_{\square} \mathrm{Tr}\{U_{\square}\}$
- Higgs length $\rho^2 = \frac{1}{V} \sum_x \rho^2(x)$
- Gauge-invariant links
  $L_\phi = \frac{1}{8V} \sum_{x,\mu} \mathrm{Tr}\{\phi^\dagger(x) U_\mu(x) \phi(x+\hat{\mu})\}$
  $L_\alpha = \frac{1}{8V} \sum_{x,\mu} \mathrm{Tr}\{\alpha^\dagger(x) U_\mu(x) \alpha(x+\hat{\mu})\}$
- Susceptibilities $\langle \chi_O \rangle = \langle (O - \langle O \rangle)^2 \rangle$

- Hybrid Monte Carlo code developed in Julia for GPUs
- Results independently checked with Metropolis algorithm
- First study with full Higgs potential (10 real coupling constants)

$N_h = 1$ phase diagram

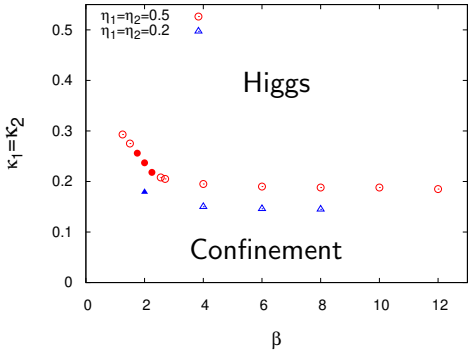
Single Higgs doublet

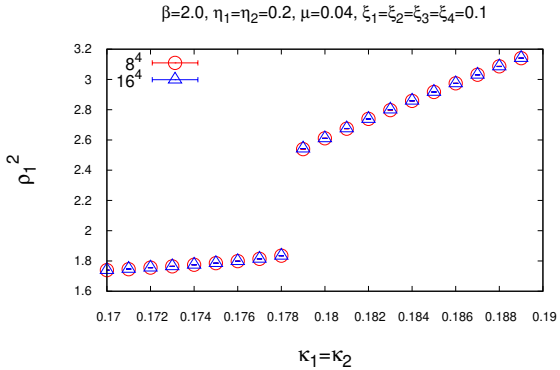


[M. Wurtz, R. Lewis, T.G. Steele, Phys.Rev.D 79 (2009) 074501]

- $N_h = 1$ case well-known
- Confinement and Higgs phases analytically connected
- first-order PT at finite β and λ
- crossover at small β and large λ
- $N_h = 2$ symmetric Higgs case showed first-order surface covers the entire phase diagram

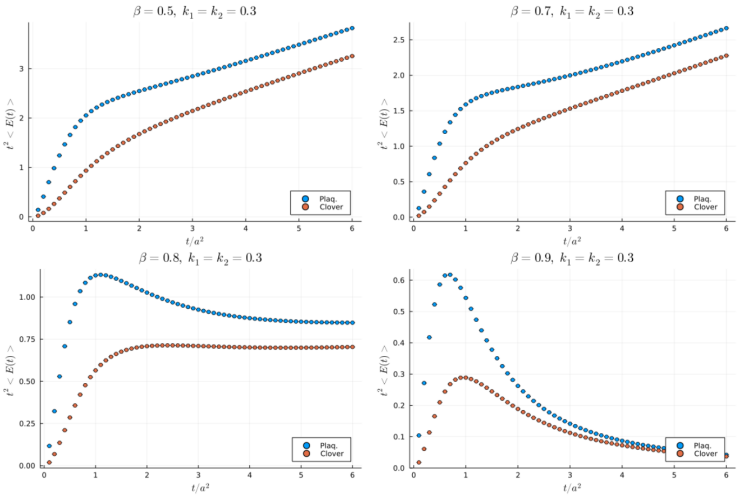
Preliminary phase diagram





Some evidence of first-order transition

Scale setting with gradient flow [Preliminary]



- only gauge fields have been flowed
- confinement and higgs phases show different behavior
- lattice spacing increases with β at fixed κ in Higgs phase

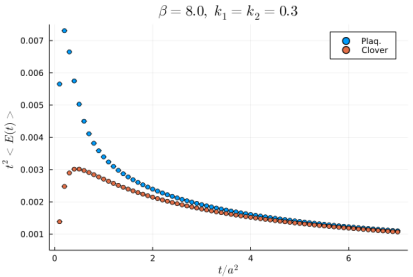
Scale setting with gradient flow [Preliminary]

Renormalized coupling at the scale of W boson mass
$g^2 \equiv 4\pi\alpha_W \simeq 0.5$

choice for scale setting

$$t^2 \langle E(t) \rangle = \frac{9g^2}{128\pi^2}(1 + \mathcal{O}(g^2)) \simeq 0.0036 \Big|_{\mu=1/\sqrt{8t}=m_W}$$

- code for m_W and m_H currently being implemented
- find lines of constant physics



Conclusions and Outlook

$3d$ SU(2) gauge theory with 4 adjoint Higgs

- First-time study of full Higgs potential revealing different broken phases
- Preliminary numerical results qualitatively confirm the mean-field expectations
- Future investigation of the critical point and topological orders

$4d$ SU(2) gauge theory with 2 fundamental Higgs doublet

- First-time study of the complete potential with real couplings in 4d
- Preliminary study of the phase diagram and scale setting
- m_W and m_H mass measurement code currently being implemented

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Thank you for your attention!