Quantum chaos in supersymmetric Yang-Mills-like model

Pavel Buividovich (University of Liverpool)

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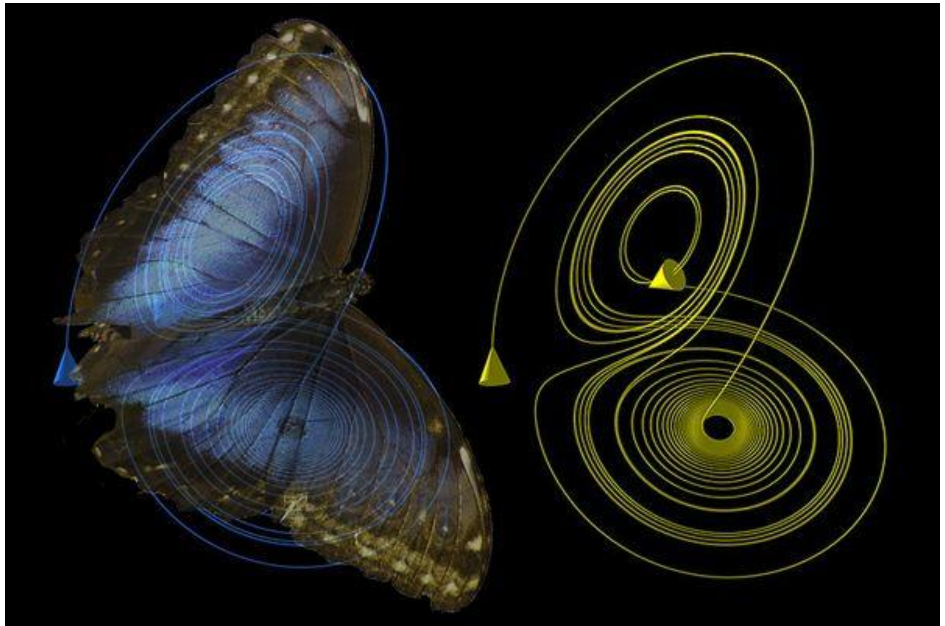
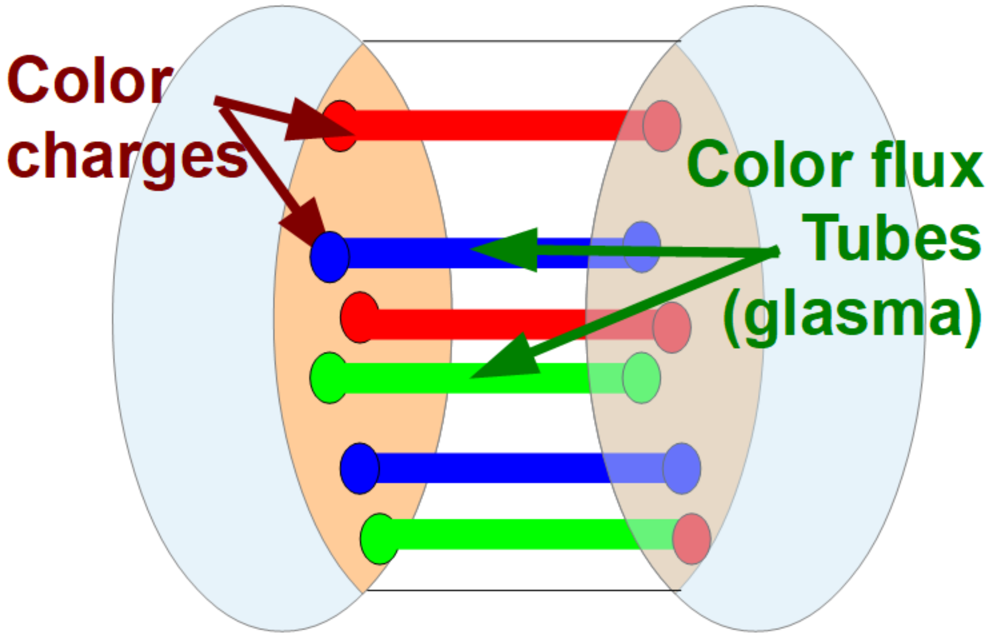


Why are we interested in quantum chaos?

- Classical dynamics of Yang-Mills theory is chaotic [Saviddy'1984]
- In the glasma regime, classical chaos/plasma instability can (partially) account for fast thermalization/hydrodynamization quark-gluon plasma [e.g. Kunihiro *et al.* ArXiv:1008.1156]
- How quantum effects affect classical chaotic dynamics?



Why are we interested in quantum chaos in gauge theories?

- Thermalization in supersymmetric gauge theory = formation of a black hole in a dual string theory (**AdS/CFT**)
- Super-Yang-Mills is a microscopic model of black hole dynamics
- Once a black hole is formed, how quickly it can **"scramble" information**? Black holes are "Fast scramblers", [Sekino,Susskind,0808.2096]
- Equivalent: how small perturbations evolve in super-Yang-Mills theory?

At high temperatures: classical dynamics …

Lyapunov instability and Poisson brackets

- Lyapunov exponent:

$$\frac{\partial x_i(t)}{\partial x_j(0)} \sim e^{\lambda_L t}$$

Impossible to get the initial state from the final one!

- In terms of Poisson brackets:

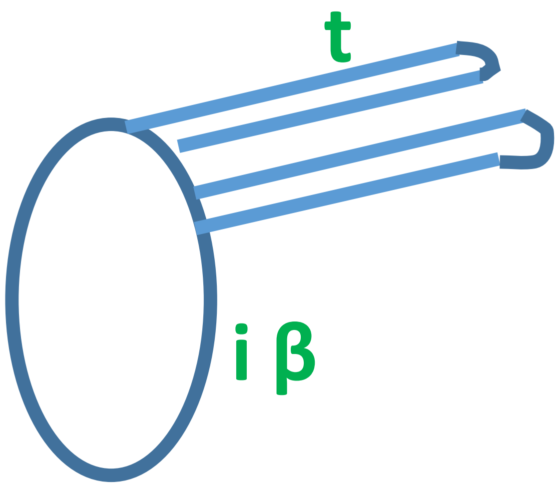
$$\frac{\partial x_i(t)}{\partial x_j(0)} = \{x_i(t), p_j(0)\} =$$

$$= \sum_k \frac{\partial x_i(t)}{\partial x_k(0)} \frac{\partial p_j(0)}{\partial p_k(0)} - \frac{\partial x_i(t)}{\partial p_k(0)} \frac{\partial p_j(0)}{\partial x_k(0)}$$

- Averaging over an ensemble of initial conditions (thermal):

$$e^{2\lambda_L t} \sim \langle \{x_i(t), p_j(0)\}^2 \rangle$$

Quantum generalization: Out-of-Time-Order Correlators



$$e^{2\lambda_L t} \sim \langle \{x_i(t), p_j(0)\}^2 \rangle$$

$$\{x_i(t), p_j(0)\} \to -i\,[\hat{x}_i(t), \hat{p}_j(0)]$$

$$\langle \mathcal{O}(x,p) \rangle \to \mathrm{Tr}\left(\hat{\rho}\,\hat{\mathcal{O}}\right)$$

$$e^{2\lambda_L t} \sim -\mathrm{Tr}\left(\hat{\rho}\,[\hat{x}_i(t), \hat{p}_j(0)]^2\right) =$$

$$= 2\mathrm{Re}\,\mathrm{Tr}\left(\hat{\rho}\,\hat{p}_j(0)\hat{x}_i^2(t)\hat{p}_j(0)\right) -$$

$$-2\mathrm{Re}\,\mathrm{Tr}\left(\hat{\rho}\,\hat{x}_i(t)\hat{p}_j(0)\hat{x}_i(t)\hat{p}_j(0)\right)$$

Conventional thermal correlator

This part is not time-ordered (out-of-time-order)

Universal bound on chaos and AdS/CFT

Reasonable physical assumptions
Analyticity of OTOCs
[Maldacena Shenker Stanford'15]

$$\boxed{\lambda_L \leq 2\pi T}$$

(QGP λ_L^{-1}~0.1 fm/c)

Holographic models with black hole backgrounds saturate the bound

Sachdev-Ye-Kitaev (SYK):

$$\hat{H}_{SYK} = -\frac{1}{4!}\sum_{abcd} J_{abcd}\hat{\psi}_a \hat{\psi}_b \hat{\psi}_c \hat{\psi}_d$$

Resembles the **η/s→1/(4π)** story…

A bridge near CERN

- Holographic dual to AdS_3 space
- Saturates the MSS bound at low T

BFSS Model: Classically chaotic system with a holographic dual

N=1 Supersymmetric Yang-Mills in D=1+9:

Reduce to a single point = BFSS matrix model

[Banks, Fischler, Shenker, Susskind'1997]

$$\hat{H}_{BFSS} = \frac{1}{2N}\mathrm{Tr}\,\hat{E}_i^2 - \frac{N}{4}\mathrm{Tr}\left[\hat{A}_i, \hat{A}_j\right]^2 + \frac{\sigma_i^{\alpha\beta}}{2}\mathrm{Tr}\left(\hat{\psi}_\alpha\left[\hat{A}_i, \hat{\psi}_\beta\right]\right)$$

