Adjoint fermions at large-N_c on the lattice

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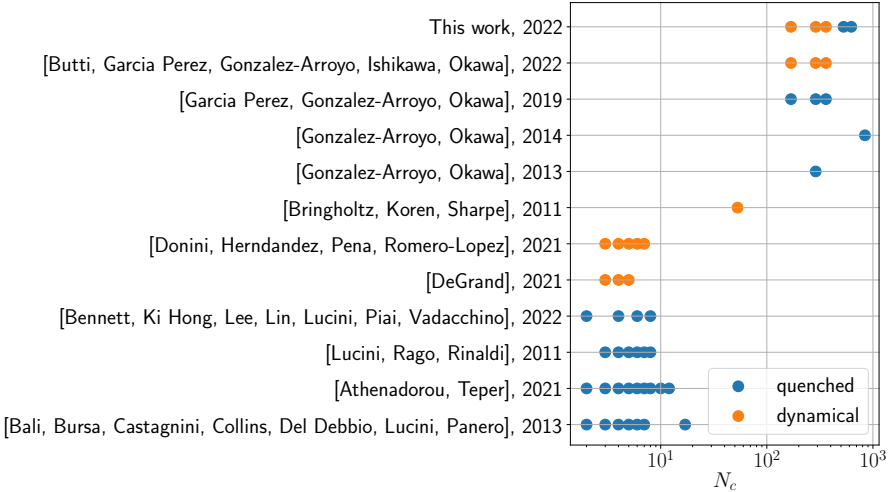
Large-N_c on the lattice



1. **Twisted reduction at large-N_c**

2. **Scale setting w/ Wilson flow**

3. **Some results for adjoint fermions**

Twisted BC + volume reduction (gauge field)

Apply tBC + volume reduction [González-Arroyo, Okawa 1983,2010] ($b = \frac{1}{g^2 N_c}$)

$$S_w = bN \sum_{n,\mu\neq\nu} \mathrm{tr}\left[\mathbb{1} - U_\mu(n) U_\nu(n+\mu) U_\mu^\dagger(n+\nu) U_\nu^\dagger(n)\right]$$

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- Twist-"eaters": $\Gamma_\mu \Gamma_\nu = z_{\nu\mu} \Gamma_\nu \Gamma_\mu$ with $z_{\mu\nu} = e^{\frac{2\pi i k}{\sqrt{N_c}} \epsilon_{\mu\nu}}$ [Talk by J. Da Silva yesterday]

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- "Effective"-lattice $V = (\sqrt{N_c})^4 \quad p_\mu = \frac{2\pi}{\sqrt{N_c}}[1, \ldots, \sqrt{N_c}]$
- Volume independence: $z(\mathcal{C}) \langle V(\mathcal{C}) \rangle_{\mathrm{TEK}} \xrightarrow{N\to\infty} \langle W(\mathcal{C}) \rangle_{V\to\infty}$

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- Apply tBC + volume reduction to both $U_\mu(n)$ and $\Psi(n)$

$$D_w = \mathbb{1} - \kappa_{\text{adj}} \sum_\mu \left[(\mathbb{1} - \gamma_\mu) U_\mu^{\text{adj}} + (\mathbb{1} + \gamma_\mu)(U_\mu^{\text{adj}})^\dagger \right] \quad \text{with} \quad U_\mu^{\text{adj}} \Psi = U_\mu \Psi U_\mu^\dagger$$

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- Spectroscopy can be performed using [González-Arroyo, Okawa, 2013]

$$\mathcal{C}_{AB}(t) = \frac{1}{2N_c^{\frac{3}{2}}} \sum_{n_0=1}^{\sqrt{N_c}-1} e^{-i(at)\frac{\pi n_0}{a\sqrt{N_c}}} \operatorname{tr}\left(A D_w^{-1}(0) B D_w^{-1}(p_0) \right)$$

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- Several exploratory studies for different number of flavors

  $N_f = 0$: (fundamental) Meson spectrum at large-N_c [JHEP-04(2021)230]

