What is the best way to quantize non-linear electrodynamics?

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Background

- Outdated problems with quantization of Maxwell's electrodynamics: self energy of a point particle infinite

- Interest in non-linear electrodynamics [Born '34, Born&Infeld '34, Schwinger '51, Schwinger '54, Plebanski '69]

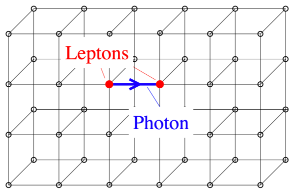
- Born-Infeld [BI] as a classical theory of strings and branes [Fradkin&Tseytlin '85, Agnanic et. al '96]

- Quantization of NL extensions of electrodynamics (~~Dirac~~ $\to$ symplectic quantization)

- Goal: theoretical groundwork for non perturbative explorations of such theories

- Numerical explorations using lattice techniques

Motivation

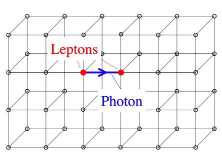
- Simplest non-linear extension of QED: BI theory [Born ’34, Born&Infeld ’34]
- Born Infeld on the 

[Sinclair&Kogut ’05, Sinclair&Kogut ’06]
- Further extensions incl. derivative self-interactions of the vector field: Generalized Proca (GP) or Vector Galileon theories [Tasinato ‘14, Heisenberg’14]
- Applications in lattice theories, optics, and cosmology: coupling of non-linear electrodynamics to gravity as a dark energy explanation [De Felice et al. ’16]
- Axiomatic approach and extension to multi-field GP [Errasti Diez et al. ‘19]
- For the first time experiments probing non-linear regime of QED: PVLAS, LUXE, SwissFEL …

Multiple options for extending (Q)ED

[Maxwell (QED)]

$$\mathscr{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



[Maxwell; 1850-1865]

*many talks at LATT22

[Non-linear (NLE)]

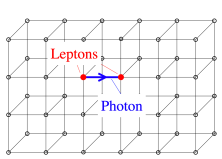
$$\mathscr{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + f(F_{\mu\nu})$$

[Born & Infeld; 1934]

[Schwinger; 1951-1953]

[Proca (QEDm)]

$$\mathscr{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$$



[Proca; 1936-1938]

Contribution #359 at LATT22 [Tsang/Shindler, poster]

[Generalized Proca (GP)]

$$\mathscr{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tilde{f}(A_\mu, F_{\mu\nu}, S_{\mu\nu})$$

[Tasinato; 2014] – [Heisenberg; 2014]

[Beltran Jimenez & Heisenberg; 2016]

[Errasti Diez, MKM; 2021]

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$S_{\mu\nu} = \partial_\mu A_\nu + \partial_\nu A_\mu$$

*In the quantum regime: massless theory is restored safely from massive one [A. Hell '21]

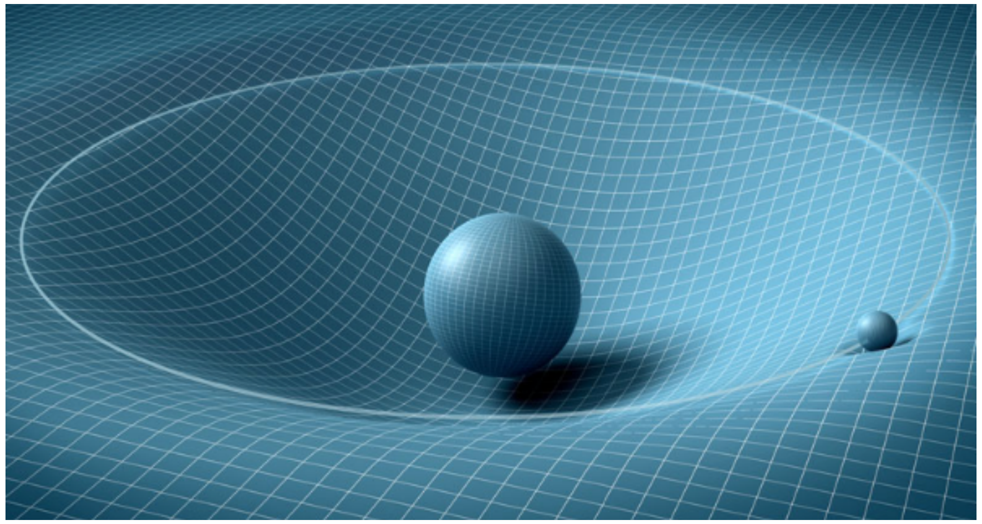
Constraints analysis in a nutshell

[Dirac-Bergmann ’50-’58]