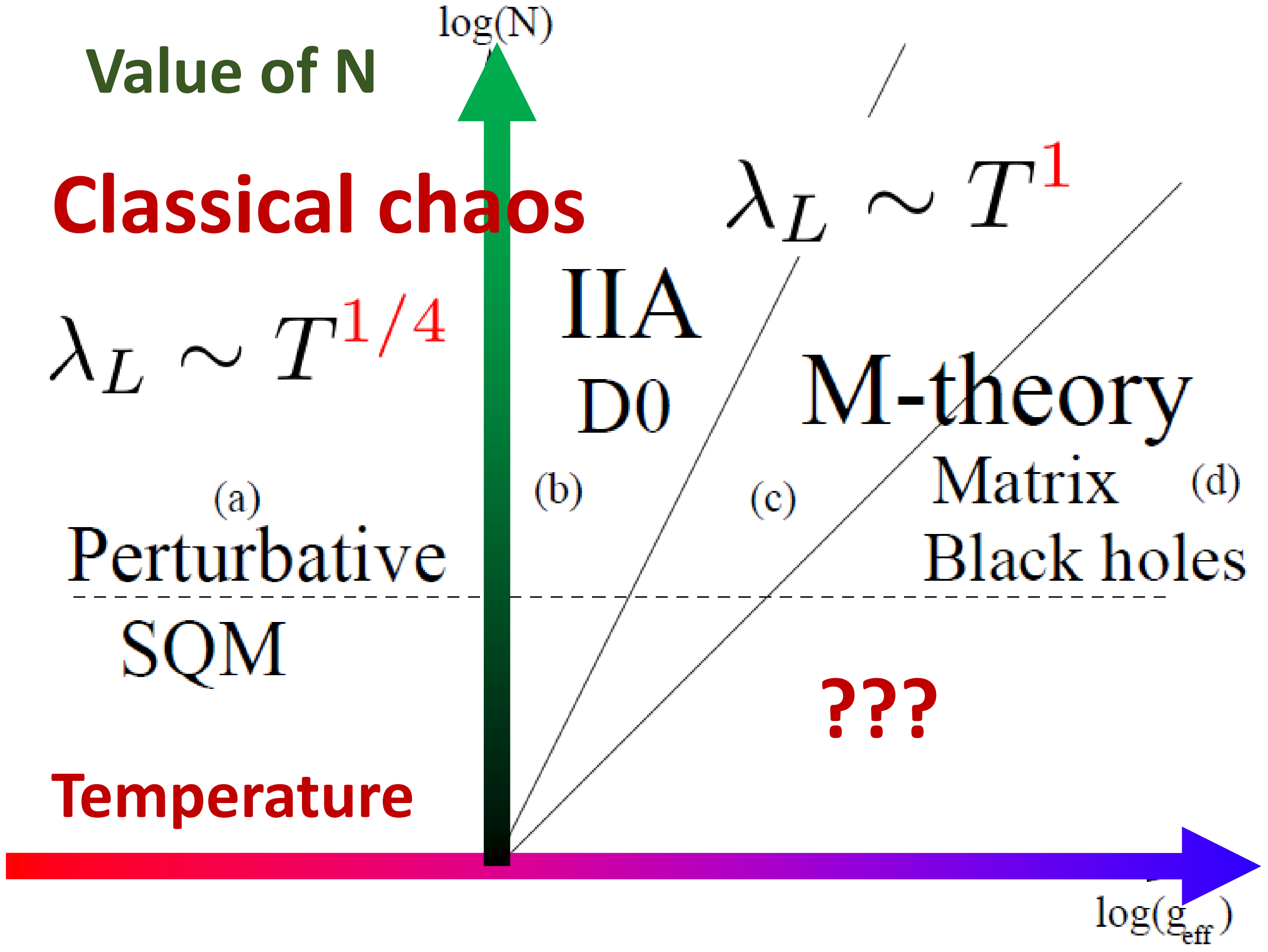
N x N hermitian matrices

Majorana-Weyl fermions, N x N hermitian

- Dual to system of *N* D0 branes joined by open strings [Witten'96]
- A^{ii}_{μ} = D0 brane positions
- A^{ij}_{μ} = open string excitations

[Similar model: talk by M. Hirasawa]

Phase diagram of BFSS model [Itzhaki et al., hep-th/9802042]



- [Berkowitz et al., 1606.04951] agreement with **black D0-brane** down to lowest temperature (N→∞ at fixed T)
- [Bergner et al., 2110.01312] signatures of **M-theory** phase in metastable states
- Both phases are **"black"** in the **dual theory** and feature **event horizons**
- **MSS bound** at **low T**?

What mechanism is responsible for quantum chaos at low T?

Of course, real-time dynamics is very difficult ...

“Minimal models” of Yang-Mills and super-Yang-Mills dynamics

$$\hat{H}_{QM} = \frac{1}{2N}\mathrm{Tr}\,\hat{E}_i^2 - \\ -\frac{N}{4}\mathrm{Tr}\,\left[\hat{A}_i, \hat{A}_j\right]^2$$

SU(2), zero angular momentum

$$\boxed{\hat{H}_B \sim \hat{p}_1^2 + \hat{p}_2^2 + \hat{x}_1^2\,\hat{x}_2^2}$$

$$\hat{H}_{SYM} = \hat{H}_{YM} + \\ + \int \frac{i\sigma^k_{\alpha\beta}}{2}\mathrm{Tr}\,\left(\hat{\psi}_\alpha \mathcal{D}_k \hat{\psi}_\beta\right) + \ldots$$

$$\boxed{\hat{H}_S = \hat{H}_B + \hat{x}_1\,\sigma_1 + \hat{x}_2\,\sigma_2}$$

[de Wit, M. Luscher, and H. Nicolai‘1984]

- Pauli matrices act on a 2-dim fermionic Hilbert space

“Minimal model” of a supersymmetric matrix model

Minimal Yang-Mills-like Hamiltonian:

$$\boxed{\hat{H}_B = \hat{p}_1^2 + \hat{p}_2^2 + \hat{x}_1^2\, \hat{x}_2^2}$$

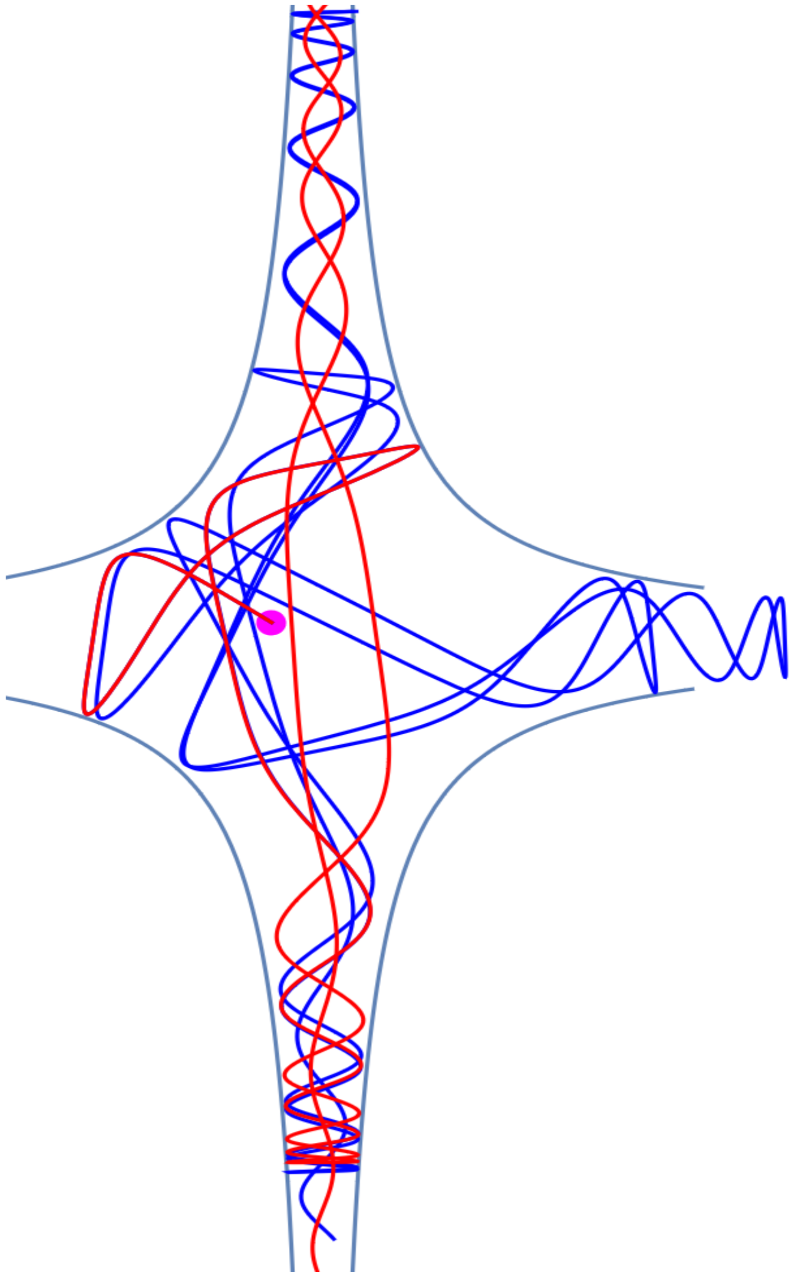
Pauli matrices are “fermionic” operators

$$\boxed{\hat{H}_S = \hat{H}_B + \hat{x}_1\, \sigma_1 + \hat{x}_2\, \sigma_2}$$

Supersymmetry generator: $\hat{H}_S = \hat{Q}^2$

$$\hat{Q} = \hat{x}_1 \hat{x}_2 \otimes \sigma_3 + \hat{p}_1 \otimes \sigma_1 + \hat{p}_2 \otimes \sigma_2$$