  $N_f = \frac{1}{2}$: $\mathcal{N} = 1$ SUSY Yang-Mills [Butti et al. JHEP-07(2022)074]

  $N_f = 1, 2$: [Phys.Rev. D 88(2013)], [JHEP-08 (2015)034]

Wilson flow on a twisted-reduced lattice

- Define **flowed energy density** in the twisted-reduced lattice

$$E = -\frac{1}{128}\sum_{\mu,\nu}\mathrm{tr}\left[z_{\nu\mu}(V_\nu V_\mu V_\nu^\dagger V_\mu^\dagger + V_\mu V_\nu^\dagger V_\mu^\dagger V_\nu + V_\nu^\dagger V_\mu^\dagger V_\nu V_\mu + V_\mu^\dagger V_\nu V_\mu V_\nu^\dagger - \mathrm{h.c.})\right]^2$$

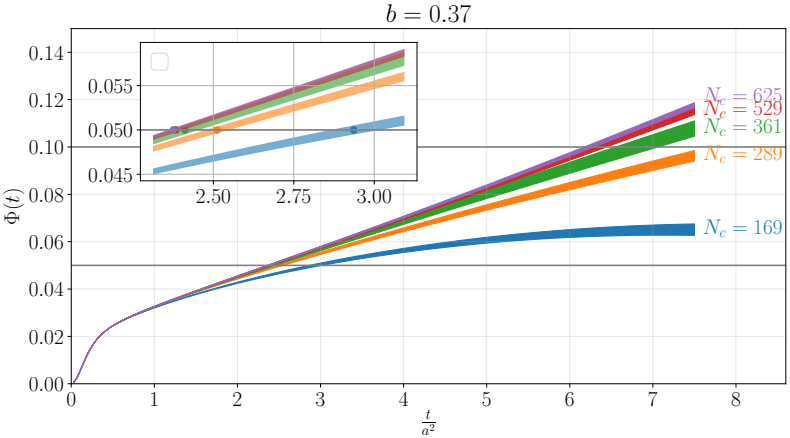
- Evolve gauge field according to the evolution equation

$$\frac{\mathrm{d}A_\mu(t)}{\mathrm{d}t} = D_\nu G_{\mu\nu}(t)$$

- Compute $\Phi(t) = \left\langle \frac{t^2 E(t)}{N_c} \right\rangle$ and solve

$$\Phi(t)\Big|_{t=t_c} = c \quad \text{and} \quad t\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t)\Big|_{t=w_c^2} = c$$

Wilson flow on a twisted-reduced lattice



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- On the reduced torus $V = 1^4$ with twisted BC [Ibañez, García Pérez, 2019]

$$\hat{\lambda}(\mu) = \frac{1}{\mathcal{N}\left(\frac{\sqrt{8t}}{\sqrt{N_c}}\right)}\Phi(t, N_c) \xrightarrow{N_c\to\infty} \lambda_{\rm gf}(\mu)$$

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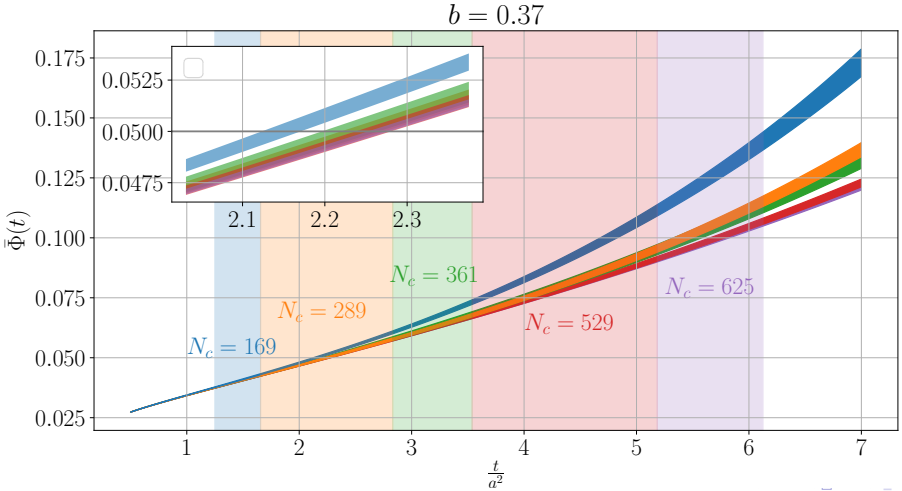
- We define [Butti et al. JHEP-07(2022)074]