- **Write a Lagrangian of the theory of interest: matter content + symmetries**

- $\mathscr{L} = \mathscr{L}(Q^1, \ldots, Q^N)$ **subject to constraints if** $n_{\text{d.o.f.}} < N$

- **Systematic iterative procedure to find a subspace of the configuration space that does admit a symplectic 2-form (physically meaningful space)**



Constraints analysis in a nutshell

[Dirac-Bergmann ’50-’58]

- Constraint analysis already at the classical level: avoid ghosts (Ostrogradski instabilities) [Ostrogradski 1850, Pais&Uhlenbeck ‘51]

- Additional challenge: consistent quantization of any system prone to Ostograndski instabilities is not guaranteed

- The same applies to the multi-field gauge systems

- Usual suspects for additional constraints, in order that it admits quantization of the theory

The easiest approach to quantization

[Faddeev & Jackiw '92], [Jackiw '93] [Barcelos-Neto & Wotzasek; 92-93], [Liao & Huang '07], [Toms '15]

[Errasti Diez, M.K.M., '21]

- **Symplectic instead of Dirac's quantization**

- **Changing to Hamiltonian formulation cumbersome; non-local algebra of commutators**

1. **Start by bringing Langrangian to the following form:**

$$\mathscr{L} = \theta \dot{Q} + \hat{\mathscr{L}}; \qquad \{Q\} = \{A_\mu, p^\mu, \lambda\}$$

2. **Compute:** $$\Omega_{\mathrm{mn}} = \frac{\delta\theta_{\mathrm{n}}}{\delta \mathrm{Q}^{\mathrm{m}}} - \frac{\delta\theta_{\mathrm{m}}}{\delta \mathrm{Q}^{\mathrm{n}}}$$

3. **Obtain new constraints:** $\gamma \dfrac{\delta \hat{\mathscr{L}}}{\delta \mathrm{Q}} = 0$; iteratively repeat until closure!

The easiest approach to quantization

[Faddeev & Jackiw '92], [Jackiw '93] [Barcelos-Neto & Wotzasek; 92-93], [Liao & Huang '07], [Toms '15]

[Errasti Diez, M.K.M., '21]

- **New, simple way to read off a partition function for the quantized system:**

$$Z = \int d\sigma \, e^{i \int_{\mathscr{M}} d^4x \mathscr{L}}; \qquad d\sigma = \det(\Omega) \prod_m d[\mathrm{Q}^m]$$

- **Ready for analytical and numerical computations, keep in mind conclusions:**

  1. **Not all non-linear massive ED versions can be quantized**

  2. **Among those that can be quantized, degenerate behaviour may happen**

- **One exception: Non-linear QED (Born-Infeld) does not have degenerate behaviour** [Bialyncki-Birula '83]

NLE on the lattice

[Sinclair,Kogut '05, Sinclair,Kogut '06]

- **Outlined quantization procedure can be applied to any NL-extension of ED**

- **Preliminary results with Born Infeld action in four spacetime dimensions**

$$S = b^2 \int d^4x \left(\sqrt{1 + \frac{1}{2b^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{16b^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} - 1 \right)$$

- **Simple Metropolis update** [Sinclair,Kogut '05, Sinclair,Kogut '06]

- **Compute Polyakov loop:**

$$W(x) = e^{ie \sum_{x_0} A_0(x_0, \overrightarrow{x}) - \frac{1}{V} \sum_{\overrightarrow{y}} A_0(x_0, \overrightarrow{y})}$$