(Flat directions remain flat due to SUSY)



Numerical method

Work in the truncated basis of harmonic oscillator states

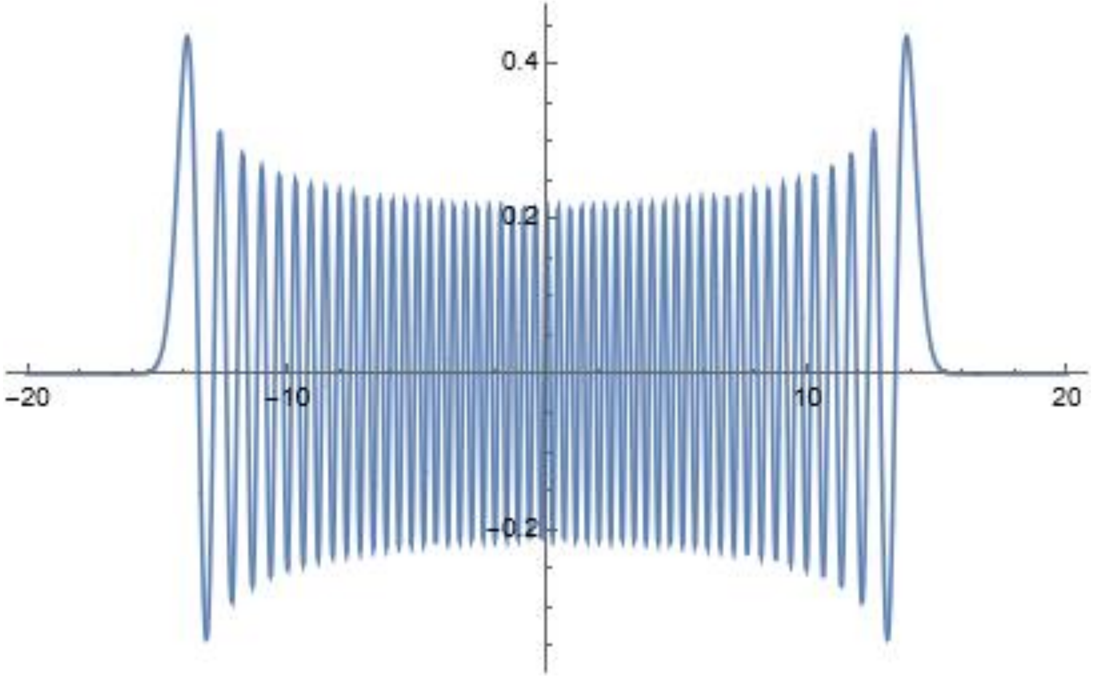
$$\Psi_{k_1,k_2}(x_1, x_2) = \psi_{k_1}(x_1)\,\psi_{k_2}(x_2)$$

$$\psi_k(x) = \frac{1}{\sqrt{2^k\,k!\,\sqrt{\pi}L}}\exp\left(-\frac{x^2}{2L^2}\right)H_k(k, x/L)$$

- Polynomial Hamiltonians

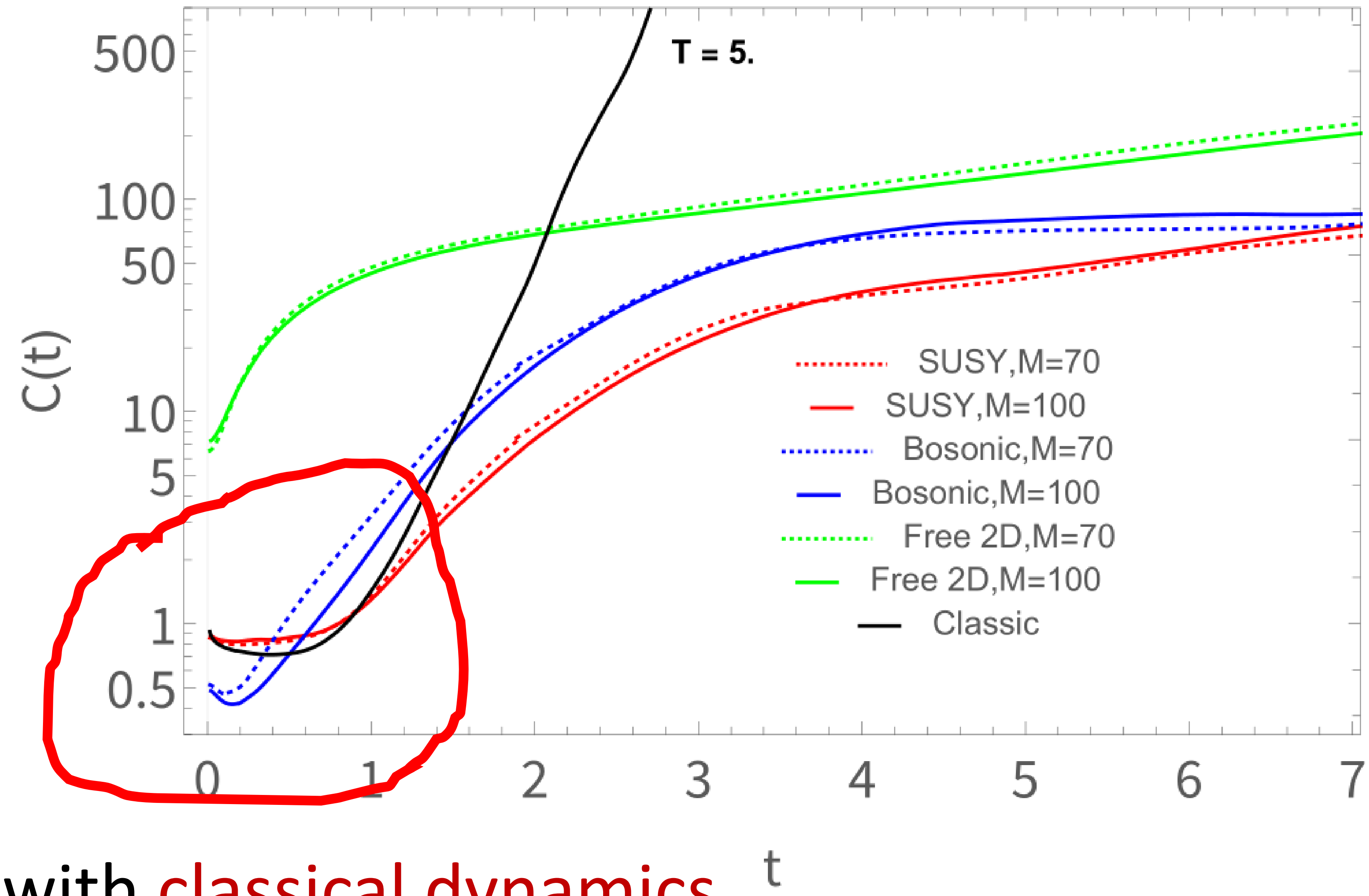
➡ sparse matrices

- Truncated at $k_1 + k_2 \leq 2M$
- M defines both UV and IR cutoffs (size ~ $M^{1/2}$)
- L tuned to min energy gap