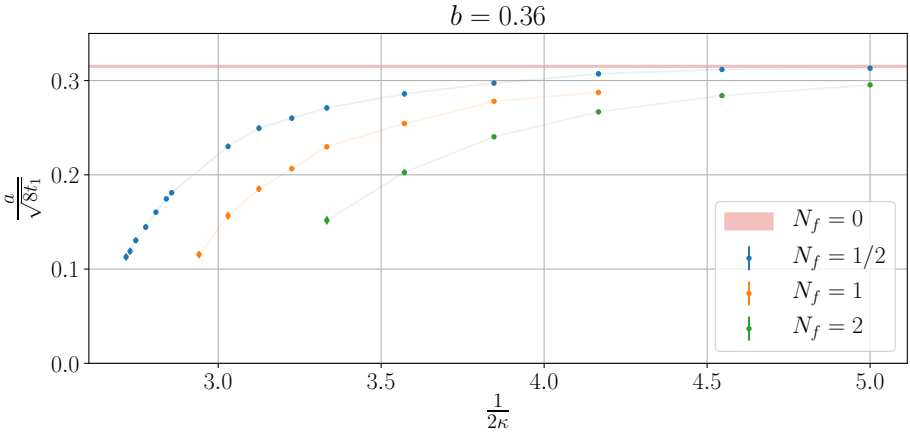
$$\hat{\Phi}(t) = \frac{\frac{3}{128\pi^2}}{\mathcal{N}_{\rm lat}(\sqrt{8t/N_c})}\Phi(t, N_c) \quad \text{for } T \in \left[1.25, \gamma^2\frac{N_c}{8}\right]$$

- Remnant finite-N_c effects are $< 3‰$ in the scaling window.

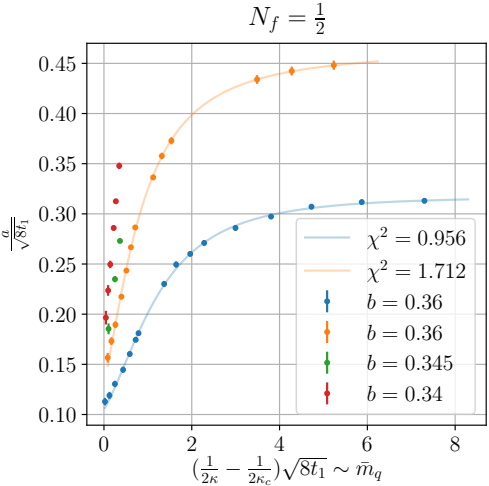
Wilson flow corrected with $\mathcal{N}_{N_c}$



The lattice spacing vs N_f



Parameterizing the dependence



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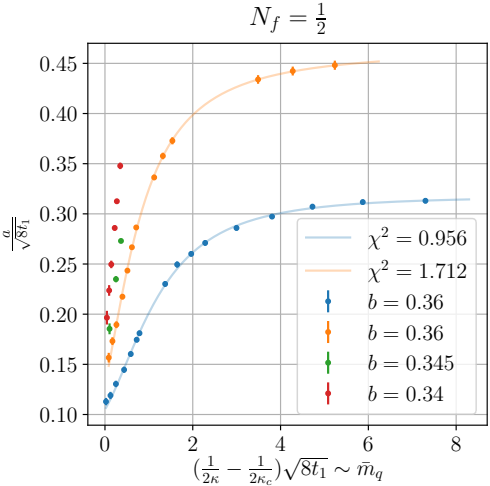
- $\bar{m}_q \to \infty \quad = \quad$ Yang-Mills (a_{YM})



