Real time evolution of scalar fields in semiclassical gravity

Phys. Rev. D **105** (2022) no.10, 105010 [arXiv:2010.13215 [gr-qc]]

Jana N. Guenther, Christian Hölbling, **Lukas Varnhorst**

11. Aug. 2022

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Introduction

Consider the collapse of a free, massless scalar field to a black hole.

Semiclassical gravity predicts Hawking radiation for late times.

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One would expect this calculation to be very messy and to depend on the detailed nature of the gravitational collapse. However, as I shall show, one can derive an asymptotic form [...] which depends only on the surface gravity of the resulting black hole.

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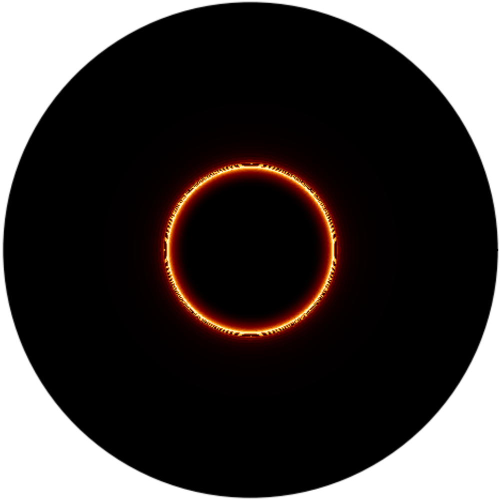

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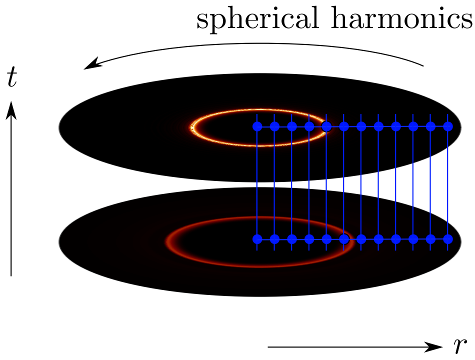


Semiclassical Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \langle \psi \mid T_{\mu\nu} \mid \psi \rangle$$

Choose state $\mid \psi \rangle$ such that:

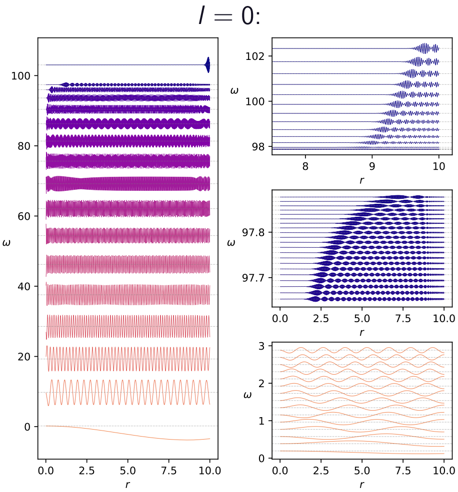
- Close to a classical state $\rightarrow$ Coherent state.
- Expectation value is spherically symmetric.

Choose spherical symmetric coordinate system [1]

$$g_{\mu\nu} = \begin{pmatrix} \alpha^2(t,r) & & & \\ & -a^2(t,r) & & \\ & & -r^2 & \\ & & & -r^2 \cos^2\theta \end{pmatrix}$$

[1] M. W. Choptuik, "Universality and scaling in gravitational collapse of a massless scalar field," Phys. Rev. Lett. **70** (1993), 9-12

Scalar field decomposition



Hamiltonian of the field can be written as

$$\mathcal{H} = \sum_{l=0}^{\infty} (2l+1) \left(\frac{\alpha a_0}{a \alpha_0} \Pi_l \Pi_l^\dagger + \phi_l^\dagger \sqrt{\frac{a\alpha_0}{\alpha a_0}} K \sqrt{\frac{a\alpha_0}{\alpha a_0}} \phi_l \right)$$