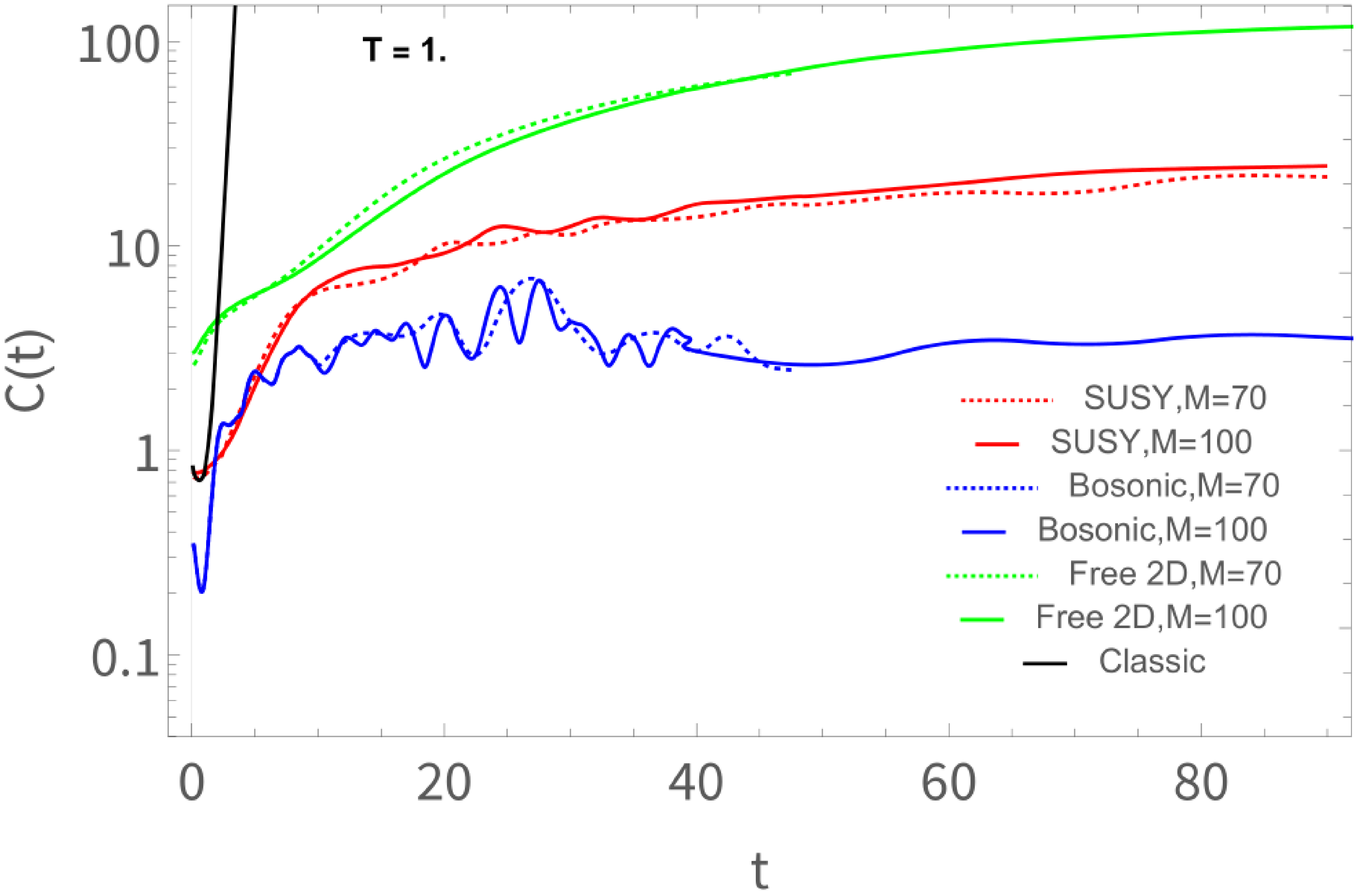
- LAPACK/ARPACK used for small/large M

Out of Time Order Correlators – moderately high temperature, T=5

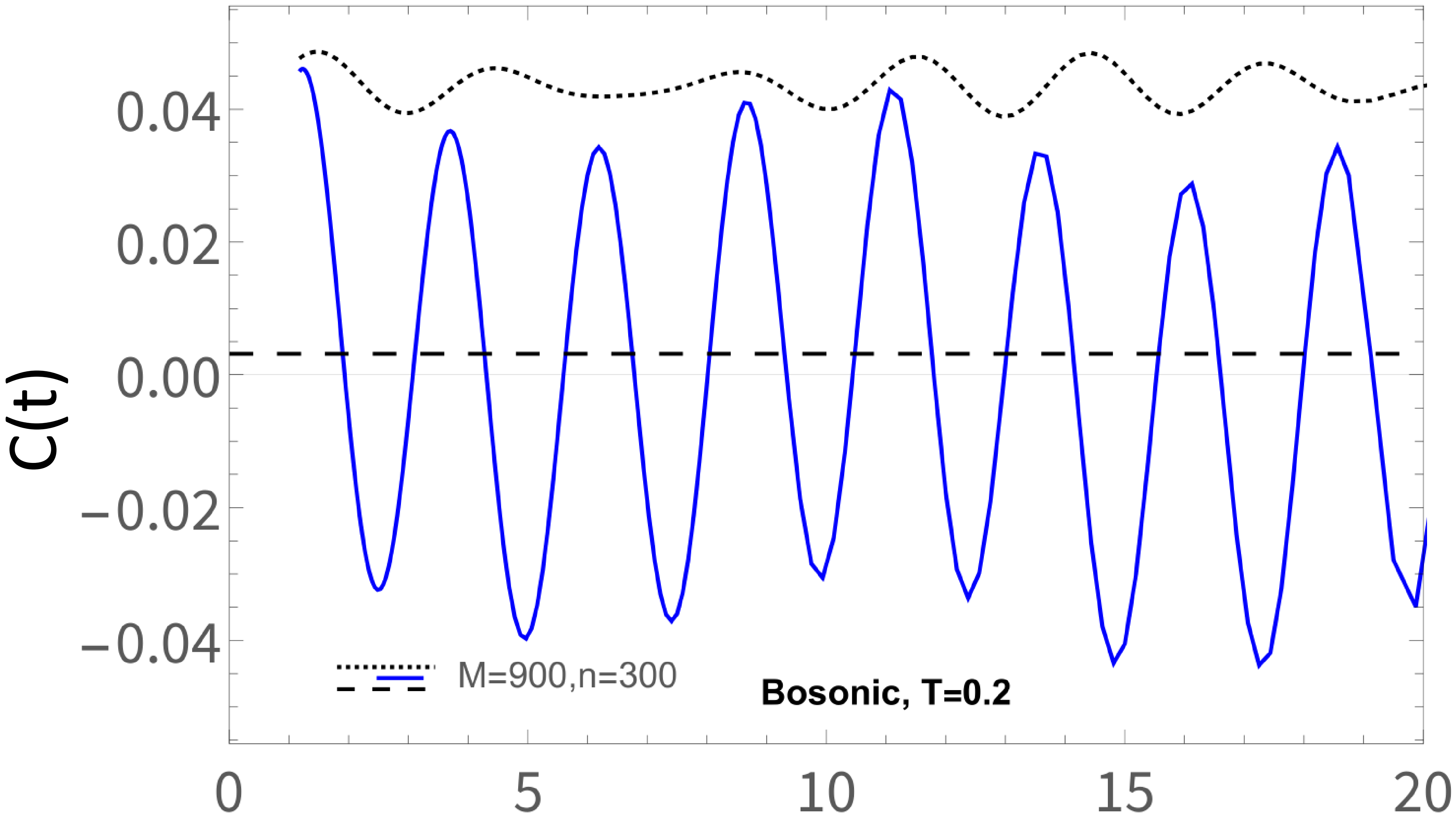


Agreement with classical dynamics
observed only for the supersymmetric Hamiltonian