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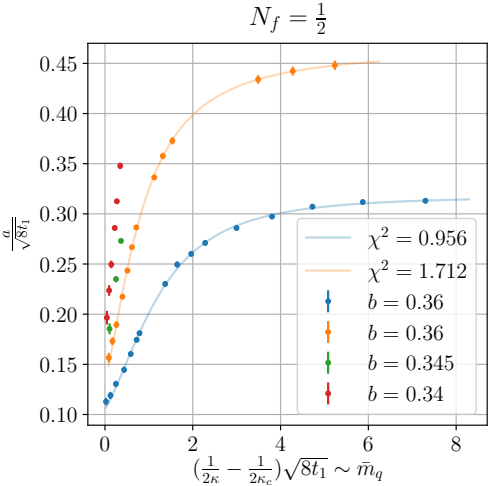

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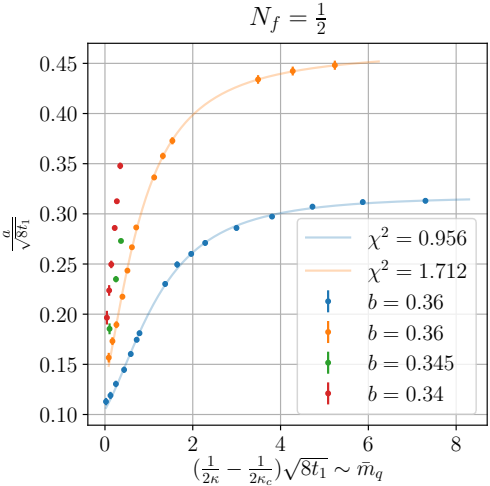
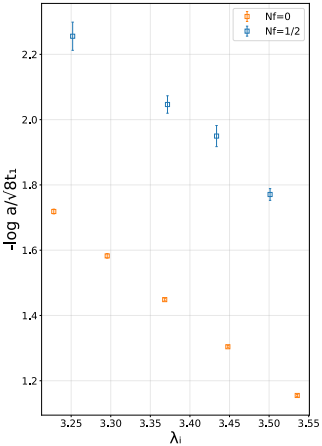
- Lattice scale depends exponentially from the log of the (subtracted) quark mass

$$a(\bar{m}_q) = \frac{p_1 + p_2 \bar{m}_q + p_3 \bar{m}_q^2}{\frac{p_1}{a_{\text{SUSY}}} + p_4 \bar{m}_q + \frac{p_3}{a_{\text{YM}}} \bar{m}_q^2}$$



β-function for SUSY



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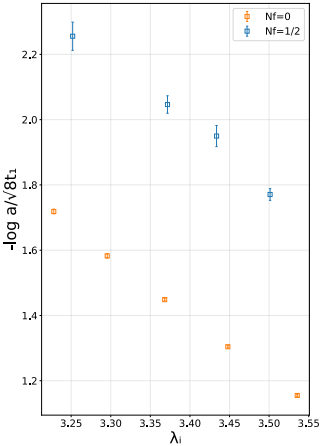

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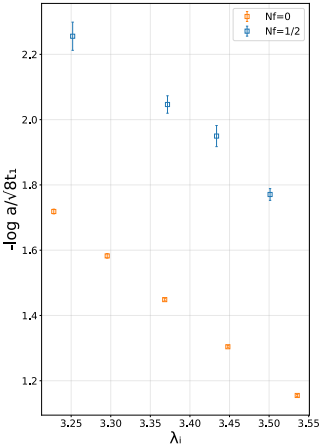
$$-\log \frac{a}{\sqrt{8t_1}} = \frac{1}{b_0 \lambda} + \frac{b_1}{b_0^2} \log \lambda + \log \Lambda$$