with

$$K = q^T q + \frac{l(l+1)}{r^2} \alpha^2$$

where

$$q = \sqrt{\frac{\alpha}{a}} r \partial_r \sqrt{\frac{\alpha}{a}} \frac{1}{r}$$

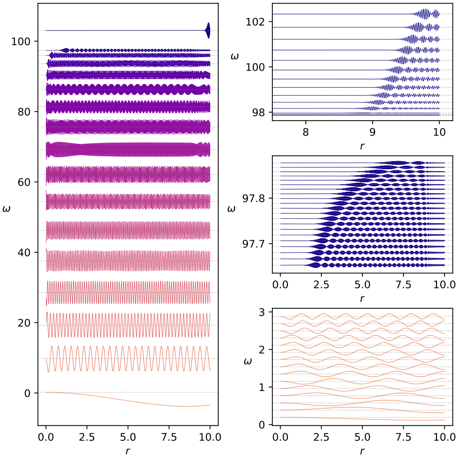
Perform SVD:

$$K = V \omega^2 V^T$$

- V: Mode functions.
- ω: Mode frequency

Scalar field quantization

$l = 0$:



Write Hamiltonian as

$$\mathcal{H} = b_+ W_r b_+^\dagger + b_-^\dagger W_r b_- + b_+ X_r b_- + b_-^\dagger X_r b_+^\dagger .$$

$b_\pm^\dagger / b_\pm$ are the creation/annihilation operators with frequencies ω.

$$b_+(t) = \frac{1}{\sqrt{2}}(b_u(t) + b_v(t)) \quad b_-(t) = \frac{1}{\sqrt{2}}(b_u^\dagger(t) - b_v^\dagger(t))$$

$$b_-^\dagger(t) = \frac{1}{\sqrt{2}}(b_u(t) - b_v(t)) \quad b_+^\dagger(t) = \frac{1}{\sqrt{2}}(b_u^\dagger(t) + b_v^\dagger(t))$$

$$b_u(t) = \frac{1}{\sqrt{2}}(b_+ u(t) + b_i^\dagger u^*(t)) V \sqrt{\omega}$$

$$b_u(t) = \frac{1}{\sqrt{2}}(b_+ v(t) - b_-^\dagger v^*(t)) V \sqrt{\omega^{-1}}$$

$\rightarrow$ Determine time evolution of $u(t)$ and $v(t)$.

Scalar field time evolution

Initialization:

$$u(0) = \frac{1}{\sqrt{\omega}} V^T \quad \text{and} \quad v(0) = \sqrt{\omega} V^T$$

As long as the metric is constant, the time evolution can be solved exactly

$$\begin{pmatrix} u(t+\Delta t) & v(t+\Delta t) \end{pmatrix} = \begin{pmatrix} u(t) & v(t) \end{pmatrix} \exp\left(-\mathrm{i} \begin{pmatrix} 0 & \sqrt{\frac{a\alpha_0}{\alpha a_0}} K \sqrt{\frac{a\alpha_0}{\alpha a_0}} \\ \frac{\alpha a_0}{a\alpha_0} & 0 \end{pmatrix} \Delta t \right)$$

Exponential can be explicitly evaluated by using SVD $K = V\omega^2 V^T$.

Time evolution is a Bogolyubov transformation.

Metric evolution

$$g_{\mu\nu} = \begin{pmatrix} \hat{\alpha}^2 \frac{d}{r} & & & \\ & -\frac{r}{d} & & \\ & & -r^2 & \\ & & & -r^2 \cos^2\theta \end{pmatrix}$$

$$\ln'(\hat{\alpha}) = \langle \psi \mid h_r^0 \mid \psi \rangle$$
$$d' + d h_r^0 = 1 - r\langle \psi \mid m_r \mid \psi \rangle$$

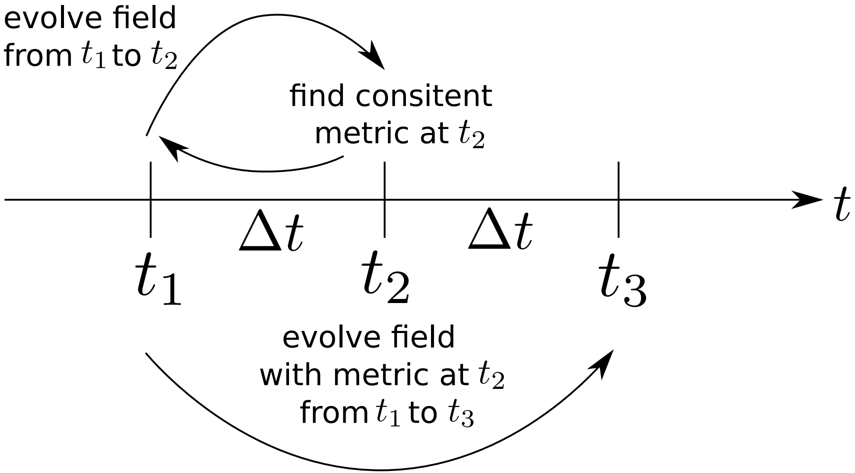
h_r^0 and m_r are field operators:

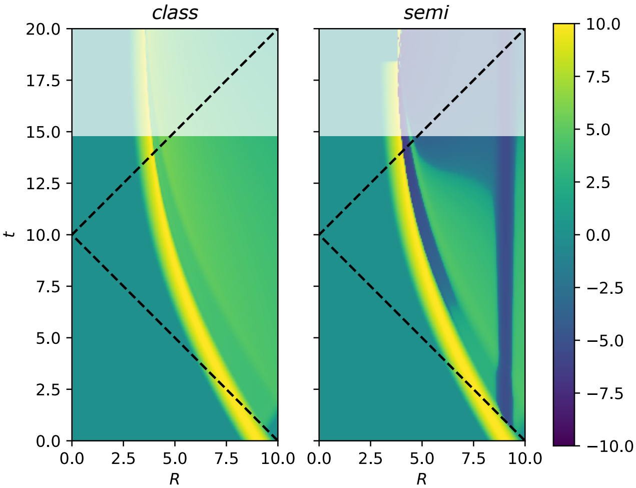
$$\langle h_r^0 \rangle_\psi = \frac{1}{d^0 \hat{a}^0} \left(|l_{ur}|^2 + |l_{vr}|^2 + \sum_{l=0}^{\infty} (2l+1) \left((v_l^\dagger v_l)_r r + (q^0 u_l^\dagger u_l q^{0\,T})_r r \right) \right)$$

$$\langle m_r \rangle_\psi = \frac{d^0 \hat{\alpha}^0}{r^2} \sum_{l=0\infty} (2l+1) \left(\frac{l(l+1)}{r^2} + M^2 \right) (u_l^\dagger u_l)_{rr}$$

- $|l_{ur}|^2 + |l_{vr}|^2$: Classical contribution
- u_l, v_l: Coefficient of creation and anihilation operators
- $q^0 = \sqrt{\hat{\alpha} d r} \partial_r \sqrt{\hat{\alpha} d r^{-3}}$ Discretized "derivative"

Combined evolution





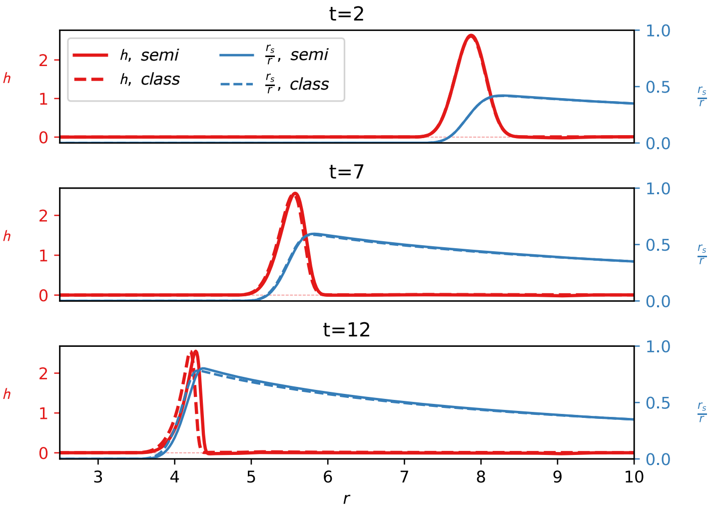
Consider only the $l = 0$ modes.

$\rightarrow$ Divergences of $\langle T_{\mu\nu} \rangle_\psi$ can be cancelled by normal ordering

Back reaction effects can be included.

Horizon seems to form earlier due to quantum effects.

$l = 0$ approximation



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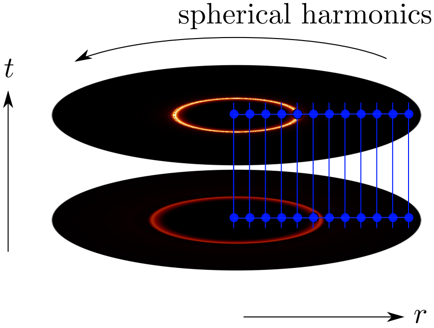
Including $l > 0$ modes



Classically, $l > 0$ modes break spherical symmetry.