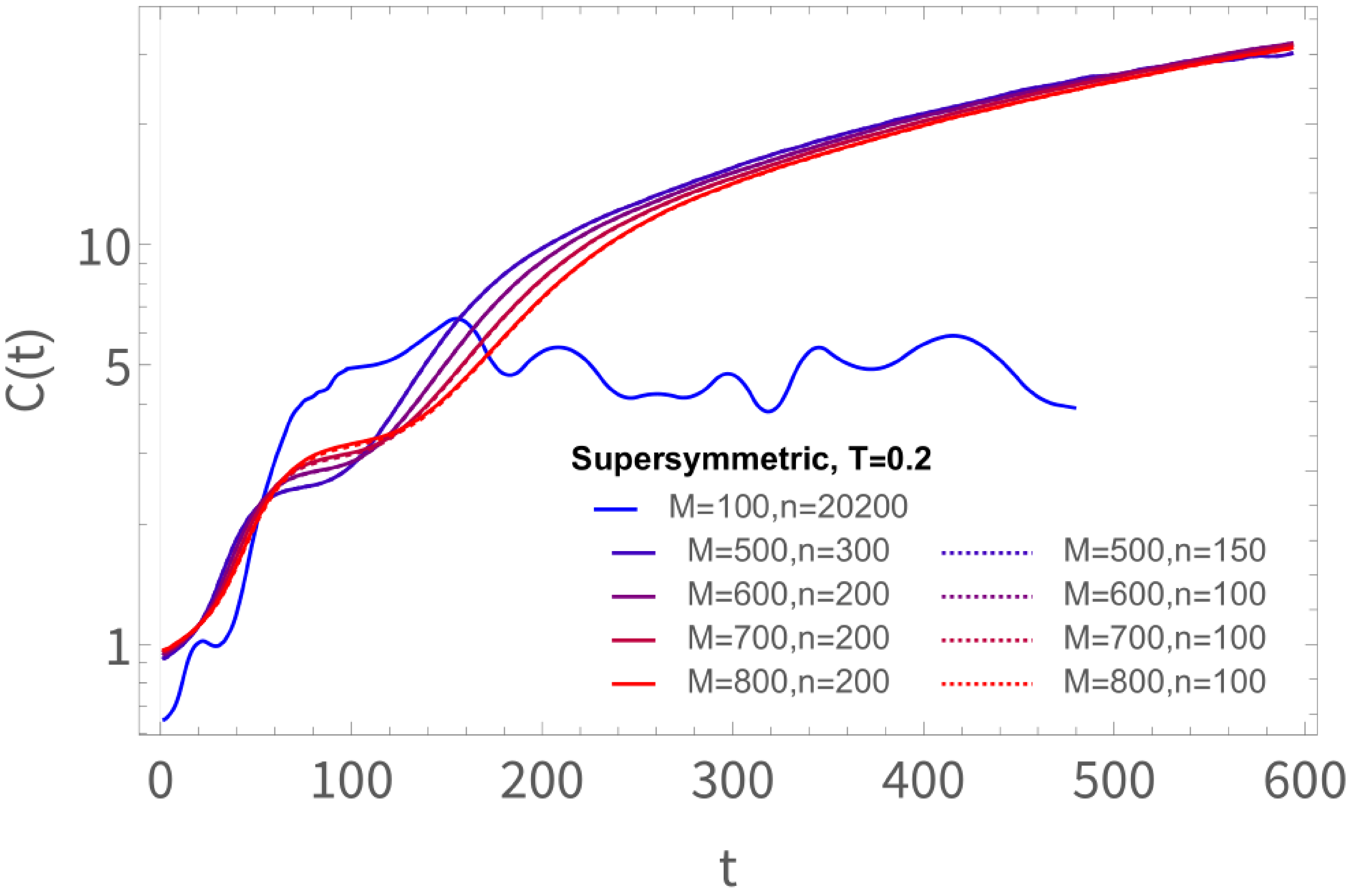
OTOCs – medium temperature, T = 1



OTOC – *T=0.2* (low-T), bosonic



OTOCs – low temperature, T = 0.2

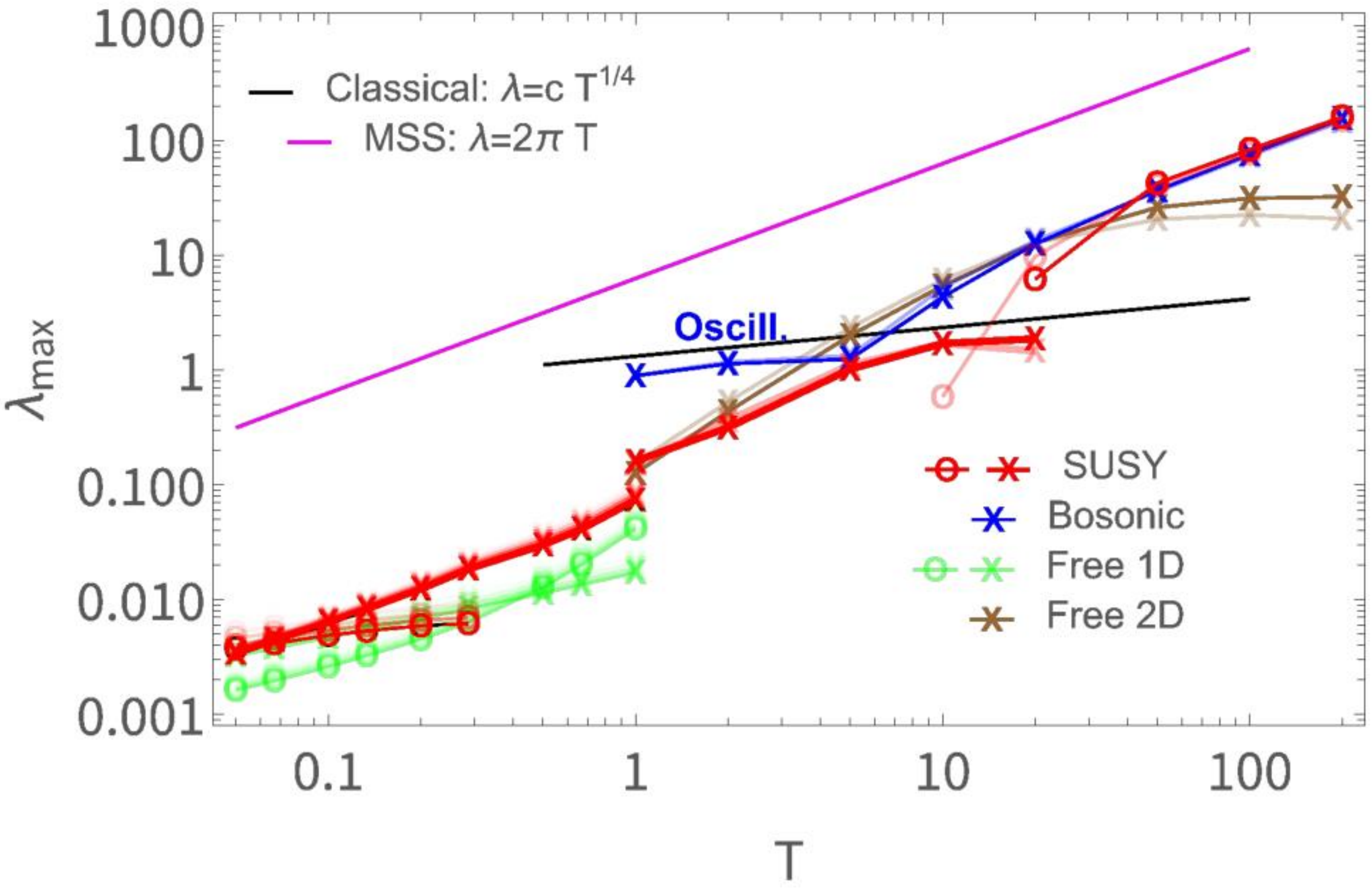


How to estimate λ_L

- Exponential OTOC growth is not clearly defined (no large N)
- Also free hamiltonians with IR cutoff exhibit some OTOC growth ➡ careful extrapolation to infinite cutoff
- Estimate an upper bound on λ_L from trajectory divergence rate