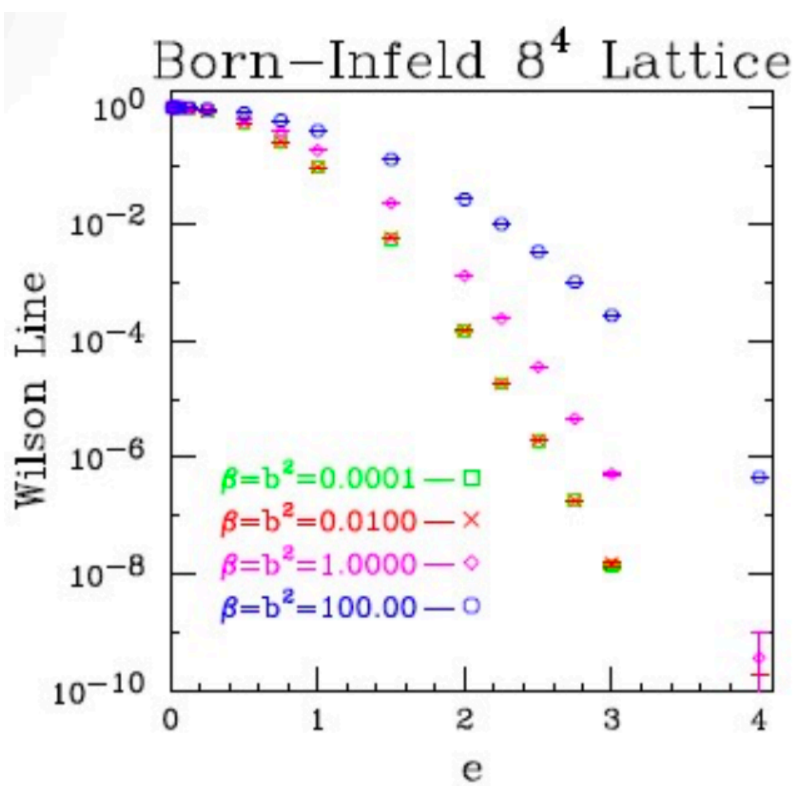
(sign problem, large statistics needed)

- **See** [Sinclair,Kogut '21] **and** contribution #73 **at LATT22** [D. Sinclair Mon, 14:00]

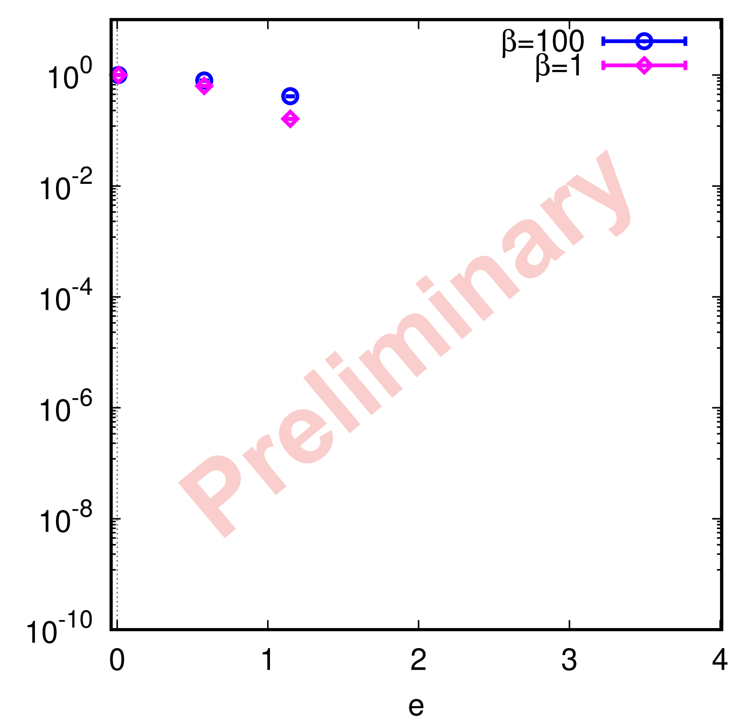
Born Infeld Theory on the lattice

[Sinclair,Kogut '05, Sinclair,Kogut '21]

- **Agreement with results in [Sinclair,Kogut '06] for small value of e on 8^4 lattice**
- **Further noise reduction techniques needed for larger e**
- **Extension to other non-linear extensions of electrodynamics in progress**



[Sinclair,Kogut '06]



[Errasti Diez, M.K.M., G. Pierini, in preparation]

Summary & Outlook

- **Many ways for NL extensions of Maxwell's electrodynamics: we have quantized all of them**

- **Safe to quantize certain extensions** (incl. derivative self-interaction)**, under specific set of conditions**

- **Given partition function; Monte Carlo methods can be used to obtain predictions**

- **Two special cases explored on the lattice: BI and Proca (**QEDm**)**

- **Implementation of more general NLE in progress (relevant for cosmology, BSM physics, Schwinger effect, vacuum birefringence …)**

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