In the quantum case, $\langle \psi \mid T_{\mu\nu} \mid \psi \rangle$ can be spherically symmetric even if $l > 0$ modes are excited.

$\rightarrow$ These modes must be included.

$$\mathcal{H} = \sum_{l=0}^{\infty} (2l+1) \left(\frac{\alpha a_0}{a \alpha_0} \Pi_l \Pi_l^\dagger + \phi_l^\dagger \sqrt{\frac{a\alpha_0}{\alpha a_0}} K \sqrt{\frac{a\alpha_0}{\alpha a_0}} \phi_l \right)$$

with

$$K = q^T q + \frac{l(l+1)}{r^2} \alpha^2$$

Additional divergence due to large l modes.

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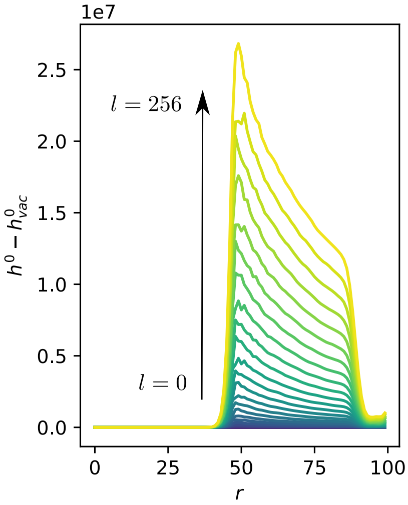

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Divergence structure

Energy momentum tensor in curved space time can be calculated from the coincidence limit of the two-point function $G(x, x')$.

For well behaved Hadamard states $\mid \psi\rangle$, the divergence structure is [1]

$$\lim_{x' \to x} \langle \psi \mid G(x, x') \mid \psi \rangle = \frac{u(x, x')}{\sigma(x, x')} + v(x, x') \ln \sigma(x, x') + w(x, x')$$

Here, $\sigma(x, x')$ is the geodesic distance between x and x' and $u(x, x')$, $v(x, x')$ are state-independent function that depend only on the metric and $w(x, x')$ is regular.

How does that translate to the sum of angular l modes?

Can one use a $1/l$ expansion to relate them? $\rightarrow$ Work in progress.

[1] S. A. Fulling, M. Sweeny and R. M. Wald, "Singularity Structure of the Two Point Function in Quantum Field Theory in Curved Space-Time," Commun. Math. Phys. **63** (1978), 257-264

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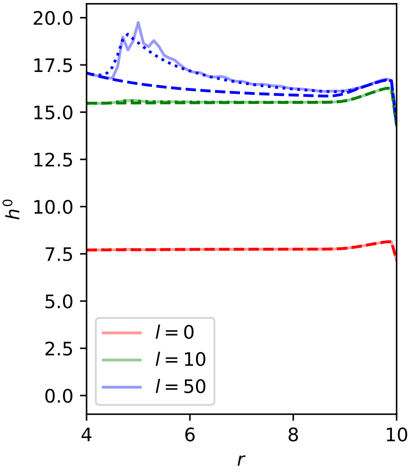

Solid lines: h^0 without subtraction.

Dashed line: Normal ordering at $t = 0$.

Dotted line: Instantaneous normal ordering (Adiabatic approximation)

Approaches:

- Normal ordering
- Point splitting in θ direction [1]
- Subtraction of adiabtic time evolution
- Subtracting the evolution with $K = \frac{l(l+1)}{r^2}\alpha^2$

[1] A. Levi and A. Ori, "Mode-sum regularization of $\langle\phi^2\rangle$ in the angular-splitting method," Phys. Rev. D **94** (2016) no.4, 044054 [arXiv:1606.08451 [gr-qc]].