$$\lambda_L = \max_t \frac{1}{2} \frac{d}{dt} \log\left(C\left(t\right)\right)$$

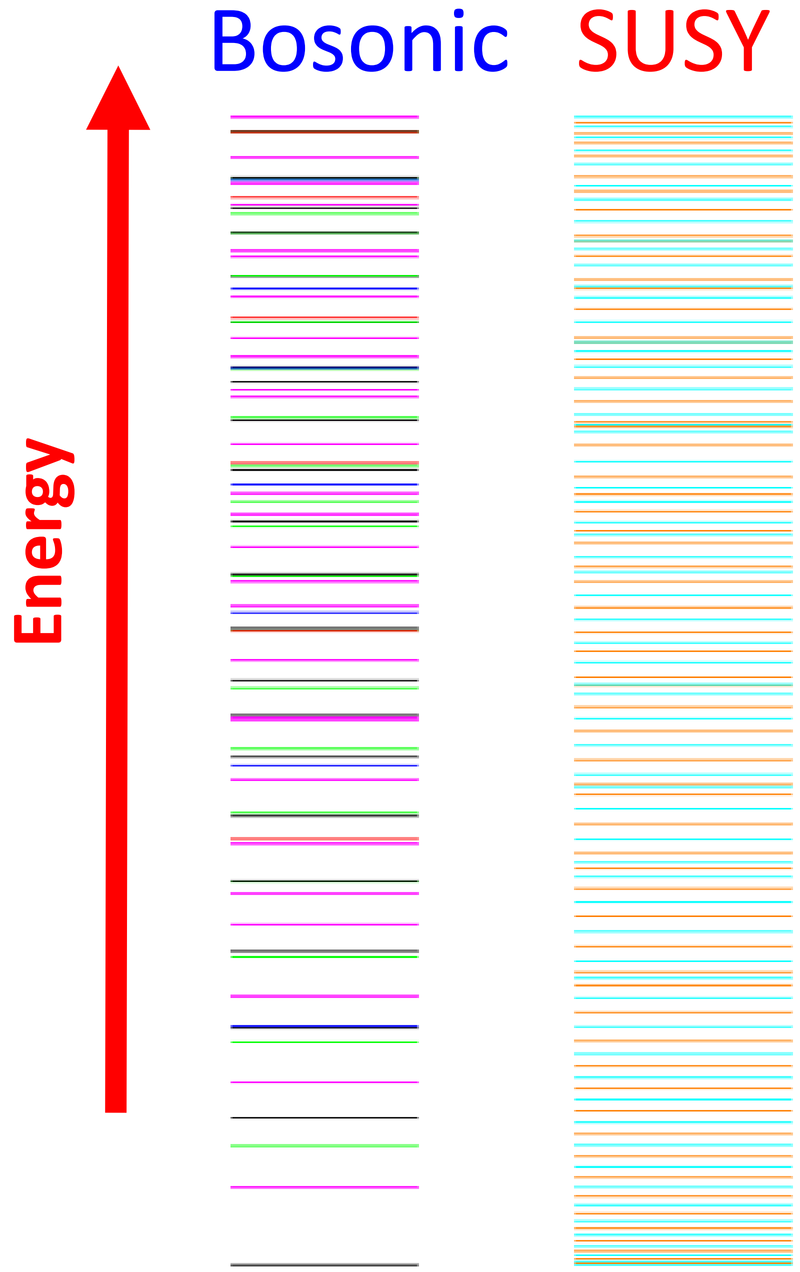
Summary of estimates of $\lambda_L(t)$



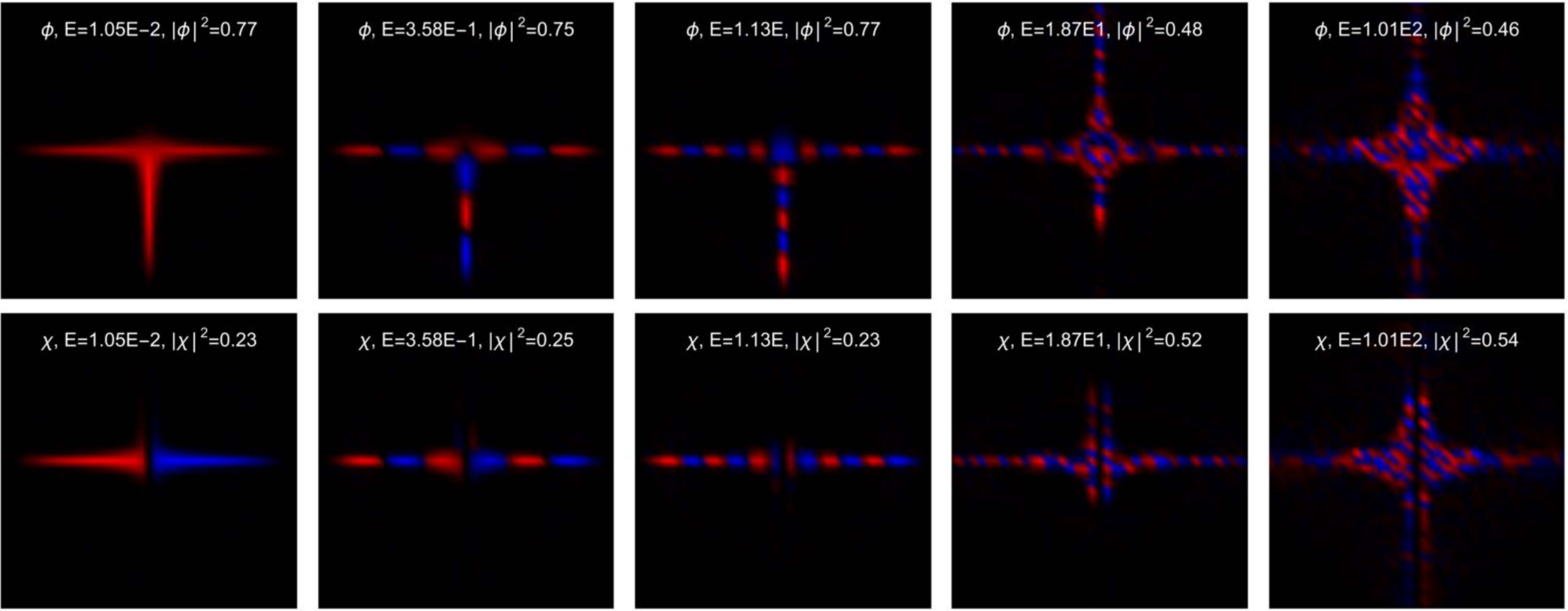
Larger IR cutoff (M) = less transparency

Global energy spectrum

- Bosonic: gapped spectrum
- Supersymmetric: narrowly spaced low-energy levels
- Continuous spectrum in the limit of infinite IR cutoff



Low-energy wave functions for the SUSY model



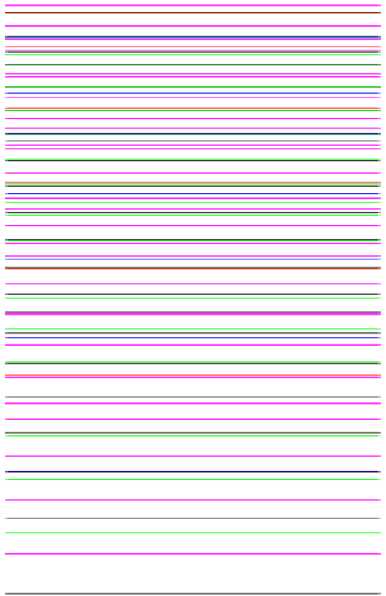
- Effectively one-dimensional states at low energies
- Parity broken due to the choice of the basis

Statistics of energy levels

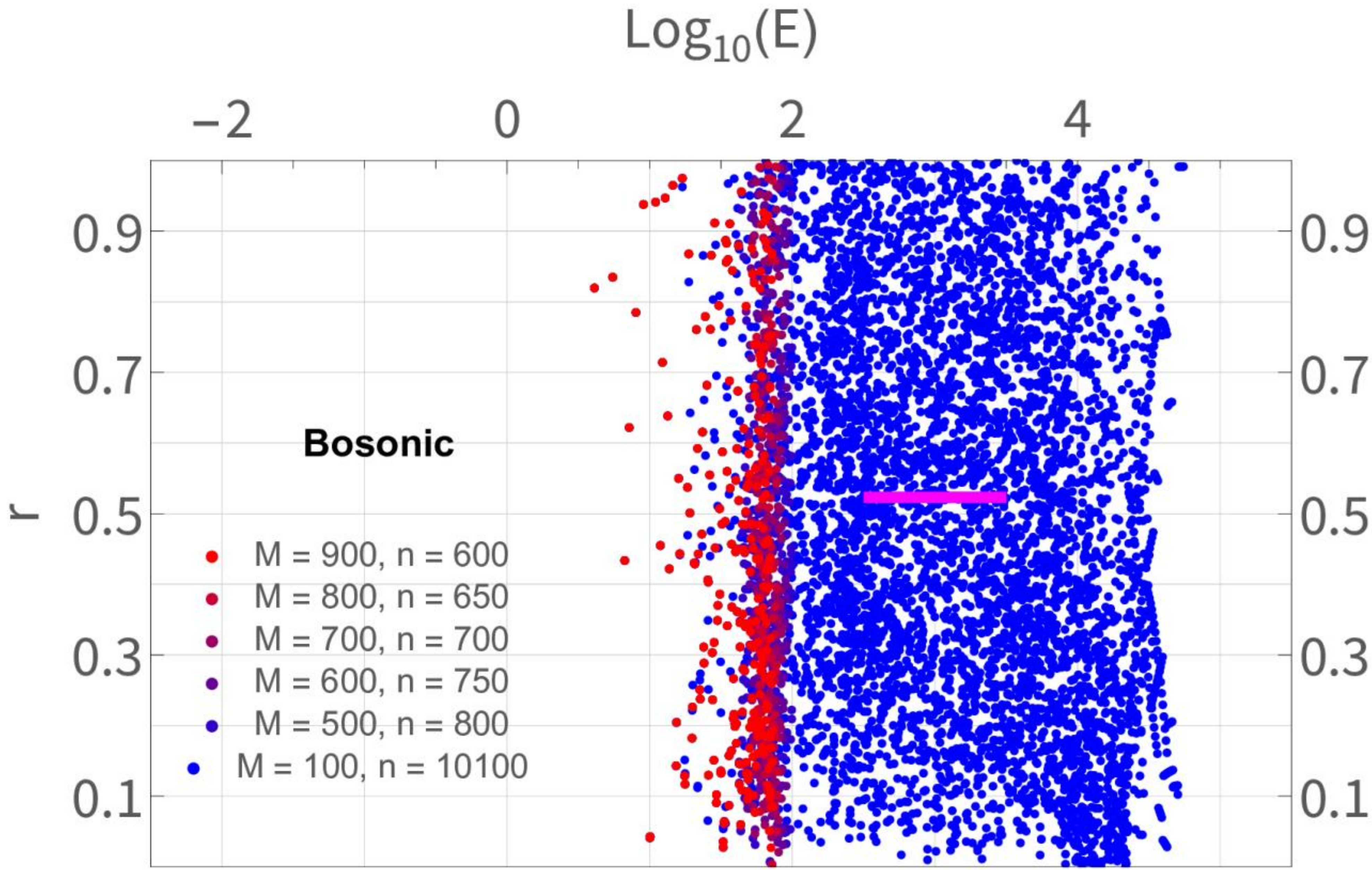
- Quantum chaos: universal energy level fluctuations [Wigner, Bohigas–Giannoni–Schmit]
- Counterpart of classical chaotic dynamics
- Described by random matrix theory (Gaussian random matrices)
- Our matrices are real → Gaussian Orthogonal Ensemble (GOE)
- Energy spectrum needs deflation in practice
- Convenient diagnostic tool: **r-ratios**

$$\Delta E_i = E_{i+1} - E_i$$

$$r_i = \frac{\min\left(\Delta E_{i-1}, \Delta E_i\right)}{\max\left(\Delta E_{i-1}, \Delta E_i\right)}$$