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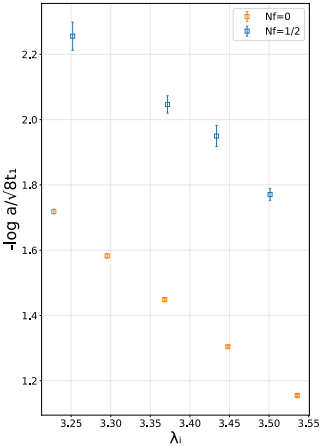
$$\lambda_l^{(1)} = \frac{1}{P_\chi(b)b} \quad \text{and} \quad \lambda_l^{(2)} = 8(1-P_\chi(b))$$



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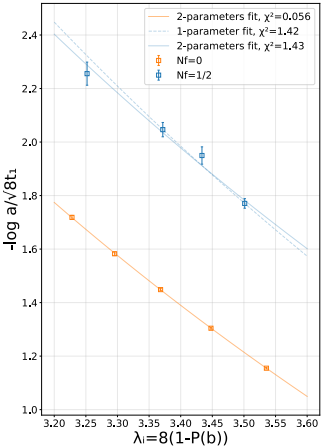
- Fit and extract b_0

$$24\pi^2 b_0 = 11-4N_f = 9\,/\,11 \;\text{ vs }\; 24\pi^2 b_0^{\text{fit}} = 9.76(80)\,/\,10.6(1)$$



Thank you!

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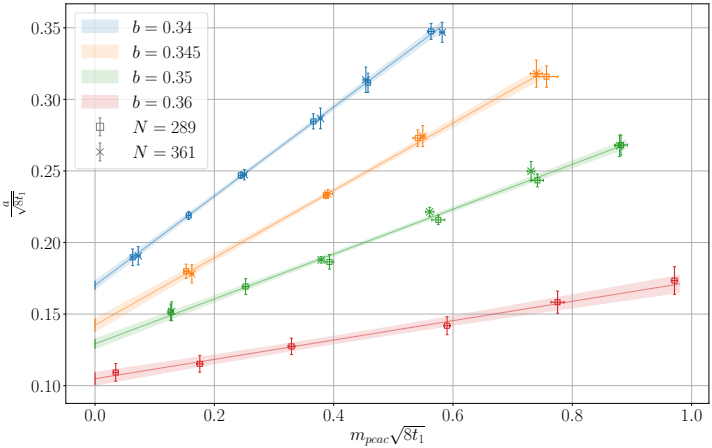
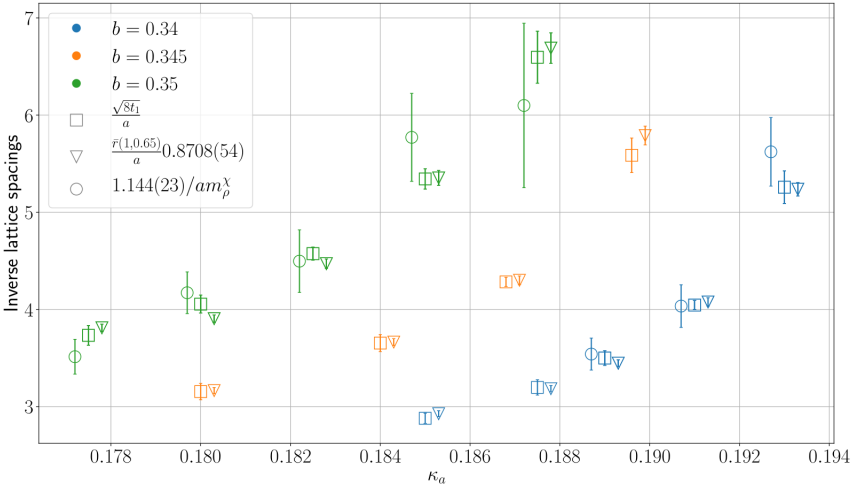
Backup material: chiral/SUSY limit



Backup material: scales comparison



Backup material: Critical mass for gluinos