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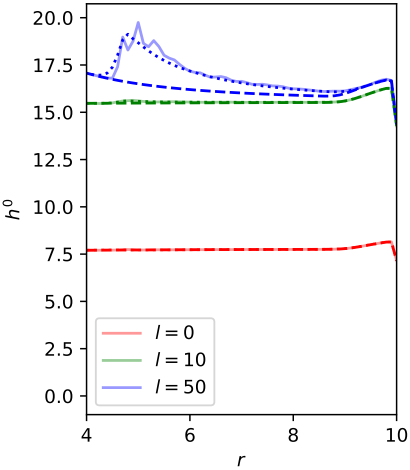

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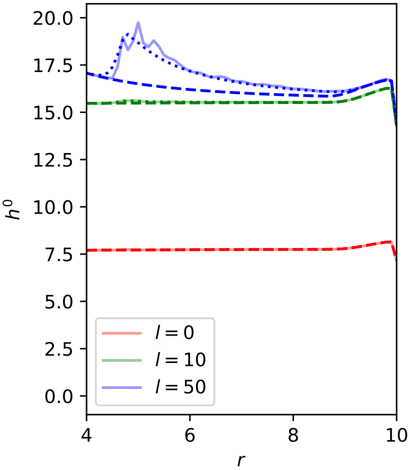

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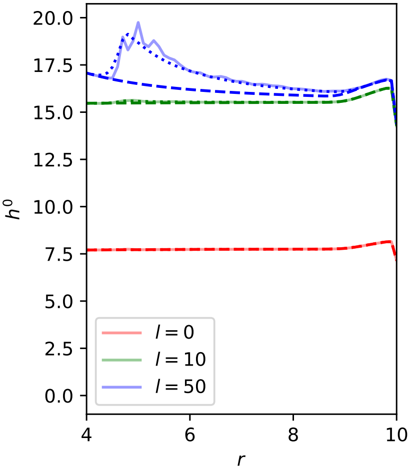

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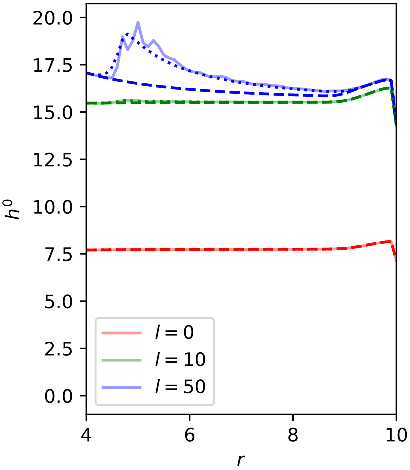

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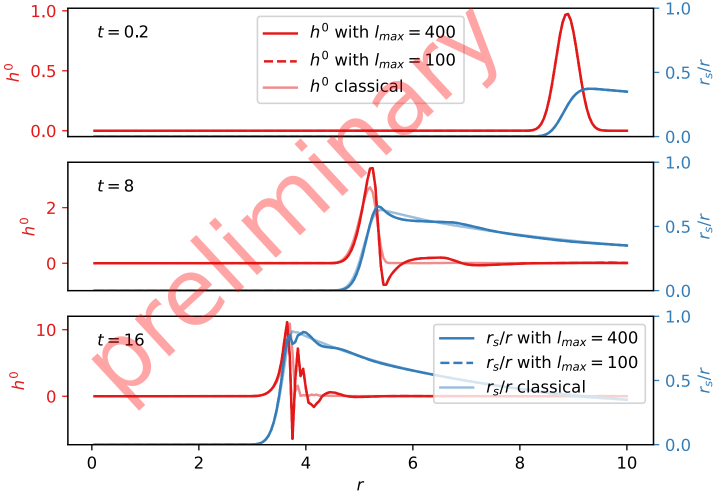
Pauli-Villars fields



As suggested by [1], using Pauli-Villars fields with alternating signs and masses

$$m_2^2 + m_4^2 = m_1^2 + m_3^2 + m_5^2$$
$$m_2^4 + m_4^4 = m_1^4 + m_3^4 + m_5^4$$

removes divergences for finite values of the masses.

Left side: $m_1 = 1$.

[1] B. Berczi, P. M. Saffin and S. Y. Zhou, "Gravitational collapse of quantum fields and Choptuik scaling," JHEP **02** (2022), 183 doi:10.1007/JHEP02(2022)183 [arXiv:2111.11400 [hep-th]].

Conclusion

- Free scalar field evolution can be solved exactly in a static metric.
- In varying metric a leap-frog-like algorithm allows to determine the evolution.
- In the $l = 0$ approximation, the back reaction can be calculated.
- Including higher l modes is work in progress, preliminary results with Pauli-Villars regularization.

Thank you!