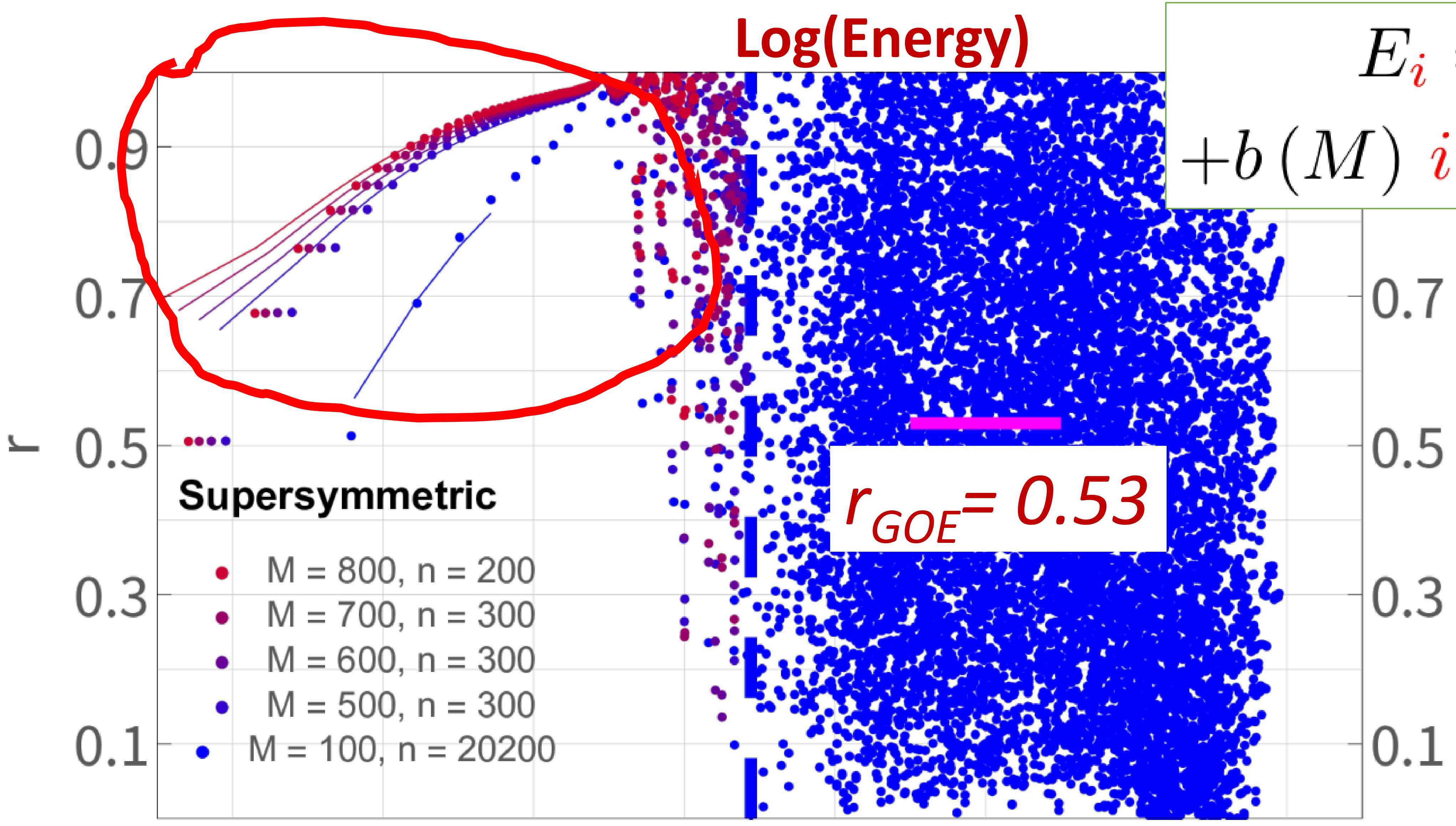
Statistics of energy levels: Bosonic model



r_{GOE}=0.53

Qualitatively like pure Yang-Mills

Statistics of energy levels: SUSY model



$$E_i = a(M) + b(M)\, i + c(M)\, i^2$$

1D Box:

$$E_i \sim \frac{(2\pi i)^2}{L^2}$$

Low-energy states are very regular

Cf. Giordano, Kovacs, Pittler, ArXiv:1312.1179

Discussion and conclusions

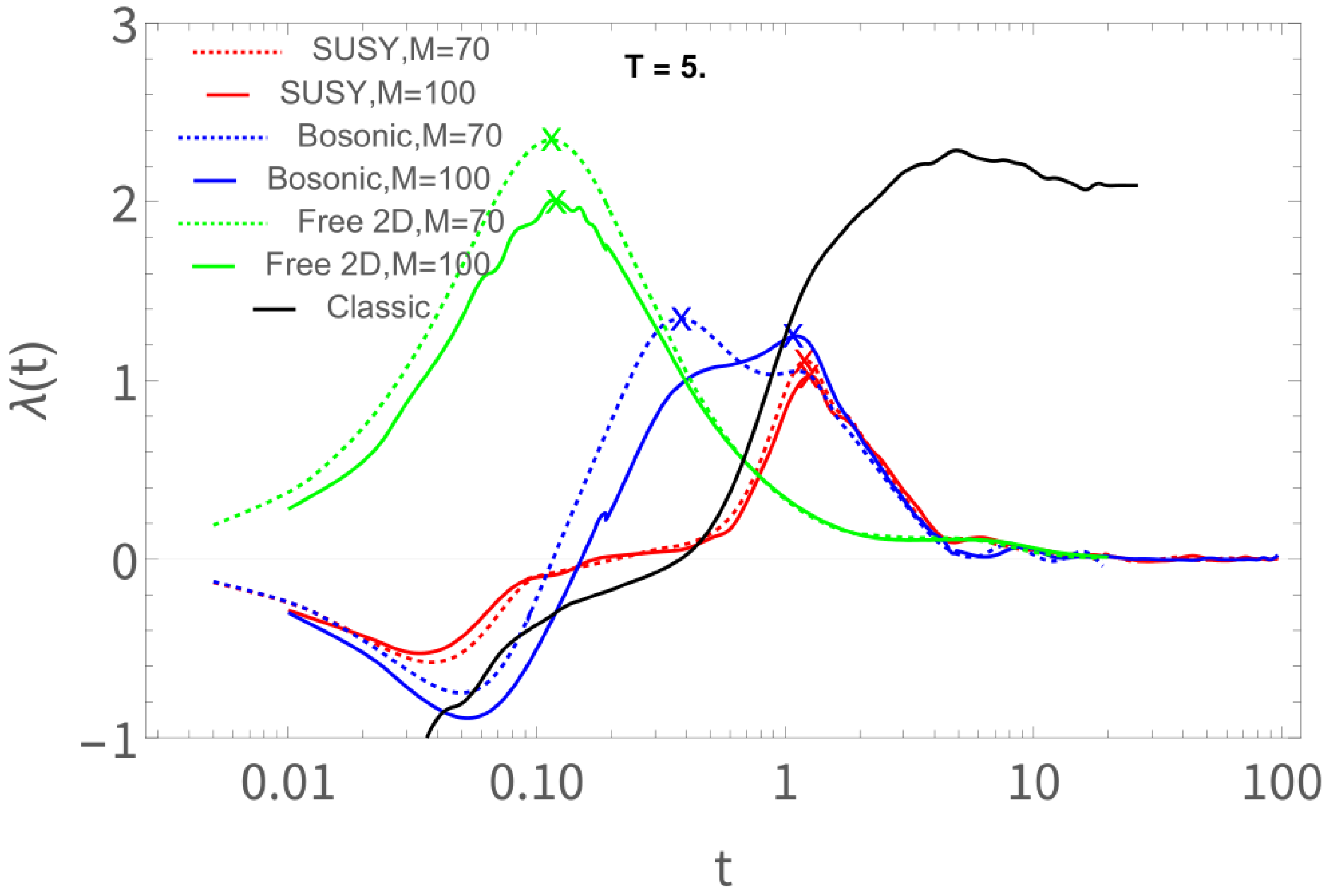
- Two parts of the spectrum for SYM-like model:
  - Chaotic high-energy bulk = classical chaos
  - Regular low-energy, low-dimensional states, absent in the bosonic model
- Sharp change between the two regimes
- Similar to **Black D0 branes – Schwarzschild black hole transition?** Cf. [Bergner et al., 2110.01312]
- OTOCs of the SUSY system grow down to lowest T, $\boldsymbol{\lambda_L \sim T}$
- Bosonic system at low T only exhibits oscillations
- At high T, classical-quantum correspondence for OTOCs only for the SUSY system

Outlook

- Simple SUSY/bosonic models can serve as a testbed for other real-time evolution methods (quantum computers?)
- Can we construct an effective model of low-energy, low-dimensional states that saturate OTOC growth at low T?
- In SYK model: zero modes due to approx. reparameterization invariance, broken down to SL(2, R) [Maldacena, Stanford'1604.07818]
- What is the holographic dual interpretation of these states?

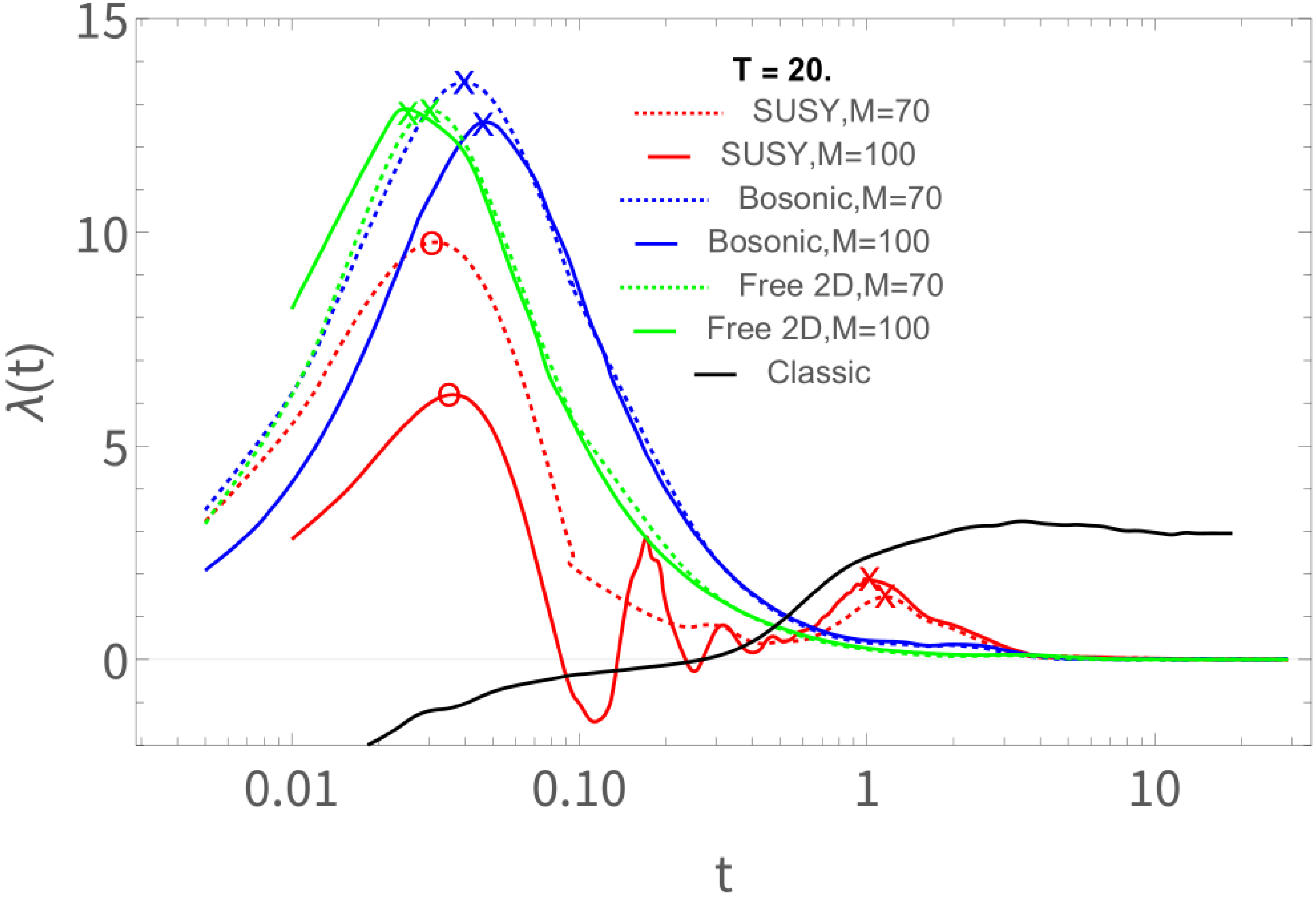
Backup slides

Estimates of $\lambda_L(t)$ – high-temperature regime



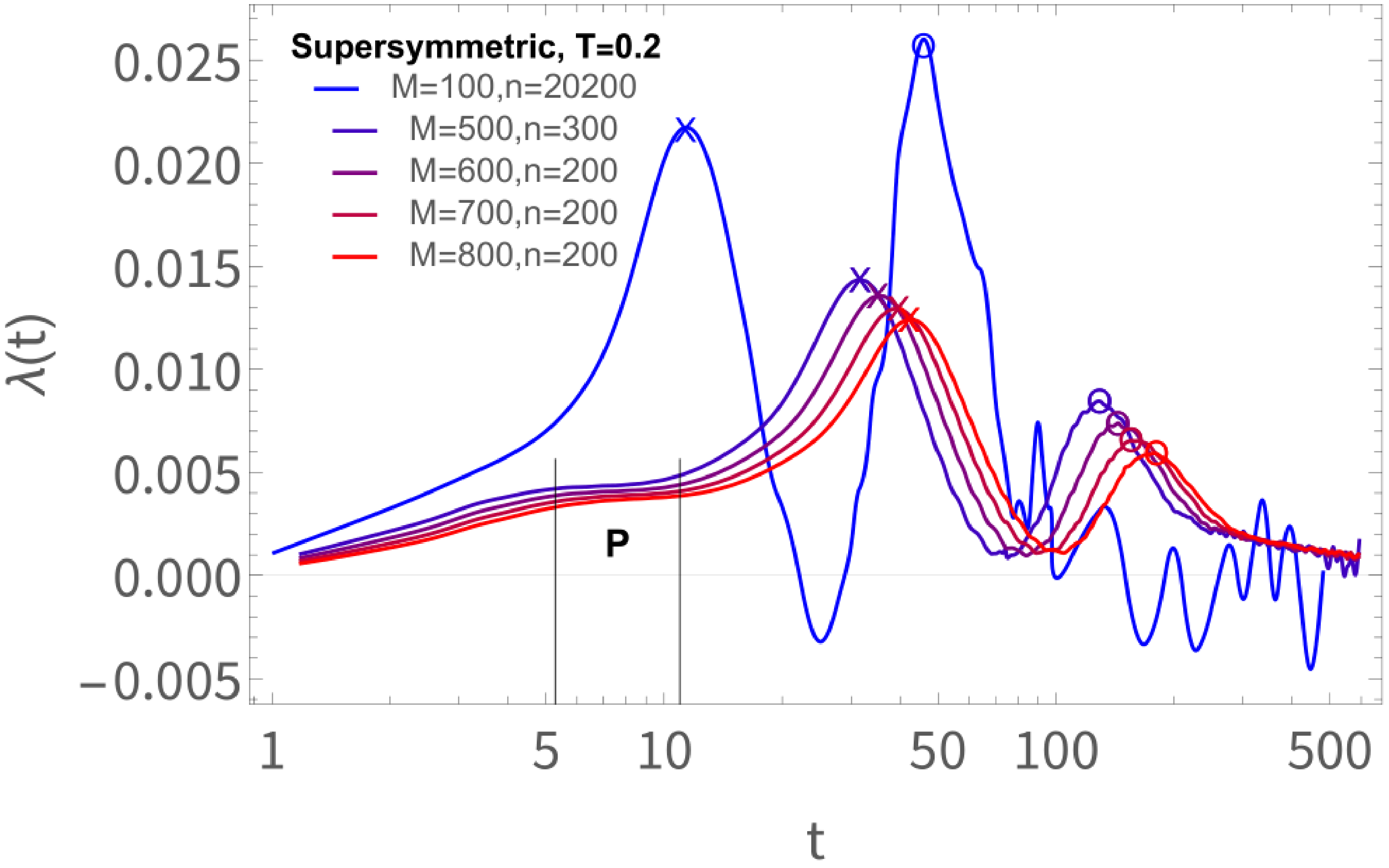
- Quite different behaviors for SUSY, Bosonic and Free
- Only SUSY exhibits some agreement with classics

Estimates of $\lambda_L(t)$ – very-high-temperature regime



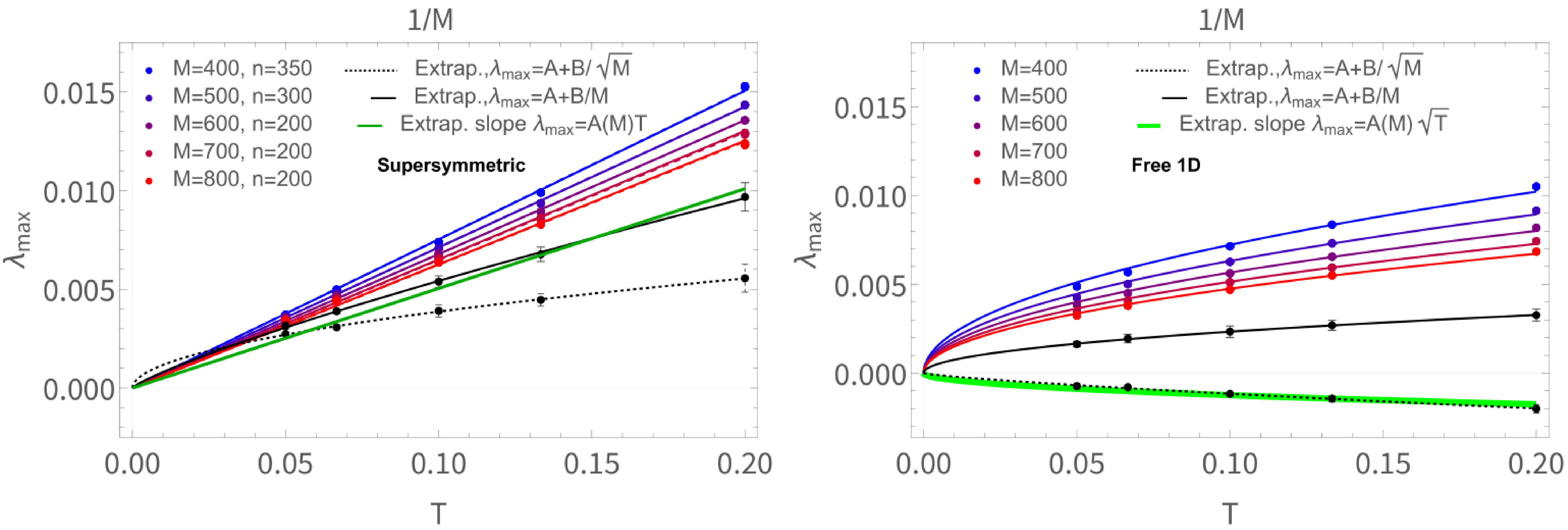
- SUSY, bosonic and free exhibit similar early-time features
- SUSY still exhibits some agreement with classics

Estimates of $\lambda_L(t)$ – SUSY, low-temperature regime



- Two characteristic maxima and a plateau
- Heights decrease with M

Extrapolating dominant low-temperature maxima to $M \to +\infty$



- Different M dependencies $\quad \lambda_{max}(M) = A + B/M,$
- Two extrapolation models: $\quad \lambda_{max}(M) = A + B/\sqrt{M},$
- Consistently higher extrapolations for SUSY